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ABSTRACT

This paper explores the dynamics of return co-movements between the largest economic sectors in South Africa, specifically with a view to shed light on the inter-sector diversification potential of domestic investors over time. It has been widely documented that investors have a home-bias when it comes to investing, and as such may be exposed to periods of increased co-movement between assets held locally across different sectors in their portfolios. Such periods of increased homogeneity in the movement of asset prices negate the benefits from diversification within the domestic financial market. The paper utilizes Dynamic Conditional Correlation (DCC) and Asymmetric-DCC Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MV-GARCH) techniques to isolate the time-varying conditional correlations from the conditional variance component. These series are then used to study whether changes in market conditions and overall sentiment influence the dynamics and aggregate level of co-movement between sectors. The results firstly suggest that using static measures of historic co-movement between asset returns across sectors in order to evaluate a portfolio's diversification potential are inaccurate. Significant leverage effects are also found in the dynamics of co-movement between the sector pairs, with negative shocks being followed in all cases by higher aggregate levels of co-movement. The results also suggest that periods of heightened global- and domestic market uncertainty magnifies the co-movements between sectors and in so doing undermines the ability of investors to diversify across local sectors.

Keywords: Conditional Variance, Multivariate GARCH, Dynamic Conditional Correlation, Sector Indices
JEL codes: C32, C51, C58, G11, G17

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1 Introduction

Over the last few years there has been growing concern amongst economists and investors of the dangers that periods of increased homogenization of asset price movements across distinct sectors and financial systems pose to our ability to effectively diversify investment portfolios. In particular, it has become clear that in times of global economic uncertainty asset markets have shown to correlate strongly beyond fundamental linkages. This is largely as a result of the interconnected design of the global financial system and the near instantaneous spreading of information, which makes coordinated actions a reality in modern markets. The high correlation between equity returns during bear markets and the dampened correlation in bull markets, for example, have been studied extensively[2].

Portfolio diversification can be achieved by investing in different asset classes, across sectors and by investing abroad, with assets contained in the portfolio ideally having low or negative correlations. This strategy enjoys clear theoretical and empirical justification, though in a globalized financial system investors must be aware that the correlation between sectors is dynamic and can change abruptly given certain trigger events. As was clearly seen in the recent global financial crisis, asset markets can at times exhibit system-wide movements that negate the benefits of diversification at a time when it is most needed.

Understanding what typically corresponds to magnified inter-sector correlation could provide investors and investment institutions with valuable insights into optimized portfolio diversification strategies. This is particularly important for portfolio managers who often rely on static estimates of past correlations to guide portfolio diversification decisions. This study focusses specifically on the dynamic nature of such co-movement in the domestic market between the main economic sectors.

The analysis is split in two main parts. In the first the time-varying conditional correlations between the different sectors will be extracted by means of Dynamic Conditional Correlation (DCC) and Asymmetric-DCC (ADCC) Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MV-GARCH) models proposed by Engle (2002) and Cappiello, *et al* (2006), respectively.

The second part will examine the dynamic structure of the time-varying conditional correlations by fitting both level and differenced equation models on the series extracted in the first part. These models will include several variables used to explain the impact that global- and domestic

[2] C.f. De Santis & Gerard (1997), Ang and Bekaert (1999), and Das and Uppal (2001).

uncertainty and market sentiment has on the co-movement of asset prices across the main domestic sectors.

The findings in this paper show the shortcomings of relying on static estimates of correlation between assets across the local economic sectors[3]. In particular this emphasizes the need to better understand the dynamics of market correlation within South Africa when designing portfolios that are well diversified across local assets.

The paper is set out as follows: The next section will provide a concise overview of the relevant literature, outlining the techniques that will be used in the study. Thereafter the technical aspects of the paper will be discussed in more detail, after which a discussion of the data and the results will follow. Finally a discussion of the implications of the findings will conclude the paper.

2 Literature overview

Modelling and explaining the dynamics of volatility in financial time-series have evolved considerably over the last two decades since the seminal work by Engle (1982) on Autoregressive Conditional Heteroskedasticity (ARCH) models. In addition to the statistical benefits of controlling for second order temporal persistence and conditional heteroskedasticity in asset return series[4], modelling the conditional correlation between assets and across sectors over time is of great practical importance. It allows for better decision making in terms of asset and derivative instrument pricing, portfolio selection and risk management. The importance of studying estimates of asset return correlations conditional upon past information (or conditional correlations for short) is also emphasized in standard Markowitzian finance theory, which suggests that investors are compensated in terms of the mean and variance-covariance structure of asset returns.

Over the last two decades there has emerged a large body of literature on MV-GARCH models, which differ in terms of the conditional volatility specifications (of which a large body of literature has evolved[5]) as well as the conditional variance-covariance matrix specifications[6]. The first MV-GARCH model explicitly measuring the conditional covariance matrix between series, the VECH model, was proposed by Bollerslev, Engle and Wooldridge (1988). The VECH approach is essentially a direct generalization of the univariate approach, and as such requires a large amount of parameters

[3] An example of this is the widely used Beta measure.

[4] Such persistence implies financial time-series data display periods of strong volatility clustering, or momentum, which is regarded by most as a stylized empirical fact.

[5] See e.g. Bollerslev's (2008) Glossary to ARCH (GARCH) for a concise overview of the universe of GARCH model specifications.

[6] For a detailed account of the MV-GARCH literature, see Silbennoinen & Terasvirta (2008) and Bauwens, Laurent, & Rombouts (2006).

to be estimated as the returns dimension grows. Subsequent efforts to make the models more parsimonious yielded, amongst others, restricted versions of Engle & Kroner (1995)'s BEKK[7]-model, which also explicitly ensures positive definiteness of the covariance matrix, as well as the Constant Conditional Correlation (CCC) model and its subsequent variants. Engle (2002) later relaxed the constancy of the correlation structure of the CCC model with the Dynamic Conditional Correlation (DCC) version, while Cappiello et al., (2006) extended it to the Asymmetric-DCC (ADCC) model to allow for leverage effects in the underlying correlation structure.

The main use in the literature of MV-GARCH techniques has been to investigate market spill-over and contagion effects, typically to illustrate the increased global interdependence of various asset classes across different financial markets. Interest in shock transmission studies initially followed the 1987 stock market crash in the US, as researchers sought to uncover spill-over effects before and after the crash (c.f. King & Wadhwani (1990)and Schwert (1990)). Subsequent work on the topic built on and refined the methodology, with notable studies including, amongst others, Bekaert & Harvey (1995), Karolyi (1995), Kaminsky & Reinhart (1999). Koutmos & Booth (1995) notably studied the difference between positive and negative shock spill-overs emanating from significant news events and how it affects the volatility linkages between equity markets, while Lin, *et al* (1990) uncovered differences in the strength of transmission between global and local shocks. De Santis & Gérard (1998) then studied, with mixed findings, the benefit of utilizing such techniques for investment purposes[8].

Following the recent global financial crisis, there has again been a growing body of literature that study the magnified inter-linkages between asset return co-movement and volatility transmission across various markets using the MV-GARCH methodology. Several studies have included a composite South African index in their list of emerging economies (c.f. Christopher, *et al* (2012), and Beirne, *et al* (2009)), mostly to study global- and regional volatility spill-over effects from Europe. Horvath, *et al* (2011), e.g., study the conditional correlation between the main sectors of several large economies, including South Africa, using the BEKK MV-GARCH approach. Christopher, *et al*. (2012) also derives time-varying conditional correlations between aggregate stock and bond market indices using the BEKK MV-GARCH framework, utilising the dynamic structure of the correlations to study its long-term relation to sovereign credit ratings using Error Correction Models (ECMs).

[7] The name is derived from the collaborative work of Baba, Engle, Kraft and Kroner on multivariate models.

[8] They found that the benefit is greatest for short term actively traded investment strategies as opposed to longer term oriented strategies.

Although the time-varying correlations between series can be extracted by using techniques that are direct generalizations of the univariate volatility models into the multivariate plane, such as the abovementioned studies that make use of the BEKK- and VECH-GARCH approaches, the main use of these models lie in studying volatility spill-over effects. As the focus in this paper will be specifically to extract the conditional correlations between the domestic sectors and study its dynamic structure, the more parsimonious DCC and ADCC MV-GARCH models will be used. These techniques are non-linear combinations of univariate GARCH models that use a two-step procedure to separate the covariance matrix into the individual univariate conditional variances and dynamic conditional correlation series.

Corsetti, *et al* (2005) and Chiang & Li (2007) use DCC MV-GARCH models to show that herding behaviour amongst investors in emerging markets, during periods of economic uncertainty, can significantly affect developing countries' capital market linkages with developed economies. Kalotychou, *et al* (2009) study inter-sector volatility correlations between Japan, the US and the UK markets, and emphasize the usefulness of studying the dynamics of asset return correlations for the purpose of portfolio allocation. They argue that there is substantial portfolio management benefits of not only timing volatility (as the multivariate model extensions of GARCH do), but also uncovering the dynamics underlying return correlations. Syriopoulos & Roumpis (2009) use the ADCC-MVGARCH techniques to investigate such dynamic correlations between the aggregate composite indices of Balkan and developed countries and convey similar sentiments from an emerging market investment perspective.

Despite several MV-GARCH studies that include South Africa in a list of other countries (typically as part of a European group of economies), none to the knowledge of the author have focussed exclusively on the structure of dynamic conditional correlations between the main domestic sectors. In fact, studies on the South African equity market return volatility dynamics in general are limited. Notable examples include Collins & Biekpe (2005), who used adjusted Pearson's correlation coefficients to study the contagion effects of the 1997 Asian crisis on stock markets in Africa, which included South Africa. Ogum (2001) used a time-varying MA-TGARCH model to study the variance structures of SA, Nigeria and Kenya for the period 1985 – 1998. Samouilhan (2006) finds evidence of market aggregate and sector level return and volatility linkages between SA and UK equity markets using univariate volatility models. Chinzara & Aziakpono (2009) study mean and volatility linkages between the South African stock market index and various other major global indices using VAR estimates and univariate GARCH techniques. Chinzara (2011) finds significant volatility spill-over effects of macroeconomic factors onto the monthly returns of the aggregate stock market index, and four other main sectors, including the financial-, retail-, mining- and industrial sectors, using univariate GARCH techniques. He also finds that in periods of economic crisis (specifically using dummy variables for the Asian and global financial crises, respectively) these effects are intensified. Duncan & Kabundi (2011) study domestic volatility co-movements between currencies, bonds and equities in South Africa using a generalized autoregressive (GVAR) model. The dynamics of their analysis relies on rolling window regressions to provide a time-varying estimation in volatility transmissions between the main domestic asset classes.

This study seeks to add to this literature by showing how international- and domestic macroeconomic uncertainty influence the dynamics of conditional co-movement between the largest domestic economic sectors[9]. This paper defines such periods of market uncertainty in terms of deviations from past aggregates of certain key macroeconomic variables available at daily frequencies. The paper therefore provides an interesting insight into the ability of domestic investors to hedge their portfolios by holding assets across the local equity market spectrum in different economic environments.

Alternative multivariate volatility models that can also be used to construct similar time-varying variance-covariance series include the Orthogonal-GARCH, EWMA[10] and Variance Sensitivity Analysis (VSA) models, which will, for the sake of brevity, not be discussed in this paper. Future research might conduct a sensitivity analysis of using these different approaches.

3 Data

The aim of this paper is to construct time-varying conditional correlations between sector pairs that reflect the co-movement of equities across the sector spectrum on an aggregate level in South Africa. The data set consists of the daily closing prices of the six largest industrial sector composite total return indices[11]. These sector indices are weighted by market capitalization and contain the majority of the equities within their respective economic groups. As such they accurately reflect the aggregate asset price behaviour of the firms within the sectors they track. Sector data was obtained from McGregor BFA and spans the period January 2 2002 to 30 April 2013, primarily on the basis of data availability. In total, 2833 observations are included in this analysis.

[9] Bekaert, *et al* (2005), Phylaktis & Xia, (2009) and Hassan & Malik (2007) are notable examples of studies that investigate equity market correlations using MV-GARCH techniques at the sector level.

[10] This refers to the Exponentially Weighted Moving Average model that is widely used in practice. For details, see J.P. Morgan (1996)'s: *RiskMetrics—A Technical Document*. Reuters, NewYork

[11] Appendix A contains the complete list of sectors studied in this paper and their market capitalizations.

The continuously compounded daily sector returns are calculated by taking the log difference of each index series, as:

$$r_{i,t} = \ln\left(\frac{p_{i,t}}{p_{i,t-1}}\right) * 100 \quad (1)$$

with $p_{i,t}$ the closing price of the sector index, *i,* at time *t.* Taking the first differences of the series is motivated by the strong rejection of the Augmented Dickey Fuller Test and the Phillips-Perron tests for unit roots by all of the series included in the analysis[12]. The graphical representation of all the indices can be viewed on figure 2 in Appendix A.

Table 1 below suggests that the sectors included show typical financial time-series behaviour [13]. This is characterized by the excess kurtosis and skewness, resulting in the rejection of the Jacque-Bera normality statistic, and the approximate leptokurtic distributions[14] common to financial time-series data. From the table we see that the Consumer Services (Basic Materials) sector displays the highest (lowest) unconditional mean returns, while the Telecommunications (Industrials) sector displays the largest (smallest) standard deviation of returns for the sample.

Table 1: Summary of statistics for the continuously compounded sector returns

	Financials	Industrials	Consumer Goods	Consumer Services	Tele-communications	Basic Materials
Mean	0.060124	0.070999	0.070094	0.096344	0.091743	0.041923
Median	0.085273	0.092317	0.072886	0.126384	0.089410	0.071116
Maximum	7.206517	6.984613	14.21184	6.407044	13.46480	11.16174
Minimum	-6.925194	-5.705644	-7.885916	-5.516482	-10.98621	-11.81173
Std. Dev.	1.231430	1.103383	1.609253	1.108236	1.913693	1.802463
Skewness	-0.014504	-0.164490	0.260229	-0.210556	0.218625	-0.052178
Kurtosis	6.059408	5.475949	7.697322	4.986533	6.336706	7.944201
Jarque-Bera	1104.967	736.4089	2636.544	486.7625	1336.798	2886.829
Probability (JB)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Observations	2833	2833	2833	2833	2833	2833

Output is from Eviews 8.

Each of the series also show significant serial autocorrelation remaining after differencing when considering the Ljung-Box Q statistics (see footnote 12 again), requiring autoregressive terms in the mean equations to be fitted[15].

As is clear from figure 3, all the series also display periods of volatility clustering, otherwise referred to as market momentum. This can be a strong indication of significant second order persistence

[12] Details on the tests and tables showing the result are omitted for the sake of brevity. This can be requested from the author.

[13] C.f. Enders, W. (2008). *Applied Econometric Time-Series*, for a discussion on the stylized facts of financial time-series data.

[14] Such distributions have fat tails and display excess peaks at the mean.

[15] Mean persistence will be controlled for using first order AR-terms in the mean equations.

remaining in the series, pointing to conditionally heterogeneity requiring explicit modelling of the variance components. Ljung Box Q-Statistics on the squared residuals and the LM-GARCH test confirm the presence of conditional heterogeneity in all the series. To control for this, Engle (1982) showed that it is possible to simultaneously model the mean and variance equations of a series using GARCH models, which will be further utilized to extract the time-varying conditional correlations in the next section.

The next step would be to test whether the sector returns are all co-integrated in order to motivate the study of their conditional correlations. As all the series are non-stationary and integrated of order 1, the Johansen (1988) co-integration test will be used to confirm whether there is at least one linear long-run relationship among all the series that yield stationary residuals. The test uses a VECM approach of the form

$$\Delta p_t = \Pi p_{t-k} + \Gamma_1 p_{t-1} + \Gamma_2 p_{t-2} + \cdots + \Gamma_{12} p_{t-12} + \mu + \delta(t) + \theta D_t + \varepsilon_t \tag{2}$$

where p_t is a $(1 \text{ x } 6)$ vector of the index closing prices in the data set at time t. The Johansen test then centres around the examination of the Π-matrix, with form $\Pi = \alpha\beta'$, where β is the k^{th}order cointegrating vector and α the adjustment parameter. The Trace and Maximum Eigenvalue tests below both consider the rank of the Π matrix using its eigenvalues, which give an indication of long-run dependence between the series. The Trace statistic tests whether the number of co-integrating vectors of the system is less than or equal to 6, while the Max-Eigenvalue statistic reflects separate tests that were used on each eigenvalue of the Π-matrix. If the tests indicate that the rank of Π can be regarded as statistically likely to be between 1 and 5, as compared to the MacKinnon-Haug-Michelis (1999) critical values, it would imply that there is a long-run relationship between the series in the study. The findings from this test are summarized in concise form below across the different data trend possibilities. It clearly suggests that across several trend possibilities using both tests there exists at least one linear long-run co-integration relationship between the different sector indices included in the study.

Table 2: Johansen Co-integration test for South African sectors

Data Trend:	None	None	Linear	Linear	Quadratic
	No Intercept	Intercept	Intercept	Intercept	Intercept
Test Type	No Trend	No Trend	No Trend	Trend	Trend
Trace	1	2	1	2	1
Max-Eig	1	2	1	1	1

Output is from Eviews 8. Critical values based on MacKinnon-Haug-Michelis (1999)

Table 3 in the appendix gives the unconditional correlation of returns between the sectors in the study. Such static estimates of historic correlation between returns are often used in practice to guide diversification decisions. As seen above, though, all the series display significant first and second order serial autocorrelation, which make such static estimates misleading if the remaining mean persistence and conditional heteroskedasticity is left uncontrolled for. It also fails to take into account the dynamic nature of the underlying correlations, conditional upon past information, which will be studied in the next section.

4 Methodology

Consider the 1x6 stochastic vector, $\{r_t\}$, of continuously compounded daily returns of the major industrial sectors mentioned in the previous section. Assuming that the returns are demeaned and follows a conditionally heteroskedastic normal distribution as described above, the following notation is used:

$$r_{it} = \mu_{it} + \varepsilon_{it} \tag{3}$$

$$\varepsilon_{it} = \sqrt{H_{it}}.\eta_i \ \ , with\ \varepsilon_{it} \sim N(0, H_t)\ \&\ \eta_i \sim N(0, I) \tag{4}$$

Here μ_t is the unconditional AR(1)-mean equation with intercept, ε_t the vector of ordinary residuals, H_t the NxN conditional covariance matrix and η_i the standardized residuals.

Various MV-GARCH models have been proposed to model the covariance process, H_t, in equation 4[16], with this study using the class of DCC models that allow the separation of the covariance matrix into the separate univariate volatility equations and the conditional correlations. Bollerslev (1990) proposed the first class of MV-GARCH models to do this, the Constant Conditional Correlation (CCC)-model. The CCC-model keeps the conditional correlations constant as the name suggests, thus making the conditional covariance matrix entries proportional to the product of the corresponding conditional standard deviations. This greatly simplifies the multivariate estimation procedure and significantly lowers the amount of parameters as compared to the VECH and BEKK techniques. These estimates provide better static correlation estimates than the unconditional estimates presented in table 2, as it controls for conditional heteroskedasticity present in all the series.

The covariance matrix can then be defined as follows in the CCC-model:

$$H_t = D_t R D_t \tag{5}$$

[16] C.f. Silbennoinen & Terasvirta (2008) for an in-depth discussion of the different types of models in the literature that measure H_t.

With $D_t = diag(\sqrt{h_{11,t}}, \dots \sqrt{h_{NN,t}})$ and $h_{ii,t}$ taking the functional form of any univariate GARCH model. $R_{ij} = \rho_{ij}$ is a positive definite symmetric matrix with ones on the diagonal. The conditional correlations are thus the off-diagonal entries in the R-matrix above, and are assumed to be constant over time. This study uses the GJR-GARCH (1,1,1) specification for the univariate conditional variance equations in D_t for the CCC-estimates, as all the series show significant threshold effects in their volatility equations[17] (as discussed later). The bivariate CCC model's covariance matrix, H_t, therefore takes the following form:

$$h_{ii,t} = \beta_0 + \beta_1 \varepsilon_{ii,t-1} + \phi . I[\varepsilon_{i,t-1} < 0] o \varepsilon_{i,t-1} + \beta_2 h_{ii,t-1} \ , \qquad (\forall i) \tag{6}$$

$$h_{jj,t} = \beta_0 + \beta_1 \varepsilon_{jj,t-1} + \phi . I[\varepsilon_{j,t-1} < 0] o \varepsilon_{j,t-1} + \beta_2 h_{jj,t-1} \ , \qquad (\forall j \neq i) \tag{7}$$

$$h_{ij,t} = \rho_{ij} \sqrt{h_{ii,t} . h_{jj,t}}, \qquad (\forall i, j \ \& \ i \neq j) \tag{8}$$

For notational purposes, let $\varepsilon_{ii,t-1}$ denote the previous period's squared residual series and $h_{ii,t}$ the univariate conditional variance equation of index i. The indicator variable $I[\varepsilon_{i,t-1} < 0] o \ \varepsilon_{i,t-1}$, is the element by element Hadamard product of the residual series if $\varepsilon_{i,t-1}$ is negative, and takes value zero otherwise. The usual GARCH restrictions apply that ensure non-negativity of the variances, i.e. that $\sum_{p=1}^{P} \alpha_{i,p} + \sum_{q=1}^{Q} \beta_{i,q} < 1$ and all the parameters are positive. These restrictions hold for all the univariate GJR-GARCH volatility equations in the study.

For this dataset the CCC model estimates 51 parameters for all the sector pairs together. To conserve space, only the constant conditional correlation entries are included in table 4 in the appendix for each sector pair in the study. As regards the mean and variance equation parameter estimates from the CCC-MVGARCH(1,1) procedure, all the series display strong significance for all the estimated auto-regressive parameters. The sector returns series also display strong auto-persistence in volatility as measured by $\beta_1 + \beta_2$ in equation 6 and 7. The Industrials sector shows the lowest conditional persistence in volatility of 0.89, with the largest being the Basic Materials sector with a volatility persistence parameter of 0.945. This is indicative of the local equity market being exposed to periods of significant asset price momentum for all the major sectors.

Assuming that financial asset returns have constant conditional correlation processes has been shown in the literature to be an inaccurate assumption[18]. In response to this shortcoming Engle (2002) generalized the CCC model to allow correlations to vary over time by using a robust two-step procedure to isolate the dynamic conditional correlation process. The first step involves using

[17] For simplicity, the threshold effects will be studied using only the GJR-specification for the CCC model above.

[18] C.f. Engle (2002)and Tse & Tsui (2002).

univariate volatility models to obtain GARCH-estimates of the respective series' conditional variances in order to standardize the residuals as follows:

$$\eta_{i,t} = \varepsilon_{i,t} / \sqrt{h_{ii,t}} \tag{9}$$

In the second step, the standardized residuals are used to estimate time-varying conditional covariances. This implies that for Engle's (2002) DCC model the variance-covariance matrix mentioned earlier can be written as:

$$H_t = D_t R_t D_t \tag{10}$$

With D_t as defined in the CCC-model and R_t now being time-varying. The dynamic conditional correlation structure is then given by the following equation:

$$Q_{ij,t} = (1 - \theta_1 - \theta_2).\bar{Q} + \theta_1(\epsilon_{i,t-1}\epsilon'_{j,t-1}) + \theta_2(Q_{ij,t-1}) \tag{11}$$

where $Q_{ij,t}$ is the unconditional variance between series i and j, $\bar{Q}$ is the unconditional covariance between the series estimated in step 1 (using the univariate GARCH specifications) and the scalar parameters θ_1 and θ_2 are non-negative and satisfy $\theta_1 + \theta_2 < 1$. The second step requires us to only estimate θ_1 and θ_2 using a likelihood function. Note that equation 11 expresses the unconditional variance matrix, $Q_{ij,t}$, as a standard GARCH-type equation, so that we can derive the dynamic conditional correlation matrix, R_t, between the two series as:

$$R_t = {Q^*_{ij,t}}^{-1}.Q_{ij,t}.{Q^*_{ij,t}}^{-1} \tag{12}$$

with $Q^*_{ij,t}$ being a diagonal matrix with the square root of the diagonal elements of $Q_{ij,t}$ as its entries, thus $Q^*_{ij,t} = Diag(Q_t)^{1/2}$. The validity of this process can be thought of intuitively as multiplying both sides of equation 5 by the inverse of Diagonal matrix D_t[19]. The dynamic conditional correlation matrix, $R_{ij},_t$ will therefore have entries in the bivariate framework as follows:

$$\begin{aligned}\rho_{ij,t} &= \frac{q_{ij,t}}{\sqrt{q_{ii,t}.q_{jj,t}}} \\ &= \frac{(1-\theta_1-\theta_2)\bar{q} + \theta_1\epsilon_{i,t-1}\epsilon'_{j,t-1} + \theta_2 q_{ij,t-1}}{((1-\theta_1-\theta_2)\bar{q_i} + \theta_1\epsilon^2_{i,t-1} + \theta_2 q_{ii,t-1})).((1-\theta_1-\theta_2)\bar{q_j} + \theta_1\epsilon^2_{j,t-1} + \theta_2 q_{jj,t-1}))}\end{aligned} \tag{13}$$

Following the methodology of Engle (2002), the DCC model is estimated by maximizing the log-likelihood function for equation 11 as:

[19] The reader is referred to Engle (2002) for formal proofs.

$$L(\theta,\phi)\,^{20} = -\frac{1}{2}\sum_{t=1}^{T}(\ln(2\pi) + \ln(|D_t R_t D_t|) + \varepsilon_t'(D_t R_t D_t)^{-1}\varepsilon_t) \tag{14}$$

by using the fact that $H_t = D_t R_t D_t$, the above equation can be simplified as:

$$L(\theta,\phi) = -\frac{T}{2}ln(2\pi) - \frac{1}{2}.\sum_{t=1}^{T}(2.\,ln|D_t| + \varepsilon_t'(D_t D_t)^{-1}\varepsilon_t) - \frac{1}{2}\sum_{t=1}^{T}(\ln|R_t| + \varepsilon_t'(R_t^{-1})\varepsilon_t) \tag{15}$$

The second step is then to maximize the correlation part by using the maximized value in 15 to solve:

$$L_C(\theta,\phi) = -\frac{1}{2}\sum_{t=1}^{T}(\ln|R_t| + \varepsilon_t'(R_t^{-1})\varepsilon_t) \tag{16}$$

The parameter estimates of the two-stage DCC estimation procedure outlined above is both consistent and asymptotically normal. According to Cappiello, et al. (2006) a clear limitation of this approach is that the dynamics of the conditional correlation do not account for asymmetric effects. This implies that although the model accounts for the magnitude of past shocks' impact on future conditional volatility and correlation, it does not differentiate between positive and negative shock effects. To account for these potential asymmetries in the conditional correlations between series, the ADCC model was proposed by Cappiello, et al.(2006). In this model, equation 11 can be extended to incorporate asymmetries as follows:

$$Q_{ij,t} = (1-\theta_1-\theta_2).\bar{Q} - g.\overline{\Psi_t} + \theta_1(\epsilon_{i,t-1}\epsilon_{j,t-1}') + \theta_2(Q_{ij,t-1}) + g.(\xi_{t-1}\xi_{t-1}') \tag{17}$$

where $\overline{\Psi_t} = E\left[\overline{\xi_{\iota t}\ \xi_{jt}}'\right]$[21] and $\overline{\xi_{\iota t}} = (I[\overline{\epsilon_{\iota t}} < 0]o\ \overline{\epsilon_{\iota t}})$, the latter being the element by element Hadamard product of the residuals if sector shocks are negative, and $\overline{\xi_t} = 0$ otherwise. Thus the asymmetric term, g, captures periods where both markets experience bad news (negative shocks), making $[\xi_{it}\ \xi_{jt}'] = I_t$. This study uses the diagonal version of the ADCC equation model, which is a special case of the Generalized ADCC (AG-DCC) model as the parameter matrices therein are replaced by scalars. In order to ensure that the $R_{ij,t}$ matrix has a unique solution, for each bi-variate case the determinant of the $({Q_{ij,t}^{*}}^{-1}.Q_{ij,t}.{Q_{ij,t}^{*}}^{-1})$ matrix will be tested for positive definiteness. These models are then estimated using quasi maximum-likelihood (QML) techniques based on the BHHH algorithm[22].

[20] Where θ is the parameters in D_t and ϕ the parameters in R_t.

[21] The sample analogue will be used for expectations throughout, implying that where indicated, $\overline{X_t} = E[x_t x_t'] = \frac{1}{T}\sum_{i=1}^{T} x_t x_t{}'$.

[22] Berndt, Hall, Hall & Hausman iterative optimization algorithm (1974).

After specifying the most appropriate mean and variance equations (see appendix) and then fitting the dynamic conditional correlation series for each bi-variate relationship, the dynamic structure of each sector pair will be explored further.

5 Results

5.1 *Asymmetry and Time-Variation in conditional correlations*

The first stage in building the DCC model framework consists of fitting the most appropriate univariate GARCH specifications to each series that best describes the return behaviour. Table 3 below contains the chosen specification and parameter values of the best GARCH model for each series based on the Bayesian Information Criterion (SBIC), Akaike criterion (AIC) and the Log Likelihood criterion. The univariate GARCH models tested include the standard GARCH (Bollerslev, 1986), GJR-GARCH (Glosten, Jagannathan and Runkle, 1993) and the EGARCH (Nelson, 1991) models[23], of which the specification details are given below the table.

Table 3: Univariate GARCH models

Sector	Model Selected	α_0	α_1	β_0	β_ε	ϕ	χ	γ	β_h
Basic Materials	**EGARCH**	0.036 (0.159)	0.058 (0.003)	-0.089 (0.000)			0.128 (0.000)	-0.053 (0.000)	0.987 (0.000)
Consumer Goods	**EGARCH**	0.086 (0.000)	-0.038 (0.05)	-0.085 (0.000)			0.121 (0.000)	-0.071 (0.000)	0.986 (0.000)
Consumer Services	**GJR-GARCH**	0.116 (0.000)	0.109 (0.000)	0.021 (0.000)	0.050 (0.000)	0.061 (0.000)			0.899 (0.000)
Financials	**GJR-GARCH**	0.067 (0.004)	0.045 (0.018)	0.022 (0.000)	0.049 (0.000)	0.079 (0.000)			0.895 (0.000)
Industrials	**GJR-GARCH**	0.085 (0.000)	0.059 (0.002)	0.035 (0.000)	0.051 (0.000)	0.077 (0.000)			0.878 (0.000)
Telecoms	**GJR-GARCH***	0.10 (0.001)	0.019 (0.298)	0.064 (0.000)			0.054 (0.000)	0.038 (0.000)	0.908 (0.000)

Source: Author's own calculations.

This table shows the optimal univariate GARCH model and its parameter estimates for each index return series based on the AIC, SBIC and Log-likelihood criteria mentioned in the text. The p-values are indicated in parentheses. The parameter entries correspond to the GARCH model specifications provided below. Series with an asterisk (*) indicate that the AIC and SBIC indicated different optimal models, with the model then chosen with the highest Log-Likelihood (there was no case where all three indicated different optimal models).

Univariate GARCH models used above include:

Mean Equation: $r_t = \alpha_0 + \alpha_1 r_{t-1} + \varepsilon_t$

Volatility Equation: $\varepsilon_t = \sqrt{h_t^2}.\eta_t \ , \ \ \eta_t \sim N(0,1)$

[23] The DCC and ADCC models are relatively insensitive to the univariate model specification ((Cappiello, et al. 2006). Nonetheless, the best univariate GARCH model will be sought as its accuracy is vital in the second stage of the model fit.

GARCH(1,1) [24]: $h_t = \beta_0 + \beta_\varepsilon \varepsilon_{t-1}^2 + \beta_h h_{t-1}$

GJR-GARCH(1,1): $h_t = \beta_0 + \beta_\varepsilon \varepsilon_{t-1}^2 + \phi . I[\varepsilon_{t-1} < 0] o \varepsilon_{t-1}^2 + \beta_h h_{t-1}$

EGARCH(1,1): $\ln(h_t) = \beta_0 + \chi \frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} + \gamma . \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \beta_h \ln(h_{t-1})$

It is interesting to note first that all the series display significant leverage effects in the conditional variance equations, requiring either GJR-GARCH or EGARCH models to be fitted. This implies that negative shocks tend to be followed by more volatility, on aggregate, than positive shocks of a similar magnitude. From the table above, the asymmetry parameters are measured by ϕ for the GJR-GARCH model and γ in the EGARCH model, respectively. This finding is consistent with Chinzara (2011), who also showed the presence of asymmetry in domestic sector returns.

All the returns series also display strong persistence in volatility, as measured by $(\alpha_1 + \beta_h)$. This is indicative of the presence of volatility clustering, or market momentum, which is a common feature of financial returns series[25]. The statistical significance of all the parameters also indicate the strong presence of conditional heteroskedasticity in all the returns series in the study. This greatly undermines the accuracy of static measures of domestic asset return correlations across sectors.

As mentioned in the methodology section, the second step is then to use the standardized residuals obtained from the estimated univariate models above to estimate the time-varying DCC and ADCC series by maximizing the log-likelihood functions mentioned before. This then provides us with estimates of the dynamic (time-varying) co-movements between sector returns, which will be studied in more detail in the next sub-section.

Figure 1 and 2 below show the bivariate conditional correlation graphs for each of the sector pairs using the DCC and ADCC MV-GARCH model estimations, respectively. It is interesting to note the heterogeneity in the dynamics of correlations between the sector pairs, showing that static estimates of co-movement can at times be misleading. Interesting to note from the figures too is that there is no clearly consistent increase or decrease in co-movements during the Global Financial Crisis (GFC) period that is shaded in the graphs. From the graphs below we also see that the DCC estimates vary more than the ADCC estimates, with both models producing similar mean levels for the conditional correlation (the exact values can be seen in tables 4 and 5 below).

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[24]Because of the numerical difficulty in estimating multivariate GARCH models, Silbennoinen & Terasvirta (2008) suggest the lag structure should be $p = q = 1$ for the univariate volatility model specifications, as used above.

[25] Cf.Koutmos & Booth (1995), Ogum (2001) and Chinzara (2011).

Figure 1 **DCC-MVGARCH graphs**

Shaded area is the Global Financial Crisis period. All the graphs below are similarly scaled between zero and one.

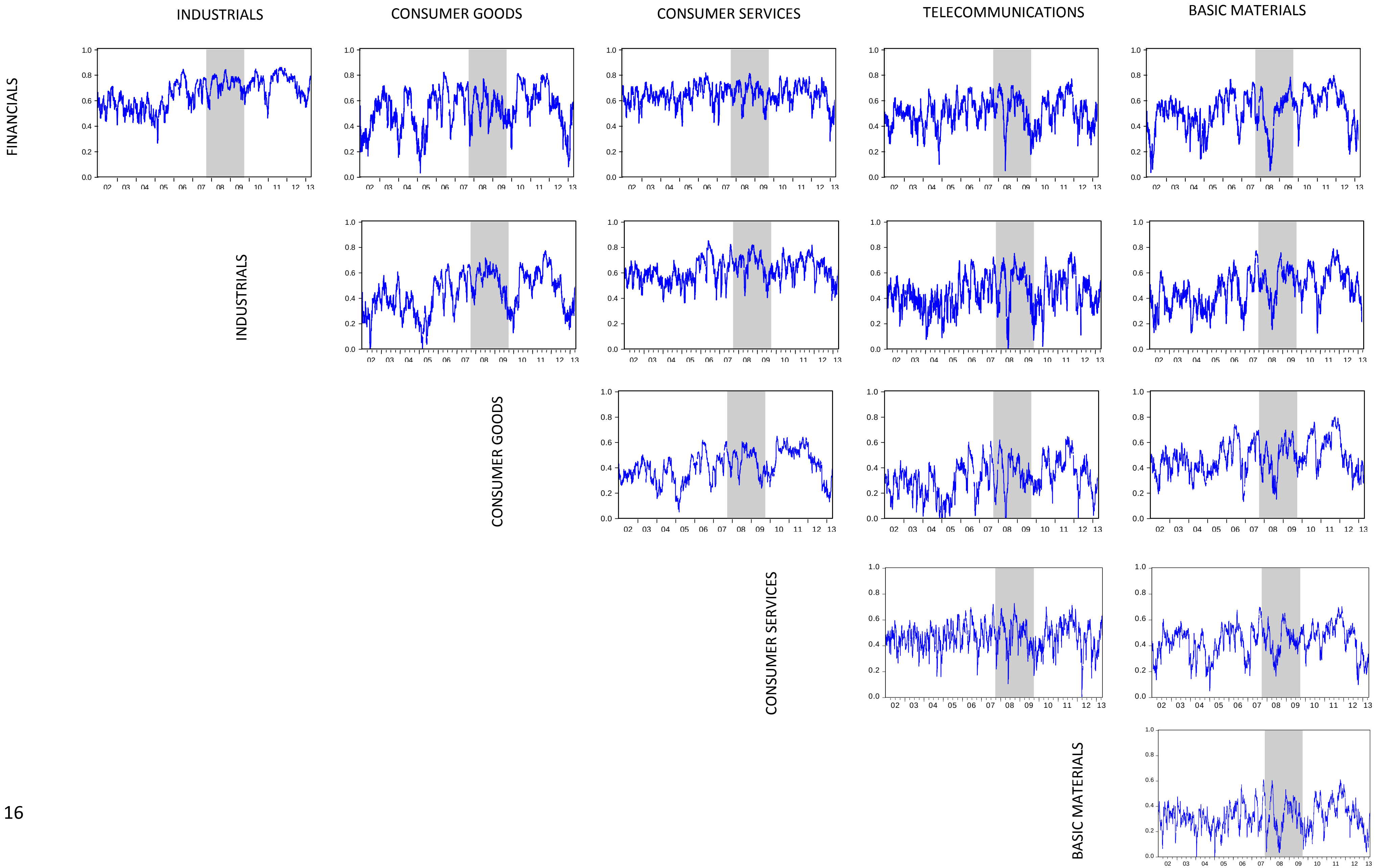
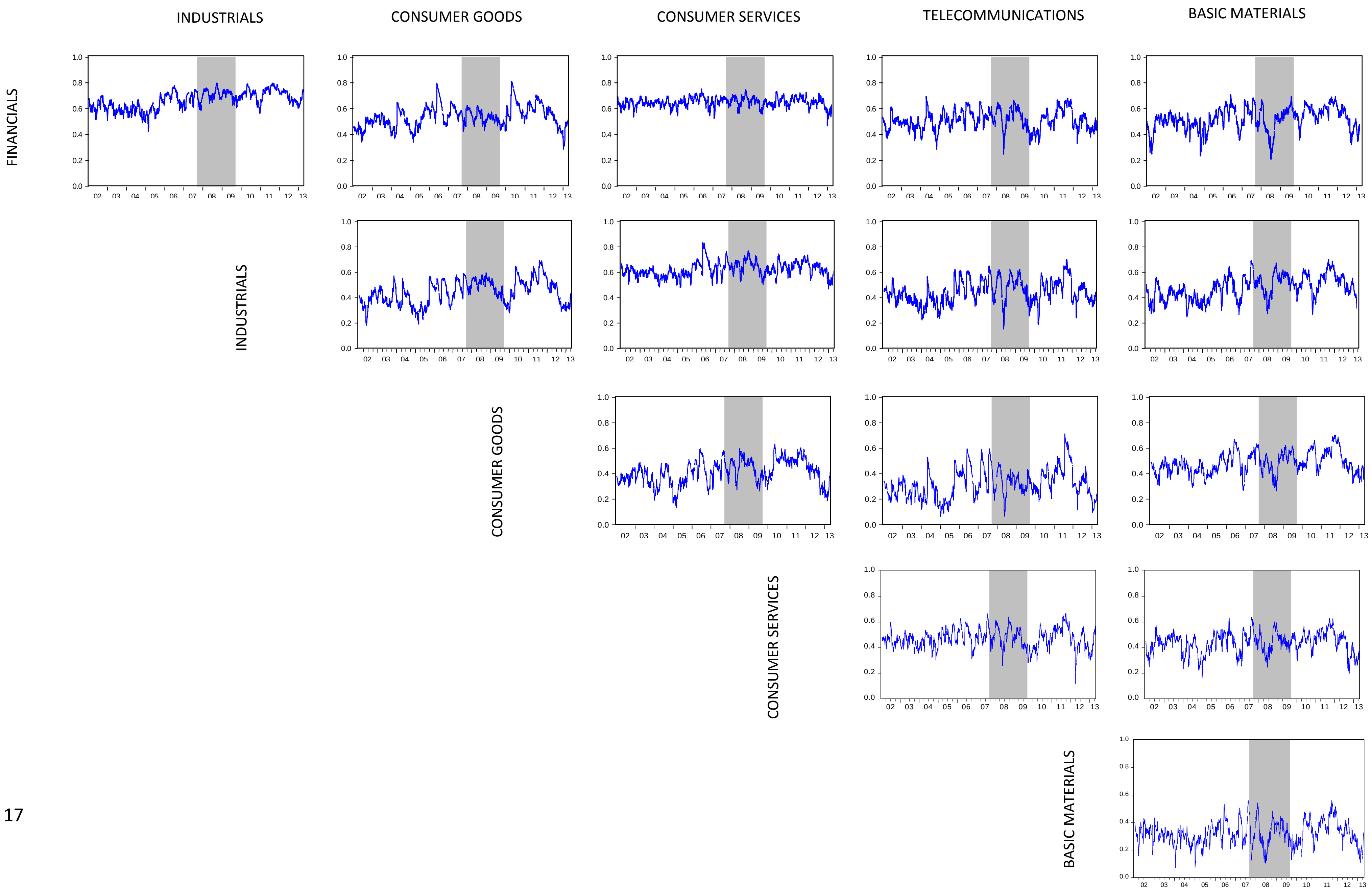


Figure 2 **ADCC-MVGARCH graphs**

Shaded area is the Global Financial Crisis period. All the graphs below are similarly scaled between zero and one.



The DCC-MVGARCH log-likelihood parameter estimates are summarized in table 4 below. The parameters measure the impact of past standardized shocks (θ_1) and lagged dynamic conditional correlations (θ_2) respectively on the current dynamic conditional correlations. The table suggests that the conditional correlations all show significant variations over time, as all the bivariate combinations have highly significant θ_1 and θ_2 parameters that are greater than zero. The necessary condition of $\theta_1 + \theta_2 < 1$ holds for all sector pairs, while the sum of the parameters is close to unity in each case. This suggests that the DCC model is adequate both at measuring time-varying conditional correlations, in that it displays mean reversion along a constant level, and controlling for the high degree of persistence in conditional volatility for all pairs of sectors in the study.

Table 5 on the next page shows the ADCC parameter estimates. Note the mean conditional correlations of both the DCC and ADCC series are very similar to each other and to the CCC model estimates in table 11 in the appendix. The mean level of correlation for nearly all the conditional estimates, however, differs significantly from the unconditional (static) correlations for most sector pairs, which again highlights the inaccuracy of assuming static inter-sectoral correlations between local assets.

Table 4: DCC MV-GARCH(1,1) parameter estimates

Sectors	Financials		Industrials		Consumer Goods		Cons Services		Telecoms	
	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$
Industrials	0.0298 (0.000)	0.964 (0.000)								
Mean, variance of R_t	$\mu_{R_t} = 0.66$ $\sigma_{Rt} = 0.113$									
Consumer Goods	0.038 (0.000)	0.952 (0.000)	0.026 (0.000)	0.967 (0.000)						
Mean, variance of R_t	$\mu_{R_t} = 0.53$ $\sigma_{Rt} = 0.155$		$\mu_{R_t} = 0.44$ $\sigma_{Rt} = 0.154$							
Consumer Services	0.034 (0.000)	0.934 (0.000)	0.034 (0.000)	0.943 (0.000)	0.019 (0.000)	0.971 (0.000)				
Mean, variance of R_t	$\mu_{R_t} = 0.64$ $\sigma_{Rt} = 0.077$		$\mu_{R_t} = 0.61$ $\sigma_{Rt} = 0.093$		$\overline{\mu_{R_t}} = 0.41$ $\sigma_{Rt} = 0.118$					
Telecoms	0.035 (0.000)	0.942 (0.000)	0.044 (0.000)	0.927 (0.000)	0.031 (0.000)	0.951 (0.000)	0.040 (0.000)	0.908 (0.000)		
Mean, variance of R_t	$\mu_{R_t} = 0.50$ $\sigma_{Rt} = 0.114$		$\mu_{R_t} = 0.44$ $\sigma_{Rt} = 0.138$		$\overline{\mu_{R_t}} = 0.32$ $\sigma_{Rt} = 0.136$		$\mu_{R_t} = 0.47$ $\sigma_{Rt} = 0.103$			
Basic Materials	0.031 (0.000)	0.959 (0.000)	0.032 (0.000)	0.956 (0.000)	0.0295 (0.000)	0.964 (0.000)	0.029 (0.000)	0.954 (0.000)	0.031 (0.000)	0.939 (0.000)
Mean, variance of R_t	$\mu_{R_t} = 0.51$ $\sigma_{Rt} = 0.148$		$\mu_{R_t} = 0.47$ $\sigma_{Rt} = 0.140$		$\overline{\mu_{R_t}} = 0.47$ $\sigma_{Rt} = 0.130$		$\mu_{R_t} = 0.43$ $\sigma_{Rt} = 0.118$		$\mu_{R_t} = 0.32$ $\sigma_{Rt} = 0.109$	

Source: Author's own calculations.

This table summarizes the estimated coefficients from the DCC-MV-GARCH model in a bivariate framework for all sector pairs in the study. The parameter values and p-values in parenthesis are reported. The log-likelihood was estimated using the Marquardt ML-technique. The first two moments $(\mu_{R_t}; \sigma_{R_t})$ of the dynamic conditional correlations between the series are given below the parameter values.

Table 5: ADCC MV-GARCH(1,1) parameter estimates

Sectors	Financials			Industrials			Consumer Goods			Cons Services			Telecoms		
	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{g}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{g}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{g}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{g}$	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{g}$
Industrials	0.135 (0.000)	0.981 (0.000)	0.021 (0.000)												
Mean, variance of R_t	$\mu_{R_t} = 0.66$; $\sigma_{Rt} = 0.07$														
Consumer Goods	0.097 (0.000)	0.982 (0.000)	0.062 (0.000)	0.096 (0.000)	0.0985 (0.000)	0.061 (0.000)									
Mean, variance of R_t	$\mu_{R_t} = 0.53$; $\sigma_{Rt} = 0.83$			$\mu_{R_t} = 0.44$; $\sigma_{Rt} = 0.097$											
Consumer Services	0.135* (0.000)	0.9963* (0.000)	-0.002* (0.999)	0.130 (0.000)	0.972 (0.000)	0.04 (0.000)	0.137 (0.000)	0.981 (0.000)	0.014 (0.000)						
Mean, variance of R_t	$\mu_{R_t} = 0.64$; $\sigma_{Rt} = 0.037$			$\mu_{R_t} = 0.61$; $\sigma_{Rt} = 0.058$			$\mu_{R_t} = 0.41$; $\sigma_{Rt} = 0.094$								
Telecoms	0.123 (0.000)	0.977 (0.000)	0.054 (0.000)	0.129 (0.000)	0.977 (0.000)	0.057 (0.000)	0.105 (0.000)	0.982 (0.000)	0.075 (0.000)	0.128 (0.000)	0.975 (0.000)	0.041 (0.000)			
Mean, variance of R_t	$\mu_{R_t} = 0.50$; $\sigma_{Rt} = 0.075$			$\mu_{R_t} = 0.44$; $\sigma_{Rt} = 0.094$			$\mu_{R_t} = 0.31$; $\sigma_{Rt} = 0.114$			$\mu_{R_t} = 0.47$; $\sigma_{Rt} = 0.075$					
Basic Materials	0.134 (0.000)	0.977 (0.000)	0.036 (0.000)	0.135 (0.000)	0.981 (0.000)	0.024 (0.000)	0.132 (0.000)	0.983 (0.000)	0.025 (0.000)	0.144 (0.000)	0.973 (0.000)	0.021 (0.000)	0.147 (0.000)	0.970 (0.000)	0.031 (0.03)
Mean, variance of R_t	$\mu_{R_t} = 0.51$; $\sigma_{Rt} = 0.090$			$\mu_{R_t} = 0.47$; $\sigma_{Rt} = 0.093$			$\mu_{R_t} = 0.47$; $\sigma_{Rt} = 0.086$			$\mu_{R_t} = 0.44$; $\sigma_{Rt} = 0.081$			$\mu_{R_t} = 0.319$; $\sigma_{Rt} = 0.083$		

Source: Author's own calculations

This table summarizes the estimated coefficients from the Asymmetric DCC-MV-GARCH model in a bivariate framework for all sector pairs in the study. The parameter values and p-values in parenthesis are reported. The log-likelihood was estimated using the BHHH ML-algorithm.

*Note: The Financials and Consumer Services Pair, had a non-zero det(QQQ) and as such cannot ensure that the conditional variance is positive. The first two moments (mean and standard deviation) of the dynamic conditional correlations between the series are given below the parameter values.

From table 5, we see that by introducing the parameter g, each sector pair displays a significantly positive impact on the strength of co-movement following negative returns to both series. This implies that periods of negative market momentum tend to reinforce co-movement between asset returns. It is also clear from the tables above that the ADCC estimates are centred more closely to the mean correlation levels as viewed by the lower standard deviations.

As the ADCC model nests both the DCC ($g = 0$) and the CCC ($g = \theta_1 = \theta_2 = 0$), we can compare the goodness of fit between the series using the Log-Likelihood statistics. Doing so, the ADCC model significantly outperforms the other two models in terms of a higher Log-Likelihood and lower AIC and SBIC statistics for all the sector pairs[26]. As such we can deduce that sector returns in this study display significant asymmetry and time-variation in its conditional correlations.

5.2 Studying the dynamics of the conditional correlations

In this subsection we explore the time-varying characteristics of the conditional correlations extracted in the previous section to understand how co-movements between sectors respond to changes in the macroeconomic environment. It is important to note, firstly, that all the DCC and ADCC series are highly persistent[27], yet do not reject the standard unit root tests. Such very high persistence could cause spurious results if left unchecked. As such, this paper follows Christopher, *et al.* (2012) in using the differenced series of each pair to explain its dynamics relative to changes in key exogenous factors. This paper, however, does not attempt to establish a long-run cointegrating relationship in order to formulate an Error Correction Model (ECM), as doing so on such high frequency co-movements seem untenable[28].

The estimated differenced equations do, however, include mean reversion elements in terms of its respective long-run mean levels. This is estimated by including the difference between the DCC and ADCC series levels with their long-run mean values, $\varepsilon_{t-1} = R_{ij,t} - \overline{R_{ij}}$, into the difference equation in order to establish whether deviations from this long-run mean is significant in driving changes in the short run co-movements. A negative parameter would suggest an opposite sign change in the next period's co-movement in response to the level of deviation from the long-run mean. This can then be interpreted as mean-reversion to the unconditional correlation following a deviation.

[26] These statistics are not included for the sake of brevity, but is available upon request from the author.

[27] The AR-terms of the DCC and ADCC series are nearly all above 0.98.

[28] Christopher, *et al*. (2012) fit an ECM using credit ratings and macroeconomic outlook indicators.

The differenced equations will also include several variables that estimate how the co-movements between sectors respond to changes in the macroeconomic environment. As such information is limited at daily frequencies, proxies will be used to measure the impact of daily fluctuations in market sentiment on the DCC and ADCC estimates of each sector pair.

The variables included in the differenced equations below are firstly the Chicago Board Options Exchange (CBOE) Volatility index (VIX), used as a proxy for global market uncertainty. Its inclusion is motivated by Connolly, *et al* (2005), who argue that increased uncertainty, as proxied for by an increase in the VIX, should raise the asset correlations between assets across sectors. Lags for this variable will be used as its information should only be fully absorbed in the domestic market with a one period delay, considering the time gap between South Africa and the US. The JSE All-Share Index (ALSI) will be used as a proxy for domestic asset market conditions, while the 10 year All Bond Index TRI closing value ($ALBI$) will be used as a measure of macroeconomic stability. Both these series reject the ADF test for stationarity[29], and as such require first differencing. We also look at the impact of the rand / dollar exchange volatility[30], $FXVOL$, as measured by the squared difference of the exchange rate. As suggested by Bracker, *et al* (1999), we expect increased $FXVOL$ to dampen inter-sector conditional correlation.

In order to capture the impact that broad market sentiment would have on the aggregate conditional correlations between the sectors, several indicator variables will be included in a level regression of the DCC and ADCC estimates. Syllignakis & Kouretas (2011) regressed their level DCC estimates on indicator variables that represent crisis periods, in order to test the impact such periods have on the aggregate correlations between Eastern European stock indices. In contrast, this study makes use of indicators that track significant changes in key macroeconomic variables as opposed to controlling for whole periods. This is done in order to isolate changes in implied market sentiment patterns in a more dynamic way. The first such variable is the VIX_{HIGH} indicator that is used to proxy for periods of high global market uncertainty. This is measured by VIX being larger than 30[31]. The $ALBI_{Low}$ and $ALSI_{LOW}$ variables correspond to periods where the $ALBI$ and the $ALSI$ dip below their respective 120 day Moving Average (MA)[32]. Using such indicators, which provide richer information in terms of changes in the sentiment, is important as the DCC and ADCC estimates are themselves dynamic.

[29] The statistics are not included for the sake of brevity, but is available upon request.

[30] The Rand / Dollar exchange is used as opposed to an aggregated exchange rate basket, as the former is observable in real time to all market participants and therefore is a better proxy for market perceptions on the daily rand exchange value.

[31] This is widely accepted as a high level of market uncertainty.

[32] Excluding weekends, this represents the 6 month MA. The results are also robust for different periods of MA.

The following differenced equation form will be used to achieve the first of the above stated goals:

$$\Delta R_{ij,t} = c + \phi(\varepsilon_{t-1}) + \beta_1.VIX(-1) + \beta_2.FXVOL(-1) + \beta_3.\Delta log(ALSI) + \beta_4.\Delta log(ALBI) \quad (18)$$

with $R_{ij,t}$ the DCC or ADCC series for the sector pair (i,j), and $\varepsilon_{t-1} = R_{ij,t} - \overline{R_{ij}}$ the deviation from the long-run mean conditional correlation. The following equations will then be used to evaluate whether market sentiment, globally and domestically, matter for the level of sector co-movements:

$$R_{ij,t} = c + \gamma_1.VIX_{Large} + \gamma_2.ALBI_{Low} + \gamma_3.ALSI_{Low} \quad (19)$$

The parameters are then fitted using OLS techniques. The first equation's estimated coefficients will thus show how sensitive the aggregate strength of correlation between each sector pair is relative to changes in each variable included, *ceteris paribus*, conditional upon past information. The second equation's coefficients estimate changes in the aggregate level of daily co-movement between sector pairs conditional upon experiencing deterioration in market sentiment as outlined above. The results for each of the sector pair differenced regressions are summarized in tables 6 and 7 below, while the level regression results are summarized in tables 8 and 9 for the DCC and ADCC techniques, respectively.

From the differenced equations in table 6 and 7 we see that there is significant mean reversion in all the sector pairs from the negative parameter values of ϕ. Also, an increase in the VIX index significantly positively impacts all the sector pair co-movements, although the size of the impact is limited in all cases. Currency volatility negatively impacts most of the sector pair co-movements, although for most pairs this effect is not statistically significant. This could be interpreted as currency volatility having similar impacts on sector returns on aggregate, so as not to change relative co-movements[33], which is an interesting finding.

The $\Delta log(ALSI)$ parameter is significant and negative for all the sector pairs, showing that an increase in the returns of the asset market as a whole leads to reduced co-movement between sectors. This is consistent with numerous other findings in the literature suggesting that during market upswings asset prices across sectors tend to reflect their fundamentals more closely. Surprisingly, the $\Delta log(ALBI)$ parameter suggests that increased domestic market stability increases the inter-sector asset co-movement, on aggregate, in all cases. The significance of this effect largely diminishes when taking into account return asymmetry in the ADCC model. A potential explanation of this might be that investors shift some part of their equity portfolio into fixed return instruments

[33] Exceptions in this regard are the Consumer Goods-, Consumer Services- and the Basic Materials sectors.

when bond prices adjust upward, leading to a slight homogenization (although mostly not significant) in the returns of assets across sectors.

Table 6*:* ***DCC Differenced Regression output:***

Sector pairs	c	ϕ	β_1	β_2	β_3	β_4
Fin & Ind	-0.002**	-0.015***	0.000***	-0.014	-0.114***	-0.016
Fin & Cons G	-0.003**	-0.019***	0.000**	-0.042**	-0.188***	0.404***
Fin & Cons S	-0.002**	-0.042***	0.000**	-0.013	-0.088***	0.093
Fin & Telecom	-0.003***	-0.028***	0.000**	-0.002	-0.107***	0.177
Fin & Basic M	-0.003***	-0.015***	0.000***	-0.041***	-0.130***	0.235**
Ind & Cons G	-0.003***	-0.012***	0.000***	-0.014	-0.159***	0.271***
Ind & Cons S	-0.003***	-0.035***	0.000***	-0.013	-0.111***	0.070
Ind & Telecom	-0.005***	-0.041***	0.000***	0.000	-0.146***	0.308*
Ind & Basic M	-0.004***	-0.022***	0.000***	-0.021	-0.180***	0.382***
Cons G & Cons S	-0.002**	-0.014***	0.000***	-0.023**	-0.099***	0.183**
Cons G & Telecom	-0.003**	-0.023***	0.000**	-0.009	-0.126***	0.138
Cons G & Basic M	-0.003***	-0.018***	0.000***	0.005	-0.176***	0.053
Cons S & Telecom	-0.002	-0.055***	0.000*	0.000	-0.087***	0.262
Cons S & Basic M	-0.004***	-0.025***	0.000***	-0.029**	-0.166***	0.342***
Telecom & Basic M	-0.004***	-0.037***	0.000***	-0.023	-0.119***	0.239*

Table 7*:* ***ADCC Differenced Regression output:***

Sector pairs	c	ϕ	β_1	β_2	β_3	β_4
Fin & Ind	-0.001**	-0.015***	0.000**	-0.005	-0.079***	0.047
Fin & Cons G	0.000	-0.014***	0.000	-0.012	-0.157***	0.067
Fin & Cons S	-0.001**	-0.047***	0.000***	-0.008	-0.048***	0.066
Fin & Telecom	-0.001**	-0.023***	0.000**	-0.002	-0.093***	0.106
Fin & Basic M	-0.002**	-0.017***	0.000***	-0.022**	-0.103***	0.151*
Ind & Cons G	-0.001**	-0.013***	0.000**	-0.010	-0.171***	0.086
Ind & Cons S	-0.002**	-0.031***	0.000***	-0.006	-0.089***	0.087
Ind & Telecom	-0.002**	-0.024***	0.000**	0.000	-0.126***	0.098
Ind & Basic M	-0.003***	-0.017***	0.000***	-0.012	-0.122***	0.195**
Cons G & Cons S	-0.002**	-0.019***	0.000***	-0.021**	-0.095***	0.113
Cons G & Telecom	-0.001	-0.015***	0.000	0.006	-0.201***	0.112
Cons G & Basic M	-0.002**	-0.016***	0.000**	0.003	-0.121***	0.064
Cons S & Telecom	-0.001	-0.027***	0.000	0.001	-0.064***	0.093
Cons S & Basic M	-0.003***	-0.030***	0.000***	-0.020*	-0.134***	0.261**
Telecom & Basic M	-0.003***	-0.034***	0.000***	-0.014	-0.101***	0.177*

Source: Author's own calculations.

Table 3 and 4 above show the parameter estimates for the following differenced regression outputs:

$$\Delta R_{ij,t} = c + \phi(\varepsilon_{t-1}) + \beta_1.VIX(-1) + \beta_2.FXVOL(-1) + \beta_3.dlog(ALSI) + \beta_4.dlog(ALBI)$$

Here $R_{ij,t}$ is the dynamic conditional correlation series as measured by the DCC- and ADCC-GARCH techniques respectively.

Note: ***, **, * denote statistical significance at the 1%, 5% and 10% level respectively.

The R^2 statistics for all the regressions are very low and indicate that there still remains many factors that need to be considered when accurately evaluating the movements of these series. As forecasting the movements are not the focus of this paper, it will not be explored in greater detail. The R^2 and any other goodness of fit statistics can be requested from the author.

Table 8*: DCC Level Regression output:*

Sector pairs	c	γ_1	γ_2	γ_3
Fin & Ind	0.634***	0.111***	0.059***	0.032***
Fin & Cons G	0.497***	0.040***	0.079***	0.107***
Fin & Cons S	0.628***	0.044***	0.019***	0.034***
Fin & Telecom	0.471***	0.085***	0.045***	0.081***
Fin & Basic M	0.507***	0.101***	0.028***	-0.004
Ind & Cons G	0.398***	0.136***	0.092***	0.092***
Ind & Cons S	0.595***	0.084***	0.032***	0.043***
Ind & Telecom	0.397***	0.150***	0.053***	0.083***
Ind & Basic M	0.456***	0.164***	0.021***	0.019***
Cons G & Cons S	0.381***	0.088***	0.039***	0.073***
Cons G & Telecom	0.282***	0.087***	0.053***	0.097***
Cons G & Basic M	0.458***	0.144***	0.069***	0.000
Cons S & Telecom	0.440***	0.055***	0.033***	0.074***
Cons S & Basic M	0.417***	0.098***	0.010*	0.038***
Telecom & Basic M	0.290***	0.101***	0.028***	0.052***

Table 9*: ADCC Level Regression outputs:*

Sector pairs	c	γ_1	γ_2	γ_3
Fin & Ind	0.644***	0.069***	0.038***	0.024***
Fin & Cons G	0.518***	0.016***	0.031***	0.053***
Fin & Cons S	0.636***	0.021***	0.009***	0.017***
Fin & Telecom	0.483***	0.062***	0.036***	0.049***
Fin & Basic M	0.510***	0.058***	0.019***	-0.002
Ind & Cons G	0.412***	0.070***	0.043***	0.075***
Ind & Cons S	0.604***	0.050***	0.023***	0.026***
Ind & Telecom	0.412***	0.108***	0.049***	0.044***
Ind & Basic M	0.462***	0.113***	0.016***	0.012***
Cons G & Cons S	0.386***	0.062***	0.029***	0.068***
Cons G & Telecom	0.291***	0.065***	0.056***	0.060***
Cons G & Basic M	0.464***	0.097***	0.045***	0.001
Cons S & Telecom	0.446***	0.055***	0.033***	0.049***
Cons S & Basic M	0.422***	0.066***	0.009**	0.030***
Telecom & Basic M	0.297***	0.078***	0.023***	0.038***

Source: Author's own calculations.

Table 5 and 6 above show the parameter estimates for the following level regression outputs:

$R_{ij,t} = c + \gamma_1.Vix_{large} + \gamma_2.ALBI_{Low} + \gamma_3.ALSI_{Low}$

Here $R_{ij,t}$ is the dynamic conditional correlation series as measured by the DCC- and ADCC-GARCH techniques respectively.

Note: ***, **, * denote statistical significance at the 1%, 5% and 10% level respectively.

The level equations given in tables 8 and 9 show strong significance for nearly all the indicator variables included. It shows firstly that for all sector pairs periods of increased global economic uncertainty raises the aggregate level of inter-sectoral asset price co-movement by approximately 10%. Its impact, however, is dampened in all cases when controlling for asymmetries in sector returns, showing then an increased level of co-movement on aggregate for the ADCC series of 6%. Periods of domestic market uncertainty, as proxied for by $ALBI_{LOW}$, show inter-sector co-movement increase by on aggregate 4.5% for the DCC and 3% for the ADCC model estimates. Periods of negative domestic asset market sentiment, as proxied for by $ALSI_{LOW}$, raises inter-sector co-movement by 5.5% and 3.5% on average for the DCC and ADCC pairs, respectively.

In summary, the results underline the need for investors to consider local and global economic conditions and levels of uncertainty when evaluating the benefits to local cross-sector diversification. Asset returns across domestic sectors tend to correlate more in periods of market uncertainty and overall asset market contraction, while negative return shocks to both sectors typically lead to higher conditional correlations between the pairs in subsequent periods (as indicated by the significant and positive parameter estimate g, in table 11). As such, the benefits to diversifying across local sectors diminish at exactly the time they are hoped by fund managers to be safeguarding the portfolio from potential losses.

Some care needs to be taken, however, in interpreting the findings from using these parametric volatility models. The strong assumption of normality in the innovations, e.g., is rejected in most financial time-series analyses[34]. This can lead to inaccurate measures, as the dynamic structure of the conditional correlations is a function of past returns. The problem of non-normality, however, is limited due to the high frequency of the data and the use of log-likelihood estimation techniques[35]. Another potential pitfall to this approach, as suggested by Silbennoinen & Terasvirta (2008), is that the long-run correlation between a pair of series is highly dependent on dynamic macroeconomic factors, responding to such factors in a non-constant way. Nonetheless, the findings are useful in providing an efficient means of studying changes in underlying dynamics of the co-movements between asset returns, and present a significant improvement on the static correlation estimates used more widely in practice. A future avenue of research will be to broaden this study to include foreign sectors and individual asset classes too.

6 Conclusion

The last few years has shown periods of intensified co-movement of asset prices separated across country- and sector borders. This has brought into question the extent to which portfolios that are diversified across local sectors shelter investors from periods of global and domestic asset return homogeneity. This paper studies the co-movements between the main economic sectors in South Africa in a dynamic framework, providing a means of differentiating between factors that influence the strength of co-movement over time.

Using DCC and ADCC MV-GARCH techniques, the time-varying conditional correlations are extracted from the variance component, to provide an estimate of dynamic sector co-movements over time. These series are then used in both differenced- and level regressions to study which factors influence the dynamics and the level of co-movement between domestic economic sectors over time. Changes in market uncertainty and -sentiment are proxied for by using indicators that represent periods where key indices deviate from past trends. As DCC and ADCC estimates are dynamic, it is necessary to include similarly dynamic indicators into the analysis. These indicators better represent changes in market conditions than, e.g., using a global financial crisis dummy.

The results show that global and domestic economic uncertainty, as well as local asset market sentiment, significantly influence both the short run dynamics and the aggregate level of co-

[34] This assumption is also rejected for all the sector returns in this study. The problem of non-normality is, however, a general financial time-series problem.

[35] High frequency financial time-series data is often considered lognormal asymptotically.

movement between local sector pairs. In particular, the results suggest that fund managers and investors should consider macroeconomic forecasts and expectations of market sentiment when evaluating the benefits in terms of diversifying domestic portfolios

The techniques used in this study are unique in its application to South African sectors. An avenue for future research will be to use these multivariate GARCH techniques to explain the conditional correlation dynamics between the South African sectors and its foreign counterparts.

Appendix A

Sectors included in the study:

Sector	**Ticker**	**Market Cap (R Million)***
Financial Sector (Financials)	J580	1 550 000
Industrial Sector (Industrials)	J520	400 000
Consumer Goods Sector (Cons G)	J530	2 390 000
Consumer Services Sector (Cons S)	J550	658 000
Telecommunications Sector (Telecoms)	J560	476 000
Basic Materials Sector (Basic M)	J510	1 590 000

*As at 30 April 2013. Data obtained from McGregor BFA.

Figure 3 **Continuously compounded sector returns:** $r_{i,t} = \ln\left(\frac{p_{i,t}}{p_{i,t-1}}\right) * 100$

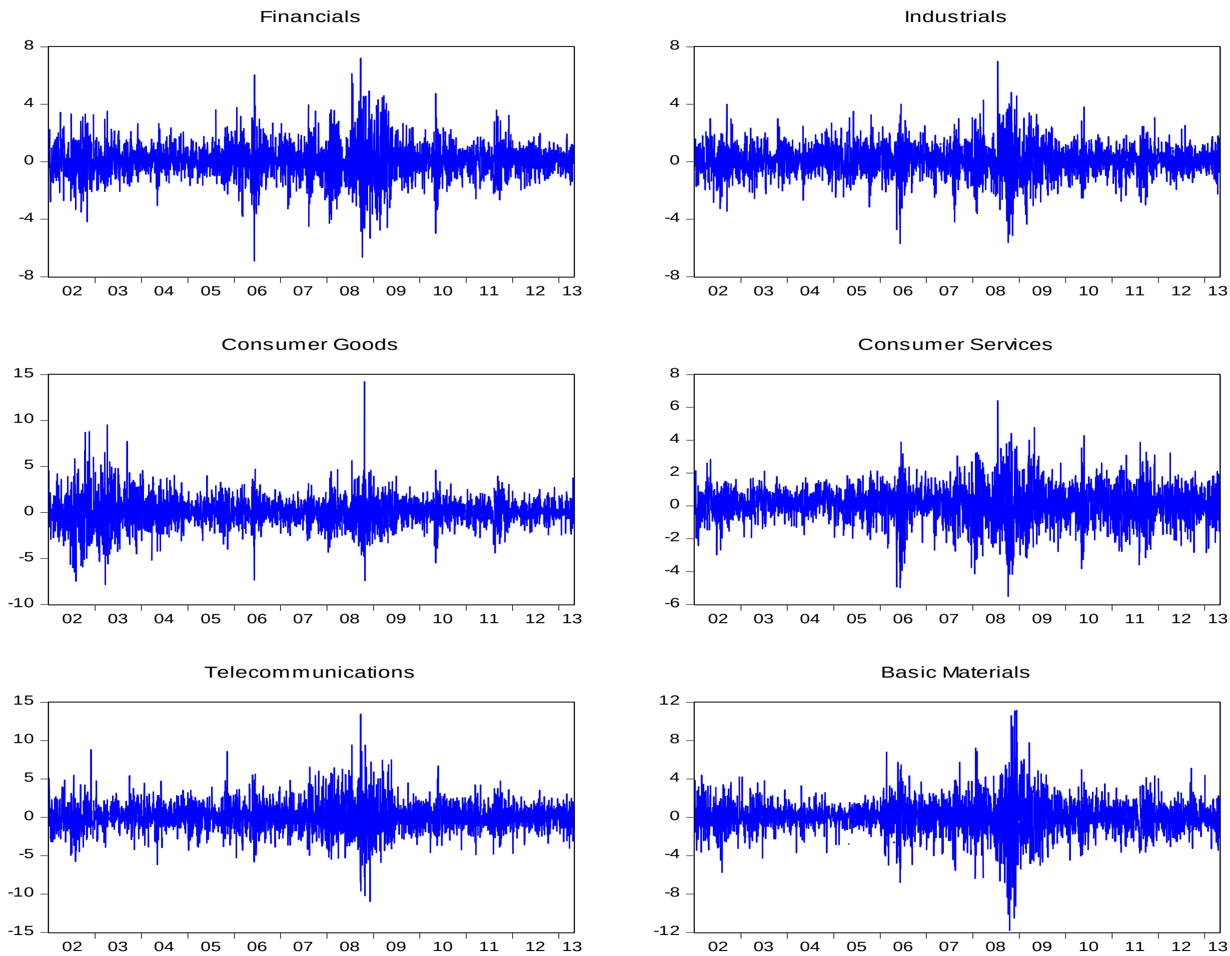


Table 10: **Unconditional correlations between South African economic sectors**

Sectors	Financials	Industrials	Consumer Goods	Consumer Services	Tele-communications
Industrials	0.708				
Consumer Goods	0.532	0.463			
Consumer Services	0.683	0.682	0.414		
Tele-communications	0.568	0.513	0.338	0.520	
Basic Materials	0.537	0.543	0.472	0.481	0.378

Source: Author's own calculations.
This table summarizes the unconditional correlation estimates between the sector returns included in this study.

Table 11: **Constant Conditional Correlations between South African economic sectors using CCC-Model estimates**

Sectors	Financials	Industrials	Consumer Goods	Consumer Services	Tele-communications
Industrials	0.672				
Consumer Goods	0.542	0.448			
Consumer Services	0.653	0.631	0.415		
Tele-communications	0.517	0.453	0.325	0.478	
Basic Materials	0.520	0.479	0.480	0.440	0.326

Source: Author's own calculations.
This table summarizes the conditional correlation estimates between the sector returns included in this study using the CCC-MVGARCH approach. The estimates above correspond to the off-diagonal entries of the R_{ij} matrix described in equation 5 in the text. The parameter estimates are all highly significant in the estimation outputs.

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