Kurt Gödel, Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme, I.

Published in the *Monatshefte für Mathematik und Physik*, Vol. 38, No. 1 (December, 1931), pp. 173-198.

The paper is broken up into four sections:

* [Section 1](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Godel1931Notes.html#SectionOne) (pgs 145-151)  
  a brief informal overview of the rest of the paper
* [Section II](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Godel1931Notes.html#SectionTwo) (pgs 151-181)  
  the main payload of the paper, leading to Theorem VI; it can be broadly divided into four parts:
  + pgs 151-157: description of the basic system of predicate logic
  + pgs 157-173: definition of the concept "recursive function"
  + pgs 163-171: definition of forty-five actual recursive functions
  + pgs 173-181: Proof of Theorem VI (173-177) and discussion
* [Section III](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Godel1931Notes.html#SectionThree) (pgs 181-191)  
  some corollaries and developments from Theorem VI, leading up to Theorem IX/X (pgs 187-191) relating recursive functions to the restricted functional calculus
* [Section IV](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Godel1931Notes.html#SectionFour) (pgs 191-195)  
  the "second incompleteness theorem", Theorem XI

Primary sources (incl. translations of the theorem)

Each of these contains a translation of the incompleteness theorem:

* *Collected Works: Volume I: Publications 1929-1936* by Kurt Gödel, Oxford University Press, 1986.  
  This has all of Gödel's papers from his "early" period, including the completeness and incompleteness theorems, in German and in English translation. The introductions to the papers are worth reading in themselves.  
  **Note:** this is the version we'll be working from in the lectures.
* *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931*, by Jean van Heijenoort, Harvard University Press, 2002.  
  This is a set of original papers covering the development of modern logic *before* the incompleteness theorem, including work by Russell, Hilbert and all the others.
* *The Undecidable: Basic Papers on Undecidable Propositions, Unsolvable Problems and Computable Functions* by Martin Davis, Dover Publications, 1984.  
  This is another set of original papers, this time starting with the incompleteness theorem and going *forward* to cover the main papers from the foundations of Computer Science, including works by Church and Turing.

Secondary sources - on-line

* Wikipedia: [Gödel's Incompleteness theorems](http://en.wikipedia.org/wiki/G%C3%B6del's_incompleteness_theorems)
* The *Stanford Encyclopedia of Philosophy* (SEP) is generally good as a more academic reference than Wikipedia: see their entry on [Kurt Gödel](http://plato.stanford.edu/entries/goedel/)
* A video lecture by Professor Mark Colyvan of the Sydney Centre for the Foundations of Science on *Kurt Gödel and the limits of mathematics*: [Part 1 (24 mins)](https://www.youtube.com/watch?v=bYpSVSGBxis), [Part 2 (21 mins)](https://www.youtube.com/watch?v=CCac2oP4XB8)
* The April 2006 *Notices of the American Mathematical Society* was [a special issue on Gödel](http://www.ams.org/notices/200604/index.html) with a number of nice articles and photos
* Peter Smith maintains an on-line blog called [Logic Matters](http://www.logicmatters.net/) that covers issues related to the theorem, including [Gödel Without Tears](http://www.logicmatters.net/igt/godel-without-tears/)
* For the historical context of the theorem, try [On the development of logic between the two world wars](http://www.jstor.org/stable/2321754) by Ivor Grattan-Guinness, from *The American Mathematical Monthly*, Vol. 88, No. 7 (Aug. - Sep., 1981), pp. 495-509.  
  (Accessible from within NUIM)
* [The Gödel theorem](http://projecteuclid.org/euclid.ndjfl/1093956833) by Norwood Russell Hanson, *Notre Dame J. Formal Logic* Volume 2, Number 2 (1961), 94-110.

Secondary sources - paper

These books are *about* the theorem, but do not contain an actual copy of the text itself

* *Engines of Logic: Mathematicians and the Origin of the Computer* by Martin Davis, W. W. Norton & Company, 2002.  
  This presents a light and readable narrative history of the development of modern Computer Science "from Leibniz to Turing". (This book has also been published under the title "The Universal Computer").
* *Gödel's Proof* by Ernest Nagel and James Newman, NYU Press, 2008.  
  A short rehearsal of the main points of the incompleteness theorem without getting bogged down in too much detail.
* *Gödel, Escher, Bach: An Eternal Golden Braid* by Douglas R Hofstadter, Penguin, 2000  
  This is a very roundabout treatment of the topic with an AI sub-theme; amusing if you have time on your hands.
* *Gödel's Theorem: An Incomplete Guide to Its Use and Abuse* by Torkel Franzén, A.K. Peters Ltd., 2005.  
  As the title suggests this book's focus is on what Gödel's theorem means in a wider context - also has a useful comparison with Gödel's *completeness* theorem.
* *An Introduction to Gödel's Theorems* by Peter Smith, Cambridge University Press, 2007.  
  A comprehensive step-by-step guide: basically a whole course designed around the theorems.

A note on page numbers

In what follows, page references refer to the page numbers in *Collected Works, Vol 1*. This is a translation, with the German on even page numbers and the English on odd pages: hence page references here for Gödel's paper will always be for *odd* page numbers.

Line numbers are given either:

* counting from the top (starting at 1), as in 185[12] (page 185, 12th line from the top)
* or from the bottom, as in 185[-10], (page 185, 10th line from the bottom)
* or a line range, as in 185[5:15] (page 185, lines 5 through 15, counting from the top)

*(This syntax is borrowed from Python's list notation...)*