Alan M. Turing, On Computable Numbers, with an Application to the *Entscheidungsproblem*

Published in the *Proceedings of the London Mathematical Society*, Vol 42 (series 2), No. 1 (1937) pp 230-65.

Available from lots of places, but officially from [the journal's website](http://plms.oxfordjournals.org/content/s2-42/1).

Structure of the paper

The paper is broken into an introduction, eleven sections and an appendix:

* Introduction: [(pp. 230-231)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionIntroduction)
* Section 1: [Computing Machines (pp. 231-232)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionOne)
* Section 2: [Definitions (pp. 232-233)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionTwo)
* Section 3: [Examples of computing machines (pp. 233-235)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionThree)
* Section 4: [Abbreviated tables (pp. 235-239)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionFour)
* Section 5: [Enumeration of computable sequences (pp. 239-241)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionFive)
* Section 6: [The universal computing machine (pp. 241-243)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionSix)
* Section 7: [Detailed description of the universal machine (pp. 243-246)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionSeven)
* Section 8: [Application of the diagonal process (pp. 246-248)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionEight)
* Section 9: [The extent of the computable numbers (pp. 249-254)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionNine)
* Section 10: [Examples of large classes of numbers which are computable (pp. 254-258)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionTen)
* Section 11: [Application to the Entscheidungsproblem (pp. 259-263)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionEleven)
* Appendix: [Computability and effective calculability (pp. 263-265)](https://2017.moodle.maynoothuniversity.ie/pluginfile.php/547197/mod_resource/content/3/Turing1937Notes.html#SectionAppendix)

Some other references include

* *The Annotated Turing: A Guided Tour Through Alan Turing's Historic Paper on Computability and the Turing Machine* by Charles Petzold, Wiley, 2008.   
  This is a whole book dedicated to just this paper: it goes through the paper line-by-line, with plenty of discussion.
* [Computable Numbers and the Turing Machine](http://www.turing.org.uk/scrapbook/machine.html) at the Turing website maintained by Andrew Hodges.   
  (His book *Alan Turing: The Enigma* is the standard Turing biography).
* For Turing's other work, a good starting point is *The Essential Turing* by B. Jack. Copeland, Clarendon Press, 2004.  
  This has a selection of Turing's papers (with introductions) going from 1936 up to his work on AI and morphogenesis.