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A memory guided sine cosine algorithm for global optimization

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ABSTRACT

Real-world optimization problems demand an algorithm which properly explores the search space to find a good solution to the problem. The sine cosine algorithm (SCA) is a recently developed and efficient optimization algorithm, which performs searches using the trigonometric functions sine and cosine. These trigonometric functions help in exploring the search space to find an optimum. However, in some cases, SCA becomes trapped in a sub-optimal solution due to an inefficient balance between exploration and exploitation. Therefore, in the present work, a balanced and explorative search guidance is introduced in SCA for candidate solutions by proposing a novel algorithm called the memory guided sine cosine algorithm (MG-SCA). In MG-SCA, the number of guides is decreased with increase in the number of iterations to provide a sufficient balance between exploration and exploitation. The performance of the proposed MG-SCA is analysed on benchmark sets of classical test problems, IEEE CEC 2014 problems, and four well known engineering benchmark problems. The results on these applications demonstrate the competitive ability of the proposed algorithm as compared to other algorithms.

1. Introduction

Optimization can be defined as the process of selecting the best option from an available set of alternatives. Many challenging problems in science, economics, business, and engineering can be formulated as an optimization problem. The general form of a single objective optimization problem can be stated as follows:

$$Min \quad f\left(\vec{x}\right) \qquad \vec{x} = \left(x_1, x_2, \dots, x_D\right) \in \mathbb{R}^D \tag{1}$$

$$s.t. \quad g_i\left(\vec{x}\right) \leq 0 \qquad i = 1, 2, \dots, I \tag{2}$$

$$h_j\left(\vec{x}\right) = 0 \qquad j = 1, 2, \dots, J \tag{3}$$

$$\vec{x} \in [\vec{x}_{\min}, \vec{x}_{\max}] \tag{4}$$

where, $\vec{x} = \left(x_1, x_2, \dots, x_D\right)$ is a decision vector in D-dimensional space. The functions $f, g_i(i = 1, 2, \dots, I)$ and $h_j(j = 1, 2, \dots, J)$ are real valued functions, respectively defined as the objective function, inequality constraints, and equality constraints. The variables I and J denote the number of inequality and equality constraints, respectively. The expression given in Eq. (4) is known as a bound constraint.

In the literature, several deterministic methods are available to find solutions to optimization problems. Most of these deterministic methods utilize derivative information of the functions being optimized (Himmelblau, 1972; Boyd and Vandenberghe, 2004; Yang, 2014). However, in real-life, it may not always be possible that the functions involved in the problems are differentiable, continues or convex, for example: economic dispatch problem (Niknam, 2010), sizing design of truss structures (Schutte and Groenwold, 2003), electromagnetic optimization problems (Grimaccia et al., 2007). Therefore, derivative-free stochastic methods were developed which utilize only the objective function to evaluate the quality of candidate solutions and which do not require that the function has mathematical properties such as being continuous, differentiable and convex. The success of stochastic optimization methods is very much attributed to the fact that the search for a good solution starts from multiple randomly selected positions in the search space, that global information about the search space is used to guide the search, and due to the ability to both explore and exploit the search space. Diversification, or exploration, is the ability of searching new regions of the available search space in order to locate promising regions of the search space that may locate a global optimal, or at least a good local optimal solution. Exploitation is the ability to refine the search in promising regions found during the exploration process. For a successful algorithm, a proper balance between these two conflicting objectives is essential.

Stochastic methods treat the optimization problem as a black box as they do not utilize the problem information. They just evaluate the value of functions at decision variables. Stochastic methods are iterative and introduce a random component into the search process. Most of

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these stochastic methods are nature-inspired, based on some phenomena from nature. Stochastic methods are also referred to as meta-heuristic algorithms. Nature inspired stochastic optimization methods include genetic algorithms (GAs) (Holland, 1992), particle swarm optimization (PSO) (Eberhart and Kennedy, 1995), differential evolution (DE) (Storn and Price, 1997) and artificial bee colony (ABC) (Karaboga and Basturk, 2007), amongst others. Some recently developed and efficient algorithms are the grey wolf optimizer (GWO) (Mirjalili et al., 2014), moth-flame optimization (MFO) algorithm (Mirjalili, 2015), whale optimization algorithm (WOA) (Mirjalili and Lewis, 2016), and many others.

The reason for developing new stochastic search algorithms which exhibit new foundational principles can be answered with the 'No Free Lunch Theorem (NFL)' (Wolpert and Macready, 1995). The NFL states that no single optimizer can be designed that is the best at solving all problems. In other words, a particular algorithm may show very efficient results on some set of problems, but the same algorithm may show poor performance on a different problem set. Thus the NFL has made the field of stochastic algorithms highly active and supports the development of new search algorithms and improvement of current available algorithms. Based on the nature of the search space of optimization problems, an algorithm requires a varying amount of exploration and exploitation during the search to locate an optimum solution. When we modify the search strategy of an algorithm, the main objective is to establish a sufficient amount of exploration and an appropriate balance between them, so that a wide range of optimization problems can be solved. The requirement of varying levels of exploration and exploitation for different optimization problems with different difficulty levels is the main issue in meta-heuristic algorithms and an imbalance between exploration and exploitation creates the problem of stagnation at a local optimum and premature convergence (Eiben and Schippers, 1998; Črepinšek et al., 2013; Del Ser et al., 2019).

The SCA (Mirjalili, 2016) is one of the recently developed stochastic algorithms to solve continuous optimization problems. Although the SCA is rich in diversification strength, it is poor in intensifying the search (Nenavath et al., 2018). Thus, an insufficient balance between intensification and diversification has been observed in the standard SCA (Nenavath et al., 2018). In SCA, each iteration updates the candidate solutions randomly and preserves the best solution. The remaining solutions are again updated using the search equation. In this process, since each candidate solution leaves its position and attains a new position, the useful information about the search history has been lost by that candidate solution. All these shortcomings emaciate the performance of SCA. Therefore, to handle such situations, several attempts have been made in literature to enhance the performance of the standard SCA. For example, Meshkat and Parhizgar (2017) introduced a weighted update position mechanism in SCA. Elaziz et al. (2017b) proposed an improved opposition-based SCA to provide better exploration ability to the candidate solutions. Sindhu et al. (2017) hybridized the SCA with an elitist strategy to solve the feature selection problem. Elaziz et al. (2017a) hybridized DE and SCA to enhance the capability of SCA to avoid local optima. To reduce susceptibility to local optima, Rizk-Allah (2018) combined SCA with multi-orthogonal search. Turgut (2017) developed a hybrid algorithm of the SCA and backtracking search (BSA) for thermal and economical optimization of a shell and tube evaporator. Li et al. (2017) and Attia et al. (2018) have used Lévy-flight search in SCA to avoid local optima. An improved version of SCA was used by Bairathi and Gopalani (2017) to train feed-forward neural networks. This improved version is based on opposition based learning which enhances the exploration ability of the algorithm. A hybrid version of SCA and GWO was proposed by Singh and Singh (2017) to capitalize on the exploitation ability of GWO and the exploration ability of SCA.

In the present paper, as an alternative approach to enhance the search performance of the standard SCA, a memory guided sine cosine algorithm (MG-SCA) is proposed in order to remove the above issues from the standard SCA and to increase the search efficiency of the algorithm. The proposed memory based strategy helps to locally explore the promising regions around the personal best state of candidate solutions. To validate the performance of MG-SCA, two benchmark sets, i.e. a classical set of 13 problems (Yao et al., 1999) and the standard IEEE CEC 2014 set of problems (Liang et al., 2013) have been selected. The MG-SCA is also applied to solve four well-known engineering optimization problems. The experiments and comparisons are supported by different metrics and statistical validations.

The rest of the paper is organized as follows. Section 2 discusses the standard SCA. Section 3 provides a detailed description of the proposed algorithm (MG-SCA). Section 4 presents the experimentation and comparison on benchmark problems and on four well-known engineering optimization problems. Finally, Section 5 concludes the work and suggests some future directions.

2. Overview of the Sine Cosine algorithm

The SCA was proposed by Mirjalili (2016) in 2014. As for other meta-heuristic algorithms, the SCA generates new solutions via random sampling around current solutions. In the population of size N, a new solution corresponding to the candidate solution $\vec{x}_i^t$, $(i = 1, 2, \dots, N)$ is generated as follows

$$x_{i,j}^{t+1} = \begin{cases} \dot{x}_{i,j}^{t+1} & if\ r_{i,j} < 0.5 \\ \tilde{x}_{i,j}^{t+1} & otherwise \end{cases} \tag{5}$$

where

$$\dot{x}_{i,j}^{t+1} = x_{i,j}^t + r_{1,j}^t \times \sin\left(r_{2_{i,j}}^t\right) \times |r_{3_{i,j}}^t y_j^t - x_{i,j}^t| \tag{6}$$

$$\tilde{x}_{i,j}^{t+1} = x_{i,j}^t + r_{1,j}^t \times \cos\left(r_{2_{i,j}}^t\right) \times |r_{3_{i,j}}^t y_j^t - x_{i,j}^t| \tag{7}$$

and $r_{i,j}$ is uniformly distributed random number from the interval (0,1) corresponding to the jth$(j = 1, 2, \dots, d)$ dimension of the solution $\vec{x}_i$. The random number, $r_{i,j}$, equally switches between the sine and cosine components to maintain randomness during the search between exploration and exploitation. The states of a solution $\vec{x}_i$ at the tth and the $(t+1)$th iteration in dimension j are represented by $x_{i,j}^t$ and $x_{i,j}^{t+1}$ respectively. The best solution obtained so far is represented by y_j^t for dimension j. The best solution obtained so far in SCA is referred to as the destination point. Random vector, $\vec{r}_1^t$, decides the search area for diversification $\left(r_{1,j}^t > 1\right)$ or intensification $(r_{1,j}^t < 1)$. The random vector, $\vec{r}_{2_i}^t$, describes the movement of the solution either towards the destination point $\vec{y}^t$ or outwards from the destination point $\vec{y}^t$. The value of $r_{2_{i,j}}^t$ for each $j = 1, 2, \dots, D$ is limited to the interval $(0, 2\pi)$, which is the domain of one cycle for the sine and cosine functions.

The random number $r_{3_{i,j}}^t \sim U(0,2)$ for each $j = 1, 2, \dots, D$, where $U(0,2)$ stands for uniform distribution with bounds 0 and 2. This means that the $r_{3_{i,j}}^t$ is uniformly distributed random number from the interval $(0, 2)$. The random vector $\vec{r}_{3_i}$ also contributes to the diversification and intensification.

In order to balance the exploration and exploitation during the search process, the vector $\vec{r}_1$ allows for exploration during the first half of the search process, after which $\vec{r}_1$ facilitates exploitation. In order to achieve this behaviour, $\vec{r}_1$ is adapted as follows:

$$r_{1,j}^t = 2 - \frac{2t}{T}, \forall j = 1, 2, \dots, D \tag{8}$$

where t represents the current iteration and T is the maximum number of iterations. The effect of the random number $r_{1,j}^t$ on the components $r_{1,j}^t \times \sin\left(r_{2_{i,j}}^t\right)$ and with different values of t and random values of $r_{2_{i,j}}^t$ is illustrated in Fig. 1. The random vector $\vec{r}_{3_i}$ promotes diversification at the end of the search, when $r_{1,j}$ approaches a value near to zero and fails to diversify the search space. SCA oscillates between exploration and exploitation using the adaptive ranges in the sine and cosine functions.

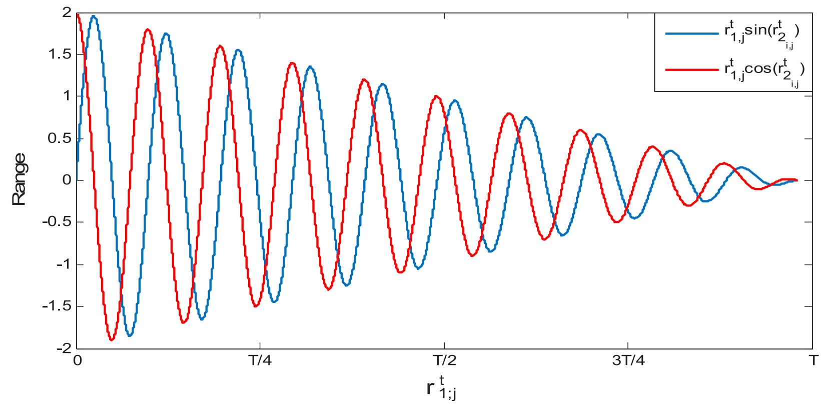

Fig. 1. Effect of the random number $r^t_{1,j}$ on the components $r^t_{1,j} \times \sin\left(r^t_{2_{i,j}}\right)$ and with different values of t and random values of $r^t_{2_{i,j}}$ (Mirjalili, 2016).

Thus, the Eqs. (5)–(7) allow the movement of a candidate solution from its current state to some other place based on the direction provided by the elite solution of the population, transition parameter r_1 and random vectors $\vec{r}_2, \vec{r}_3$ and $\vec{r}$. Pseudo code for the standard SCA is provided in Algorithm 1.

Algorithm 1. Standard Sine Cosine Algorithm

1. Randomly initialize the candidate solutions uniformly within the search bounds.
2. Evaluate the quality of each candidate solution using an objective function
3. Select the destination point $\vec{y}_t$ with best objective function value
4. Initialize the iteration count $t = 0$
5. Initialize the parameter $\vec{r}_1$ using equation (8)
6. Initialize the vectors $\vec{r}_2$ and $\vec{r}_3$
7. **while** $t < T$
8. **for** each candidate solution
9. Update the state using equation (5)
10. Evaluate the quality of updated candidate solution
11. **end of for**
12. Update the destination point $\vec{y}_t$
13. Update the vector $\vec{r}_1$ using equation (8)
14. $t = t + 1$
15. **end of while**
16. Return the destination point $\vec{y}_t$

Although, in the literature, various efforts have been made to enhance the search strategy of the standard SCA, in some cases these modified variants are still unable to find good solutions due to the insufficient ability of balancing exploration and exploitation of the search space. Therefore, as an alternative, the present paper proposes a modified version of the SCA, named MG-SCA, presented in the next section.

3. Proposed algorithm

This section proposes the memory guided sine cosine algorithm (MG-SCA). The MG-SCA is motivated in Section 3.1, and the search strategy is discussed in Section 3.2.

3.1. Motivation

The efficiency of meta-heuristic algorithms in terms of search ability depends on how well the meta-heuristic achieves a balance between diversification and intensification during the course of the search. SCA has shown an insufficient balance between exploration and exploitation (Nenavath et al., 2018). Results of the standard SCA on multimodal problems (Gupta and Deep, 2019b,a) demonstrate the problem of high diversification without sufficient exploitation. In the standard SCA, the previous set of solutions is replaced in each iteration by new updated solutions and only the best solution is saved for the next iteration. The individual solutions do not contribute their ability to provide a search direction and this may be the reason for inefficient exploration within the algorithm. Therefore, in the present paper, collective guidance in terms of a personal best history for each individual solution is used to update the solutions and to provide good exploration.

This paper proposes the memory guided sine cosine algorithm (MG-SCA) to help exploration of the search space in multiple good directions, thereby providing a better balance of exploitation and exploration. In MG-SCA, a memory matrix of personal best solutions is used to guide the candidate solutions. To enhance the exploitation ability of the algorithm, the number of guides that proceed with the search process is decreased over the duration of the search process. A large number of guided directions during the initial search iterations favours exploration, and a small number of guided directions favours exploitation.

3.2. Memory guided Sine Cosine algorithm

The MG-SCA initializes the memory matrix to contain all the initial solutions. The memory matrix is defined as follows:

$$MG = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,D} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,D} \\ \vdots & \vdots & \vdots & \vdots \\ m_{N,1} & m_{N,2} & \cdots & m_{N,D} \end{bmatrix} \tag{9}$$

where N is the number of candidate solutions that participates in the search process, D represents the dimension of the search space, and MG is the memory matrix. The array $\vec{m}_i = [m_{i,1}, m_{i,2}, \ldots, m_{i,D}]$ in the memory matrix MG represents the memory of the ith candidate solution $\vec{x}_i$. At each iteration, the ith memory $\vec{m}_i$ of the memory matrix is replaced with updated solution $\vec{x}_i^{t+1}$ if $f\left(\vec{x}_i^{t+1}\right) < f\left(\vec{m}_i\right)$. The number of guide solutions $\left(\Delta_t\right)$ at iteration t, is calculated as:

$$\Delta_t = round\left[N - (N-1)\frac{t}{T}\right] \tag{10}$$

From Eq. (10), it can be seen that Δ_t is decreasing with t. In the initial iteration of the algorithm, $r_{1,j}\,(>1)\,\forall j = 1, 2, \ldots, D$ supports exploration, and the continuous replacement of solutions during each iteration results in high exploration. Therefore, in such situations the balance between exploration and exploitation can be maintained by decreasing the number of guided candidate solutions. The search equations which are used to update the ith candidate solution are as follows:

$\boldsymbol{for\ i = 1:\ N}$
  $\boldsymbol{for\ j = 1:\ D}$
    $\boldsymbol{if\ i < \Delta_t}$

$$x_{i,j}^{t+1} = \begin{cases} y_j^t + r_{1,j}^t \times \sin\left(r_{2_{i,j}}^t\right) \times \left| r_{3_{i,j}}^t m_{i,j}^t - x_{i,j}^t \right| & if\ r_{i,j} < 0.5 \\ y_j^t + r_{1,j}^t \times \cos\left(r_{2_{i,j}}^t\right) \times \left| r_{3_{i,j}}^t m_{i,j}^t - x_{i,j}^t \right| & otherwise \end{cases} \tag{11}$$

$$\textit{else}$$

$$x_{i,j}^{t+1} = \begin{cases} y_j^t + r_{1,j}^t \times \sin\left(r_{2_{i,j}}^t\right) \times \left| r_{3_{i,j}}^t y_j^t - x_{i,j}^t \right| & if\ r_{i,j} < 0.5 \\ y_j^t + r_{1,j}^t \times \cos\left(r_{2_{i,j}}^t\right) \times \left| r_{3_{i,j}}^t y_j^t - x_{i,j}^t \right| & otherwise \end{cases} \tag{12}$$

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From Eqs. (11) and (12), when the number of guide solutions are greater than the index of the current solution, the candidate solution is updated using the memory matrix; otherwise, the candidate solution updates its solution using the destination point $\vec{y}_t$. The term $\left| r_{3_{i,j}}^t m_{i,j}^t - x_{i,j}^t \right|$ is used in the search equation as a cognitive component to maintain a balance between exploration and exploitation, due to the component $\vec{r}_1^t$ which provides too much exploration. The current solution $\vec{x}_i^t$ is replaced with the best solution $\vec{y}_t$ obtained so far to locally explore the search regions around $\vec{y}_t$. In the later iterations, when the component $r_{1,j}^t < 1 \forall j = 1, 2, \ldots, D$, the search space is exploited. A small number of candidate solutions update their states with the guidance of memory. Thus, by the end of the search process, promising regions are discovered around the destination point $\vec{y}_t$. The cognitive component $\left| r_{3_{i,j}}^t m_{i,j}^t - x_{i,j}^t \right|$ used in the search equation is similar to the cognitive component of PSO, which facilitates exploration of the search regions around the personal best history. The term $\left| r_{3_{i,j}}^t y_j^t - x_{i,j}^t \right|$ refers to the social component which is similar to the social component of PSO, helping with exploration around the destination point. The functions sine and cosine also allow search in areas distant from the destination and personal best history of the candidate solutions.

The pseudo code of the proposed algorithm is presented in Algorithm 2.

Algorithm 2. Memory Guided Sine Cosine Algorithm (MG-SCA)

1. Initialize the candidate solutions uniformly within the boundaries
2. Evaluate the quality of each candidate solution using objective function
3. Initialize the parameters $T, \vec{r}_1, \vec{r}_2$ and $\vec{r}_3$
4. Determine the destination point $\vec{y}_t$
5. Initialize the memory MG of guide solutions
6. Initialize the iteration count $t = 0$
7. **while** $t < T$
8. Calculate the number of guide solutions (Δ_t) using equation (10)
9. **for** each candidate solution i
10. **if** $i < \Delta_t$
11. Update the position using equation (11)
12. **else**
13. Update the position using equation (12)
14. **end of if**
15. Update the memory i of matrix MG
16. **end of for**
17. Update the destination point $\vec{y}_t$
18. Update the parameter $\vec{r}_1^t$
19. $t = t + 1$
20. end of while
21. Return the destination point $\vec{y}_t$

4. Results and discussion

This section conducts an empirical analysis of the performance of MG-SCA. The first experiment analyses performance on a standard benchmark of classical problems, while the second experiment analyses the performance on the standard benchmark of IEEE CEC 2014. The performance of the MG-SCA on four different classes of engineering problems is also discussed.

4.1. Benchmark functions

Standard benchmark problems are used by many researchers (Mirjalili et al., 2014; Mirjalili, 2015; Mirjalili and Lewis, 2016) to evaluate their algorithms. For ease of comparison with published results, the same benchmark problems are used to compare the standard SCA and the proposed MG-SCA. The standard set of benchmark problems are summarized in Table 1, and the IEEE CEC 2014 benchmark problems are given in Liang et al. (2013). The IEEE CEC 2014 benchmark set consists of various categories of problems such as unimodal, multimodal, hybrid and composite.

For the standard set of benchmark problems, 30 candidate solutions, 1000 iterations and 30 independent runs are used. For the IEEE CEC 2014, $10^4 \times D$ function evaluations and 51 runs are used as recommended in Liang et al. (2013). To compare the performance of MG-SCA with standard SCA, various statistical metrics (minimum, mean, median, worst, and standard deviation of objective function values) are calculated, and convergence analysis, statistical analysis, diversity analysis have been done in the present paper. The diversity analysis has been done by comparing the diversity curves of SCA and MG-SCA. These diversity curves are drawn by considering the average distance between the solutions in each iteration (Olorunda and Engelbrecht, 2008).

The MG-SCA is also compared with PSO (Shi and Eberhart, 1998), Moth-flame Optimization (MFO) (Mirjalili, 2015), Grey Wolf Optimizer (GWO) (Mirjalili et al., 2014), Whale Optimization Algorithm (WOA) (Mirjalili and Lewis, 2016), modified SCA (m-SCA) (Nayak et al., 2018), Opposition-based Sine Cosine Algorithm (OBSCA) (Elaziz et al., 2017b), Covariance Matrix Adaptation Evolution Strategy (CMA-ES) (Hansen and Ostermeier, 2001), Improved Differential Evolution (IDE) (Hongfeng, 2018), Sinusoidal Differential Evolution (SinDE) (Draa et al., 2015) and Beta Differential Evolution (BDE) (Bhandari, 2018). The control parameter values of these algorithms are adopted as given in the original papers.

4.2. Analysis of the results

Tables 2 and 3 present the results on the performance of SCA and MG-SCA for the 10 and 30-dimensional standard benchmark problems respectively. In these tables, the minimum, median, average, maximum and standard deviation of absolute error in objective function values obtained over 30 independent runs are listed. The best results are indicated in bold. A statistical comparison between the results of SCA and MG-SCA is performed using a non-parametric Mann–Whitney U test at 5% significance level, and the outcomes are presented in Table 4. In this table, '+/=/-' symbols are used to indicate that the MG-SCA is significantly better, the same, or worse than the SCA. The tables clearly indicate that the MG-SCA outperforms the SCA on all of the unimodal problems in 10 and 30 dimensions.

For all of the multimodal problems, the MG-SCA provides better optimization results as compared to standard SCA. The outcomes of the Mann–Whitney U test in Table 4 indicate that the MG-SCA is superior to the standard SCA. Thus, for all the unimodal and multimodal problems the MG-SCA provides better results as compared to SCA.

From the diversity curves drawn in Figs. 2 and 3, it can be observed that, after an initial exploration phase (where both algorithms maintained the same diversity level), MG-SCA exploited faster, with a fast reduction in the diversity. Despite of this, MG-SCA performance in terms of accuracy is best, indicating that MG-SCA efficiently exploits good areas of the search space. Thus, from the diversity analysis it can be concluded that the proposed memory-guided search strategy has enhanced the search efficiency of the candidate solutions in the MG-SCA by establishing an appropriate balance between exploration and exploitation.

The second experiment uses the problems that are selected from the IEEE CEC 2014 (Liang et al., 2013) benchmark set. The results

Table 1
Classical benchmark test problems.

Test problems	Category	Range of search space	F_{min}
$F1(x) = \sum_{i=1}^{d} x_i^2$	Unimodal	[−100, 100]	0
$F2(x) = \sum_{i=1}^{d} \lvert x_i \rvert + \prod_{i=1}^{d} \lvert x_i \rvert$	Unimodal	[−10, 10]	0
$F3(x) = \sum_{i=1}^{d} (\sum_{j-1}^{i} x_i)^2$	Unimodal	[−100, 100]	0
$F4(x) = max_i\{\lvert x_i \rvert, 1 \le i \le d\}$	Unimodal	[−100, 100]	0
$F5(x) = \sum_{i=1}^{d-1} [100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2]$	Unimodal	[−30, 30]	0
$F6(x) = \sum_{i=1}^{d} ([x_i + 0.5])^2$	Unimodal	[−100, 100]	0
$F7(x) = \sum_{i=1}^{d} i \cdot x_i^4 + rand[0, 1)$	Unimodal	[−1.28, 1.28]	0
$F8(x) = \sum_{i=1}^{d} -x_i sin(\sqrt{\lvert x_i \rvert})$	Multimodal	[−500, 500]	−418.9829×D
$F9(x) = \sum_{i=1}^{d} [x_i^2 - 10cos(2\pi x_i) + 10]$	Multimodal	[−5.12, 5.12]	0
$F10(x) = \sum_{i=1}^{d} -20exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right) - exp(\frac{1}{d}\sum_{i=1}^{d} cos(2\pi x_i)) + 20 + e$	Multimodal	[−32, 32]	0
$F11(x) = \frac{1}{4} \times 10^{-3} \sum_{i=1}^{d} x_i^2 - \prod_{i=1}^{d} cos\left(x_i/\sqrt{i}\right) + 1$	Multimodal	[−600, 600]	0
$F12(x) = \frac{\pi}{d}\{10sin(\pi y_1) + \sum_{i=1}^{d-1} (y_i - 1)^2 \left[1 + 10\sin 2(\pi y_{i+1})\right] + (y_n - 1)^2\} + \sum_{i=1}^{d} u(x_i, 10, 100, 4)$ $y_i = \frac{x_i+5}{4}$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & if\ x_i > a \\ k(-x_i - a)^m & if\ x_i < -a \\ 0 & otherwise \end{cases}$	Multimodal	[−50, 50]	0
$F13(x) = 0.1 \times \left\{\sin 2(3\pi x_1) + \sum_{i=1}^{d} (x_i - 1)^2 \left[1 + \sin 2(1 + 3\pi x_i)\right] + (x_d - 1)^2 \left[1 + \sin 2(2\pi x_n)\right]\right\} + \sum_{i=1}^{d} u(x_i, 5, 100, 4)$	Multimodal	[−50, 50]	0

produced by MG-SCA and the standard SCA on this problem set are presented in Tables 5 and 6 corresponding to dimensions 10 and 30 respectively. These tables provide the minimum, median, average, maximum and standard deviation of absolute error values over the 51 independent runs. The better results are highlighted in bold. The statistical comparison between the results is presented in Tables 7 and 8. The statistical outcomes indicate that, for all of the unimodal problems (F1–F3), the MG-SCA significantly outperforms SCA. For all of the multimodal problems (F4–F16), the MG-SCA provides statistically significantly better optimization results as compared to standard SCA in all the problems. Similarly, for all of the hybrid (F17–F22) and the composite (F23–F30) problems, the MG-SCA significantly outperforms SCA.

4.3. Convergence analysis

In order to analyse the convergence rate of the MG-SCA, Figs. 4 and 5 provide the convergence plots for the classical benchmark problems in 30 dimensions. It can be observed from these curves that for all the problems, the MG-SCA shows faster convergence as compared to the standard SCA.

4.4. Computational complexity comparison

In this section, the worst time computational complexity in terms of big-O notation is calculated for the standard SCA and the proposed MG-SCA. This complexity is calculated based on the pseudo-code of the algorithms provided in Algorithm 1 and Algorithm 2, respectively. The obtained complexities for both the algorithms are the same, namely $O(N \times D \times T)$, where N is the population size, D represents the dimension of the problem and T is the maximum number of iterations.

The average CPU time (in seconds) for both the algorithms is also recorded and presented in Tables 9 and 10. From these tables, it can be observed that both the algorithms provide the solution in approximately the same time for all of the benchmark test problems.

4.5. Comparison with other algorithms

In this section, the performance of the MG-SCA is compared with that of other meta-heuristic algorithms. PSO (Shi and Eberhart, 1998), Moth-flame Optimization (MFO) (Mirjalili, 2015), Grey Wolf Optimizer (Mirjalili et al., 2014), Whale Optimization Algorithm (WOA) (Mirjalili and Lewis, 2016), standard SCA (Mirjalili, 2016), modified SCA (m-SCA) (Nayak et al., 2018), Opposition based Sine Cosine Algorithm (OBSCA) (Elaziz et al., 2017b), Covariance Matrix Adaptation Evolution Strategy (CMA-ES) (Hansen and Ostermeier, 2001), Improved Differential Evolution (IDE) (Hongfeng, 2018), Sinusoidal Differential Evolution (SinDE) (Draa et al., 2015) and Beta Differential Evolution (BDE) (Bhandari, 2018) are selected for the comparison with the MG-SCA. The control parameters of these algorithms are set as in the original papers. Tables 11 and 12 summarize the results for

Table 2
Minimum, Median, Average, Maximum and Standard Deviation (STD) of error in objective function values obtained by standard SCA and proposed algorithm (MG-SCA) for 10-dimensional classical set of benchmarks.

Test problem	Algorithm	Minimum	Median	Average	Maximum	STD
F1	SCA	6.03E−36	7.47E−30	8.92E−28	1.60E−26	3.02E−27
	MG-SCA	**5.74E−262**	**4.77E−250**	**6.70E−239**	**1.98E−237**	**0**
F2	SCA	2.05E−23	1.98E−20	3.81E−19	5.71E−18	1.11E−18
	MG-SCA	**4.68E−146**	**2.60E−141**	**6.69E−138**	**1.92E−136**	**3.51E−137**
F3	SCA	2.46E−17	5.39E−12	8.04E−10	5.19E−09	1.56E−09
	MG-SCA	**1.31E−106**	**4.58E−95**	**2.80E−90**	**6.59E−89**	**1.20E−89**
F4	SCA	5.50E−13	3.86E−09	1.32E−06	3.22E−05	5.90E−06
	MG-SCA	**8.18E−64**	**5.49E−58**	**1.59E−55**	**3.14E−54**	**5.84E−55**
F5	SCA	6.47E+00	7.19E+00	7.14E+00	8.16E+00	**4.24E−01**
	MG-SCA	**5.06E+00**	**6.06E+00**	**5.84E+00**	**6.29E+00**	4.44E−01
F6	SCA	1.79E−01	3.67E−01	3.99E−01	7.71E−01	1.58E−01
	MG-SCA	**6.11E−07**	**2.43E−06**	**1.38E−02**	**2.38E−01**	**5.32E−02**
F7	SCA	2.19E−04	1.43E−03	1.69E−03	6.81E−03	1.52E−03
	MG-SCA	**1.39E−04**	**6.01E−04**	**6.15E−04**	**1.94E−03**	**4.55E−04**
F8	SCA	1.55E+03	1.95E+03	1.94E+03	2.27E+03	**4.36E+03**
	MG-SCA	**7.72E+02**	**1.34E+03**	**1.30E+03**	**1.73E+03**	4.42E+03
F9	SCA	**0**	**0**	2.67E−12	7.97E−11	1.45E−11
	MG-SCA	**0**	**0**	**0**	**0**	**0**
F10	SCA	**4.44E−15**	**4.44E−15**	2.09E−09	6.27E−08	1.15E−08
	MG-SCA	**4.44E−15**	**4.44E−15**	**4.44E−15**	**4.44E−15**	**0**
F11	SCA	**0**	**0**	4.52E−02	3.99E−01	1.15E−01
	MG-SCA	**0**	**0**	**1.20E−02**	**7.86E−02**	**2.02E−02**
F12	SCA	4.46E−02	8.25E−02	8.39E−02	1.86E−01	3.01E−02
	MG-SCA	**1.39E−07**	**8.49E−07**	**3.87E−03**	**1.99E−02**	**7.87E−03**
F13	SCA	7.79E−02	2.57E−01	2.65E−01	4.71E−01	1.16E−01
	MG-SCA	**1.44E−06**	**6.45E−06**	**2.68E−02**	**1.10E−01**	**4.53E−02**

Table 3
Minimum, Median, Average, Maximum and Standard Deviation (STD) of error in objective function values obtained by standard SCA and proposed algorithm (MG-SCA) for 30-dimensional classical set of benchmarks.

Test problem	Algorithm	Minimum	Median	Average	Maximum	STD
F1	SCA	7.22E−07	4.38E−04	6.70E−03	6.17E−02	1.54E−02
	MG-SCA	**2.32E−112**	**9.04E−109**	**1.17E−104**	**3.33E−103**	**6.08E−104**
F2	SCA	3.10E−08	3.12E−06	4.91E−05	8.94E−04	1.69E−04
	MG-SCA	**3.66E−73**	**1.78E−69**	**5.80E−68**	**5.05E−67**	**1.24E−67**
F3	SCA	7.95E+01	2.63E+03	3.74E+03	1.11E+04	3.62E+03
	MG-SCA	**6.00E−28**	**9.56E−22**	**7.92E−20**	**1.46E−18**	**2.68E−19**
F4	SCA	1.11E+00	2.07E+01	2.08E+01	5.66E+01	1.18E+01
	MG-SCA	**1.78E−17**	**3.77E−15**	**6.79E−14**	**1.11E−12**	**2.08E−13**
F5	SCA	2.86E+01	4.03E+01	3.35E+02	2.75E+03	7.14E+02
	MG-SCA	**2.52E+01**	**2.67E+01**	**2.68E+01**	**2.88E+01**	**9.70E−01**
F6	SCA	3.53E+00	4.55E+00	4.54E+00	5.55E+00	4.86E−01
	MG-SCA	**4.89E−01**	**1.38E+00**	**1.34E+00**	**2.27E+00**	**4.67E−01**
F7	SCA	7.05E−03	2.25E−02	3.42E−02	1.15E−01	2.76E−02
	MG-SCA	**4.79E−04**	**1.43E−03**	**1.77E−03**	**6.38E−03**	**1.33E−03**
F8	SCA	7.64E+03	8.69E+03	8.63E+03	9.23E+03	**1.30E+04**
	MG-SCA	**4.47E+03**	**5.75E+03**	**5.76E+03**	**6.93E+03**	1.32E+04
F9	SCA	2.49E−08	8.20E+00	2.13E+01	7.02E+01	2.38E+01
	MG-SCA	**0**	**0**	**0**	**0**	**0**
F10	SCA	2.88E−03	2.01E+01	1.68E+01	2.03E+01	**6.46E+00**
	MG-SCA	**7.99E−15**	**1.15E−14**	**8.69E+00**	**2.01E+01**	1.01E+01
F11	SCA	2.19E−03	3.74E−01	3.64E−01	1.00E+00	2.89E−01
	MG-SCA	**0**	**0**	**3.21E−03**	**3.81E−02**	**8.51E−03**
F12	SCA	3.79E−01	1.37E+00	2.09E+00	8.32E+00	2.09E+00
	MG-SCA	**3.38E−02**	**5.89E−02**	**7.07E−02**	**1.43E−01**	**3.16E−02**
F13	SCA	2.29E+00	3.68E+00	2.73E+02	6.83E+03	1.25E+03
	MG-SCA	**7.36E−01**	**1.41E+00**	**1.42E+00**	**1.85E+00**	**2.57E−01**

the standard and the IEEE CEC 2014 benchmark sets, respectively. The results are ranked based on the difference between the wins and losses corresponding to each run. A win occurs when the obtained mean objective function value is better for the MG-SCA as compared to the competing algorithm. Similarly, a loss occurs when the mean objective function value is worse for the MG-SCA as compared to the

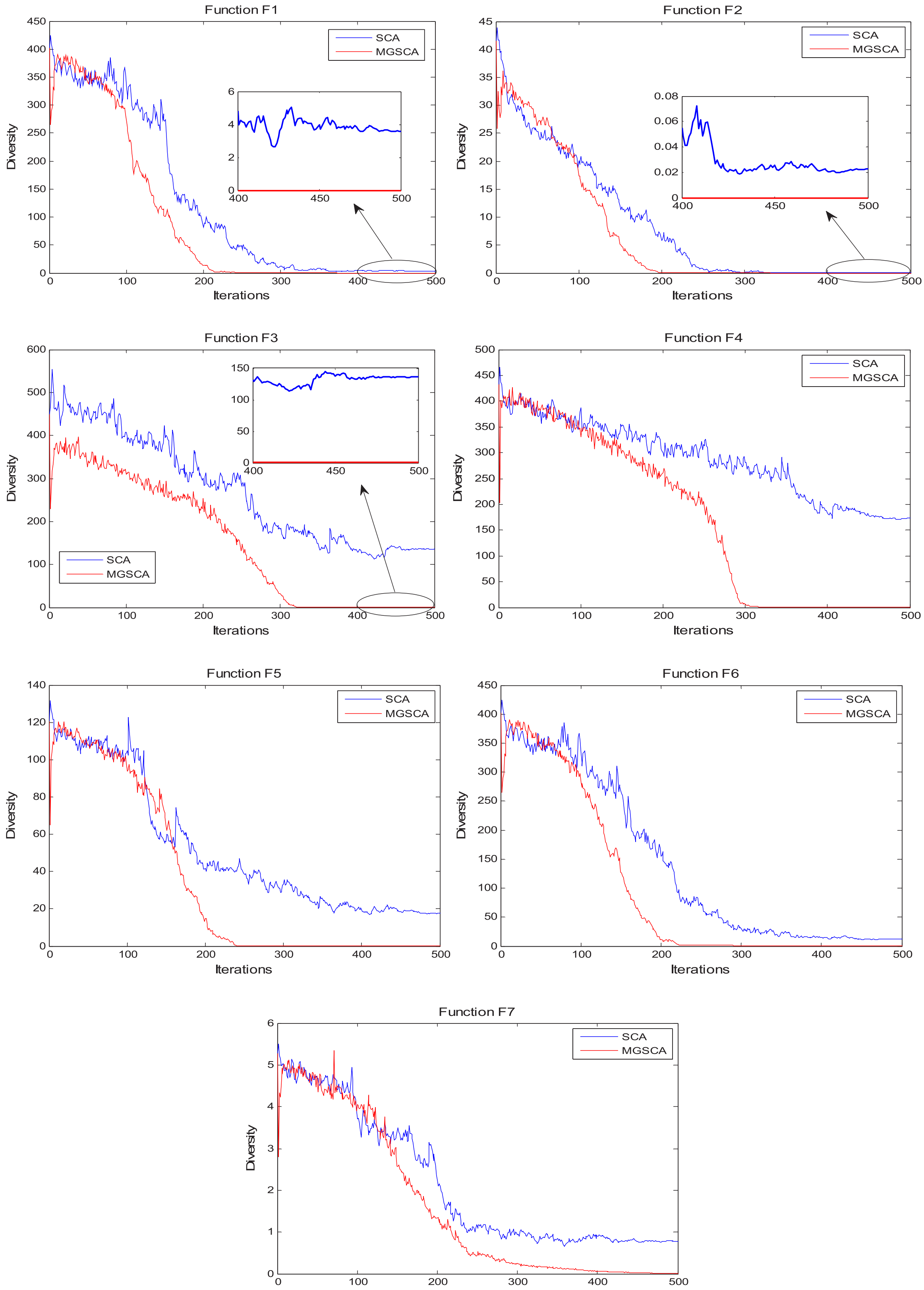

Fig. 2. Diversity plot for unimodal problems.

competing algorithm. After calculating the rank for each algorithm, the average and overall ranks of all the algorithms are calculated, which are provided at the end of the tables. The average and overall ranks show that the proposed MG-SCA has outperformed all the algorithms except CMA-ES on standard benchmark set and except for CMA-ES, IDE, SinDE and BDE on IEEE CEC 2014 benchmark set.

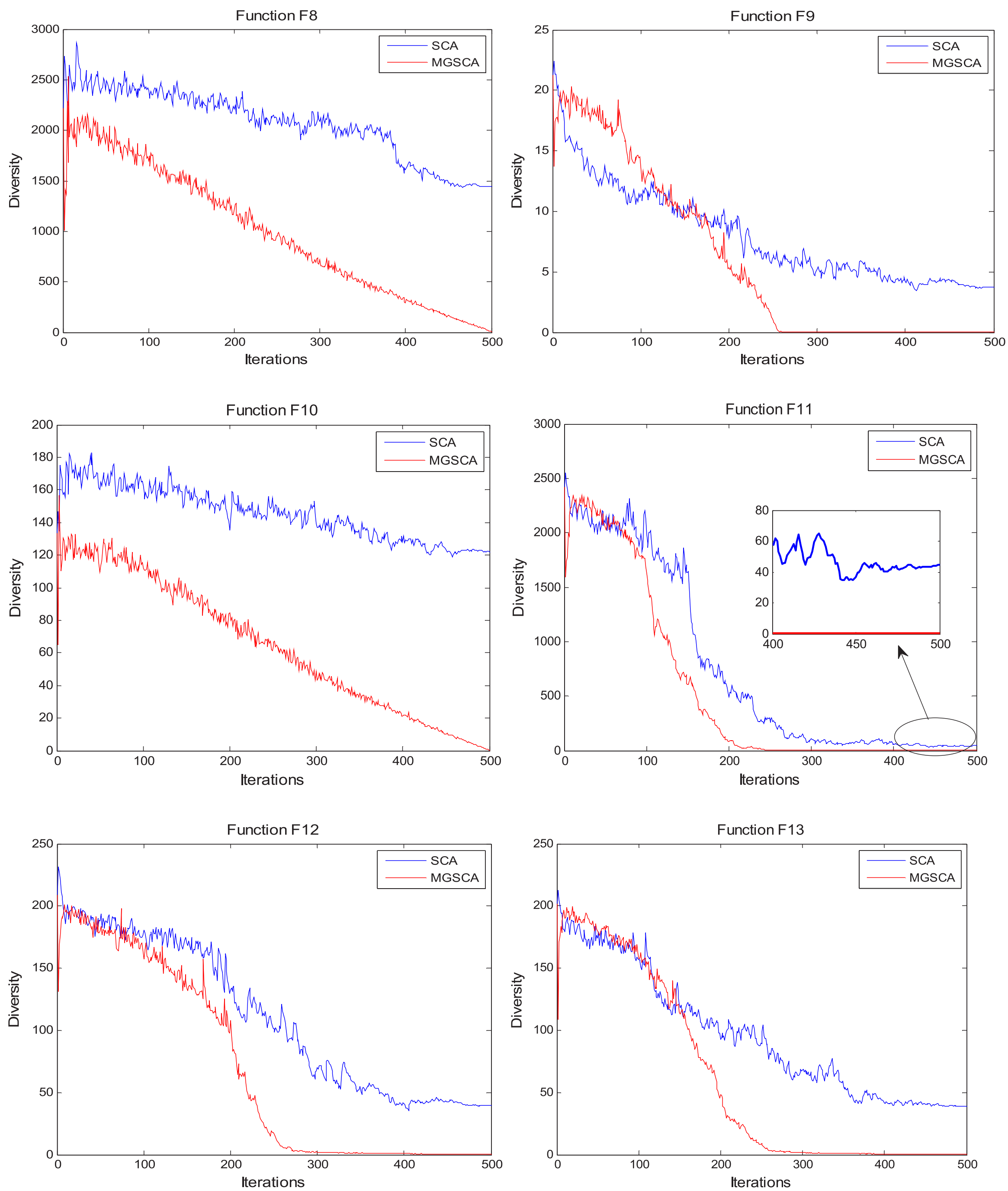

Fig. 3. Diversity plot for multimodal problems.

4.6. *Application of the memory guided sine cosine algorithm to engineering optimization problems*

Although the proposed algorithm has shown to be efficient in solving the benchmark problems, an analysis of its efficiency on real-world optimization problems is necessary. Therefore, in the present section, well-known engineering optimization problems with various difficulties such as constraints and discrete-valued decision variables are used to analyse the search efficiency of the proposed algorithm. To handle discrete search spaces, each element of a candidate solution is rounded to the nearest integer. To deal with constraints, a simple and natural selection approach is used, based on constraint violation (Deb, 2000). For a general optimization problem as defined in Eqs. (1)–(4), the constraint violation for any solution $\vec{x}$ is defined as follows:

$$V(\vec{x}) = \sum_{i=1}^{I} G_i(\vec{x}) + \sum_{j=1}^{J} H_j(\vec{x})$$

where $G_i(\vec{x}) = \begin{cases} g_i(\vec{x}) & if\ g_i(\vec{x}) > 0 \\ 0 & otherwise \end{cases}$ and $H_j(\vec{x}) = \begin{cases} \left|h_j(\vec{x})\right| & if\ \left|h_j(\vec{x})\right| - \epsilon > 0 \\ 0 & otherwise \end{cases}$

$\in$ is a predefined tolerance parameter, fixed at 10^{-4}.

On the constraint violation of each candidate solution, the best candidate solution is chosen in the following steps:

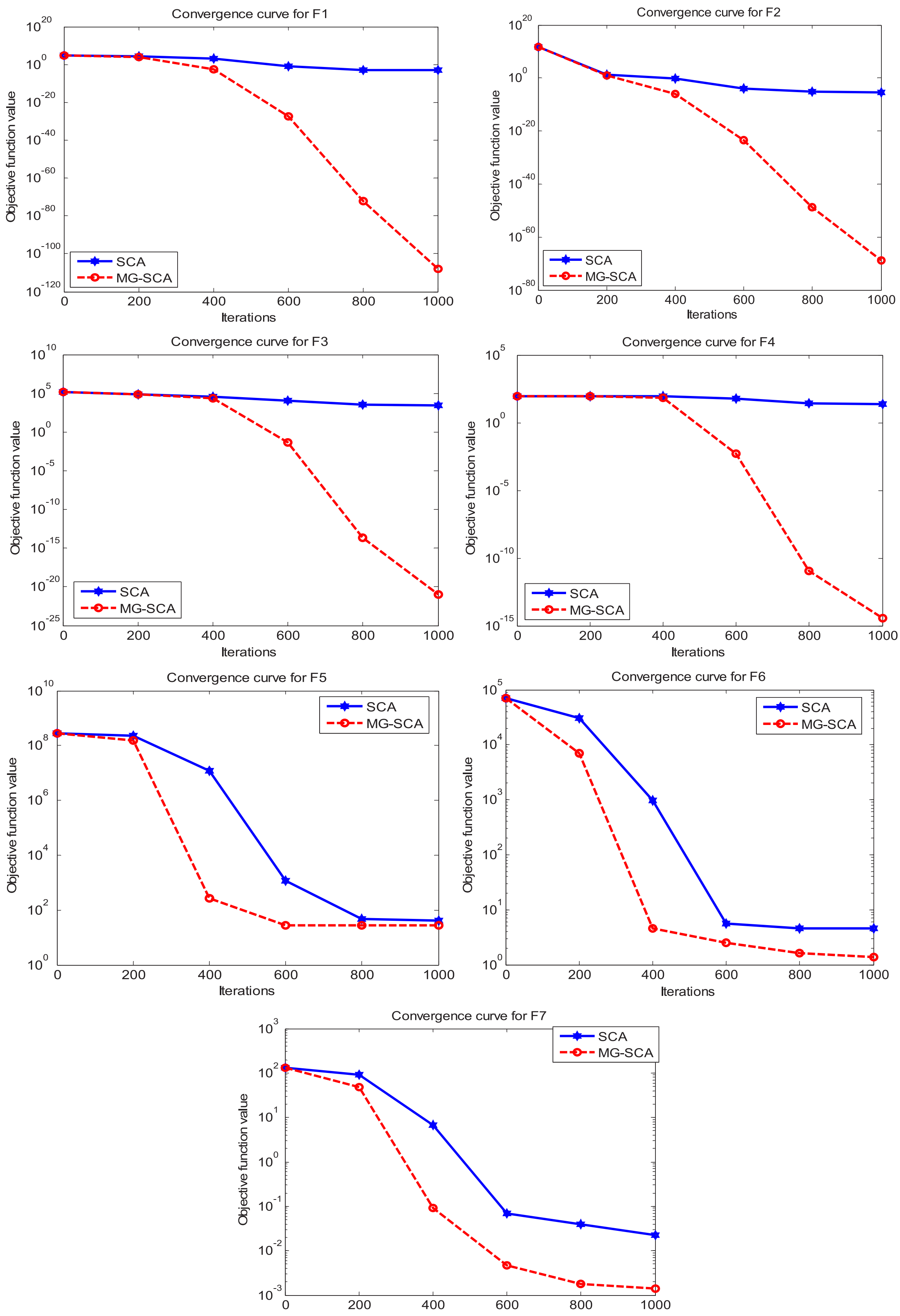

Fig. 4. Convergence curves for unimodal problems.

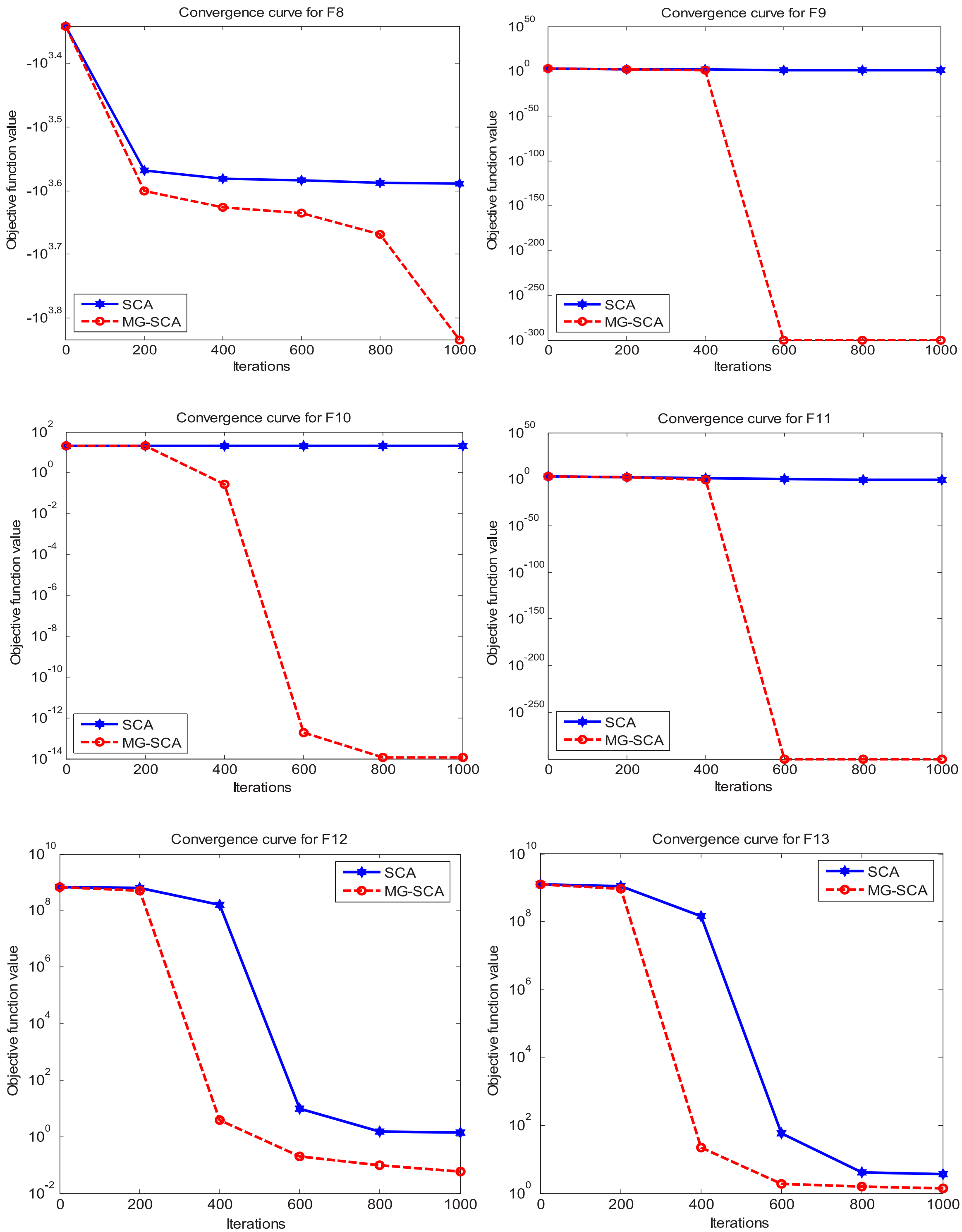

Fig. 5. Convergence curves for multimodal problems.

1. Sort the set of candidate solutions according to increasing constraint violation.
2. In the obtained sorted list, arrange the feasible candidate solutions in increasing order of objective function value (for minimization problems).

Now, the best candidate solution (destination point $\vec{y}$) can be selected from the sorted list of candidate solutions.

The above described mechanism is a very simple and natural way of picking the best candidate solution from the set of candidate solutions for constrained problems.

4.6.1. Gear train design problem

The gear train design problem (Sandgren, 1990) is an unconstrained case study with discrete-valued decision variables. These decision variables represent gears of a train. Mathematically, the problem is stated

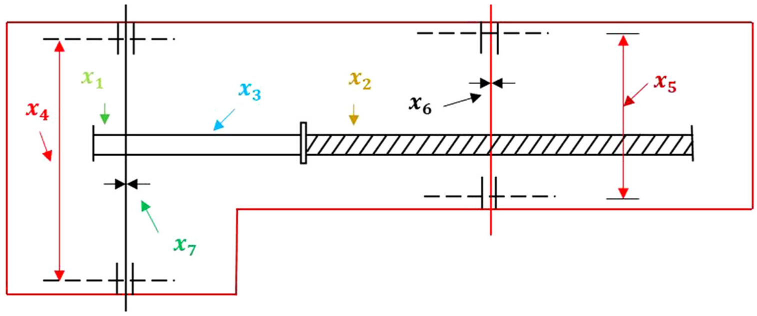

Fig. 6. Speed reducer design (Gandomi et al., 2013).

Table 4
Statistical results obtained from Mann–Whitney U test on experiment 1 with dimension 10 and 30.

Test problem	Dimension 10		Dimension 30	
	p-value	Decision	*p*-value	Decision
F1	3.02E−11	+	3.02E−11	+
F2	3.02E−11	+	3.02E−11	+
F3	3.02E−11	+	3.02E−11	+
F4	3.02E−11	+	3.08E−11	+
F5	3.02E−11	+	4.08E−11	+
F6	4.98E−11	+	3.02E−11	+
F7	1.25E−04	+	3.02E−11	+
F8	4.98E−11	+	3.02E−11	+
F9	1.61E−01	=	1.21E−12	+
F10	6.58E−05	+	1.40E−05	+
F11	5.94E−01	=	6.53E−11	+
F12	3.02E−11	+	3.02E−11	+
F13	7.39E−11	+	3.02E−11	+

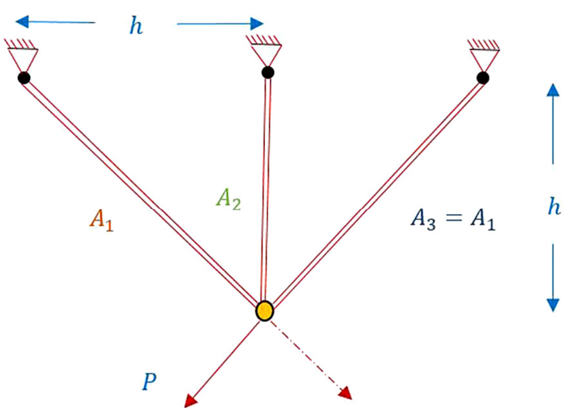

Fig. 7. Three bar truss design (Gandomi et al., 2013).

as:

$$Minimize \quad f_1(\vec{x}) = \left(\frac{1}{6.931} - \frac{\eta_C \eta_B}{\eta_A \eta_D}\right)^2, \quad \vec{x} = (\eta_A, \eta_B, \eta_C, \eta_D)$$
$$s.t. \quad 12 \le \eta_A, \eta_B, \eta_C, \eta_D \le 60$$

The best optimal solutions obtained by the MG-SCA and the standard SCA are presented in the Table 13. The results are recorded in the table by conducting 30 independent trials using 20 candidate solutions and 800 function evaluations. In the table, the results obtained from various algorithms such as PSO (Shi and Eberhart, 1998), MFO (Mirjalili, 2015), GWO (Mirjalili et al., 2014), WOA (Mirjalili and Lewis, 2016), m-SCA (Nayak et al., 2018), OBSCA (Elaziz et al., 2017b), CMA-ES (Hansen and Ostermeier, 2001), IDE (Hongfeng, 2018), SinDE (Draa et al., 2015) and BDE (Bhandari, 2018) are also provided to compare the performance of MG-SCA with that of the other algorithms. The outcomes of the Mann–Whitney U test, which are shown in the table demonstrate that the MG-SCA has significantly outperformed SCA, PSO, MFO, GWO, WOA, OBSCA, CMA-ES, IDE, SinDE and BDE, and performed similar to the m-SCA.

4.6.2. Parameter estimation for frequency-modulated

This problem estimates the decision variables of a frequency-modulated synthesizer (Das and Suganthan, 2010). This problem is unconstrained, highly multimodal and complex with strong epistasis. The problem contains six decision variables, $a_1, \eta_1, a_2, \eta_2, a_3$ and η_3. The problem is formulated as

$$Minimize \quad f_2(x) = \sum_{t=1}^{100} \left(X(t,\vec{x}) - X_0(t,\vec{x})\right)^2,$$
$$\vec{x} = (a_1, \eta_1, a_2, \eta_2, a_3, \eta_3) \quad \text{and}$$
$$\vec{x} = (1, 5, 1.5, 4.8, 2, 4.9)$$
$$s.t. \quad -6.4 \le a_1, \eta_1, a_2, \eta_2, a_3, \eta_3 \le 6.35$$

where

$$X(t,\vec{x}) = a_1 \ \sin(\eta_1 t\theta + a_2 \sin(\eta_2 t\theta + a_3 \ \sin(\eta_3 t\theta)))$$

The expression for target sound waves is given by

$$X_0(t,\vec{x}) = \sin\left(5t \times \frac{2\pi}{100} + 1.5 \sin\left(4.8t \times \frac{2\pi}{100} + 2 \ \sin\left(4.9t \times \frac{2\pi}{100}\right)\right)\right)$$

The best obtained solution by the MG-SCA along with different statistics are presented in Table 14. The results are recorded in the table by conducting 30 independent trials using 20 candidate solutions and 10^5 function evaluations. The table compares the obtained MG-SCA results with PSO (Shi and Eberhart, 1998), MFO (Mirjalili, 2015), GWO (Mirjalili et al., 2014), WOA (Mirjalili and Lewis, 2016), m-SCA (Nayak et al., 2018), OBSCA (Elaziz et al., 2017b), CMA-ES (Hansen and Ostermeier, 2001), IDE (Hongfeng, 2018), SinDE (Draa et al., 2015) and BDE (Bhandari, 2018). In the table, the outcomes of the Mann–Whitney U test are listed. The results indicate that the MG-SCA has significantly outperformed PSO, MFO, GWO, WOA, m-SCA, CMA-ES and BDE, and performed similar to the SCA and OBSCA.

4.6.3. Speed reducer design problem

This problem is constrained and non-linear in nature. The objective of this problem (Gandomi and Yang, 2011) is to minimize the total weight of a speed reducer. The problem has seven decision variables, namely face width (b), module of teeth (m), number of teeth on pinion (z), length of shaft 1 between bearings (l_1), length of shaft 2 between bearings (l_2), diameter of shaft 1(d_1), and diameter of shaft 2(d_2). The problem is illustrated in Fig. 6. Mathematically, the problem is defined as follows:

$$Minimize \quad f_3(\vec{x}) = 0.7854 x_1 x_2^2 \left(14.9334 x_3 + 3.3333 x_3^2 - 43.0934\right) +$$
$$7.4777\left(x_6^3 + x_7^3\right) - 1.508 x_1 \left(x_6^2 + x_7^2\right) + 0.7854(x_4 x_6^2 + x_5 x_7^2)$$

$$\vec{x} = (x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (b, m, z, l_1, l_2, d_1, d_2)$$

s.t.

$$g_1(\vec{x}) = \frac{397.5}{x_1 x_2^2 x_3^2} \le 1$$

Table 5
Minimum, Median, Average, Maximum and Standard Deviation (STD) of error obtained in objective function by standard SCA and proposed algorithm (MG-SCA) for 10-dimensional IEEE CEC 2014 benchmark problems.

Test problem	Algorithm	Minimum	Median	Average	Maximum	STD
F1	SCA	5.46E+06	1.54E+07	2.06E+07	8.09E+07	1.42E+07
	MG-SCA	**2.95E+04**	**3.54E+05**	**8.19E+05**	**4.21E+06**	**1.00E+06**
F2	SCA	3.54E+08	2.25E+09	3.01E+09	8.90E+09	2.15E+09
	MG-SCA	**5.43E+02**	**4.94E+03**	**3.48E+06**	**6.84E+07**	**1.43E+07**
F3	SCA	1.96E+03	1.52E+04	2.12E+04	7.26E+04	1.79E+04
	MG-SCA	**3.65E+01**	**1.02E+03**	**1.01E+03**	**2.09E+03**	**4.98E+02**
F4	SCA	6.39E+01	2.42E+02	3.88E+02	1.55E+03	3.31E+02
	MG-SCA	**5.60E−01**	**3.51E+01**	**2.69E+01**	**3.85E+01**	**1.37E+01**
F5	SCA	2.02E+01	2.04E+01	2.04E+01	2.06E+01	**1.03E−01**
	MG-SCA	**5.31E+00**	**2.01E+01**	**1.98E+01**	**2.03E+01**	2.07E+00
F6	SCA	6.05E+00	1.00E+01	9.76E+00	1.15E+01	**1.22E+00**
	MG-SCA	**7.10E−01**	**3.03E+00**	**3.10E+00**	**5.81E+00**	**1.22E+00**
F7	SCA	6.66E+00	7.58E+01	6.62E+01	1.19E+02	3.19E+01
	MG-SCA	**1.27E−01**	**5.10E−01**	**8.53E−01**	**2.89E+00**	**7.90E−01**
F8	SCA	4.27E+01	7.10E+01	7.41E+01	1.09E+02	1.58E+01
	MG-SCA	**4.98E+00**	**9.95E+00**	**1.12E+01**	**2.61E+01**	**4.73E+00**
F9	SCA	2.60E+01	8.36E+01	7.95E+01	1.18E+02	2.11E+01
	MG-SCA	**3.98E+00**	**1.59E+01**	**1.63E+01**	**2.80E+01**	**5.25E+00**
F10	SCA	6.91E+02	1.33E+03	1.32E+03	1.76E+03	2.21E+02
	MG-SCA	**2.76E+01**	**2.43E+02**	**2.63E+02**	**7.69E+02**	**1.53E+02**
F11	SCA	1.20E+03	1.72E+03	1.64E+03	2.02E+03	2.52E+02
	MG-SCA	**1.18E+02**	**6.62E+02**	**6.67E+02**	**1.35E+03**	**2.25E+02**
F12	SCA	8.00E−01	1.45E+00	1.53E+00	2.65E+00	4.37E−01
	MG-SCA	**3.87E−02**	**2.43E−01**	**2.60E−01**	**1.08E+00**	**1.74E−01**
F13	SCA	5.74E−01	2.85E+00	2.80E+00	4.81E+00	1.25E+00
	MG-SCA	**9.93E−02**	**2.32E−01**	**2.49E−01**	**5.15E−01**	**8.01E−02**
F14	SCA	7.00E−01	1.48E+01	1.45E+01	2.77E+01	7.30E+00
	MG-SCA	**7.33E−02**	**2.62E−01**	**3.37E−01**	**9.32E−01**	**2.17E−01**
F15	SCA	6.49E+00	7.92E+03	1.11E+04	4.67E+04	1.23E+04
	MG-SCA	**5.68E−01**	**1.78E+00**	**1.80E+00**	**3.06E+00**	**6.34E−01**
F16	SCA	2.92E+00	3.98E+00	3.90E+00	4.37E+00	**3.16E−01**
	MG-SCA	**1.63E+00**	**2.75E+00**	**2.75E+00**	**3.53E+00**	3.57E−01
F17	SCA	3.82E+03	1.71E+05	3.44E+05	1.88E+06	3.96E+05
	MG-SCA	**6.05E+02**	**7.08E+03**	**7.31E+03**	**1.93E+04**	**5.93E+03**
F18	SCA	8.46E+03	5.83E+05	2.81E+06	3.87E+07	6.49E+06
	MG-SCA	**1.86E+02**	**6.69E+03**	**8.10E+03**	**3.67E+04**	**6.31E+03**
F19	SCA	6.50E+00	1.10E+01	1.12E+01	2.27E+01	3.55E+00
	MG-SCA	**1.13E+00**	**1.92E+00**	**2.17E+00**	**4.09E+00**	**6.03E−01**
F20	SCA	3.78E+02	5.15E+04	1.61E+05	1.10E+06	2.42E+05
	MG-SCA	**2.57E+01**	**1.24E+02**	**7.81E+02**	**9.39E+03**	**1.79E+03**
F21	SCA	1.89E+03	5.03E+04	1.23E+05	8.77E+05	1.90E+05
	MG-SCA	**2.19E+02**	**8.50E+02**	**1.42E+03**	**1.23E+04**	**1.97E+03**
F22	SCA	5.39E+01	2.32E+02	2.27E+02	5.03E+02	1.12E+02
	MG-SCA	**2.23E+01**	**2.71E+01**	**2.86E+01**	**4.62E+01**	**5.32E+00**
F23	SCA	3.40E+02	4.30E+02	4.32E+02	5.36E+02	4.62E+01
	MG-SCA	**3.29E+02**	**3.30E+02**	**3.31E+02**	**3.38E+02**	**2.17E+00**
F24	SCA	1.48E+02	2.03E+02	2.01E+02	2.37E+02	2.43E+01
	MG-SCA	**1.10E+02**	**1.24E+02**	**1.25E+02**	**1.45E+02**	**6.97E+00**
F25	SCA	1.70E+02	2.07E+02	2.06E+02	2.22E+02	**7.83E+00**
	MG-SCA	**1.17E+02**	**1.43E+02**	**1.60E+02**	**2.02E+02**	3.54E+01
F26	SCA	1.01E+02	1.02E+02	1.02E+02	1.04E+02	8.56E−01
	MG-SCA	**1.00E+02**	**1.00E+02**	**1.00E+02**	**1.00E+02**	**5.82E−02**
F27	SCA	1.23E+02	4.61E+02	4.45E+02	6.11E+02	**1.16E+02**
	MG-SCA	**2.34E+00**	**4.00E+02**	**2.15E+02**	**4.33E+02**	2.01E+02
F28	SCA	4.14E+02	5.49E+02	5.49E+02	7.39E+02	7.74E+01
	MG-SCA	**3.57E+02**	**3.70E+02**	**3.73E+02**	**3.84E+02**	**6.65E+00**
F29	SCA	5.59E+03	4.72E+04	1.52E+05	1.83E+06	3.14E+05
	MG-SCA	**3.60E+02**	**6.57E+02**	**7.76E+02**	**2.34E+03**	**3.94E+02**
F30	SCA	1.29E+03	4.88E+03	8.13E+03	4.71E+04	9.54E+03
	MG-SCA	**4.85E+02**	**6.07E+02**	**6.76E+02**	**1.62E+03**	**2.24E+02**

Table 6
Minimum, Median, Average, Maximum and Standard Deviation (STD) of error obtained in objective function by standard SCA and proposed algorithm (MG-SCA) for 30-dimensional IEEE CEC 2014 benchmark problems.

Test problem	Algorithm	Minimum	Median	Average	Maximum	STD
F1	SCA	1.57E+08	7.55E+08	7.38E+08	1.57E+09	3.45E+08
	MG-SCA	**5.82E+06**	**2.50E+07**	**2.92E+07**	**1.19E+08**	**2.07E+07**
F2	SCA	2.00E+10	7.54E+10	7.08E+10	1.34E+11	3.41E+10
	MG-SCA	**1.99E+08**	**2.02E+09**	**2.26E+09**	**5.74E+09**	**1.69E+09**
F3	SCA	3.55E+04	1.65E+05	1.50E+05	2.84E+05	7.45E+04
	MG-SCA	**6.47E+03**	**1.62E+04**	**1.77E+04**	**3.35E+04**	**6.63E+03**
F4	SCA	1.34E+03	1.54E+04	1.59E+04	3.83E+04	9.46E+03
	MG-SCA	**1.63E+02**	**2.69E+02**	**2.76E+02**	**4.93E+02**	**6.55E+01**
F5	SCA	2.08E+01	2.10E+01	2.10E+01	2.11E+01	**7.23E−02**
	MG-SCA	**2.01E+01**	**2.04E+01**	**2.04E+01**	**2.08E+01**	1.44E−01
F6	SCA	3.46E+01	4.19E+01	4.14E+01	4.50E+01	**2.26E+00**
	MG-SCA	**1.29E+01**	**1.97E+01**	**1.94E+01**	**2.54E+01**	2.89E+00
F7	SCA	2.09E+02	9.04E+02	8.32E+02	1.36E+03	2.82E+02
	MG-SCA	**2.78E+00**	**1.72E+01**	**1.99E+01**	**5.61E+01**	**1.18E+01**
F8	SCA	2.61E+02	4.63E+02	4.31E+02	5.24E+02	8.01E+01
	MG-SCA	**5.99E+01**	**1.07E+02**	**1.07E+02**	**1.60E+02**	**2.14E+01**
F9	SCA	2.71E+02	5.40E+02	5.11E+02	6.15E+02	1.05E+02
	MG-SCA	**8.43E+01**	**1.39E+02**	**1.39E+02**	**1.94E+02**	**2.56E+01**
F10	SCA	6.35E+03	7.58E+03	7.54E+03	8.45E+03	**4.95E+02**
	MG-SCA	**1.32E+03**	**2.77E+03**	**2.82E+03**	**4.48E+03**	6.83E+02
F11	SCA	6.76E+03	7.91E+03	7.85E+03	8.77E+03	**4.69E+02**
	MG-SCA	**2.14E+03**	**3.26E+03**	**3.30E+03**	**4.45E+03**	6.26E+02
F12	SCA	2.23E+00	3.16E+00	3.15E+00	4.92E+00	5.24E−01
	MG-SCA	**1.09E−01**	**5.71E−01**	**6.33E−01**	**1.63E+00**	**3.36E−01**
F13	SCA	2.23E+00	8.67E+00	7.99E+00	9.75E+00	1.82E+00
	MG-SCA	**3.73E−01**	**5.41E−01**	**5.51E−01**	**7.39E−01**	**8.94E−02**
F14	SCA	8.04E+01	3.20E+02	3.00E+02	4.47E+02	9.65E+01
	MG-SCA	**2.11E−01**	**8.40E−01**	**2.34E+00**	**1.30E+01**	**3.31E+00**
F15	SCA	2.86E+03	1.24E+07	1.44E+07	3.60E+07	1.08E+07
	MG-SCA	**1.53E+01**	**4.66E+01**	**8.72E+01**	**5.62E+02**	**1.01E+02**
F16	SCA	1.21E+01	1.36E+01	1.36E+01	1.41E+01	**3.84E−01**
	MG-SCA	**9.78E+00**	**1.17E+01**	**1.16E+01**	**1.32E+01**	6.91E−01
F17	SCA	4.86E+06	5.33E+07	5.97E+07	1.44E+08	3.33E+07
	MG-SCA	**4.67E+04**	**8.75E+05**	**9.56E+05**	**5.37E+06**	**7.62E+05**
F18	SCA	7.04E+07	3.05E+09	3.13E+09	9.21E+09	2.00E+09
	MG-SCA	**6.71E+02**	**1.17E+04**	**1.48E+05**	**6.44E+06**	**9.00E+05**
F19	SCA	6.12E+01	5.45E+02	5.68E+02	1.41E+03	2.45E+02
	MG-SCA	**9.04E+00**	**1.94E+01**	**2.28E+01**	**8.71E+01**	**1.43E+01**
F20	SCA	1.31E+04	7.87E+05	1.26E+06	5.24E+06	1.40E+06
	MG-SCA	**2.89E+02**	**3.91E+03**	**4.24E+03**	**1.52E+04**	**3.82E+03**
F21	SCA	1.26E+06	2.30E+07	2.69E+07	1.03E+08	2.07E+07
	MG-SCA	**1.19E+04**	**1.77E+05**	**2.35E+05**	**1.03E+06**	**2.39E+05**
F22	SCA	1.03E+03	2.10E+03	2.08E+03	3.39E+03	5.58E+02
	MG-SCA	**6.46E+01**	**3.21E+02**	**3.39E+02**	**8.07E+02**	**1.78E+02**
F23	SCA	3.89E+02	1.21E+03	1.23E+03	1.84E+03	3.77E+02
	MG-SCA	**3.22E+02**	**3.29E+02**	**3.29E+02**	**3.38E+02**	**4.03E+00**
F24	SCA	2.02E+02	5.16E+02	4.78E+02	5.37E+02	9.29E+01
	MG-SCA	**2.00E+02**	**2.00E+02**	**2.00E+02**	**2.00E+02**	**1.56E−03**
F25	SCA	2.25E+02	3.22E+02	3.14E+02	3.60E+02	3.45E+01
	MG-SCA	**2.05E+02**	**2.11E+02**	**2.11E+02**	**2.22E+02**	**2.82E+00**
F26	SCA	1.04E+02	1.09E+02	1.09E+02	1.11E+02	1.70E+00
	MG-SCA	**1.00E+02**	**1.01E+02**	**1.01E+02**	**1.01E+02**	**1.53E−01**
F27	SCA	5.92E+02	1.47E+03	1.38E+03	1.67E+03	2.60E+02
	MG-SCA	**4.33E+02**	**8.22E+02**	**8.19E+02**	**1.00E+03**	**9.17E+01**
F28	SCA	1.94E+03	3.77E+03	3.73E+03	5.10E+03	6.26E+02
	MG-SCA	**8.20E+02**	**9.51E+02**	**9.68E+02**	**1.36E+03**	**1.06E+02**
F29	SCA	1.66E+07	7.43E+07	7.49E+07	1.47E+08	2.64E+07
	MG-SCA	**3.91E+03**	**1.54E+04**	**1.19E+06**	**1.10E+07**	**3.25E+06**
F30	SCA	2.79E+05	1.39E+06	1.64E+06	4.18E+06	9.69E+05
	MG-SCA	**5.30E+03**	**1.77E+04**	**1.92E+04**	**4.36E+04**	**8.25E+03**

Table 7
Statistical decision based on Mann–Whitney U test on experiment 2 with dimension 10.

Test function	p-value	Decision	Test function	p-value	Decision
F1	3.30E−18	+	F16	1.69E−17	+
F2	3.30E−18	+	F17	2.43E−16	+
F3	4.18E−18	+	F18	3.37E−17	+
F4	3.30E−18	+	F19	3.30E−18	+
F5	5.29E−18	+	F20	3.01E−17	+
F6	3.30E−18	+	F21	7.51E−18	+
F7	3.30E−18	+	F22	3.30E−18	+
F8	3.30E−18	+	F23	3.30E−18	+
F9	3.94E−18	+	F24	3.30E−18	+
F10	3.50E−18	+	F25	3.72E−15	+
F11	5.29E−18	+	F26	3.30E−18	+
F12	5.29E−18	+	F27	1.50E−12	+
F13	3.30E−18	+	F28	3.30E−18	+
F14	4.43E−18	+	F29	3.30E−18	+
F15	3.30E−18	+	F30	4.43E−18	+

Table 8
Statistical decision based on Mann–Whitney U test on experiment 2 with dimension 30.

Test function	p-value	Decision	Test function	p-value	Decision
F1	3.30E−18	+	F16	7.51E−18	+
F2	3.30E−18	+	F17	3.50E−18	+
F3	3.30E−18	+	F18	3.30E−18	+
F4	3.30E−18	+	F19	3.72E−18	+
F5	3.30E−18	+	F20	3.50E−18	+
F6	3.30E−18	+	F21	3.30E−18	+
F7	3.30E−18	+	F22	3.30E−18	+
F8	3.30E−18	+	F23	3.30E−18	+
F9	3.28E−18	+	F24	3.05E−18	+
F10	3.30E−18	+	F25	3.30E−18	+
F11	3.30E−18	+	F26	3.30E−18	+
F12	3.30E−18	+	F27	2.35E−14	+
F13	3.30E−18	+	F28	3.30E−18	+
F14	3.30E−18	+	F29	3.30E−18	+
F15	3.30E−18	+	F30	3.30E−18	+

$$g_2\left(\vec{x}\right) = \frac{27}{x_1 x_2^2 x_3} \leq 1$$

$$g_3\left(\vec{x}\right) = \frac{1.93x_5^3}{x_2 x_3 x_7^4} \leq 1$$

$$g_4\left(\vec{x}\right) = \frac{1.93x_4^3}{x_2 x_3 x_6^4} \leq 1$$

$$g_5\left(\vec{x}\right) = \frac{\sqrt{1.57\times 10^8 + \left(\frac{745x_5}{x_2x_3}\right)^2}}{85x_7^3} \leq 1$$

$$g_6\left(\vec{x}\right) = \frac{\sqrt{1.69\times 10^7 + \left(\frac{745x_4}{x_2x_3}\right)^2}}{110x_6^3} \leq 1$$

$$g_7\left(\vec{x}\right) = \frac{x_1}{12x_2} \leq 1$$

$$g_8\left(\vec{x}\right) = \frac{x_2x_3}{40} \leq 1$$

$$g_9\left(\vec{x}\right) = \frac{5x_2}{x_1} \leq 1$$

$$g_{10}\left(\vec{x}\right) = \frac{1.5x_6 + 1.9}{x_4} \leq 1$$

$$g_{11}\left(\vec{x}\right) = \frac{1.1x_7 + 1.9}{x_5} \leq 1$$

$2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 7.3 \leq x_4 \leq 8.3, 7.8 \leq x_5 \leq 8.3$
$2.9 \leq x_6 \leq 3.9, 5 \leq x_7 \leq 5.5.$

This problem is solved by the MG-SCA using 50 candidate solutions and 5000 iterations and the obtained results are presented in Table 15. The obtained MG-SCA results are compared with PSO (Shi and Eberhart, 1998), MFO (Mirjalili, 2015), GWO (Mirjalili et al., 2014), WOA (Mirjalili and Lewis, 2016), m-SCA (Nayak et al., 2018), OBSCA (Elaziz et al., 2017b), CMA-ES (Hansen and Ostermeier, 2001), IDE (Hongfeng, 2018), SinDE (Draa et al., 2015), BDE (Bhandari, 2018). In the table, the outcomes of the Mann–Whitney U test are listed. The results conclude that the MG-SCA has significantly outperformed

Table 9
Computational time to obtain the solution for experimental benchmark set 1.

Function	Dimension 10		Dimension 30		Function	Dimension 10		Dimension 30	
	SCA	MG-SCA	SCA	MG-SCA		SCA	MG-SCA	SCA	MG-SCA
F1	0.16	0.16	0.31	0.29	F8	0.46	0.45	0.42	0.41
F2	0.24	0.24	0.30	0.30	F9	0.45	0.46	0.41	0.39
F3	0.89	0.91	1.73	1.73	F10	0.52	0.53	0.46	0.44
F4	0.39	0.39	0.34	0.33	F11	0.55	0.55	0.47	0.46
F5	0.47	0.46	0.39	0.38	F12	1.14	1.13	1.05	1.03
F6	0.42	0.42	0.36	0.35	F13	1.12	1.13	1.04	1.02
F7	0.55	0.55	0.55	0.54					

Table 10
Computational time to obtain the solution for experimental benchmark set 2.

Function	Dimension 10		Dimension 30		Function	Dimension 10		Dimension 30	
	SCA	MG-SCA	SCA	MG-SCA		SCA	MG-SCA	SCA	MG-SCA
F1	0.98	1.07	5.32	5.52	F16	0.96	1.04	5.31	5.54
F2	0.95	1.02	5.18	5.40	F17	1.03	1.10	5.76	5.98
F3	0.90	0.97	4.69	4.88	F18	0.97	1.04	5.19	5.40
F4	0.89	0.97	4.69	4.89	F19	2.70	2.78	21.07	21.33
F5	0.96	1.03	5.14	5.36	F20	0.99	1.06	5.09	5.31
F6	9.59	9.66	83.68	83.47	F21	1.03	1.10	5.41	5.63
F7	0.96	1.03	5.24	5.45	F22	1.42	1.49	8.97	9.17
F8	0.90	0.98	4.18	4.39	F23	1.79	1.85	14.04	14.40
F9	0.94	1.01	5.13	5.36	F24	1.55	1.63	12.42	10.60
F10	1.03	1.11	5.51	5.78	F25	1.64	1.73	12.04	12.27
F11	1.95	2.08	6.47	6.69	F26	10.74	10.78	96.05	95.67
F12	7.57	7.65	45.17	45.61	F27	10.69	10.80	93.98	94.31
F13	0.90	0.98	4.71	5.00	F28	2.03	2.10	16.19	16.36
F14	0.96	1.04	4.78	4.99	F29	3.37	3.44	30.99	26.88
F15	0.96	1.03	5.25	5.46	F30	2.04	2.12	17.12	14.92

Table 11
Performance comparison of proposed MG-SCA with other algorithms on standard set of benchmark problems.

Test problem	Results	PSO	MFO	GWO	WOA	SCA	m-SCA	OBSCA	CMA-ES	IDE	SinDE	BDE	MG-SCA
F1	Mean	1.60E−08	2.33E+03	1.07E−46	2.80E−74	6.70E−03	1.38E−05	1.88E−18	9.65E−48	5.43E−10	6.83E−11	3.86E−02	1.17E−104
	STD	1.54E+03	1.46E+04	4.69E+03	1.02E+04	1.25E+04	1.42E+04	7.66E−18	9.98E−48	3.21E−10	3.36E−10	1.97E−01	6.08E−104
	Rank	8	12	4	2	10	9	5	3	7	6	11	1
F2	Mean	3.33E−01	3.23E+01	1.94E−08	1.20E−50	4.91E−05	8.36E−05	3.66E−17	3.66E−22	1.20E−06	1.19E−08	6.97E−01	5.80E−68
	STD	1.12E+01	2.05E+11	1.34E+04	2.49E+08	9.54E+04	3.52E+05	5.92E−17	3.11E−22	4.48E−07	4.55E−08	3.34E+00	1.24E−67
	Rank	10	12	6	2	8	9	4	3	7	5	11	1
F3	Mean	8.62E+02	1.88E+04	1.09E+01	4.29E+04	3.74E+03	1.11E+03	1.35E+01	5.87E−32	3.06E+04	3.27E+03	1.80E+03	7.92E−20
	STD	5.41E+03	2.71E+04	1.23E+04	2.64E+04	2.20E+04	2.10E+04	6.01E+01	8.64E−32	4.18E+03	1.39E+03	2.88E+03	2.68E−19
	Rank	5	10	3	12	9	6	4	1	11	8	7	2
F4	Mean	4.02E+00	6.94E+01	1.59E−07	4.74E+01	2.08E+01	3.38E+01	7.07E−01	7.52E−17	9.27E−01	3.77E+00	4.80E+01	6.79E−14
	STD	6.69E+00	8.16E+00	9.37E+00	2.89E+01	1.69E+01	1.68E+01	2.11E+00	1.10E−16	2.05E−01	3.32E+00	1.09E+01	2.08E−13
	Rank	7	12	3	10	8	9	4	1	5	6	11	2
F5	Mean	6.17E+03	2.69E+06	2.80E+01	2.79E+01	3.35E+02	5.76E+01	2.83E+01	9.60E+00	4.74E+01	4.27E+01	2.11E+04	2.68E+01
	STD	9.66E+05	5.76E+07	1.87E+07	3.61E+07	5.63E+07	5.63E+07	3.23E−01	1.29E+00	2.74E+01	2.71E+01	1.06E+05	9.70E−01
	Rank	10	12	4	3	9	8	5	1	7	6	11	2
F6	Mean	2.73E−09	1.99E+03	9.31E−02	3.83E−01	4.54E+00	4.91E+00	4.60E+00	3.38E−31	6.32E−10	4.07E−12	4.37E+01	1.34E+00
	STD	2.05E+03	1.47E+04	4.74E+03	1.03E+04	1.26E+04	1.42E+04	3.14E−01	8.18E−32	3.74E−10	1.11E−11	2.38E+02	4.67E−01
	Rank	4	12	5	6	8	10	9	1	3	2	11	7
F7	Mean	2.81E−02	5.43E+00	6.34E−03	2.77E−03	3.42E−02	9.62E−02	2.15E−03	6.22E−01	3.11E−02	1.59E−02	3.68E−01	1.77E−03
	STD	3.57E−01	3.05E+01	1.45E+00	6.38E+00	2.93E+01	2.93E+01	1.32E−03	2.75E−01	7.57E−03	3.58E−03	9.56E−01	1.33E−03
	Rank	6	12	4	3	8	9	2	11	7	5	10	1
F8	Mean	−8.71E+03	−8.53E+03	−6.24E+03	−1.07E+04	−3.94E+03	−3.88E+03	−3.76E+03	−1.19E+02	−9.48E+03	−1.06E+04	−9.81E+03	−6.81E+03
	STD	1.19E+03	1.36E+03	1.87E+03	1.95E+03	4.03E+02	4.41E+02	2.69E+02	3.68E+00	5.53E+02	4.70E+02	5.94E+02	6.62E+02
	Rank	5	6	8	1	9	10	11	12	4	2	3	7
F9	Mean	4.53E+01	1.75E+02	2.17E+01	0.00E+00	2.13E+01	2.16E−03	5.26E−11	1.55E+01	9.87E+01	2.19E+01	5.50E+01	0.00E+00
	STD	4.16E+01	6.19E+01	5.25E+01	6.37E+01	7.50E+01	8.05E+01	2.84E−10	4.11E+00	8.45E+00	4.82E+00	3.26E+01	0.00E+00
	Rank	9	12	7	2	6	4	3	5	11	8	10	1
F10	Mean	1.29E−01	1.52E+01	2.68E−14	3.61E−15	1.68E+01	2.02E+01	1.27E+01	7.52E−15	7.19E−06	2.55E−07	1.67E+01	8.69E+00
	STD	2.86E+00	7.65E+00	3.55E+00	3.60E+00	3.77E+00	7.21E+00	9.84E+00	1.23E−15	2.37E−06	5.38E−07	6.08E+00	1.01E+01
	Rank	6	9	3	1	11	12	8	2	5	4	10	7
F11	Mean	1.22E−02	2.11E+01	2.71E−03	6.76E−03	3.64E−01	2.45E−01	4.34E−03	0.00E+00	8.16E−08	2.47E−04	4.83E−01	3.21E−03
	STD	1.40E+01	1.32E+02	4.24E+01	9.17E+01	1.13E+02	1.28E+02	2.30E−10	0.00E+00	2.20E−07	1.35E−03	1.40E+00	8.51E−03
	Rank	8	12	4	7	10	9	6	1	2	3	11	5
F12	Mean	4.49E−02	4.08E−01	1.41E−02	2.39E−02	2.09E+00	2.05E+00	4.94E−01	2.91E−31	2.87E−10	9.92E−11	8.87E+04	7.07E−02
	STD	5.03E+05	1.74E+08	3.68E+07	1.25E+08	1.74E+08	1.63E+08	1.25E−01	3.02E−31	1.55E−10	3.87E−10	1.88E+05	3.16E−02
	Rank	6	8	4	5	11	10	9	1	3	2	12	7
F13	Mean	2.20E−03	2.21E−01	1.78E−01	5.18E−01	2.73E+02	3.96E+00	2.46E+00	6.16E−31	1.40E−09	2.93E−12	4.05E+05	1.42E+00
	STD	7.94E+05	3.10E+08	6.29E+07	2.19E+08	2.64E+08	3.10E+08	1.56E−01	3.66E−32	9.80E−10	8.16E−12	1.33E+06	2.57E−01
	Rank	4	6	5	7	11	10	9	1	3	2	12	8
Average rank		6.77	10.38	4.62	4.69	9.08	8.85	6.08	3.31	5.77	4.54	10.00	3.92
Overall rank		**8**	**12**	**4**	**5**	**10**	**9**	**7**	**1**	**6**	**3**	**11**	**2**

Table 12
Performance comparison of proposed algorithm (MG-SCA) with other meta-heuristic algorithms on 30 dimensional CEC2014 problems.

Test problem	Results	PSO	MFO	GWO	WOA	SCA	m-SCA	OBSCA	CMA-ES	IDE	SinDE	BDE	MGSCA
F1	Mean	8.19E+07	7.59E+07	6.90E+07	3.16E+07	7.38E+08	2.12E+08	4.65E+08	2.70E−14	3.81E+07	1.51E+06	1.25E+07	2.92E+07
	Std	8.27E+09	9.77E+07	5.38E+07	1.38E+07	3.45E+08	5.56E+07	1.22E+08	1.13E−14	7.92E+06	1.19E+06	7.47E+07	2.07E+07
	Rank	9	8	7	5	12	10	11	1	6	2	3	4
F2	Mean	2.81E+04	1.36E+10	2.18E+09	3.10E+06	7.08E+10	1.57E+10	2.20E+10	5.29E−14	0.00E+00	0.00E+00	9.25E+03	2.26E+09
	Std	5.30E+02	8.42E+09	2.47E+09	2.12E+06	3.41E+10	2.48E+09	4.34E+09	2.11E−14	0.00E+00	0.00E+00	6.19E+04	1.69E+09
	Rank	4	8	6	5	11	9	10	2	1	1	3	7
F3	Mean	2.09E+01	8.99E+04	3.12E+04	3.52E+04	1.50E+05	3.97E+04	5.28E+04	1.07E−13	0.00E+00	3.13E−11	5.27E+02	1.77E+04
	Std	2.77E+01	4.98E+04	8.60E+03	2.18E+04	7.45E+04	7.56E+03	1.29E+04	4.31E−14	1.80E−14	1.54E−10	1.11E+03	6.63E+03
	Rank	4	11	7	8	12	9	10	2	1	3	5	6
F4	Mean	9.01E+03	1.14E+03	2.62E+02	1.95E+02	1.59E+04	9.86E+02	1.61E+03	1.07E−13	4.81E+01	4.24E+00	5.76E+02	2.76E+02
	Std	1.85E+02	1.13E+03	8.96E+01	4.74E+01	9.46E+03	3.02E+02	5.26E+02	5.11E−14	4.31E+01	1.27E+01	4.29E+01	6.55E+01
	Rank	11	9	6	5	12	8	10	1	3	2	4	7
F5	Mean	2.19E+02	2.04E+01	2.09E+01	2.04E+01	2.10E+01	2.09E+01	2.09E+01	2.00E+01	2.07E+01	2.05E+01	2.02E+01	2.04E+01
	Std	6.01E+03	1.75E−01	4.80E−02	1.64E−01	7.23E−02	3.78E−02	5.64E−02	1.57E−02	6.70E−02	4.61E−02	1.33E−01	1.44E−01
	Rank	12	5	9	4	11	8	10	1	7	6	2	3
F6	Mean	6.78E+03	2.40E+01	1.39E+01	3.60E+01	4.14E+01	3.35E+01	3.05E+01	4.20E+01	2.38E+01	3.48E+00	2.38E+01	1.94E+01
	Std	2.46E+00	3.33E+00	2.24E+00	4.05E+00	2.26E+00	2.58E+00	1.20E+00	1.06E+01	1.66E+00	2.01E+00	4.39E+00	2.89E+00
	Rank	12	6	2	9	10	8	7	11	4	1	5	3

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Table 12 (*continued*).

Test problem	Results	PSO	MFO	GWO	WOA	SCA	m-SCA	OBSCA	CMA-ES	IDE	SinDE	BDE	MGSCA
F7	Mean	1.61E+00	1.17E+02	1.84E+01	9.99E−01	8.32E+02	1.15E+02	1.83E+02	2.66E−03	1.07E−09	1.93E−04	1.50E+00	1.99E+01
	Std	2.67E+01	6.91E+01	1.44E+01	7.38E−02	2.82E+02	1.98E+01	3.19E+01	4.65E−03	7.61E−09	1.38E−03	6.08E+00	1.18E+01
	Rank	6	10	7	4	12	9	11	3	1	2	5	8
F8	Mean	1.10E+03	1.43E+02	7.77E+01	1.93E+02	4.31E+02	2.36E+02	2.53E+02	4.29E+02	1.05E+01	9.99E−01	6.12E+01	1.07E+02
	Std	1.24E+01	3.81E+01	1.86E+01	4.43E+01	8.01E+01	1.48E+01	1.94E+01	9.03E+01	1.06E+01	1.48E+00	2.60E+01	2.14E+01
	Rank	12	6	4	7	11	8	9	10	2	1	3	5
F9	Mean	2.30E+06	2.23E+02	9.62E+01	2.26E+02	5.11E+02	2.75E+02	3.04E+02	6.37E+02	1.41E+02	3.29E+01	1.10E+02	1.39E+02
	Std	8.06E+07	6.06E+01	2.59E+01	5.27E+01	1.05E+02	1.61E+01	1.61E+01	1.59E+02	1.17E+01	7.50E+00	4.69E+01	2.56E+01
	Rank	12	6	2	7	10	8	9	11	5	1	3	4
F10	Mean	4.43E+01	3.47E+03	2.13E+03	4.04E+03	7.54E+03	5.99E+03	4.54E+03	5.15E+03	1.03E+02	5.93E+01	1.75E+03	2.82E+03
	Std	7.82E+03	8.85E+02	6.62E+02	6.57E+02	4.95E+02	4.53E+02	4.87E+02	8.13E+02	7.48E+01	7.09E+01	7.28E+02	6.83E+02
	Rank	1	7	5	8	12	11	9	10	3	2	4	6
F11	Mean	6.50E+05	4.15E+03	2.81E+03	4.82E+03	7.85E+03	7.00E+03	5.96E+03	5.14E+03	5.49E+03	2.43E+03	3.90E+03	3.30E+03
	Std	6.35E+02	6.90E+02	9.41E+02	9.06E+02	4.69E+02	3.42E+02	4.31E+02	7.61E+02	2.91E+02	5.14E+02	7.38E+02	6.26E+02
	Rank	12	5	2	6	11	10	9	7	8	1	4	3
F12	Mean	3.73E+02	4.33E−01	1.52E+00	1.63E+00	3.15E+00	2.44E+00	2.12E+00	2.82E−01	1.24E+00	4.10E−01	3.77E−01	6.33E−01
	Std	2.76E+02	2.64E−01	1.19E+00	4.22E−01	5.24E−01	3.52E−01	3.10E−01	2.61E−01	1.83E−01	1.58E−01	1.71E−01	3.36E−01
	Rank	12	4	7	8	11	10	9	1	6	3	2	5
F13	Mean	2.14E+02	2.21E+00	4.42E−01	5.27E−01	7.99E+00	2.94E+00	3.24E+00	2.35E−01	3.54E−01	1.36E−01	5.13E−01	5.51E−01
	Std	1.58E+02	1.34E+00	2.94E−01	1.26E−01	1.82E+00	3.72E−01	3.29E−01	5.87E−02	3.60E−02	3.42E−02	1.41E−01	8.94E−02
	Rank	12	8	4	6	11	9	10	2	3	1	5	7
F14	Mean	9.05E+02	3.54E+01	4.54E+00	2.55E−01	3.00E+02	4.42E+01	5.23E+01	3.60E−01	3.05E−01	2.45E−01	4.41E−01	2.34E+00
	Std	1.92E+03	2.47E+01	8.41E+00	4.84E−02	9.65E+01	7.81E+00	1.13E+01	7.16E−02	7.85E−02	1.03E−01	2.46E−01	3.31E+00
	Rank	12	8	7	2	11	9	10	4	3	1	5	6
F15	Mean	1.04E+07	2.23E+05	1.25E+02	7.00E+01	1.44E+07	1.92E+03	2.12E+04	3.52E+00	1.44E+01	3.89E+00	3.78E+01	8.72E+01
	Std	6.54E+04	5.77E+05	3.19E+02	1.99E+01	1.08E+07	1.43E+03	1.11E+04	1.06E+00	9.90E−01	9.19E−01	9.26E+01	1.01E+02
	Rank	11	10	7	5	12	8	9	1	3	2	4	6
F16	Mean	2.49E+07	1.27E+01	1.09E+01	1.25E+01	1.36E+01	1.28E+01	1.27E+01	1.43E+01	1.20E+01	1.01E+01	1.26E+01	1.16E+01
	Std	2.21E+09	5.33E−01	5.81E−01	4.48E−01	3.84E−01	2.24E−01	3.54E−01	3.67E−01	2.27E−01	5.32E−01	5.92E−01	6.91E−01
	Rank	12	8	2	5	10	9	7	11	4	1	6	3
F17	Mean	1.26E+04	3.39E+06	1.40E+06	3.56E+06	5.97E+07	5.37E+06	1.31E+07	1.65E+03	1.54E+06	1.32E+05	2.80E+04	9.56E+05
	Std	3.30E+02	4.07E+06	1.56E+06	2.52E+06	3.33E+07	2.76E+06	5.63E+06	3.68E+02	6.94E+05	1.24E+05	2.24E+04	7.62E+05
	Rank	2	8	6	9	12	10	11	1	7	4	3	5
F18	Mean	4.81E−02	5.19E+06	9.21E+06	1.50E+04	3.13E+09	1.43E+08	2.36E+08	1.35E+02	2.65E+03	9.53E+02	3.54E+04	1.48E+05
	Std	1.43E+00	3.61E+07	1.98E+07	5.01E+04	2.00E+09	8.38E+07	1.19E+08	4.10E+01	2.05E+03	1.64E+03	1.63E+05	9.00E+05
	Rank	1	8	9	5	12	10	11	2	4	3	6	7
F19	Mean	2.91E+01	7.36E+01	4.34E+01	5.07E+01	5.68E+02	9.42E+01	1.27E+02	9.88E+00	8.33E+00	4.76E+00	1.11E+01	2.28E+01
	Std	1.48E+01	5.32E+01	2.62E+01	3.83E+01	2.45E+02	2.63E+01	4.42E+01	1.95E+00	7.27E−01	8.44E−01	1.08E+01	1.43E+01
	Rank	6	9	7	8	12	10	11	3	2	1	4	5
F20	Mean	1.34E+01	5.67E+04	1.34E+04	2.29E+04	1.26E+06	9.51E+03	2.14E+04	2.63E+02	2.85E+02	2.17E+01	3.02E+03	4.24E+03
	Std	4.16E+02	4.34E+04	9.03E+03	1.95E+04	1.40E+06	3.62E+03	7.96E+03	1.11E+02	1.12E+02	3.05E+01	7.27E+03	3.82E+03
	Rank	1	11	8	10	12	7	9	3	4	2	5	6
F21	Mean	3.88E+02	7.83E+05	7.16E+05	1.17E+06	2.69E+07	1.48E+06	2.14E+06	1.10E+03	2.11E+05	1.81E+04	5.08E+04	2.35E+05
	Std	2.69E−01	1.18E+06	1.26E+06	9.72E+05	2.07E+07	6.95E+05	1.45E+06	3.28E+02	1.01E+05	2.57E+04	1.24E+05	2.39E+05
	Rank	1	9	8	10	12	11	6	2	5	3	4	7
F22	Mean	7.35E−01	8.67E+02	3.78E+02	8.13E+02	2.08E+03	7.54E+02	8.91E+02	3.10E+02	1.12E+02	5.91E+01	6.81E+02	3.39E+02
	Std	1.19E+01	2.29E+02	1.65E+02	2.70E+02	5.58E+02	1.46E+02	1.69E+02	1.79E+02	6.33E+01	4.84E+01	2.11E+02	1.78E+02
	Rank	1	10	6	9	12	8	11	4	3	2	7	5
F23	Mean	2.94E+03	3.71E+02	3.35E+02	3.35E+02	1.23E+03	3.66E+02	3.82E+02	3.15E+02	3.15E+02	3.15E+02	3.15E+02	3.29E+02
	Std	3.05E−01	3.98E+01	1.04E+01	9.16E+00	3.77E+02	1.16E+01	2.70E+01	1.71E−13	1.36E−12	1.35E−12	1.20E−01	4.03E+00
	Rank	12	9	7	6	11	8	10	1	3	2	4	5
F24	Mean	1.05E+06	2.76E+02	2.00E+02	2.06E+02	4.78E+02	2.00E+02	2.00E+02	2.98E+02	2.25E+02	2.26E+02	2.46E+02	2.00E+02
	Std	4.44E+07	2.73E+01	8.23E−04	5.12E+00	9.29E+01	6.94E−02	1.68E−03	3.80E+02	2.75E+00	5.40E+00	5.69E+00	1.56E−03
	Rank	12	9	1	5	11	4	2	10	6	7	8	3
F25	Mean	1.64E+01	2.14E+02	2.10E+02	2.16E+02	3.14E+02	2.26E+02	2.25E+02	2.04E+02	2.12E+02	2.04E+02	2.08E+02	2.11E+02
	Std	1.52E+04	7.65E+00	5.34E+00	1.60E+01	3.45E+01	8.89E+00	1.43E+01	2.99E+00	1.71E+00	9.08E−01	4.51E+00	2.82E+00
	Rank	1	8	5	9	12	11	10	3	7	2	4	6
F26	Mean	2.83E+05	1.03E+02	1.47E+02	1.00E+02	1.09E+02	1.02E+02	1.04E+02	1.02E+02	1.00E+02	1.00E+02	1.01E+02	1.01E+02
	Std	1.99E+02	1.50E+00	5.02E+01	1.10E−01	1.70E+00	5.30E−01	5.51E−01	1.38E+01	4.35E−02	4.28E−02	1.17E+00	1.53E−01
	Rank	12	8	11	3	10	7	9	6	2	1	5	4
F27	Mean	2.28E+01	9.21E+02	6.16E+02	1.08E+03	1.38E+03	8.28E+02	5.07E+02	4.17E+02	5.06E+02	3.51E+02	8.82E+02	8.19E+02
	Std	2.15E+00	2.23E+02	1.20E+02	3.71E+02	2.60E+02	3.39E+02	3.13E+01	1.75E+02	1.08E+02	4.30E+01	2.03E+02	9.17E+01
	Rank	1	10	6	11	12	8	5	3	4	2	9	7
F28	Mean	2.59E+00	1.12E+03	1.20E+03	2.38E+03	3.73E+03	1.98E+03	1.47E+03	3.98E+03	8.39E+02	8.35E+02	1.29E+03	9.68E+02
	Std	5.98E+01	1.57E+02	2.62E+02	4.88E+02	6.26E+02	2.96E+02	7.60E+01	3.09E+03	2.71E+01	2.48E+01	2.88E+02	1.06E+02
	Rank	1	5	6	10	11	9	8	12	3	2	7	4

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Table 12 (*continued*).

Test problem	Results	PSO	MFO	GWO	WOA	SCA	m-SCA	OBSCA	CMA-ES	IDE	SinDE	BDE	MGSCA
F29	Mean	2.71E+02	3.06E+06	1.29E+06	4.85E+06	7.49E+07	1.04E+07	9.45E+06	8.01E+02	1.98E+05	9.27E+05	4.80E+06	1.19E+06
	Std	5.04E+02	3.62E+06	4.28E+06	4.82E+06	2.64E+07	5.39E+06	4.42E+06	9.49E+01	1.18E+06	2.61E+06	4.83E+06	3.25E+06
	Rank	1	7	6	9	12	11	10	2	3	4	8	5
F30	Mean	1.18E+07	5.89E+04	5.20E+04	8.38E+04	1.64E+06	2.38E+05	3.17E+05	2.55E+03	2.89E+03	3.11E+03	1.76E+04	1.92E+04
	Std	8.87E+04	5.40E+04	3.15E+04	6.11E+04	9.69E+05	1.01E+05	9.86E+04	6.60E+02	1.08E+03	1.24E+03	3.18E+04	8.25E+03
	Rank	12	7	6	8	11	9	10	1	2	3	4	5
Average rank		7.27	7.90	5.87	6.87	11.37	8.87	9.10	4.37	3.83	2.27	4.70	5.23
Overall rank		8	9	6	7	12	10	11	**3**	**2**	**1**	**4**	**5**

Table 13
Results for the gear train design problem.

Algorithm	Optimal decision parameters				Optimum value ($f_{1,\min}$)
	η_A	η_B	η_C	η_D	
MG-SCA	43	16	19	49	**2.7009E−12**
SCA	55	17	14	30	1.3616E−09(+)
PSO	54	17	22	48	1.1661E−10(+)
MFO	51	16	23	50	1.1834E−09(+)
GWO	54	17	22	48	1.1661E−10(+)
WOA	55	14	17	30	1.3616E−09(+)
m-SCA	53	13	20	34	2.3078E−11(=)
OBSCA	52	15	30	60	2.3576E−09(+)
CMA-ES	60	12	12	12	3.1048E−03(+)
IDE	53	13	30	51	2.3078E−11(+)
SinDE	53	13	20	34	2.3078E−11(+)
BDE	59	15	21	37	3.0676E−10(+)

Table 14
Results for the parameter estimation for frequency-modulated.

Algorithm	Best	Average	Worst	STD	Statistical decision
MG-SCA	0.001949	12.25	19.91	4.58	
SCA	10.31	14.53	22.20	3.48	=
PSO	25.24	27.63	29.65	1.17	+
MFO	11.58	21.57	26.82	4.41	+
GWO	8.42	14.77	25.10	5.16	+
WOA	11.38	18.12	25.10	4.42	+
m-SCA	10.90	15.94	22.84	4.15	+
OBSCA	4.48	9.65	21.15	5.12	=
CMA-ES	24.52	28.45	29.46	1.02	+
IDE	0.00	5.69	16.15	5.98	−
SinDE	0.00	5.60	16.96	6.45	−
BDE	14.81	22.29	28.38	3.27	+

SCA, MFO and CMA-ES, and performed similar to the GWO, WOA, m-SCA and OBSCA. The table also shows that the performance of the MG-SCA is worse than the PSO.

4.6.4. Three bar truss design problem

This case study considers a three-bar planner truss structure as shown in Fig. 7, and originally presented by Nowcki (Nowcki, 2011). This problem is a non-linear and fractional optimization problem and therefore difficult to solve. In this problem, the volume of a statically loaded three-bar truss is minimized with respect to stress constraints on each truss component. The objective of the problem is to minimize the optimal cross sectional area, $f_4(A_1, A_2)$. The problem is defined as follows:

$$
\begin{aligned}
Minimize \quad & f_4\left(A_1, A_2\right) = l \times (2\sqrt{2A_1} + A_2) \\
s.t \quad & .g_1\left(A_1, A_2\right) = \frac{\sqrt{2}A_1 + A_2}{\sqrt{2}A_1^2 + 2A_1A_2} P \le \sigma \\
& g_2\left(A_1, A_2\right) = \frac{A_2}{\sqrt{2}A_1^2 + 2A_1A_2} P \le \sigma \\
& g_3\left(A_1, A_2\right) = \frac{1}{A_1 + \sqrt{2}A_2} P \le \sigma \\
& 0 \le A_1, A_2 \le 1
\end{aligned}
$$

where $l = 100cm.$, $P = 2KN/cm^2$ and $\sigma = 2KN/cm^2$

The best solution recorded in 30 independent trials by the MG-SCA is presented in Table 16. The same number of candidate solutions and function evaluations are utilized in Gandomi et al. (2013). Table 16 also presents the results obtained from various algorithms such as PSO (Shi and Eberhart, 1998), MFO (Mirjalili, 2015), GWO (Mirjalili et al., 2014), WOA (Mirjalili and Lewis, 2016), m-SCA (Nayak et al., 2018), OBSCA (Elaziz et al., 2017b), CMA-ES (Hansen and Ostermeier, 2001), IDE (Hongfeng, 2018), SinDE (Draa et al., 2015) and BDE (Bhandari, 2018). In the table, the outcomes of the Mann–Whitney U test are presented which conclude that the MG-SCA has significantly outperformed SCA, MFO, WOA, m-SCA and CMA-ES, and performed similar to the PSO, GWO and OBSCA.

5. Conclusions

This paper presented the Memory Guided Sine Cosine Algorithm (MG-SCA) which is an improved version of the standard SCA. The MG-SCA focuses on a better balance between exploration and exploitation, to prevent becoming stuck in local optima. A good balance is obtained by using multiple search guides. The number of search guides are decreased with increase in the number of iterations for the transition from the diversification phase to the intensification phase. The MG-SCA is evaluated on 13 standard benchmark problems and 30 standard benchmark problems of IEEE CEC 2014 problem set, and compared with

Table 15
Results for the speed reducer design problem.

Algorithm	Optimal decision parameters							Optimum value ($f_{3,\min}$)
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
MG-SCA	3.6	0.7	20.2513	8.3	8.3	3.30665	5.0	**3002.7385**
SCA	3.51889	0.7	17	7.3	8.3	3.35899	5.30519	3028.8657(+)
PSO	3.50662	0.7	17	7.44735	7.88468	3.2725	5.38998	3003.7983(−)
MFO	3.59093	0.70554	19.7972	8.08267	7.84181	3.70621	5.48167	3836.2164(+)
GWO	3.6	0.8	28	7.3	8.3	2.9	5.0	3020.2331(=)
WOA	3.52111	0.7	17	7.3	7.8	3.35021	5.29533	3010.1480(=)
m-SCA	3.52394	0.7	17	7.3	7.8	3.3628	5.32467	3033.2845(=)
OBSCA	3.00576	0.72755	21.8423	7.30835	8.15455	3.36452	5.25164	3027.5130(=)
CMA-ES	3.6	0.7	17	8.3	8.3	3.9	5.5	3363.8734(+)
IDE	3.5	0.7	17	7.3	7.8	3.35021	5.28668	2996.3482(−)
SinDE	3.5	0.7	17	7.3	7.8	3.35021	5.28668	2996.3482(−)
BDE	3.5	0.7	17	7.3	7.8	3.35021	5.28668	2996.3482(−)

Table 16
Results for the three bar truss design problem.

Algorithm	Decision variables		Objective function value ($f_{4,\min}$)
	A_1	A_2	
MG-SCA	1.00000	0.42715	263.8986
SCA	0.78394	0.42219	263.9506(+)
PSO	0.58959	0.20568	263.8994(=)
MFO	0.78901	0.44025	267.1922(+)
GWO	0.54083	0.41266	263.9497(=)
WOA	0.79995	0.37725	263.9858(+)
m-SCA	0.79491	0.39113	263.9481(+)
OBSCA	0.77457	0.46647	263.9463(=)
CMA-ES	0.78623	0.41608	263.9867(+)
IDE	0.78867	0.40825	263.8958(−)
SinDE	0.78867	0.40825	263.8958(−)
BDE	0.78867	0.40825	263.8958(−)

standard SCA, PSO, MFO, GWO, WOA, m-SCA, OBSCA, CMA-ES, IDE, SinDE and BDE. The results show that the decreasing nature of number of guidance has meaningful impact on enhancing the exploration ability and in maintaining an appropriate balance between exploration and exploitation. Besides, MG-SCA is faster in terms of convergence rate and provides significantly better results as compared to the standard SCA. The comparison with other state-of-the-art algorithms also favours the competitive search ability of the MG-SCA. The performance of the MG-SCA was also evaluated on engineering optimization problems, and compared with other meta-heuristics. The results show better search efficiency and solution accuracy of the candidate solutions by the MG-SCA for most of the problems as compared to the standard SCA and most of the other meta-heuristics.

Several research directions can be followed in future work. First, the performance of the MG-SCA can be evaluated using clustering datasets and problems that need in-depth exploration and exploitation trends. Also, multi-objective and binary variants of the MG-SCA can be used to solve several engineering problems such as power dispatch problems, vehicle scheduling problems, travelling salesman problems, knapsack problems etc.

CRediT authorship contribution statement

Shubham Gupta: Conceptualization, Methodology, Validation, Visualization, Writing original draft, Writing - review & editing. **Kusum Deep:** Writing - review & editing, Visualization, Investigation, Supervision. **Andries P. Engelbrecht:** Writing - review & editing, Visualization, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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