# ReAssignment Module 6 - Transportation Model Dual

Dibakar Bhowal

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```
library(lpSolve)
library(lpSolveAPI)
library(tinytex)
```

$$Question\ 1 - \ Primal\ Formulation\ and\ Solution$$

$$Min\ Z = \sum_{i=1}^{4}\sum_{j=1}^{4} c_{ij}x_{ij}$$

$$Where: (x_{ij})\ is\ the\ number\ of\ products\ transported\ from\ plant\ (i)\ to\ destination\ (j)$$
$$(c_{ij})\ is\ the\ cost\ of\ producing + shipping\ one\ unit\ from\ plant\ (i)\ to\ destination\ (j)$$

Supply Constraints:

$$x_{i1} + x_{i2} + x_{i3} \leq \text{capacity of plant } i$$

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 100$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 75$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 85$$

$$x_{41} + x_{42} + x_{43} + x_{44} \leq 40$$

Demand Constraints:

$$x_{11} + x_{21} + x_{31} + x_{41} \geq 80$$

$$x_{12} + x_{22} + x_{32} + x_{42} \geq 90$$

$$x_{13} + x_{23} + x_{33} + x_{43} \geq 70$$

The dummy destination absorbs the surplus supply:

$$x_{14} + x_{24} + x_{34} + x_{44} \geq 60$$

Non-negativity Constraints:

$$x_{ij} \geq 0$$

$$Where: i = 1, 2, 3, 4 \ and \ j = 1, 2, 3, 4$$

```
costs <- matrix(c(
  305, 306, 312, 0,
  203, 205, 210, 0,
  256, 252, 256, 0,
  153, 154, 160, 0
), nrow = 4, byrow = TRUE)

supply <- c(100, 75, 85, 40)

demand <- c(80, 90, 70, 60)

solution <- lp.transport(costs, "min", row.signs = rep("<=", 4), row.rhs = supply,
                         col.signs = rep(">=", 4), col.rhs = demand)

print(solution)
```

```
## Success: the objective function is 55320
```

```
optimal_transportation <- solution$solution
print(optimal_transportation)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   40    0   60
## [2,]   75    0    0    0
## [3,]    0   15   70    0
## [4,]    5   35    0    0
```

$$Question\ 2 - \ Dual\ Formulation\ and\ Solution$$

$$Max\ V = 80p_1 + 90p_2 + 70p_3 + 60p_4 - 100c_1 - 75c_2 - 85c_3 - 40c_4$$

$$Where: (p_1, p_2, p_3)\ are\ prices\ for\ demands\ at\ destinations\ 1,\ 2,\ 3$$

$$(p_4)\ is\ the\ price\ for\ the\ dummy\ destination\ (surplus\ capacity)$$

$$Where : (c_1, c_2, c_3, c_4)\ are\ unit\ cost\ of\ production\ for\ supplies\ at\ source\ 1,\ 2,\ 3,\ 4$$

Dual Constraints :

For Plant 1:

$$p_1 - c_1 \geq 5$$

We know, $c_1 = 300$

$$p_1 \geq 305$$

$$p_2 - c_1 \geq 6$$

We know, $c_1 = 300$

$$p_2 \geq 306$$

$$p_3 - c_1 \geq 12$$

We know, $c_1 = 300$

$$p_3 \geq 312$$

$$p_4 - c_1 \geq 0$$

We know, $c_1 = 300$

$$p_4 \geq 300$$

For Plant 2:

$$p_1 - c_2 \geq 3$$

We know, $c_2 = 200$

$$p_1 \geq 203$$

$$p_2 - c_2 \geq 5$$

We know, $c_2 = 200$

$$p_2 \geq 205$$

$$p_3 - c_2 \geq 10$$

We know, $c_2 = 200$

$$p_3 \geq 210$$

$$p_4 - c_2 \geq 0$$
We know, $c_2 = 200$
$$p_4 \geq 200$$

For Plant 3:

$$p_1 - c_3 \geq 6$$
We know, $c_3 = 250$
$$p_1 \geq 256$$

$$p_2 - c_3 \geq 2$$
We know, $c_3 = 250$
$$p_2 \geq 252$$

$$p_3 - c_3 \geq 6$$
We know, $c_3 = 250$
$$p_3 \geq 256$$

$$p_4 - c_3 \geq 0$$
We know, $c_3 = 250$
$$p_4 \geq 250$$

For Plant 4:

$$p_1 - c_4 \geq 3$$
We know, $c_4 = 150$
$$p_1 \geq 153$$

$$p_2 - c_4 \geq 4$$
We know, $c_4 = 150$
$$p_2 \geq 154$$

$$p_3 - c_4 \geq 10$$
We know, $c_4 = 150$
$$p_3 \geq 160$$

$$p_4 - c_4 \geq 0$$

We know, $c_4 = 150$

$$p_4 \geq 150$$

Non Negativity Constraint:

$$p_j \geq 0$$

$$c_i \geq 0$$

where j=1,2,3,4 and i=1,2,3,4

Economic Interpretation of the Dual:

$$MR = MC \quad Rule$$

$$Our\ dual\ constraint\ is\ p_j - c_i \geq TC_{ij}\ (Where,\ TC_{ij} = Transportation\ Cost)$$

$$This\ means\ p_j \geq TC_{ij} + c_i$$

$$To\ be\ more\ specific,\ For\ Plant\ 4:$$

$$p_2 \geq c_4 + 4$$

We know, $c_4 = 150$

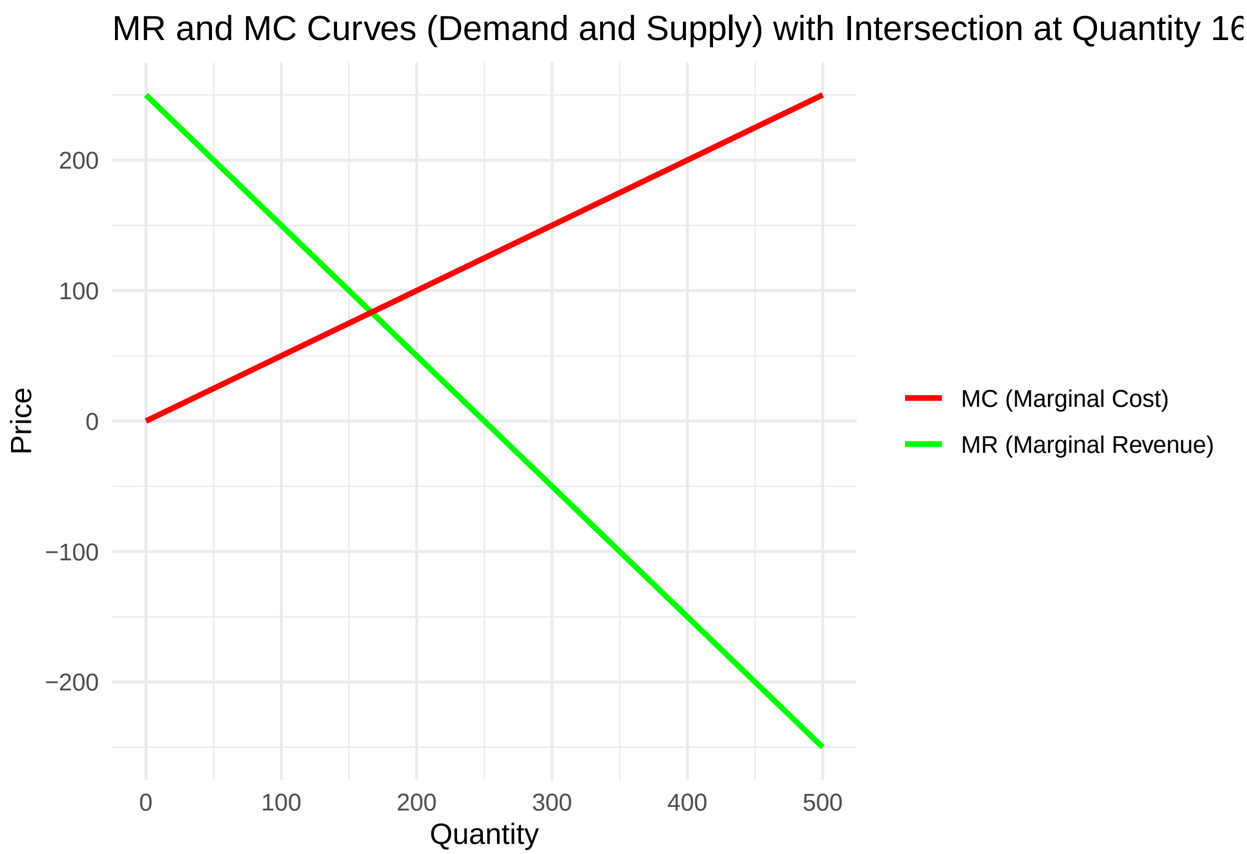
$$p_2 \geq 154$$

##Which means $154 is the per unit cost of production and shipping from Plant 4 to destination 2, since the per unit cost of production in Plant 4 is given $150. The left side is the per unit revenue received by selling one unit of the product. This is what we call MR (marginal revenue) in economics. The right side is the per unit cost of making and transporting good. This is called MC (marginal cost). Plant 4 keeps on increasing production and shipping to the destinations 2 as long as price is over $154, that is as long as MR is greater then MC. On the opposite,Plant 4 reduces production and shipping if MR is less than MC. These both are dynamic situations where either production increases or decreases. The first scenario occurs at the left side of the intersection of MR and MC curve while the later case occurs to the right of the intersection. When MR=MC, the producer neither increases production nor decreases it. This is what we call equilibrium for profit maximization. This equality of MR and MC occus at the intersection of MR and MC curve in the following graph. Thus, transportation cost minimization problem is equivalent to profit maximization in the dual and which ends up with MR=MC.

##MR=MC, the universal rule of economics for profit maximization.

```r
library(ggplot2)

Quantities <- seq(0, 500, by = 1)

MR <- function(quantity) {250 - 1 * quantity}
MC <- function(quantity) {0.5 * quantity}

Data <- data.frame(
  Quantity = Quantities,
  MR = MR(Quantities),
  MC = MC(Quantities)
)

ggplot(Data, aes(x = Quantity)) +
  geom_line(aes(y = MR, color = "MR (Marginal Revenue)"), size = 1) +
  geom_line(aes(y = MC, color = "MC (Marginal Cost)"), size = 1) +
  labs(
    title = "MR and MC Curves (Demand and Supply) with Intersection at Quantity 160",
    x = "Quantity",
    y = "Price"
  ) +
  scale_color_manual(values = c("MR (Marginal Revenue)" = "green", "MC (Marginal Cost)" = "red")) +
  theme_minimal() +
  theme(legend.title = element_blank())
```

```
## Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
## i Please use `linewidth` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```


MR and MC Curves (Demand and Supply) with Intersection at Quantity 16
Price
200
100
0
−100
−200
0
100
200
300
400
500
Quantity
MC (Marginal Cost)
MR (Marginal Revenue)