Assignment Module 6 - Transportation Model Dual

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```r
library(lpSolve)
library(lpSolveAPI)
library(tinytex)
```

$$Question\ 1 - \ Primal\ Formulation\ and\ Solution$$

$$Min\ Z = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij}$$

$$Where: (x_{ij})\ is\ the\ number\ of\ AEDs\ transported\ from\ plant\ (i)\ to\ warehouse\ (j)$$
$$(c_{ij})\ is\ the\ cost\ of\ producing + shipping\ one\ unit\ from\ plant\ (i)\ to\ warehouse\ (j)$$

Supply Constraints:

$$x_{i1} + x_{i2} + x_{i3} + x_{i4} \leq \text{capacity of plant } i$$

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 100$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 125$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 150$$

Demand Constraints:

$$x_{11} + x_{21} + x_{31} \geq 80$$

$$x_{12} + x_{22} + x_{32} \geq 90$$

$$x_{13} + x_{23} + x_{33} \geq 70$$

The dummy warehouse absorbs the surplus supply:

$$x_{14} + x_{24} + x_{34} \geq 135$$

Non-negativity Constraints:

$$x_{ij} \geq 0$$

$$Where: i = 1, 2, 3 \ and \ j = 1, 2, 3, 4$$

```r
costs <- matrix(c(
  420, 414, 425, 0,
  312, 315, 314, 0,
  510, 512, 515, 0
), nrow = 3, byrow = TRUE)

supply <- c(100, 125, 150)

demand <- c(80, 90, 70, 135)

solution <- lp.transport(costs, "min", row.signs = rep("<=", 3), row.rhs = supply,
                         col.signs = rep(">=", 4), col.rhs = demand)

print(solution)
```

```
## Success: the objective function is 88250
```

```r
optimal_transportation <- solution$solution
print(optimal_transportation)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   10   90    0    0
## [2,]   55    0   70    0
## [3,]   15    0    0  135
```

$$Question\ 2 - \ Dual\ Formulation\ and\ Solution$$

$$Max\ W = 80v_1 + 90v_2 + 70v_3 + 135v_4$$

$$Where: (v_1, v_2, v_3)\ are\ shadow\ prices\ for\ demands\ at\ warehouses\ 1,\ 2,\ 3$$
$$(v_4)\ is\ the\ shadow\ price\ for\ the\ dummy\ warehouse\ (surplus\ capacity)$$

Dual Constraints :

For Plant A:

$$u_1 + v_1 \leq 420$$

$$u_1 + v_2 \leq 414$$

$$u_1 + v_3 \leq 425$$

$$u_1 + v_4 \leq 0$$

For Plant B:

$$u_2 + v_1 \leq 312$$

$$u_2 + v_2 \leq 315$$

$$u_2 + v_3 \leq 314$$

$$u_2 + v_4 \leq 0$$

For Plant C:

$$u_3 + v_1 \leq 510$$

$$u_3 + v_2 \leq 512$$

$$u_3 + v_3 \leq 515$$

$$u_3 + v_4 \leq 0$$

```r
dual_objective <- c(80, 90, 70, 135)


dual_constraints <- matrix(c(
  1, 0, 0, 0, 1, 0, 0, 0,
  1, 0, 0, 0, 0, 1, 0, 0,
  1, 0, 0, 0, 0, 0, 1, 0,
  1, 0, 0, 0, 0, 0, 0, 1,
```

```
  0, 1, 0, 0, 1, 0, 0, 0,
  0, 1, 0, 0, 0, 1, 0, 0,
  0, 1, 0, 0, 0, 0, 1, 0,
  0, 1, 0, 0, 0, 0, 0, 1,

  0, 0, 1, 0, 1, 0, 0, 0,
  0, 0, 1, 0, 0, 1, 0, 0,
  0, 0, 1, 0, 0, 0, 1, 0,
  0, 0, 1, 0, 0, 0, 0, 1
), nrow = 12, byrow = TRUE)

dual_rhs <- c(420, 414, 425, 0, 312, 315, 314, 0, 510, 512, 515, 0)

dual_directions <- rep("<=", 12)

dual_solution <- lp("max", dual_objective, dual_constraints, dual_directions, dual_rhs)

print(solution)
```

```
## Success: the objective function is 88250
```

```
dual_optimal <- dual_solution$solution
print(dual_optimal)
```

```
## [1]   0 315   0   0
```

Question 3 – Economic Interpretaion

Plant A sends 10 units to warehouse 1 and 90 units to warehouse 2(Fully satisfying the demand of warehouse 2)

Plant B sends 55 units to warehouse 1 and 70 units to warehouse 3(Fully satisfying the demand of warehouse 3)

Plant C sends 15 units to warehouse 1(Completing warehouse 1's demand) and 135 units to the dummy warehouse.

Warehouse 2 is the critical constraint because it's shadow price is 315 while for warehouse 1,3 and dummy it's 0

Indicating increasing demand at warehouse 2 will increase the overall cost by 315 per unit while increasing demand at warehouse 1,3 and dummy wouldn't cause any additional cost.

So, warehouse 2 is a bottleneck to the production process.