Definitions

Finite difference:

$f(x + \Delta x) = f(x) + \Delta f(x)$

Limit of a sequence:

$|a - S_n| < \epsilon$

$\lim_{n\to\infty} S_n = a$

Limit of a function:

$|f(x) - l| \leq \epsilon$

$\lim_{x\to x_0} f(x) = l$

Infinite sum:

$\sum_{n=0}^{\infty} a_n = \lim_{n\to\infty} \sum_{n=0}^{\infty} a_n$

Discrete derivative:

$\frac{\Delta f}{\Delta x} = \frac{f(x_n+\Delta x)-f(x_n)}{\Delta x}$

Derivative:

$\frac{df(x)}{dx} = \lim_{\Delta x\to 0} \frac{\Delta f(x)}{\Delta x} = \lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

Asymptotic relation:

Sequences:

$\lim_{n\to\infty} \frac{A_n}{B_n} = 1$

$A \sim B$

Functions:

$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{g(x)}{f(x)} = 1$

Discrete integral:

$\sum_{n=0}^{N} f(x_n) \cdot \Delta x$

Taylor expansion:

$f_k(x) = \sum_{j=0}^{K} \frac{1}{j!} \cdot \frac{d^j f(x_0)}{dx^j}(x - x_0)^j$

Integral:

$\int_a^b f(x)dx = \lim_{N\to\infty} \sum_{n=0}^{N-1} f(x_n)\Delta x_n = \lim_{N\to\infty} \sum_{n=0}^{N-1} f(x_n)(x_{n+1} - x_n)$

$\Delta x = \frac{b-a}{N}$

Definite integral with relation to a differential:

$\int_a^b f(x)dg(x) = \lim_{N\to\infty} \sum_{n=1}^{N-1} f(x_n)\Delta g(x_n) = \lim_{N\to\infty} \sum_{n=1}^{N-1} f(x_n)(g(x_{n+1}) - g(x_n))$

Fundamental theorem of calculus:

$f(x)dx = \frac{dF(x)}{dx}dx = dF(x)$

Rules

Derivative rules:

Product rule:

$\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot \frac{d}{dx}g(x) + g(x) \cdot \frac{d}{dx}f(x)$

Example:

$\frac{d}{dx}(x^3 \cdot sin(x)) = x^3 \cdot \frac{d}{dx}sin(x) + (sin(x)) \cdot \frac{d}{dx}x^3 = x^3 \cdot cos(x) + 3x^2 \cdot sin(x)$

Chain rule:

$\frac{dh(x)}{dx} = \frac{df(u(x))}{du} \cdot \frac{du(x)}{dx}$

Example:

$\frac{d}{dx}ln(x^2 + 1) = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot 2x = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

Quotient rule:

$\frac{d}{dx}\frac{f(x)}{g(x} = \frac{g(x)\cdot\frac{d}{dx}f(x)-f(x)\cdot\frac{d}{dx}g(x)}{(g(x))^2}$

Example:

$\frac{d}{dx}\frac{2+3x}{x+1} = \frac{(x+1)\cdot 3-(2+3x)\cdot 1}{(x+1)^2} = \frac{1}{(x+1)^2}$

Integral rules:

Integration by parts:

$\int_a^b g(x)\frac{df(x)}{dx}dx = (f(b)g(b) - f(a)g(a)) - \int_a^b f(x)\frac{dg(x)}{dx}dx$
Example:
$\int_0^1 xe^x dx = \int_0^1 x\frac{de^x}{dx} = (e^1 \cdot 1 - e^0 \cdot 0) - \int_0^1 e^x \frac{dx}{dx}dx = e - \int_0^1 e^x dx = e - (e^1 - e^0) = e - e + 1 = 1$
Integration by substitution:
$\int_a^b f(g(x))(\frac{dg(x)}{d(x)})dx = \int_a^b f(g(x))dg(x)$
Example:
TODO
ABC formula:
$ax^2 + bx + c = 0 \iff x = \frac{-b \pm \sqrt{b^2 - 4a \cdot c}}{2a}$
Example:
TODO

Examples

Answers in the expected notation:
Limits:
$\lim_{n \to 0} \frac{3n^3+1}{4n^3+n^3+n} = \lim_{n \to 0} \frac{3n^3}{4n^3} = \lim_{n \to \infty} \frac{3}{4} = \frac{3}{4}$
or: $\lim_{x \to 0} \frac{2x^6+3x^2-6x}{7x^6-6x} \sim \frac{-6x}{-6x} = 1$

Derivatives by definition:
$\frac{df(x^2)}{dx} = \frac{(x+dx)^2-x^2}{dx} = \frac{x^2+2xdx+(dx)^2-x^2}{dx} = \frac{2xdx}{dx} = 2x$

Other options for derivatives: $(1/x, e^x, e^{2x}, x^3)$

Integrals by definition:
$\int_0^b x^2 dx = \lim_{N \to \infty} \sum_{n=0}^{N-1} x_n^2 \Delta x$
where: $\Delta x = \frac{b-0}{N} = \frac{b}{N}$ and $x_n = n\Delta x = \frac{n}{N}b$
rewrite: $\sum_{n=0}^{N-1} x_n^2 \Delta x = \sum_{n=0}^{N-1} \frac{n^2}{N^3} b^3 = \frac{b^3}{N^3} \sum_{n=0}^{N-1} n^2$
we use the following sum: $\sum_{n=0}^{N-1} n^2 = \frac{(N-1)N(2N-1)}{6}$
our sum becomes: $\sum_{n=0}^{N-1} x^2 \Delta x = b^3 \frac{(N-1)N(2N-1)}{6N^3} = b^3 \frac{2N^3-3N^2+6}{6N^3}$
take the limit: $\int_0^b x^2 dx = \lim_{N \to \infty} b^3 \frac{2N^3-3N^2+6}{6N^3} = b^3 \lim_{N \to \infty} \frac{2N^3}{6N^3} = \frac{1}{3}b^3$
thus: $\int_0^b x^2 dx = \frac{1}{3}b^3$