Definitions

Finite difference:
$f(x + \Delta x) = f(x) + \Delta f(x)$

Limit of a sequence:
$|a - S_n| < \epsilon$
$\lim\limits_{n\to\infty} S_n = a$
where a is the limit
such that for all n > N, $|a - S_n| < \epsilon$

Limit of a function:
$|f(x) - l| \leq \epsilon$
$\lim\limits_{x\to x_0} f(x) = l$
where l is the limit such that for all n > N, $|f(x) - l| \leq \epsilon$

Infinite sum:
$\sum_{n=0}^{\infty} a_n = \lim\limits_{N\to\infty} \sum_{n=0}^{N} a_n$

Discrete derivative:
$\frac{\Delta f}{\Delta x} = \frac{f(x_n+\Delta x)-f(x_n)}{\Delta x}$

Derivative:
$\frac{df(x)}{dx} = \lim\limits_{\Delta x\to 0} \frac{\Delta f(x)}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

Asymptotic relation:
Sequences:
$\lim\limits_{n\to\infty} \frac{A_n}{B_n} = 1$,then $A \sim B$
Functions:
$\lim\limits_{x\to x_0} \frac{f(x)}{g(x)} = 1$,then $f(x) \sim g(x)$.

Discrete integral:
$\sum_{n=0}^{N} f(x_n) \cdot \Delta x$

Integral:
$\int_a^b f(x)dx = \lim\limits_{N\to\infty} \sum_{n=0}^{N-1} f(x_n)\Delta x_n = \lim\limits_{N\to\infty} \sum_{n=0}^{N-1} f(x_n)(x_{n+1} - x_n)$
$\Delta x = \frac{b-a}{N}$

Definite integral with relation to a differential:
$\int_a^b f(x)dg(x) = \lim\limits_{N\to\infty} \sum_{n=1}^{N-1} f(x_n)\Delta g(x_n) = \lim\limits_{N\to\infty} \sum_{n=1}^{N-1} f(x_n)(g(x_{n+1}) - g(x_n))$

Fundamental theorem of calculus:
$f(x)dx = \frac{dF(x)}{dx}dx = dF(x)$

Critical points:
1. Compute the derivative
2. Find the points where the derivative = 0
3. Compute the second derivative
4. Find the value at the point in the second derivative
5. If the value > 0 then minimum, if < 0, maximum

If the value is 0, then we have a saddle point.
In this case we look at the 3rd derivative etc.

Rules

Derivative rules:
Product rule:
$\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot \frac{d}{dx}g(x) + g(x) \cdot \frac{d}{dx}f(x)$
Example:
$\frac{d}{dx}(x^3 \cdot sin(x)) = x^3 \cdot \frac{d}{dx}sin(x) + (sin(x)) \cdot \frac{d}{dx}x^3 = x^3 \cdot cos(x) + 3x^2 \cdot sin(x)$
Chain rule:
$\frac{dh(x)}{dx} = \frac{df(u(x))}{du} \cdot \frac{du(x)}{dx}$
Example:
$\frac{d}{dx}ln(x^2 + 1) = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot 2x = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$
Quotient rule:
$\frac{d}{dx}\frac{f(x)}{g(x} = \frac{g(x)\cdot\frac{d}{dx}f(x) - f(x)\cdot\frac{d}{dx}g(x)}{(g(x))^2}$
Example:
$\frac{d}{dx}\frac{2+3x}{x+1} = \frac{(x+1)\cdot 3-(2+3x)\cdot 1}{(x+1)^2} = \frac{1}{(x+1)^2}$

Integral rules:
Integration by parts:

$\int_a^b g(x)\frac{df(x)}{dx}dx = (f(b)g(b) - f(a)g(a)) - \int_a^b f(x)\frac{dg(x)}{dx}dx$
Example:
$\int_0^1 xe^x dx = \int_0^1 x\frac{de^x}{dx}dx = (e^1 \cdot 1 - e^0 \cdot 0) - \int_0^1 e^x \frac{dx}{dx}dx = e - \int_0^1 e^x dx = e - (e^1 - e^0) = e - e + 1 = 1$
Integration by substitution:
$\int_a^b f(g(x))(\frac{dg(x)}{d(x)})dx = \int_a^b f(g(x))dg(x)$
Example:
TODO
ABC formula:
$ax^2 + bx + c = 0 \iff x = \frac{-b\pm\sqrt{b^2-4a\cdot c}}{2a}$

Examples

Answers in the expected notation:
Limits:
$\lim\limits_{n\to\infty} \frac{3n^3+1}{4n^3+n^3+n} = \lim\limits_{n\to\infty} \frac{3n^3}{4n^3} = \lim\limits_{n\to\infty} \frac{3}{4} = \frac{3}{4}$ **or:** $\lim\limits_{x\to 0} \frac{2x^6+3x^2-6x}{7x^6-6x} \sim \frac{-6x}{-6x} = 1$

Derivatives:
$f(x) = x^3 \cdot cos(x^2 + 3x)$
$\frac{df(x)}{dx} = 3x^2 \cdot (cos(x^2 + 3x)) + x^3 \cdot (-sin(x^2 + 3x)) \cdot 2x + 3) = x^2(3cos(x^2 + 3x) - x\ sin(x^2 + 3x) \cdot 2x + 3)$

Derivatives by definition:
$\frac{df(x^2)}{dx} = \lim\limits_{\Delta x\to 0} \frac{(x+\Delta x)^2-x^2}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{x^2+2x\Delta x+(\Delta x)^2-x^2}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{2x\Delta x}{\Delta x} = 2x$
$\frac{df(\frac{1}{x})}{dx} = \lim\limits_{\Delta x\to 0} \frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x} = \lim\limits_{\Delta x\to 0} -\frac{\Delta x}{\frac{x(\Delta x+x)}{\Delta x}} = \lim\limits_{\Delta x\to 0} -\frac{\Delta x}{x(\Delta x+x)\Delta x} = \lim\limits_{\Delta x\to 0} -\frac{1}{x(\Delta x+x)} = -\frac{1}{x^2}$
$\frac{df(e^x)}{dx} = \lim\limits_{\Delta x\to 0} \frac{e^{x+\Delta x}-e^x}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{e^{\Delta x+x}}{1} = \lim\limits_{\Delta x\to 0} e^{\Delta x+x} = e^x$
$\frac{df(e^{x+1})}{dx} = \lim\limits_{\Delta x\to 0} \frac{e^{x+\Delta x+1}-e^{x+1}}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{e^{x+1}e^{\Delta x}-e^{x+1}}{\Delta x} = e^{x+1} \cdot \lim\limits_{\Delta x\to 0} \frac{e^{\Delta x}-1}{\Delta x} = e^{x+1} \cdot \lim\limits_{\Delta x\to 0} \frac{\Delta x}{\Delta x} = e^{x+1} \cdot 1 = e^{x+1}$
$\frac{df(e^{2x})}{dx} = \lim\limits_{\Delta x\to 0} \frac{e^{2(x+\Delta x)}-e^{2x}}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{2e^{2(\Delta x+x)}}{1} = \lim\limits_{\Delta x\to 0} 2e^{2(\Delta x+x)} = \lim\limits_{\Delta x\to 0} 2e^{2(\Delta x+x)} = 2e^{2x}$
$\frac{df(x^3)}{dx} = \lim\limits_{\Delta x\to 0} \frac{(x+\Delta x)^3-x^3}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{\Delta x^2+2\Delta x+x^2+\Delta x+x^2+x^2}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{3x^2+3\Delta x+\Delta x^2}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{3x^2+3\Delta x+\Delta x^2}{\Delta x} = 3x^2$

Integrals by definition:
$\int_0^b x^2 dx = \lim\limits_{N\to\infty} \sum_{n=0}^{N-1} x_n^2 \Delta x$
where: $\Delta x = \frac{b-0}{N} = \frac{b}{N}$ and $x_n = n \cdot \Delta x = \frac{n\cdot b}{N}$
rewrite: $\lim\limits_{N\to\infty} \sum_{n=0}^{N-1} x_n^2 \cdot \Delta x = \lim\limits_{N\to\infty} \sum_{n=0}^{N-1} (\frac{n\cdot b}{N})^2 \cdot \frac{b}{N} = \lim\limits_{N\to\infty} \sum_{n=0}^{N-1} \frac{n^2\cdot b^3}{N^3} = \lim\limits_{N\to\infty} \frac{b^3}{N^3} \sum_{n=0}^{N-1} n^2$
we use the following sum: $\sum_{n=0}^{N-1} n^2 = \frac{(N-1)N(2N-1)}{6}$
our sum becomes: $\lim\limits_{N\to\infty} \frac{b^3}{N^3} \cdot \frac{(N-1)N(2N-1)}{6} = \lim\limits_{N\to\infty} b^3 \cdot \frac{2-\frac{3}{N}+\frac{1}{N}^2}{6}$
take the limit: $\int_0^b x^2 dx = \lim\limits_{N\to\infty} b^3 \cdot \frac{2-\frac{3}{N}+\frac{1}{N}^2}{6} = b^3 \cdot \frac{2-0+0}{6} = \frac{1}{3}b^3$
thus: $\int_0^b x^2 dx = \frac{1}{3}b^3$

Integration by parts:
Example:
$\int_{-1}^1 (xe^x + x^3)dx = \int_{-1}^1 xe^x dx + \int_{-1}^1 x^3 dx$
$= (xe^x - e^x + \frac{1}{4}x^4) - (xe^x - e^x + \frac{1}{4}x^4)$
$= (1 \cdot e^1 - e^1 + \frac{1}{4} \cdot 1^4) - (-1 \cdot e^{-1} - e^{-1} + \frac{1}{4} \cdot (-1)^4) = \frac{2}{e}$