Languages and Automata

Assignment 4, Tue 3rd Mar, 2020

Handing in your answers: There are two options:

1. Brightspace. Before submitting, make sure:
   - the file is a PDF document
   - your name and student number are included in the document (they might be printed).
2. Post box, located in the Mercator building on the ground floor. There will be boxes labelled with *LnA* and the corresponding group teacher's name. Put your work in the post box corresponding to your group. Before putting your solutions in the post box make sure:
   - your name and student number are written clearly on the document.

   There will be 1 box, the *Uitleverbak*, for work that hasn't been picked up at the exercise hours.

Deadline: Fri 6th Mar, 2020, 17:00 (in Nijmegen!). This deadline is strict: submission in brightspace will close at that time.

Goals: After completing these exercises successfully you should be able to recognise a regular language and, if it is not, then show this by using non-regularity of other languages or by using distinguishable words. Further, you should be able to minimise an automaton.

There are 3 mandatory exercises, worth **10 points** in total. There is 1 more, extra hard, exercise. Be aware that this exercise is just for fun, you cannot earn any points with it.

1 Non-regular Languages

Let A be the alphabet $\{a, b, c\}$, and let

$$L_1 = \{a^m b^n \in A^* \mid m \text{ is even if and only if } n \text{ is odd}\}$$
$$L_2 = \{c^n a^p c^m b^p \mid n, m, p \in \mathbb{N}\}$$
$$L_3 = \{a^n b^m \mid n \neq m\}$$

Decide the following statements. **Give in each case an appropriate explanation**, in which you may use languages that have been shown to be non-regular in the lecture, and the techniques that have been presented there (see last slide). Tip: Try to use simpler techniques first.

a) Is L_1 regular? **(1pt)**

b) Is L_2 regular? **(1pt)**

c) Is L_3 regular? **(1pt)**

d) Consider the following claim: "Let L, K be languages over A. If L is regular, then $\overline{L} \cap K$ must also be regular". Is this claim correct? If it is, give an explanation, if it's not, give a counterexample. **(1pt)**

2 Non-Regular Languages via Distinguishable Words

Show the following languages over $\{a, b, c\}$ to be non-regular using infinite collections of distinguishable words.

a) **(2pt)**

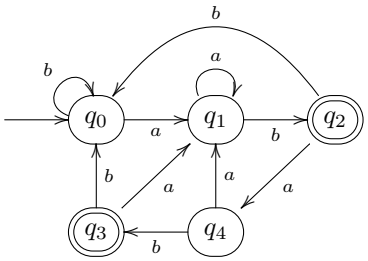
$$L = \{a^m b^n \mid m, n \in \mathbb{N}, m \geq n\}$$

b) **(2pt)**

$L = \{ucv \mid u, v \in \{a, b\}^*$, u appears as subword in v **or** v appears as subword in u$\}$

3 Minimisation

Consider the following automaton: **(2pt)**



Minimise this automaton, using the construction from the lecture.

4 Fun Exercises

1. Show that the following language L over $A = \{a, b\}$ is not regular.

$$L = \{a^n b^m \mid n = km, \text{ for some } k \in \mathbb{N}\}$$

2. Show that the following language L over $A = \{a\}$ is not regular.

$$L = \{a^{n^2} \mid n \in \mathbb{N}\}.$$