Languages and Automata

Assignment 5, Tuesday 10^{th} March, 2020

Update 12 March: please hand in via brightspace. Before submitting, make sure:

- the file is a PDF document
- your name and student number are included in the document (they might be printed).

Deadline: Friday 13^{th} March, 2020, 17:00 (in Nijmegen!). This deadline is strict: submission in brightspace will close at that time.

Goals: After completing these exercises successfully you should be able to read context-free grammars, write down grammars for context-free languages and regular languages, and work with basic closure properties of context-free languages. The total number of points is 10.

There are 4 mandatory exercises, worth **10 points** in total. There are 3 more, extra hard, exercises. Be aware that these exercises are just for fun, you cannot earn any points with them.

1 Ambiguous grammars

Let $\Sigma = \{a, b\}$. Consider the following context-free grammar

$$G_1 = \boxed{S \quad \to \quad a\,S \mid S\,b \mid a\,b \mid S\,S}$$

a) Show that the grammar is ambiguous by giving two left-most derivations of $a\,a\,b\,b$. **(1pt)**

b) Give a regular expression for $L_1 := \mathcal{L}(G_1)$. **(1pt)**

c) Give a non-ambiguous regular grammar for L_1. **(1pt)**

2 Constructing context-free grammars

For each of the following languages construct a context-free grammar that generates the language, and **explain why your answer is correct**. **(4pt)**

$$\begin{array}{lcl} L_1 & = & \{a^n b^{n+m} a^m \mid n, m \geq 0\} \\ L_2 & = & \{w \in \{a, b, r\}^* \mid \text{every } a \text{ (in } w\text{) is followed directly by } b\text{, and } w \text{ ends with } brrr\} \end{array}$$

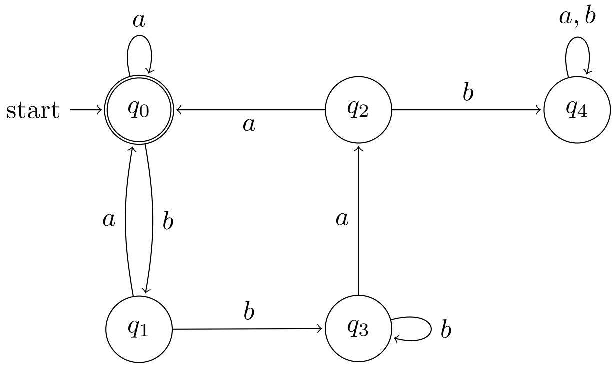
3 Closure properties

For each of the following statements, decide whether or not they are correct. Justify your answer with a proof (if the statement is correct) or a counterexample (if it is not correct). **(2pt)**

1. If L is context-free, then the complement $\overline{L}$ is *not* context-free.
2. If L is context-free and K is regular, then $L \cap K$ is regular.

4 From DFA to regular grammar

Consider the following DFA M:



Construct a context-free grammar that generates $\mathcal{L}(M)$. **(1pt)**

Fun Exercises

1. Construct a context-free grammar that generates the following language:

$$L_5 = \{w \in \{a, b\}^* \mid |w| = 2k + 1 \text{ and } w_1 = w_{k+1}\},$$

where w_i denotes the i-th symbol in a word w. That is, L_5 consists of all words of odd length that have the same symbol in the first and middle positions.

2. $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \times, +, (,)\}$. Construct a context-free grammar that generates the language

$$L_6 = \{w \in \Sigma^* \mid w \text{ is a well-formed arithmetical expressions}\}$$

NB. $2+3+4\times 5$ and $((2+3)+4)\times 5$ and $(((2+3)))+4\times 5$ are well-formed. $2+(3+4\times 5$ and $(2+3)+4)\times 5$ and $)($ are not.

3. Suppose that L and L' are context-free languages. Show that both L^* and $L\,L'$ are context-free languages.