Languages and Automata Assignment 5

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a)

First construction:
S $\rightarrow$ aS $\rightarrow$ a aS $\rightarrow$ a a SS $\rightarrow$ a a Sb $\rightarrow$ a a b b
Second construction:
S $\rightarrow$ aS $\rightarrow$ a SS $\rightarrow$ a ab S $\rightarrow$ a a b b

b)

$(a^* \cup ab)^* b^*$

c)

S $\rightarrow$ aS | Sb | SS

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$\mathcal{L}_1$

S $\rightarrow$ aN | λ
N $\rightarrow$ bL
M $\rightarrow$ aS
L $\rightarrow$ aMS

This grammar describes the language $\mathcal{L}_1$ because if we take an a we also need to take n+m b and then another a before we can again take an a.

$\mathcal{L}_2$

S $\rightarrow$ abS | rS | bS | bR
R $\rightarrow$ rrr

This grammar describes the language $\mathcal{L}_2$ because we can only take an a if we also take a b, thus giving the rule that every a should be followed directly by a b. Besides this we can put any number of r's and b's before we have to end the word using the rule bR, which ensures the word is ended with brrr.

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1)

This does not hold since there are languages where the complement of the language is context-free as well. An example of such a language is the simple language L = $\{a^n\}$, where we have $\Sigma = \{a, b\}$. Then we have the $\bar{L} = \{b^n\}$.

2)

This does not hold since we can find languages that are regular and their intersection is not regular. An example of is the language L = $\{a^n b^n c^m | m, n \geq 0\}$ and K = $\{a^m b^n c^n | m, n \geq 0\}$ which leads to L $\cap$ K = $\{a^n b^n c^n | n \geq 0\}$ which is known not to be regular.

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We first label the states in the DFA to non-terminals in the grammar.
$q_0 = S, q_1 = Q, q_2 = M, q_3 = L, q_4 = N$

Then we can define the following grammar:
S $\rightarrow$ aS | bQ | λ
Q $\rightarrow$ aS | bL
L $\rightarrow$ bL | aM
M $\rightarrow$ aS | bN
N $\rightarrow$ aN | bN