

Languages and Automata Assignment 1

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1

a)

$$\begin{aligned}f : A^* &\rightarrow A^* : \\f(\lambda) &= \lambda \\f(a \cdot w) &= f(w) \\f(b \cdot w) &= b f(w)\end{aligned}$$

b)

$$\forall w \in A^* [f(f(w)) = f(w)]$$

$$\begin{aligned}f(\lambda) &= \lambda = f(f(w)) \checkmark \\ \text{We assume } f(w) \text{ holds. Thus } f(f(w)) &= f(w) \quad (\text{IH}) \\ f(f(x \cdot w)) &= f(f(x)) \cdot f(f(w)) \quad (\text{rewrite as concatenation}) \\ &= f(f(x)) \cdot f(w) \quad (\text{IH}) \\ &= f(x) \cdot f(w) \quad (\text{by definition of } f) \\ &= f(x \cdot w) \quad (\text{By def of concatenation}) \checkmark\end{aligned}$$

2

a)

The word "abba" is in all languages.

The word "b" is in none of the languages since it always starts with an a or the word "bba" which means that it is impossible to have just "b" as a word.

b)

L_2 is different from L_1 since "a" $\notin L_1$.

L_3 is different from L_1 and L_2 since "aaa" $\notin L_2$ or L_1 .

c)

Yes the language L is equal to a, b^* .

$\lambda \in L$ since we can choose 0 for both stars.

We are also able to choose any arbitrary configuration of a's and b's since we can use the part $(b^*a)^*$ to choose as many a's as we want and then we can use b^* to choose as many b's as we want and repeat this because of the outer star. In this way we can make any word that is also in a, b^* .

3

a)

$$L = \mathcal{L}(a^*(bba \cup baba \cup ba^*b \cup \lambda)^*a^*)$$

Since we want to have an even number of b's we need to give all the possible options that give an even amount of b's and an a at the end. Since 0 is even we also need to include λ as an option. Besides this we should allow any number of a's around and in between the b's.

(I used \cup instead of $+$ to indicate the union.)

b)

$$L = \mathcal{L}(((DU \cup UD)^* \cup D \cup U) \cup ((DU \cup UD)^*D) \cup ((DU \cup UD)^*U))$$

The first part allows the elevator to go up and down one and then return to the ground floor. The part $UD \cup U$ allows paths of length 1.

The part after that allows the path to stop at the upper floor or the basement, which the first part did not allow.