Problem session 3

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4

4.1

We can transform a array that represents a max-heap into a array representing a mini-heap by following process:

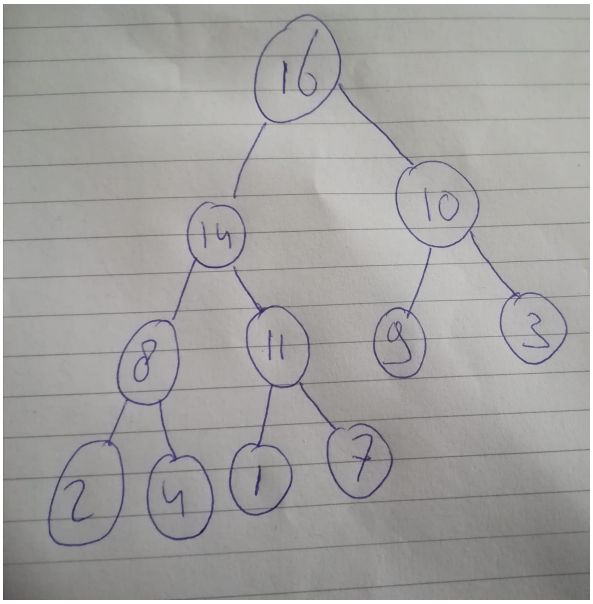
For each (initially) non-leaf node N, starting at the first non-leaf node moving towards the root of the tree (index in array = $\lfloor n/2 \rfloor - 1$ where n is number of nodes/array length):

Check which of its child nodes is the lowest, swap places with that node. Now check if node N has any child nodes, and if so: is any of these child nodes lower then N? If so, swap places with that node. Keep continuing this process until node N has no more conflicting child nodes (regarding the min-heap property).

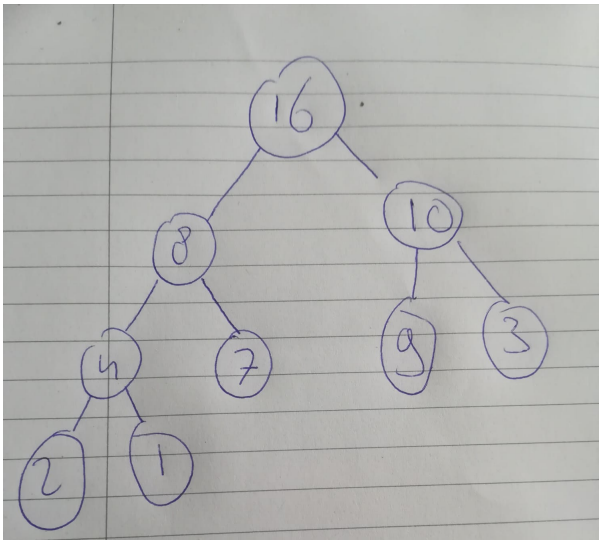
After this is done for all initial non-leaf nodes, the array/tree should be a min-heap. The complexity of this process is $\mathcal{O}(n \cdot \log(n))$, namely for 0.5 of the nodes we have to start swapping down levels (in worst case swapping $\log(n)$ times, as $\log(n)$ is the number of levels in the tree).
For some reason we are able to reduce this $\mathcal{O}(n \dot{log}\ n)$ complexity to $\mathcal{O}(n)$ by doing some math, the professor showed this in the lecture.

4.2

1)



2)

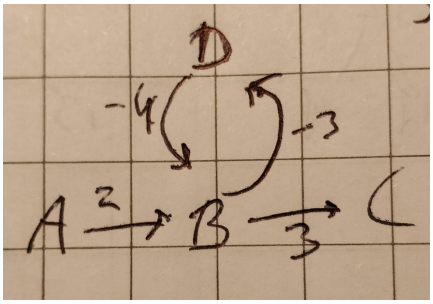


4.3

Q:	0	1	2	3	4	5
	0	30	40	∞	∞	∞
			40	50	80	∞
				50	70	∞
					70	110

4.4

The following graph will not find the shortest path since the theoretical shortest path is $-\infty$.
This is caused by the loop from B -> D that has a negative weight. In this case it will try to find the shortest path by looping through this loop infinitely many times.



4.5

We can use a slightly altered version of Dijkstra for this problem, lets call it Dijkstra*. With Dijkstra* you do the normal Dijkstra routine, but you don't just keep track of the distance cost to a certain node, you add the number of hops needed to reach that distance.
Normally if Dijkstra finds a different route to a node, it checks 'This route cost $<$ Known route cost?' With Dijkstra* this check would become: '(This route cost $<$ Known route cost) OR ((This route cost $==$ known route cost) AND (This route number hops $<$ known route hops)).
If this check is true, then the cost/hop info should be updated. If we do Dijkstra* for every vertex, we can collect all the easiest arrays by looking at the hop counts.

There might be optimizations possible so we don't have to run the full Dijkstra* for every vertex. If the graph is undirected, we could use the information we already know. For example, if we already did Dijkstra* with node A as starting point, and easiest A->B turned out to be 2, we can already fill in this hop count of 2 for Dijkstra* with B as starting point and we don't have to actually check any routes from B to A, we already know the shortest one is with hop count 2.

4.6

To solve this problem we first run Dijkstra with the chosen v as the source node. This will give us the shortest path from v to all other vertices.
Then we run Dijkstra for all the other vertices and note the shortest path from every vertex to v.
After this we can add those paths to get the shortest path from any chosen vertex to any other vertex via v.
The complexity of this algorithm is $|V| \cdot |V|^2 = |V|^3$ when using arrays.
This complexity can be improved to $|V| \cdot |E| + |V|lg|V|$ when using a Fibonacci heap.

4.7

We can solve this problem using the following algorithm:
Network of roads G = (V,E)

```
1    For each r in list:
2    {
3        Include r in a copy of G
```

```
4           Run dijkstra from starting point s
5       }
```

Then you are able to compare the results, and check in which version the cost (from s) to t is the lowest. The extra road r in this version is the best to build if the goal is to minimize cost s->t

Note that we assumed that a road is undirected, if the roads are directed then we should run Dijkstra from both s as starting point and t. Then take the average of these results or something to see what extra road gives us the best overall improvement.