Weekly Assignment 6

9th October 2019

Exercise 1. *Weight: 25%*

We want to choose scientific experiments among n experiments $E_1, \dots E_n$ that must be done in outer space. Each experiment needs tools that should be chosen among p tools $I_1, ..., I_p$. Each tool may be used by multiple experiments. By conducting the experiment E_i, we earn p_i euros, but carrying the tool I_j into outer space costs c_j euros.

The goal is to maximize the profits.

Let us consider the following graph: for each experiment E_i, we add a vertex E_i. For each tool I_j, we add a vertex I_j. There is also a source s and a sink t. An edge of capacity p_i goes from s to E_i and an edge of capacity c_j goes from I_j to t. Finally, if the experiment E_i uses the tool I_j , then there is an edge of infinite capacity from E_i to I_j.

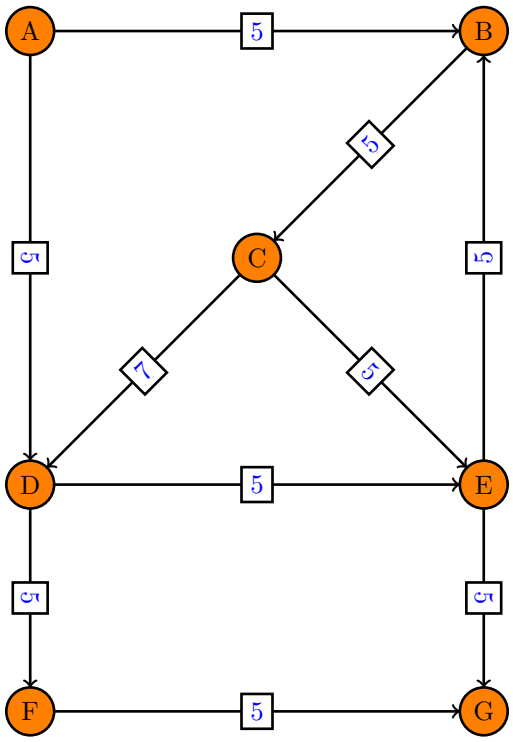
1. What is the graph associated to the following experiments?

	I_1	I_2	I_3	I_4	I_5	I_6
E_1	x	x	x			
E_2	x			x	x	
E_3			x			x
E_4		x		x	x	x

2. How can we find the solution (that is to say, the set of experiments) that maximizes the profits? How can we compute the total profits?

Exercise 2. *Weight: 15%*

Run the Edmonds Karp algorithm (the instance of the Ford-Fulkerson algorithm which selects the shortest augmenting path) on the following flow graph to find out the max-flow, using vertex A as the source and G as the sink.



Exercise 3. *Weight: 25%*

Consider a set of mobile computing *clients* in a certain town who each need to be connected to one of several possible *base stations*. There are n clients, with the position of each client specified by its (x, y) coordinates in the plane. There are also k base stations; the position of each of these is specified by (x, y) coordinates as well.

We wish to connect each client to exactly one of the base stations. Our choice of connection is constrained in the following ways. There is a *range parameter* r — a client can only be connected to a base station that is within distance r. There is also a *load parameter* L — no more than L clients can be connected to any single station.

Design a polynomial-time algorithm for the following problem. Given the positions of a set of clients and a set of base stations, as well as the range and load parameters, decide whether every client can be connected simultaneously to a base station, subject to the conditions from the previous paragraph.

Exercise 4. *Weight: 10%*

Decide whether you think the following state is true or false. If it is true, give a short explanation. If it is false give a counter-example. Let G be an arbitrary flow network, with a source s, a sink t, and a positive integers capacity c_e on every edge, and let (A, B) be a minimum $s-t$ cut with respect to these capacities. Now suppose we add 1 to every capacity, then (A, B) is still a minimum $s - t$ cut with respect to these new capacities.

Exercise 5. *Weight: 25%*

The managers of a popular website have identified k distinct *demographic groups* $G_1, G_2, \ldots, G_k$. These groups may overlap; for example G_1 can be equal to all residents of Gelderland, and G_2 can be equal to all people with a degree in computer science. The site has contracts with m different *advertisers*, to show a certain number of copies of their ads to users of the site. Here's what the contract with the i^{th} advertiser looks like:

- For a subset $X_i \subseteq \{G_1, \ldots, G_n\}$ of the demographic groups, advertiser i wants its ads shown only to users who belong to at least one of the groups in the set X_i.
- Advertiser i wants its ads shown to at least r_i users each minute, for some number r_i.

Now consider the problem of designing a good *advertising policy* — a way to show a single ad to each user of the site. Suppose at a given minute, there are n users visiting the site. Because we have registration information on each of these users, we know that user j belongs to a subset $U_j \subseteq \{G_1, \ldots, G_k\}$ of the demographic groups. Is there a way to show a single ad to each user so that the site's contracts with each of the m advertisers is satisfied for this minute?

Give an efficient algorithm to decide if this is possible, and if so, to actually choose an ad to show to each user.