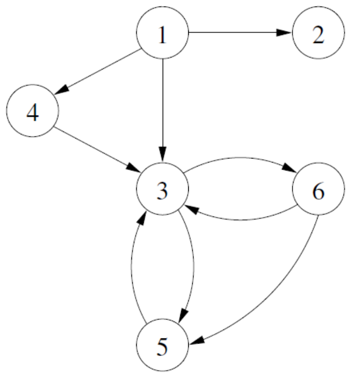
3. Werkcollege 3

3.1. Apply DFS to the graph below. Give values of $d[u]$ and $f[u]$ for all vertices u. Draw the DFS tree. Start with vertex 1, and when choosing an edge, choose the one leading to the vertex with the lowest number.



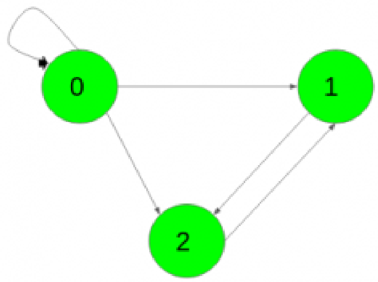
3.2. Suppose a Computer Science curriculum consists of n courses. Each course is mandatory. The prerequisite graph for the curriculum is a directed, acyclic graph (DAG) G that has a node for each course, and an edge from course v to course w if and only if v is a prerequisite for w. In such a case, a student is not allowed to take course w in the same or an earlier semester than she takes course v. A student can take any number of courses in a single semester.

Design an algorithm to find out the minimum number of semesters necessary to complete the curriculum. The running time of your algorithm should be linear. Your algorithm should consider the extreme case also when there are no pre-requisites at all

3.3. Most graph algorithms that take an adjacency-matrix representation as input require time $\mathcal{O}(|V|^2)$, but there are some exceptions. Show how to determine whether a directed graph $G = (V, E)$ contains a *universal sink*, i.e., a vertex with in-degree $n - 1$ and out-degree 0, in time $\mathcal{O}(|V|)$ given an adjacency matrix for G.

3.4. Design an algorithm to find all the mother vertices in a directed graph $G = (V, E)$. A mother vertex is a vertex v such that all other vertices in G can be reached by a path from v.

3.5. Given a directed, unweighted graph with N vertices, represented as an adjacency matrix, and an integer k, find out the number of paths of length k from u to v, for each pair of vertices (u, v).



For the above sample input graph and sample value of $k = 2$, the output should be the following adjacency matrix:

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Number of paths from 0 to 0 of length k is 1: $(0- > 0- > 0)$
- Number of paths from 0 to 1 of length k is 2: $(0- > 0- > 1, 0- > 2- > 1)$
- Number of paths from 0 to 2 of length k is 2: $(0- > 0- > 2, 0- > 1- > 2)$
- Number of paths from 1 to 1 of length k is 1: $(1- > 2- > 1)$
- Number of paths from 2 to 2 of length k is 1: $(2- > 1- > 2)$