1 Programming with expressions and values

Exercise 1.1. Describe what each of the following Haskell functions mean:

```
f1           :: Int
f1           = 1 + 5

f2           :: Int → Int → Int
f2 m n
| m < n      = m
| otherwise = n

f3           :: String → Int → String
f3 s n
| n ≤ 0      = ""
| otherwise = s ++ f3 s (n-1)

f4           :: Int → Int → Int
f4 x 0       = x
f4 x y       = f4 y (x mod y)

f5           :: (Int,Int) → Int
f5 x         = fst x + snd x

f6           :: (a,b) → (b,a)
f6 (a,b)     = (b,a)

f7           :: (a,a) → (a,a)
f7 x         = f6 (f6 x)
```

Exercise 1.2. Determine the result of the expressions below by rewriting. First add parentheses `()` in the expressions illustrating the priorities of the operations correctly. Then apply rewrites as shown in the lecture: place every rewrite step on a new line, and <u>underline</u> the part of the expression that you rewrite (the *redex*, *reducible expression*).

```
e1 = 42
e2 = 1 + 125 * 8 / 10 - 59
e3 = not True || True && False
e4 =  1 + 2  ==  6 - 3
e5 = "1 + 2" == "6 - 3"
e6 = "1111 + 2222" == "1111" ++ " + " ++ "2222"
```

Exercise 1.3 (Warm-up: definitions, `Database.hs`). Consider the following definitions, which introduce a type of persons and some sample data.

```
type Name              =  String
type Age               =  Integer
type FavouriteCourse   =  String
frits, peter, ralf :: Person
```

```
frits   =  ("Frits",  33,  "Algorithms and Data Structures")
peter   =  ("Peter",  57,  "Imperative Programming")
ralf    =  ("Ralf",   33,  "Functional Programming")
students  ::  [Person]
students  =  [frits, peter, ralf]
```

1. Add your own data and/or invent some additional entries. In particular, add yourself to the list of students.

2. The function `age` defined below extracts the age from a person, e.g. `age ralf = 33`. (In case you wonder why some variables have a leading underscore see the hint)

   ```
   age :: Person → Age
   age (_n, a, _c)  =  a
   ```

   Define functions

   ```
   name             ::  Person → Name
   favouriteCourse  ::  Person → FavouriteCourse
   ```

   that extract name and favourite course, respectively.

3. Define a function `showPerson :: Person → String` that returns a string representation of a person. You may find the predefined operator `++` useful, which concatenates two strings e.g. `"hello, " ++ "world" = "hello, world"`. *Hint:* `show` converts a value to a string e.g. `show 4711 = "4711"`.

4. Define a function `twins :: Person → Person → Bool` that checks whether two persons are twins. (For lack of data, we agree that two persons are twins if they are of the same age.)

5. Define a function `increaseAge :: Person → Person` which increases the age of a given person by one e.g.

   ```
    ⋙ increaseAge ralf
   ("Ralf",34,"Functional Programming")
   ```

6. The function `map` takes a function and a list and applies the function to each element of the list e.g.

   ```
    ⋙ map age students
   [33,57,33]
    ⋙ map (\ p → (age p, name p)) students
   [(33,"Frits"),(57,"Peter"),(33,"Ralf")]
   ```

   The function `filter` applied to a predicate and a list returns the list of those elements that satisfy the predicate e.g.

   ```
    ⋙ filter (\ p → age p > 50) students
   [("Peter",57,"Imperative Programming")]
    ⋙ map (\ p → (age p, name p)) (filter (\ p → age p > 50) students)
   [(57,"Peter")]
   ```

Create expressions to solve the following tasks:

- increment the age of all students by two;
- promote all of the students (attach `"dr "` to their name);
- find all students named Frits;
- find all students whose favourite course is Functional Programming;
- find all students who are in their twenties;
- find all students whose favourite course is Functional Programming and who are in their twenties;
- find all students whose favourite course is Imperative Programming or who are in their twenties.

Exercise 1.4 (Pencil and paper: evaluation). 1. Recall the implementation of Insertion Sort from §0.4 (listed below, with some minor modifications).

```
insertionSort :: [Integer] → [Integer]
insertionSort []        =  []
insertionSort (x : xs)  =  insert x (insertionSort xs)

insert :: Integer → [Integer] → [Integer]
insert a []     =  a : []
insert a (b : xs)
  | a ≤ b  =  a : b : xs
  | a > b  =  b : insert a xs
```

The function `insert` takes an element and an ordered list and inserts the element at the appropriate position e.g.

`insert 7 (2 : (9 : []))`
⇒ {definition of `insert` and `7 > 2`}
`2 : (insert 7 (9 : []))`
⇒ {definition of `insert` and `7 ≤ 9`}
`2 : (7 : (9 : []))`

Recall that Haskell has a very simple computational model: an expression is evaluated by repeatedly replacing equals by equals. Evaluate the expression

`insertionSort (7 : (9 : (2 : [])))`

by hand, using the format above. (We have not yet discussed lists in any depth, but I hope you will be able to solve the exercise anyway. The point is that evaluation is a purely mechanical process - this is why a computer is able to perform the task.)

2. The function twice applies its first argument twice to its second argument.

```
twice f x = f (f x)
```

(Like `map` and `filter`, it is an example of a higher-order function as it takes a function as an argument.) Evaluate `twice (+ 1) 0` and `twice twice (* 2) 1` by hand. Use the computer to evaluate

```
twice ("|" ++) ""
twice twice ("|" ++) ""
twice twice twice ("|" ++) ""
twice twice twice twice ("|" ++) ""
twice (twice twice) ("|" ++) ""
twice twice (twice twice) ("|" ++) ""
twice (twice (twice twice)) ("|" ++) ""
```

Is there any rhyme or rhythm? Can you identify any pattern?

Exercise 1.5 (Pencil and paper: λ-expressions).

1. An alternative definition of `twice` builds on λ-expressions.

   ```
   twice' = \ f → \ x → f (f x)
   ```

   Re-evaluate `twice (+ 1) 0` and `twice twice (* 2) 1` using this definition. You need to repeatedly apply the evaluation rule for λ-expressions (historically known as the β-rule).

   ```
   (\ x → body) arg  ⇒  body {x := arg}
   ```

   A function applied to an argument reduces to the body of the function where every occurrence of the formal parameter is replaced by the actual parameter e.g.

   `(\ x → x + x) 47 ⇒ x + x { x := 47} ⇒ 47 + 47 ⇒ 94`.

2. It is perhaps slightly worrying that you can apply a function to itself (as in `twice twice (* 2) 1 = ((twice twice) (* 2)) 1`). Can you guess the type of `twice`?

Exercise 1.6 (Worked example: prefix and infix notation).

1. Haskell features both alphabetic identifiers, written *prefix* e.g. `sin pi`, and symbolic identifiers, written *infix* e.g. `2 + 7`. The use of infix notation for addition is traditional. (The symbol "+" is a simplification of "et", Latin for "and".) Most programming languages (with the notable exception of LISP) have adopted infix notation. But is this actually a wise thing to do? What are the advantages and disadvantages of infix over prefix (or postfix) notation. Discuss!

2. Infix notation is inherently ambiguous: `x ⊗ y ⊗ z`. What does this mean: `(x ⊗ y) ⊗ z` or `x ⊗ (y ⊗ z)`? To disambiguate without parentheses, operators may *associate* to the left or to the right. Subtraction associates to the left: `5 - 4 - 2 = (5 - 4) - 2`. Why? Concatenation of strings associates to the right: `"F" ++ "P" ++ "1" = "F" ++ ("P" ++ "1")`. Why? Haskell allows the programmer to specify the *association* of an operator using a *fixity declaration*:

   ```
   infixl -
   infixr ++
   ```

   Function application can be seen as an operator ("the space operator") and associates to the left: `f a b` means `(f a) b`. (On the other hand, the "function type" operator associates to the right: `Integer → Integer → Integer` means `Integer → (Integer → Integer)`.) Haskell also features an explicit operator for function application, which associates to the right: `f $ g $ a` means `f $ (g $ a) = f (g a)`. Can you foresee possible use-cases?

3. The operator

```
infixl **
a ** b  =  2 * a + b
```

can be used to capture binary numbers e.g. `1 ** 0 ** 1 ** 1` ⟹ `11` and `(1 ** 1 ** 0) + 4711` ⟹ `4717`. The fixity declaration determines that `**` associates to the left. Why this choice? What happens if we declare `infixr **`?

4. Association does not help when operators are mixed: `x ## y ** z`. What does this mean: `(x ## y) ** z` or `x ## (y ** z)`? To disambiguate without parentheses, there is a notion of *precedence* (or binding power), e.g. `*` has higher precedence (binds more tightly) than `+`.

```
infixl 7 *
infixl 6 +
```

The precedence level ranges between `0` and `9`. Function application ("the space operator") has the highest precedence (ie `10`), so `square 3 + 4 = (square 3) + 4`. Find out about the precedence levels of the various operators and *fully* parenthesize the expression below.

```
f x ≥ 0 && a || g x y * 7 + 10 == b - 5
```

Exercise 1.7 (Programming). Define the string

```
thisOldMan :: String
```

that produces the following poem (if you type `putStr thisOldMan`).

This old man, he played one,
He played knick-knack on my thumb;
With a knick-knack paddywhack,
Give the dog a bone,
This old man came rolling home.

This old man, he played two,
He played knick-knack on my shoe;
With a knick-knack paddywhack,
Give the dog a bone,
This old man came rolling home.

This old man, he played three,
He played knick-knack on my knee;
With a knick-knack paddywhack,
Give the dog a bone,
This old man came rolling home.

This old man, he played four,

He played knick-knack on my door;
With a knick-knack paddywhack,
Give the dog a bone,
This old man came rolling home.

This old man, he played five,
He played knick-knack on my hive;
With a knick-knack paddywhack,
Give the dog a bone,
This old man came rolling home.

This old man, he played six,
He played knick-knack on my sticks;
With a knick-knack paddywhack,
Give the dog a bone,
This old man came rolling home.

This old man, he played seven,
He played knick-knack up in heaven;
With a knick-knack paddywhack,
Give the dog a bone,
This old man came rolling home.

This old man, he played eight,
He played knick-knack on my gate;
With a knick-knack paddywhack,
Give the dog a bone,
This old man came rolling home.

This old man, he played nine,
He played knick-knack on my spine;
With a knick-knack paddywhack,
Give the dog a bone,
This old man came rolling home.

This old man, he played ten,
He played knick-knack once again;
With a knick-knack paddywhack,
Give the dog a bone,
This old man came rolling home.

Try to make the program as short as possible by capturing recurring patterns. Define a suitable function for each of those patterns.

Exercise 1.8 (Programming, `Shapes.hs`). The datatype Shape defined below captures simple geometric shapes: circles, squares, and rectangles.

```
data Shape
  =  Circle Double              -- radius
```

```
    |  Square Double              -- length
    |  Rectangle Double Double   -- length and width
    deriving (Show)
```

Examples of concrete shapes include `Circle (1/3)`, `Circle 2.1`, `Square pi`, and `Rectangle 2.0 4.0`.

The function `showShape` illustrates how to define a function that consumes a shape. A shape is one of three things. Correspondingly, `showShape` consists of three equations, one for each kind of shape.

```
showShape :: Shape → String
showShape (Circle r)        =  "circle of radius " ++ show r
showShape (Square l)        =  "square of length " ++ show l
showShape (Rectangle l w)   =  "rectangle of length " ++ show l
                                  ++ " and width " ++ show w
```

Use the same definitional scheme to implement the functions

```
area         :: Shape → Double
perimeter    :: Shape → Double
center       :: Shape → (Double, Double)  -- |x|- and |y|-coordinates
boundingBox  :: Shape → (Double, Double)  -- width and height
```

(The names are hopefully self-explanatory.)

Exercise 1.9. The *greatest common divisor* function is defined in roughly the following way:

```
gcd     :: Int → Int → Int
gcd x y =  gcd' (abs x) (abs y)
            where gcd' a 0  =  a
                  gcd' a b  =  gcd' b (a 'rem' b)
```

Rewrite the following expressions using this definition:

```
gcd -42 0
gcd 0 -42
gcd 18 42
gcd 123456789 987654321
```

Exercise 1.10 (Strings, `Strings.hs`). The standard environment of Haskell provides various functions for working with texts of type `String`. In this exercise you are only allowed to use the following functions as `String` operations:

- `length`: returns the number of characters of the given `String`.
  Example: `length ""` returns 0.
  Example: `length "0123456789"` returns 10.

- `!!`: returns the `Char` value on the given `Int` index in a string. The indices of a non-empty string *s* go from 0 up to and including `(size s)-1`. The empty string `""` does not have any valid indices.
  Example: `"0123456789" !! 4` returns `'4'`.
  Example: `"0123456789" !! -1` returns *** *Exception: Prelude.!!: negative index.*
  Example: `"0123456789" !! 10` returns *** *Exception: Prelude.!!: index too large.*

Using only the above functions, write the following ones:

1. Write your own implementation of `head :: String → Char` and `tail :: String → String` that given a non-empty `String` return the first and the remaining elements of the `String` respectively. If the argument is the empty `String`, it should show an error.
   Example: `head "Madam, I'm Adam" = 'M'`.
   Example: `tail "Madam, I'm Adam" = "adam, I'm Adam"`.

2. Write a function `is_equal` that determines the equality of two `String` arguments. This means that for every pair s_1, s_2 of type `String` the following should hold: `is_equal` s_1 s_2 = s_1 == s_2.

3. Write a function `is_substring` that returns a `Bool` when given two `String` arguments. The result should be `True` if and only if the first argument is a *substring* of the second argument.
   Example: `is_substring "there" "Is there anybody out there?"`[1] returns `True`, after all: `"Is there anybody out there?"`.
   Example: `is_substring "there" "Just for the record"`[2] returns `False` because there is a space between `the` and `re`.

4. Write a function `is_sub` that returns a `Bool` when given two `String` arguments. The results should be true `True` if and only if the characters of the first argument appear in the same order in the second argument. Characters of the second argument may be skipped.
   Example: `is_sub "there" "Just for the record"` returns `True` because the space will be skipped.
   Example: `is_sub "she and her" "Is there anybody in there?"` returns `True`, after all: `"Is there anybody in there?"`.
   Example: `is_sub "There there"` [3] `"Is there anybody in there?"` returns `False` because `T` is no part of the second `String`.

5. Write a recursive function `is_match` that returns a `Bool` when given two `String` arguments (*pattern* and *source*). The function determines whether the *pattern* can be applied on the source (the pattern *matches* the source). The *pattern* may contain *wildcard* characters:

   - `.`: this matches the next, arbitrary single character in the *source*.
   - `*`: this matches zero or more consecutive characters in the *source*.

   All other characters in the *pattern* have to match exactly with the corresponding characters in the *source*.
   Example: `is_match "here" "here"` returns `True`.
   Example: `is_match "here" " here"` returns `False` because the first characters of the two strings do not match.
   Example: `is_match "here" "here "` returns `False` because the second text has one extra character.
   Example: `is_match ".." "AB"` returns `True` because the second text consists of exactly

[1] Pink Floyd – The Wall (1979)
[2] Marillion – Clutching at straws (1987)
[3] Radiohead – Hail to the thief (2003)

two characters.
Example: `is_match "*?" "Is there anybody in there?"` returns `True` because the second text ends with a `'?'` character.
Example: `is_match "*?*!*?*!" "Answers? Questions! Questions? Answers!"` [4] returns `True`.
Example: `is_match "*.here*.here*." "Is there anybody in there?"` returns `True`.
Example: `is_match ".here.here." "Is there anybody in there?"` returns `False`.

Exercise 1.11 (Determining prime numbers). Write the *recursive* function

```
isPrime :: Int → Bool
```

that determines whether the argument is a prime number. A prime number is a positive integer divisible by exactly two *different* positive integers; namely itself and 1 (hence the reason why 1 is not a prime itself). Test the function with `[x | x ← [1 .. 1000], isPrime x]`. This should return all primes in the range of one up to and including a thousand. The result should be:

```
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73,
 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163,
 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251,
 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349,
 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443,
 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557,
 563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617, 619, 631, 641, 643, 647,
 653, 659, 661, 673, 677, 683, 691, 701, 709, 719, 727, 733, 739, 743, 751, 757,
 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839, 853, 857, 859, 863,
 877, 881, 883, 887, 907, 911, 919, 929, 937, 941, 947, 953, 967, 971, 977, 983,
 991, 997]
```

Note: Use the function `rem` for integer division with remainder. The function `mod` is practically an alias of this function.

Exercise 1.12 (Prime factors). Every integer $x > 1$ can be decomposed in prime factors in a canonical way. This means that you can find a range of prime numbers $p_0 \dots p_k$ where $k \geq 0$ for which $p_i < p_j$ for all $i < j$ and as many positive numbers $n_0 \dots n_k$ in such a way that:

$$x = p_0^{n_0} \cdot \cdots \cdot p_k^{n_k} = \prod_{i=0}^{k} p_i^{n_i}$$

Examples:			
	36	=	$2^2 \cdot 3^2$
	52	=	$2^2 \cdot 13$
	133	=	$7 \cdot 19$
	123456789	=	$3^2 \cdot 3607 \cdot 3803$

Write the function `primeFactors :: Int → String` that decomposes an integer into its prime factors. The prime factors found should be glued together in the resulting `String`.

[4] Focus – Focus III (1973)

Examples:	primeFactors 52	=	"2*2*13"
	primeFactors 36	=	"2*2*3*3"
	primeFactors 133	=	"7*19"
	primeFactors 123456789	=	"3*3*3607*3803"

Use the function isPrime from one of the previous exercises.

The following operations on Strings and Ints are useful: s_1 ++ s_2 glues String s_2 to String s_1; toString n creates a String value from an Int.

Exercise 1.13 (Adding numbers). Write the function numberSum :: Int → Int that sums the digits of a positive integer.
Example: numberSum 9876543 = 9+8+7+6+5+4+3 = 42.
Example: numberSum 1000 = 1+0+0+0 = 1.

Exercise 1.14 (O Tannenbaum). Write the function triangle that receives an Int argument n. This function will *draw* a triangle as shown on the right with $n = 5$. The *drawing* is actually returning a String of which every *line* ends with a *newline*.

Hence, the string corresponding with the image to the right is:

```
"    *\n   ***\n  *****\n *******\n*********\n"
```

The output will be:

```
    *
   ***
  *****
 *******
*********
```

Write the function christmasTree that receives an Int argument n. This function *draws* a Christmas tree as a String (as shown on the right with $n = 4$). For every sub triangle the function should use a generalized version of the above defined triangle function.

```
   *
   *
  ***
   *
  ***
 *****
   *
  ***
 *****
*******
```

The following operations on Strings and Chars are useful: s_1 ++ s_2 glues String s_2 to String s_1; toString c creates a String value from a Char c. '\n' is the *newline* character.

Exercise 1.15 (Trajectory). An object, such as a ball, which is shot away from a flat surface with an angle of $\theta_0 (0 < \theta_0 \leq \frac{\pi}{2})$ [5] and initial velocity $v_0 (0 < v_0)$, follows an arclike trajectory influenced by gravitational acceleration [6]. If we ignore effects such as air resistance and wind, the trajectory of the ball is a curve defined in a vertical flat surface in which the `x`-axis shows the distance to the starting position on the ground and the `x`-axis shows the height of the ball.

Because we ignore air resistance, the velocity of the ball in the `x`-direction ($v_x(t)$) is constant. The velocity of the ball in the `x`-direction ($v_y(t)$) depends on the time t and the gravitational acceleration $g = 9.81\frac{m}{s^2}$:

$$\begin{aligned} v_x(t) &= v_0 \cdot \cos(\theta_0) \\ v_y(t) &= v_0 \cdot \sin(\theta_0) - g \cdot t \end{aligned}$$

You can calculate the distance $x(t)$ and the height $y(t)$ of the ball on time t as follows:

$$\begin{aligned} x(t) &= v_0 \cdot \cos(\theta_0) \cdot t \\ y(t) &= v_0 \cdot \sin(\theta_0) \cdot t - \frac{1}{2} \cdot g \cdot t^2 \end{aligned}$$

Height h, expressed in distance `x` is:

$$h(x) = \tan(\theta_0) \cdot x - \frac{g}{2 \cdot (v_0 \cdot \cos(\theta_0))} \cdot x^2$$

The ball reaches the highest point at time $t_{\max y}$:

$$t_{\max y} = \frac{v_0 \cdot \sin(\theta_0)}{g}$$

Thus you can find the maximal height reached at that time by plugging t_{maxy} into $y(t)$. The ball takes just as much time to reach the highest point as it takes to get back on the ground. Therefore the ball is airborne for $t_2 = 2 \cdot t_{\max y}$ seconds. You can find the distance the ball travels by plugging t_2 into $x(t)$. The velocity of the ball when touching the ground can be found by plugging t_2 into $v_x(t)$ and $v_y(t)$. Then, the combined value is:

$$v_1 = \sqrt{v_x(t_2)^2 + v_y(t_2)^2}$$

Functions Implement the aforementioned functions v_x, v_y, `x`, `x` and h in Haskell with the respective names `v_x`, `v_y`, `x_at`, `y_at` and `h`. The equations given above are parameterized with the initial velocity v_0 and angle θ_0 and need to be passed as parameters to the Haskell functions.

Best angle Write a *recursive* function `best_angle` that, given an initial velocity v_0, calculates the angle $\theta \in \{\frac{1}{100}\pi, \frac{2}{100}\pi \ldots \frac{50}{100}\pi\}$ in such a way that when the ball is shot away with velocity v_0 and angle θ it travels the *greatest* distance.

Experiment with different values for v_0. Does this result in the same return value?

[5] Angle in radians.

[6] Source: Halliday, Resnick. *Physics, Parts I and II, combined edition*, Wiley International Edition, 1966, ISBN 0 471 34524 5

Exercise 1.16 (Stringnum, `Stringnum.hs`). In the module `Stringnum` you develop functions to work with `String`s representing arbitrarily large non-negative integers. The functions are:

- `isAStringNum` tests whether the given `String` represents a correct positive integer (including zero).

- The functions `smaller`, `bigger` and `equal` implement the operations `<`, `>` and `==`.

- The functions `plus`, `decrement`, `times` and `divide` implement the operations `+`, `-`, `*` and `/`, where `decrement` a b = `"0"` if `bigger` b a.

The following functions on `String`s and `Char`s are useful (they are in module Data.Char): `isDigit` c tests whether a `Char` c is a digit; `digitToInt` c returns the ASCII-code of `Char` c; `size` s returns the length of the `String` s; `++` glues two `String`s together; s `!!` i returns the character from s at index i;

Exercise 1.17 (Stringnum2, `Stringnum.hs`). If you can calculate with arbitrarily large non-negative integers, then, using these operations, you can also construct the implementations for all arbitrarily large integers (negative, positive, and 0). In this implementation, use the exported functions from `Stringnum` to create a new module `Stringnum2` implementing the same operations for all integers. Add the following operations besides the existing ones:

- `isNegative` tests whether the given number is negative (< 0).

- `absolute` calculates the absolute value of the argument.

- `changeSign` flips the sign of the number.

Exercise 1.18 (Time, `Time.hs`). The length of movies and music is often displayed in the $m^+ : ss$ format where m^+ is the number of minutes and ss the number of seconds. For example, the length of the movie *"The Lord of the Rings: The Fellowship of the Ring"* is 178:00, i.e. two hours and 58 minutes. The length of the song *"Tea"* of Sam Brown on the album *"Stop!"* is 0:41, i.e. 41 seconds. Moreover, time in between songs (silence) is often indicated via negative time. For instance, three seconds of silence is -0:03. Implement the following two functions:

- `showTime` n: with n the number of seconds (positive or negative), this function shows the time as explained in the above format;

- `parseTime x`: with `x` a string, this function attempts to extract the format explained above and return the corresponding (positive or negative) number of seconds. If `x` is ill formatted, then the result should be zero.

Hints to practitioners 1. Functional programming folklore has it that a functional program is correct once it has passed the type-checker. Sadly, this is not quite true. Anyway, the general message is to exploit the compiler for *static* debugging: compile often, compile soon. (To trigger a re-compilation after an edit, simply type `:reload` or `:r` in GHCi.)

We can also instruct the compiler to perform additional sanity checks by passing the option `-Wall` to GHCi e.g. call `ghci -Wall` (turn all warnings on). The compiler then checks, for example, whether the variables introduced on the left-hand side of an equation are actually used on the right-hand side. Thus, the definition `k x y = x` will provoke the warning “Defined but not used: `y`”. Variables with a leading underscore are not reported, so changing the definition to `k x _y = x` suppresses the warning.