ASSIGNMENT 3
P452- Computational Physics

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Question 3

Given in the Question

Letter Grades: [A, B, C, D, E]
Observed distribution of students: [77, 150, 210, 125, 38]
Degrees of freedom: n-1 = 4

Calculation for Expected frequencies

Starting with the standard normal distribution function,

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2}\right)$$

Here, μ is the mean grade.
We get the expected frequencies as [32.395, 145.182, 239.365, 145.182, 32.394]

χ^2-statistic

The formula for chi-squared statistics:

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Here, O is the observed frequency and E is the expected frequency.

Putting the values for the variables:

$$\begin{aligned}\chi^2 =& \frac{(77-32.395)^2}{32.395} + \frac{(150-145.182)^2}{145.182} + \frac{(210-239.365)^2}{239.365} \\ &+ \frac{(125-145.182)^2}{145.182} + \frac{(38-32.394)^2}{32.394} \\ \chi^2 &= 68.957\end{aligned}$$

Critical Values: The critical values obtained using a χ^2 distribution table with $df = 4$:

- $\chi^2_{0.05} = 9.488$ [Critical value (5% significance level)]
- $\chi^2_{0.10} = 7.779$ [Critical value (10% significance level)]

As the χ^2 statistic is lower for both the 5% and 10% significance levels, we reject both null hypotheses.

Question 4

Given in the Question

Shipment A: [4.65, 4.84, 4.59, 4.75, 4.63, 4.75, 4.58, 4.82, 4.86, 4.60, 4.77, 4.65, 4.80]
Shipment B: [4.75, 4.79, 4.74, 4.74, 4.77, 4.58, 4.81, 4.80]

The sample mean can be calculated as:

$$\bar{x}_1 = \frac{\sum_{i=1}^{n_1} x_{1i}}{n_1} = 4.7146$$
$$\bar{x}_2 = \frac{\sum_{i=1}^{n_2} x_{2i}}{n_2} = 4.74$$

The sample variance can be calculated as:

$$s_1^2 = \frac{\sum_{i=1}^{12} (x_{1i} - \bar{x}_1)^2}{12 - 1} = 0.0973$$
$$s_2^2 = \frac{\sum_{i=1}^{8} (x_{2i} - \bar{x}_2)^2}{8 - 1} = 0.0697$$

F-test

The F-statistic is calculated as $F = \frac{s_1^2}{s_2^2} = 1.9499$

Critical Value:

- For significance level $\alpha = 0.05$ and degrees of freedom $df_1 = 12$ and $df_2 = 7$, the critical value of F-statistic = 3.9999
- For significance level $\alpha = 0.10$ and degrees of freedom $df_1 = 12$ and $df_2 = 7$, the critical value of F-statistic = 2.9047

The calculated f value is lower than the critical value at both the 5% and 10% significance levels. Therefore, we fail to reject the null hypothesis, indicating that the variances are similar and not significantly different.

T-test

Given two sets of data, denoted as x_1 and x_2, representing shipments A and B respectively, our goal is to determine whether the means of these two populations exhibit a statistically significant difference.

We calculate the pooled standard deviation to estimate the common standard deviation of the two populations.

Pooled Standard Deviation:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = 0.089$$

t-Statistic: Calculate the t-statistic, which quantifies the difference between the sample means using the pooled standard deviation:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p} = -0.285$$

Critical Value:

- For significance level $\alpha = 0.05$ and degrees of freedom $df_1 = 12$ and $df_2 = 7$, the critical value of F-statistic $= 1.734$
- For significance level $\alpha = 0.10$ and degrees of freedom $df_1 = 12$ and $df_2 = 7$, the critical value of F-statistic $= 1.3304$

The obtained t value is less than the critical value for both 5% and 10% so, we fail to reject the null hypothesis and thus the means are close and not very different.
Hence, the lenses are from the same population in the two sets.