Radiative Transfer in Exoplanetary Atmospheres

Computational Physics Term Project Report

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Abstract

In this computational physics project, the main objective is to analyze the behavior of light in planetary atmospheres by solving the radiative transfer equation with a focus on single scattering. The project involves several key steps, including the analytical solution of the radiative transfer equation to obtain the intensity (I) of light at different levels of the atmosphere, specifically at the Top of Atmosphere (TOA) and Bottom of Atmosphere (BOA). The credibility of the analytical approach for single scattering is validated by its exact match with numerical methods, providing confidence in the results obtained for more complex multiple scattering scenarios. A significant aspect of the project involves the inclusion of a phase function expansion using Legendre polynomials. This expansion allows for a more detailed examination of the scattering behavior of light in the atmosphere by plotting contours of intensity (I) for variations across azimuth angle (phi) and cosine of the zenith angle (mu). This visualization provides insights into how light is scattered and absorbed as it travels through the atmosphere. Furthermore, the project includes a comparative analysis between two approaches for modeling the phase function of single scattering events. The first approach utilizes the Legendre expansion, while the second approach employs the Henyey-Greenstein (HG) phase function. By comparing the contours of intensity (I) obtained from these two methods at TOA and BOA, the project aims to assess the similarities and differences in their predictions regarding the scattering behavior of light.

Contents

1 Introduction

Light passing through the atmosphere interacts with various particles and molecules, causing it to be absorbed, emitted, or scattered, which changes its direction. The Radiative Transfer Equation (RTE) quantifies how the intensity of radiation, denoted as I, changes with respect to its position and direction within a medium. It is measured in power(W) per unit area(m^2), per unit solid angle(sr), and per unit wavelength band(nm or μm). In a plane-parallel atmosphere, the location is defined by the vertical coordinate z (height) only. Radiation passing through a medium loses intensity due to absorption and scattering. However, emission from the material and multiple scattering from other directions can enhance its intensity. We use a dimensionless optical depth τ to describe these processes from the top of the atmosphere (TOA) to a specific point along the vertical coordinate z. This optical depth combines the medium's absorption and scattering properties with the distance the light travels. The direction is specified by zenith θ and azimuth φ angles (Fig.1: left). The optical length of a segment AB (Fig.1: right) in the atmosphere is τ/μ, where $\mu = -\cos(\theta)$, with the minus sign convention due to the opposite directions of the z and τ axes. Solving the RTE involves calculating $I(\tau, \mu, \varphi)$ at each point in the 3D coordinate system using the atmosphere's optical properties and two boundary conditions: the incoming light intensity at the top of the atmosphere (TOA) and the reflection at the bottom of the atmosphere (BOA).

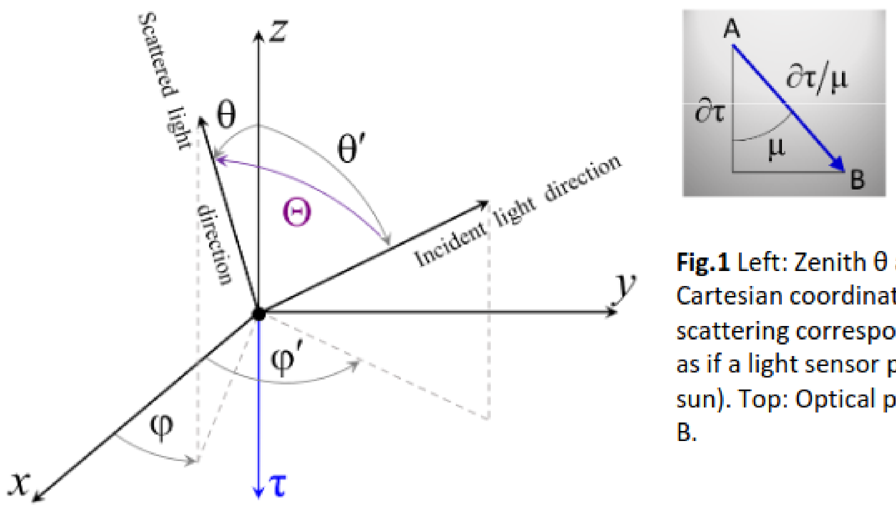

Fig.1 Left: Zenith θ and azimuth φ angles in Cartesian coordinate system. Forward scattering corresponds to zero azimuth (e.g., as if a light sensor pointing in the plane of the sun). Top: Optical path between points A and B.

Figure 1: The three base coordinates (Korkin et al. 2022)

In the scalar RTE, the atmosphere's optical properties are defined by three parameters: optical thickness τ, single scattering albedo ω_0, and phase function $p(\Theta)$. The single scattering albedo represents the ratio of scattering to total extinction (absorption + scattering). It indicates the likelihood of light surviving a scattering event: $\omega_0 = 0$ implies all interactions result in absorption, while $\omega_0 = 1$ implies all interactions result in light scattering. The phase function $p(\Theta)$ is a scattering probability distribution function dependent on the scattering angle Θ. The scattering angle is the angle between the incident and scattered light directions.

The equation describes how light intensity changes ($dI/d\tau$) as it travels in the atmosphere in a specific direction τ. Absorption and scattering reduce the intensity (first term on the right side), while scattering from all directions into the propagation direction increases it (second and third terms on the right side). This results in the RTE in integral (over θ, φ) and differential (over τ) form:

$$\mu \frac{\partial I(\tau, \mu, \varphi)}{\partial \tau} = -I(\tau, \mu, \varphi) + \frac{\omega_0}{4\pi} p\left(\mu, \mu_0, \varphi, \varphi_0\right) I_0 \exp\left(-\frac{\tau}{\mu_0}\right) + J(\tau, \mu, \varphi) \tag{1}$$

The second term in Eq.(1) accounts for the initial scattering of the direct solar beam. I_0 represents the spectral irradiance at the top of the atmosphere (TOA) in (W/m^2/nm). The solar geometry is determined by the cosine of the solar zenith angle, $\mu_0 = \cos(\theta_0)$, and the azimuth angle φ_0. Typically, the azimuth angle is assumed to be zero, $\varphi_0 = 0$, in the solar (principal) plane. The scattering integral J representing multiple scattering is given by,

$$J(\tau, \mu, \varphi) = \frac{\omega_0}{4\pi} \int_0^{2\pi} \int_{-1}^{1} p\left(\mu, \mu', \varphi, \varphi'\right) I\left(\tau, \mu', \varphi'\right) d\mu' d\varphi' \tag{2}$$

2 The Radiative Transfer Equation with Single Scattering

In a domain where the optical depth is small (e.g., $\tau \leq 0.1$), a large portion of scattering events is dominated by single scattering of the direct solar beam. This occurs in optically thin cirrus and aerosol atmospheres. This approximation is analyzed separately due to its analytical simplicity and its significance in understanding the physics of scattering, as well as for speeding up numerical simulations.

The single scattering approximation is a particular but very important solution to the RTE. Setting $J(\tau, \mu, \varphi) = 0$ in Eq. (1) gives the RTE for the single scattering approximation.

$$\mu \frac{dI(\tau, \mu, \varphi)}{d\tau} = I(\tau, \mu, \varphi) - \frac{\omega_0}{4\pi} I_0\, p\left(\mu, \mu_0, \varphi, \varphi_0\right) e^{-\tau/\mu_0} \tag{3}$$

2.1 Boundary conditions

As mentioned earlier, the RTE solution requires two boundary conditions. At the top of the atmosphere (TOA), the initial condition applies to downward radiation, where the boundary condition is zero due to the absence of scattered light from space. The bottom of the atmosphere (BOA) is more intricate, involving both surface reflectivity and the RTE solution at the surface.

Upper Boundary:	**Lower Boundary:**
$\tau = 0$ (TOA)	$\tau = \tau_*$ (BOA)
$\mu = \mu_+ > 0$ (forward hemisphere)	$\mu = \mu_- < 0$ (backward hemisphere)

Two factors influence radiation reflected at the bottom: direct reflection of the solar beam and diffuse reflection from the atmosphere (including multiple reflections between the atmosphere and surface). For Lambertian surface reflection with surface albedo ρ, the bottom boundary condition is as follows:

$$I\left(\tau_0, \mu_-, \varphi\right) = \frac{\rho}{\pi} \mu_0 I_0 \exp\left(-\frac{\tau_0}{\mu_0}\right) + \frac{\rho}{\pi} \int_0^{2\pi} \int_0^1 \mu I\left(\tau_0, \mu, \varphi\right) d\mu d\varphi \tag{4}$$

Comparing Eq.(3) with Eqs.(1) and (2), we see that the reflection of the direct solar beam is akin to single scattering in the atmosphere, while multiple bouncing (diffused light) is similar to multiple scattering. Integrating the total intensity over all directions gives irradiance (W/m^2/nm), and scaling by inverse π (1/sr) gives intensity. The factor μ (μ_0 for the direct solar beam) in the integral adjusts for the lower irradiance of oblique rays.

2.2 Solving the RTE analytically

To solve this equation, the equation is multiplied by $\frac{e^{-\tau/\mu}}{\mu}$ to give

$$\frac{dI(\tau,\mu,\varphi)}{d\tau}e^{-\tau/\mu} = I(\tau,\mu,\varphi)\frac{e^{-\tau/\mu}}{\mu} - \frac{\omega_0}{4\pi}I_0\, p\left(\mu,\mu_0,\varphi,\varphi_0\right)e^{-\tau/\mu_0}\frac{e^{-\tau/\mu}}{\mu} \tag{5}$$

which can be rearranged to

$$\frac{dI(\tau,\mu,\varphi)}{d\tau}e^{-\tau/\mu} - I(\tau,\mu,\varphi)\frac{e^{-\tau/\mu}}{\mu} = -e^{-\tau/\mu}\omega_0 I_0 e^{-\tau/\mu_0}\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu} \tag{6}$$

which can be expressed as

$$\frac{d}{d\tau}\left[I(\tau,\mu,\varphi)e^{-\tau/\mu}\right] = -e^{-\tau/\mu}\omega_0 I_0 e^{-\tau/\mu_0}\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu}$$

Integrating from the top of the atmosphere, where the optical depth τ is 0, to the surface, where the optical depth is τ^*,

$$\int_0^{\tau^*}\frac{d}{d\tau}\left[I(\tau,\mu,\varphi)e^{-\tau/\mu}\right]d\tau = -\int_0^{\tau^*} e^{-\tau/\mu}\omega_0 I_0 e^{-\tau/\mu_0}\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu}d\tau \tag{7}$$

provides the single-scattered radiance field at the top of the atmosphere, i.e.,

$$\begin{aligned} I(\tau^*,\mu,\varphi)e^{-\tau^*/\mu} - I(0,\mu,\varphi) &= \omega_0 I_0\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu}\left[\frac{1}{1/\mu+1/\mu_0}e^{-\tau(1/\mu+1/\mu_0)}\right]_0^{\tau^*} \\ &= \omega_0 I_0\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi}\frac{\mu_0}{\mu+\mu_0}\left[e^{-\tau^*(1/\mu+1/\mu_0)}-1\right] \end{aligned} \tag{8}$$

If there are no radiation sources at the bottom of the atmosphere in the direction of μ, then $I(\tau^*,\mu,\varphi)$ is zero. Consequently, the upward intensity at the Top of Atmosphere (TOA) for directions where $\mu > 0$ would be

$$I(0,\mu,\varphi) = \omega_0 I_0\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi}\frac{\mu_0}{\mu_0+\mu}\left[1-e^{-\tau^*(1/\mu+1/\mu_0)}\right]$$

Now, considering a beam propagating downwards, the radiative transfer equation can be formulated as

$$-\mu\frac{dI(\tau,\mu,\varphi)}{d\tau} = I(\tau,\mu,\varphi) - \omega_0 I_0 e^{-\tau/\mu_0}\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi}$$

The equation, after multiplying through by $e^{\tau/\mu}/\mu$, can be represented as

$$\frac{d}{d\tau}\left[I(\tau,\mu,\varphi)e^{\tau/\mu}\right] = e^{\tau/\mu}\omega_0 I_0 e^{-\tau/\mu_0}\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu}$$

Integrating from 0 to τ^* yields the single-scattered radiance field at the bottom of the atmosphere, i.e.,

$$\begin{aligned} I(\tau^*,\mu,\varphi)e^{\tau^*/\mu} - I(0,\mu,\varphi) &= \int_0^{\tau^*}\omega_0 I_0\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu}e^{\tau(1/\mu-1/\mu_0)}d\tau \\ &= \omega_0 I_0\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu}\left[\frac{1}{1/\mu-1/\mu_0}e^{\tau(1/\mu-1/\mu_0)}\right]_0^{\tau^*} \\ &= \omega_0 I_0\frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi}\frac{\mu_0}{\mu_0-\mu}\left[e^{\tau^*(1/\mu-1/\mu_0)}-1\right] \end{aligned}$$

If there is no source of radiation at the top of the atmosphere in the direction of μ, then $I(0,\mu,\varphi) = 0$. Consequently, the radiation at the Bottom of Atmosphere (BOA) in the direction of μ when $\mu > 0$ is

$$\begin{aligned} I(\tau^*,\mu,\varphi) &= \omega_0 I_0 \frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi} \frac{\mu_0}{\mu_0-\mu}\left[e^{\tau^*(1/\mu-1/\mu_0)}-1\right]e^{-\tau^*/\mu} \\ &= \omega_0 I_0 \frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi} \frac{\mu_0}{\mu_0-\mu}\left[e^{-\tau^*/\mu_0}-e^{-\tau^*/\mu}\right] \end{aligned}$$

When $\mu = \mu_0$, the equation simplifies to

$$\frac{d}{d\tau}\left[I(\tau,\mu,\varphi)e^{\tau/\mu}\right] == \omega_0 I_0 \frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu} \quad [\mu=\mu_0]$$

This can be integrated to obtain

$$I(\tau^*,\mu,\varphi)e^{\tau^*/\mu_0} - I(0,\mu,\varphi) = \omega_0 I_0 \frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu}\tau^*. \quad [\mu=\mu_0]$$

The downward scattered field can be expressed as

$$I(\tau^*,\mu,\varphi) = \omega_0 I_0 \frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu_0}\tau^* e^{-\tau^*/\mu_0}. \quad [\mu=\mu_0]$$

The final solution takes the form

$$I(\tau,\mu,\varphi) = I(\tau,\Theta) = \frac{\omega_0 I_0}{4\pi} p(\Theta) \begin{cases} \frac{\mu_0}{\mu_0-\mu}\left[\exp\left(-\frac{\tau}{\mu_0}\right)-\exp\left(\frac{\tau_0-\tau}{\mu}-\frac{\tau_0}{\mu_0}\right)\right], & \mu<0 \\ \frac{\mu_0}{\mu_0-\mu}\left[\exp\left(-\frac{\tau}{\mu_0}\right)-\exp\left(-\frac{\tau}{\mu}\right)\right], & \mu>0,\mu\neq\mu_0 \\ \frac{\tau}{\mu_0}\exp\left(-\frac{\tau}{\mu_0}\right), & \mu=\mu_0. \end{cases}$$

The overall radiance in this direction will include the unscattered solar radiance as well.

In cases of very thin atmospheres, the exponential terms in the equations can be approximated to first order in τ^*, resulting in the reflected and transmitted diffuse fields being represented by

$$\left.\begin{array}{c} I(0,\mu,\varphi) \\ I(\tau^*,\mu,\varphi) \end{array}\right\} = \omega_0 I_0 \frac{p\left(\mu,\mu_0,\varphi,\varphi_0\right)}{4\pi\mu}\tau^* \quad [\tau^* \ll 1]$$

This observation is quite helpful for understanding how diffuse radiation is distributed in thin atmospheres. The intensity of diffuse radiation is influenced by the optical thickness, adjusted by ω_0/μ. Additionally, the character of the diffuse field is shaped by the phase function.

3 Phase Function

The phase function defines how incident intensity redirects to outgoing intensity, representing the 3D scattering probability. In many applications, such as heating/cooling, photodissociation, and biological dose rates, it's often fine to ignore polarization effects. The error from this simplification is usually much smaller than errors from uncertainties in input parameters, which affect the medium's optical properties. While we focus on the scattering phase function, note that in certain remote sensing scenarios, polarization information may be crucial. For analyzing planetary atmospheres with multiple scattering and radiative transfer, we use a normalized parameter as the phase function, described in terms of the scattering angle Θ:

$$\frac{1}{4\pi}\int_{\Omega} P(\cos\Theta)\,d\Omega = 1 \tag{9}$$

$$\int_0^{2\pi}\int_0^{\pi}\frac{P(\cos\Theta)}{4\pi}\sin\Theta d\Theta d\phi = 1$$

where Θ can be written as

$$\begin{aligned}\cos\Theta &= \Omega\cdot\Omega' = \Omega_x\Omega'_x + \Omega_y\Omega'_y + \Omega_z\Omega'_z \\ &= \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi'-\phi)\end{aligned}$$

Here, the prime indicates the incident direction, while the terms without a prime indicate the direction after scattering. Forward scattering occurs when $\Theta < \pi/2$, and backward scattering occurs when $\Theta > \pi/2$.

Here are some analytical examples of the phase function:

3.1 Isotropic scattering

The intensity of scattered light from a molecule is influenced by the polarization of the incoming light. When the incident light is vertically polarized, the scattered intensity doesn't vary with the direction of the scattering plane, indicating isotropic scattering.

$$P(\Theta) = 1;$$

3.2 Rayleigh Scattering

If the light frequency is not close to a resonant frequency, the scattering of light by small particles or molecules is similar to that of an induced dipolar oscillator. If we assume that the incident radiation is unpolarized, then the normalized scattering phase function is given by,

$$p_{\text{Ray}}(\cos\Theta) = \frac{3}{4}\left(1+\cos^2\Theta\right);$$

3.3 The Mie Scattering

Scattering in planetary media occurs due to molecules and particulate matter. When the scatterer is much smaller than the wavelength, as with scattering of solar radiation by molecules, the scattering phase function is mildly anisotropic. This type of scattering poses no special challenges for solving the radiative transfer equation. In contrast, scattering of solar radiation by larger particles is characterized by strong forward scattering, with a diffraction peak in the forward direction. The Mie-Debye theory, detailed in the referenced book, has undergone extensive refinement and development by numerous researchers. While the theory's mathematical principles are well-established, implementing it numerically has been notably difficult. Ongoing efforts are dedicated to creating efficient and precise computer algorithms. However, a comprehensive discussion of this aspect is outside the scope of our project.

3.4 The Henyey-Greenstein function

Henyey and Greenstein (1941) introduced a one-parameter scattering phase function, which is

$$P(\Theta) = \frac{1-g^2}{\left(1+g^2-2g\cos\Theta\right)^{3/2}}.$$

The Henyey-Greenstein phase function, characterized by the anisotropy parameter g, is a simplified representation of actual phase functions, without a physical basis. The parameter g ranges from -1 to 1, with $g = 0$ representing isotropic or symmetric scattering.

$$g = \langle \cos\Theta \rangle = \frac{1}{4\pi} \int_{\Omega} \cos\Theta P(\Theta) d\Omega.$$

The Henyey-Greenstein phase function is popular due to its simplicity and usefulness in numerical simulations. Other phase functions can be expanded using Legendre polynomials (see, e.g., [8, 10]).

For condensate scattering, where no analytic form exists, tables from geometric optics or Mie theory are often used.

3.5 Phase function as a Legendre series expansion

Legendre polynomials are fundamental in radiative transfer (RT) numerical simulations due to their orthogonality, serving as basis functions. These polynomials, whether ordinary, associated, or generalized (for polarization), are used depending on the specific problem. Their orthogonality allows for simplification of integrals into summations, offering a balance between numerical accuracy and computational efficiency by adjusting the number of terms as needed. Here, we focus on the computation of ordinary and associated Legendre polynomials.

The ordinary Legendre polynomials, denoted as $P_k(x)$ where $x = [-1 : 1]$ represents the cosine of the zenith angle, have a single degree of freedom given by the polynomial order $k = 0, 1, 2, ..., K$. These polynomials are particularly useful for computations involving a single angle, such as zenith or scattering angles. They are applied in scenarios like computing fluxes and azimuthally independent intensity (when solar, view directions, or both coincide with the local normal), and expanding the phase function. The orthogonality of $P_k(x)$ allows for efficient computation.

$$\int_{-1}^{1} P_k(x) P_l(x) dx = \frac{2}{2k+1} \delta_{kl},$$

The Kronecker delta is denoted as δ_{kl}. The Legendre polynomials have specific forms for $k = 0$ and $k = 1$: $P_0(x) = 1$ and $P_1(x) = x$, respectively. For arbitrary values of k, the polynomials can be determined using a recurrence relation

$$(k+1)P_{k+1}(x) = (2k+1)xP_k(x) - kP_{k-1}(x).$$

In radiative transfer (RT) codes, a common practice is to adjust indices, such as $k \to k-1$ in Eq. (10), scale both sides by $1/k$, and then express the equation.

$$P_k(x) = (2 - 1/k)xP_{k-1}(x) - (1 - 1/k)P_{k-2}(x).$$

for which an analytical expression is

$$p(\mathrm{v}) = \sum_{k=0}^{K} x_k P_k(\mathrm{v}).$$

where,

$$v = \cos\Theta$$

The variable v is related to the RTE variables through this equation:

$$v = \mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\varphi - \varphi')$$

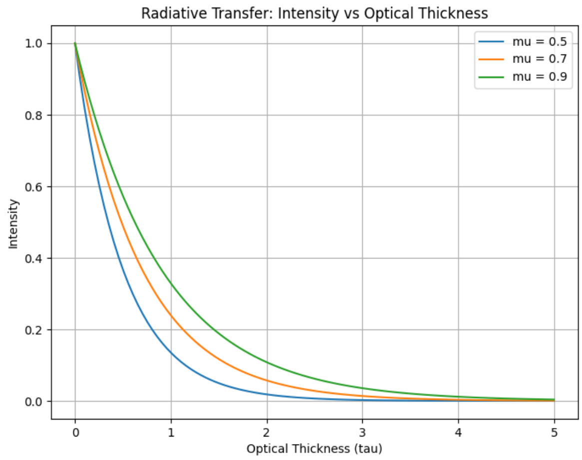

Figure 2: Variation of Intensity with optical depth for various zenith angles (μ)

4 RT without scattering

In the case when the medium is non-scattering (i.e. $\omega_0 = 0$), the second and third terms in the right part of Eq.(1) vanish, reducing the RTE to a first order differential equation whose solution is the Bouguer-Lambert-Beer (exponential) attenuation law. I have solved this equation to give I vs tau plot as shown.

5 Solving RTE with HG and legendre polynomial

We will now proceed to a detailed analysis of the radiative transfer equation with a focus on single scattering, aiming to understand the behavior of light in planetary atmospheres. The first step is to solve the radiative transfer equation analytically, incorporating the single scattering term. This analytical solution provides the intensity (I) of light at different levels of the atmosphere, specifically at the Top of Atmosphere (TOA) and Bottom of Atmosphere (BOA) as was shown in equation –. We also solved the equation numerically using the RK4 method and we find that the results are comparable as shown in figure –

5.1 RT with phase function as Legendre polynomials

Following the analytical solution, a phase function expansion using Legendre polynomials is incorporated. This expansion allows for a more detailed examination of the scattering behavior of light in the atmosphere. The first step is to compute Legendre polynomials up to a specified order, which are used to calculate the phase function values for scattering. Two functions are defined to calculate the upward and downward intensities of single scattering events, taking into account the solar zenith angle, scattering angles, and optical depth. Parameters such as single scattering albedo and surface albedo are initialized, and the intensities are computed at the Top of Atmosphere (TOA) and Bottom of Atmosphere (BOA). By plotting contours of intensity (I) for variations across azimuth angle (phi) and cosine of the zenith angle (mu), the scattering patterns of light can be visualized and analyzed. The plot is as shown in –.

5.2 RT with the HG scattering phase function

In addition to the Legendre expansion, a comparative analysis is conducted using the Henyey-Greenstein (HG) phase function. The first step is to define a function to compute the HG phase

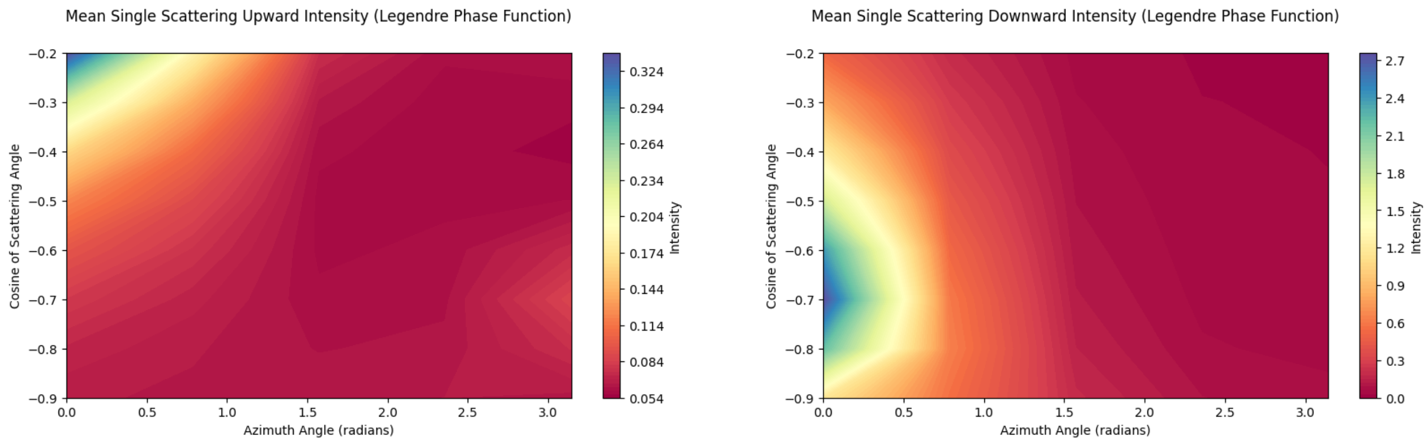

Figure 3: Legendre Polynomial Phase

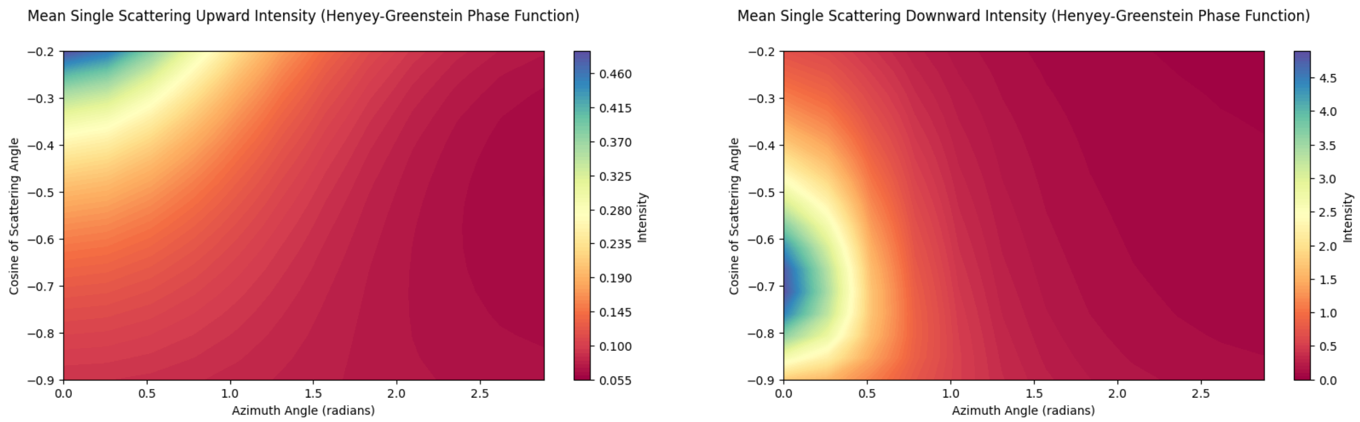

Figure 4: HG Phase Function

function for a given scattering angle and asymmetry parameter. This calculation is performed in a loop over the scattering angles, where the HG phase function is computed for each angle. Then, two functions are defined to calculate the upward and downward intensities of single scattering events using the HG phase function. Parameters such as single scattering albedo, solar zenith angle, and surface albedo are initialized, and the intensities are computed at the Top of Atmosphere (TOA) and Bottom of Atmosphere (BOA). The phase function values for each angle are calculated using the HG formula, and contours of intensity (I) are plotted for this approach as well. The plot is shown in –.

The results show that the contours of intensity (I) obtained from both the Legendre expansion and the HG phase function approach are comparable at TOA and BOA. This suggests that both methods provide similar predictions regarding the scattering behavior of light in the atmosphere.

The plots of the intensity (I) contours reveal the intricate patterns of light scattering in the atmosphere. The contours illustrate how light is scattered as it traverses through the atmosphere, providing valuable insights into the radiative processes at play.

6 Conclusion

In conclusion, the analytical solution of the radiative transfer equation, coupled with the phase function expansion using Legendre polynomials, offers valuable insights into the behavior of light in planetary atmospheres. The comparable results obtained from the Legendre expansion and the HG phase function approach indicate that both methods can be effectively used to model the scattering behavior of light in the atmosphere. Furthermore, the analytical and numerical solutions for single scattering events are found to be exactly matching. This suggests that the analytical approach is reliable for modeling single scattering, providing credibility to the results obtained from numerical methods when dealing with more complex multiple scattering scenarios.

Overall, this project enhances our understanding of radiative processes in planetary atmospheres, which is crucial for various applications such as climate modeling and remote sensing. The methods developed in this project can be further refined and applied to study other planetary atmospheres or to analyze observational data, contributing to a deeper understanding of planetary climates.

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