Calculating the mean lifetime of muons using Teachspin apparatus and FPGA programming

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Abstract: Quarks and Leptons are the basic building blocks of matter. Their interaction is through forces mediated by gauge bosons which correspond to the fundamental symmetries of the universe. The goal of this report is to measure the lifetime of one of these fundamental particles: the muon. Muons are charged leptons similar to that electrons but with a much greater mass. The mean lifetime of muon determines the value of the Fermi Constant G_F, which is the fundamental parameter to describe the strength of low energy weak interactions.

In this experiment, several cosmic ray muons are stopped each minute in a large volume of scintillator. The first part uses a Teachspin apparatus which consists of a scintillator detctor, PMT and an analyzer circuit to study the muon decay. By analyzing the spectrum of time delays between the entry of muon into the scintillator and the decay of muon, the mean lifetime is calculated.Further, the charge ratio of $\mu^+/\mu^- = 1.133 \pm 0.030$ and Fermi coupling constant $G_F = 91.065 \pm 2.373$ eV fm^{-3} are obtained from it.

The second part of the experiment includes "Altera Cyclone V FPGA (Field Programmable Gate Arrays) board" used to prepare a set-up analogous to the working of the Teachspin apparatus in terms of detecting a muon decay by applying VHDL programming language.

INTRODUCTION

Muon is a truly elementary particle. It has a unitary negative electric charge of -1 and a spin of 1/2 and a much greater mass (105 MeV/c2). The antiparticle of muon is μ^+ , with a charge state +1. Muon decay is of great importance for the study of weak interactions which is one of the fundamental forces in nature. The muon mean lifetime value is one of the most precisely known constants. With advancement for methodology and techniques, it is now measured with a precision of a few parts per million (ppm). Low-energy muons are primarily created from decays of particles which are produced in the interactions of high-energy cosmic-rays with the atmosphere of earth. About 80 % of the cosmic rays at the sea level are muons, positively or negatively charged. They have a mean lifetime of 2.197 microseconds. Laboratory experiments show that muons have much smaller lifetime than the time it takes to reach the surface of the earth. However, the fact that muons are detected on Earth surface indicates the effects of special relativity i.e their decay is slowed down since they travel at very high speeds (time dilation).

The first experiment to measure the muon lifetime was performed by Rossi, but the setup used here is similar to the one proposed by Hall, Lind and Ristinen. A muon that enters, stops, and further decays in a scintillator produces two pulses separated by a few μs. One pulse observed when it enters the liquid, and a second one when it decays in the detector. The pulses are viewed with an oscilloscope or digitized and stored on the computer. The time difference between these correlated pulses is to be measured. After many such observations, a distribution of the number of counts vs. time difference is plot.The plot is an exponential decay curve, whose decay constant is the lifetime of the muon.

Most of the muons that are incident on the detector pass straight through it except the lowest energy particles which come to rest within the scintillating liquid itself.

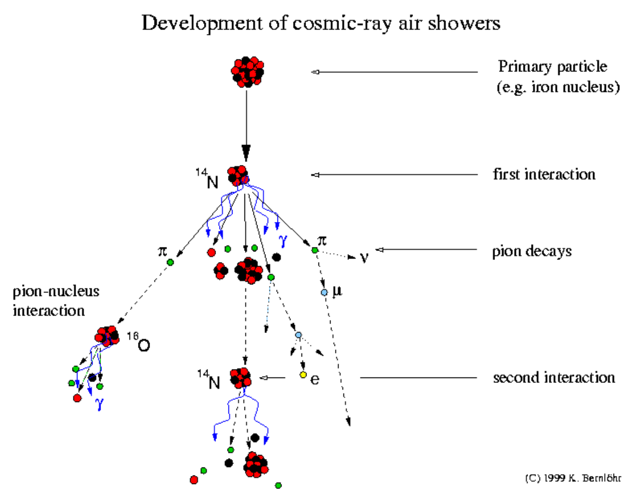

FIG. 1. *Production of muons in the higher atmosphere due to cosmic ray interaction.* [Source: Cosmic ray air showers]

THEORY

Primary cosmic rays are mainly composed of protons and heavier nuclei (about 98%) and a very small amount of electrons(around 2%). Secondary particles such as neutrons, pions (both charged and neutral),protons, kaons, photons, electrons and positrons are produced when the cosmic ray collide with the nuclei of air molecules. This is followed by a cascade process in which these secondary particles undergo electromagnetic and nuclear interactions to produce yet additional particles. Muons are produced when some pions decay from weak interaction.

Mass- $206.78\ m_e$
charge- $\pm e$
spin- $\frac{1}{2}$

Muon Decay

Muon decays via weak force. The theoretical mean life time of muon is 2.197 μs. Muon decays to an electron, an electron antineutrino, and a muon neutrino. Anti-muons, in mirror fashion, most often decay to the corresponding antiparticles: a positron, an electron neutrino, and a muon antineutrino.

Muons can decay via a few paths, but almost all of them decay into an electron and two neutrinos according to Equation below

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$
$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

The neutrinos are produced according to lepton number conservation. Due to the difference in the number of positive and negative pions in the atmosphere, the ratio of positive to negative muons is also not unity.

Let us consider muon decays to be independent events. Suppose that a muon decay during a time period dt is λdt.
If we have N(t) number of muons at time t. The probability that a muon decays in some small time interval dt is λdt. Here, λ is called the "decay rate" that characterizes how rapidly a muon decays. The change dN in number of muons is given by,

$$dN = -N(t)\lambda dt$$

$$\frac{dN}{N(t)} = -\lambda dt$$

Upon integrating, we have

$$N(t) = N_0\ exp(-\lambda t)$$

where N(t) is the number of surviving muons at some time t and N_0 is the number of muons at t = 0.

The time-dependent probability that a muon decays in the time interval between t and t + dt is given by D(t)dt. If we had started with N_0 muons, then the fraction -dN/N_0 that would on average decay in the time interval between t and t + dt is just given by differentiating the above relation:

$$-dN = N_0\lambda\ exp(-\lambda t)\ dt$$
$$\frac{-dN}{N_0} = \lambda\ exp(-\lambda t)\ dt$$

Thus

$$D(t) = \lambda\ exp(-\lambda t)$$

This is true regardless of the starting value of N_0. That is, the distribution of decay times, for new muons entering our detector, is also exponential with the very same exponent used to describe the surviving population of muons.

Some muons completely pass through the detector. This generates a single detector output pulse. Other muons are stopped inside dectector and decay into electron or positron. The later event gives two consecutive pulse. The time interval between two consecutive pulse of this second type of event gives the lifetime of the decay.

The "lifetime" τ of a muon is the reciprocal of λ,

$$\tau = \frac{1}{\lambda}$$

Time Dilation Effect of Special Relativity

Given the life time of muon to be 2.2 μ secs, they should not be able to reach the ground even at the speed of light. We measure a life time of 2.2 μs if we are at rest w.r.t muon. However, according to special theory of relativity, when muon is moving at relativistic speed, the time will be dilated w.r.t us.

For a rough calculation (not taking in to account how the muon will be affected when travelling through a medium) if time was absolute.

If we take relativity:

$$t = \gamma\tau$$
$$\implies t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}$$

Considering the velocity of muons to be 0.9999c and τ to be 2.2 μs, we get t = 140 μs. The distance travelled would be 0.9999c $\times$ 140 μs = 42 Km.

The fraction of muons that reach earth

$$\approx \exp(\frac{-140\mu s}{2.2\mu s}) = \exp(-63.63)$$

which is a negligible quantity. But in reality, we observe muon flux rate of around 1 per cm^2 per minute on the ground.

But due to great speed of muon, the length that the muon travels in its frame would also be contracted by γ. Thus, the muons would be able to reach the ground as per special relativity.

Fermi Coupling Constant

The Fermi coupling constant G_F is a measure of the strength of the weak force by which muons decay. The relationship between the muon lifetime τ and G_F (upto a good approximation) is given by:

$$\tau = \frac{192\pi^3\hbar^7}{(G_F)^2 m^5 c^4}$$

where m is the mass of the muon taken from the Particle Data Group [3] [4], and τ is measured using the apparatus.

Impedance matching

The connections from output of Discriminator and amplifier to oscilloscope are made through 50 ohm Coaxial Cable.Since the square pulses from discriminator are DC signals. The oscilloscope should be DC coupled with input impedance set to 50 ohm.Otherwise Signal loss and distortion due to reflection was observed. The reflection coefficient is given by:

$$R = \frac{Z_L - Z_0}{Z_L + Z_0}$$

If Z_L becomes very large compared to Z_0 then R $\approx$ 1. Then Signal is mostly reflected and and we get negligible transmitted signal.

METHODOLOGY

Detector physics

The detector is a plastic scintillator. The active volume of the detector is in the shape of a right circular cylinder of dimensions 15 cm x 12.5 cm put at the bottom of the black anodized tube of aluminum alloy. Plastic scintillator is transparent organic material. It is made by mixing together one or more fluors and a solid plastic solvent having an aromatic ring structure. When a charged particle passes through the scintillator, it will lose some of its kinetic energy by ionization and atomic excitation of the solvent molecules. A part of this deposited energy is then transferred to the fluor molecules. The electrons of the fluor molecules are then excited to higher states. During radiative de-excitation, there is emission of light in the blue and near-UV portion of the electromagnetic spectrum. This has a typical decay time of a few nanoseconds.

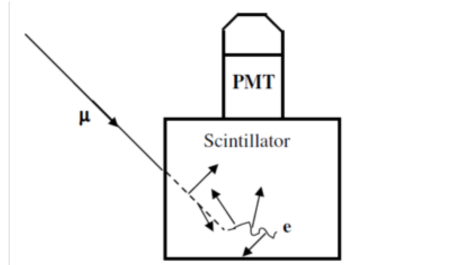

FIG. 2. *Schematic showing,* ***short arrows:*** *two light pulses being generated to determine the muon lifetime.* ***Dotted line:*** *light pulse from the slowing muon ,* ***Wavey line:*** *the light pulse from its decay into an electron or positron .* [Source: Detector physics]

To measure the muon's lifetime, we are only considering those muons that enter, slow, stop and then decay inside the plastic scintillator.

Such muons have a total energy of only about 160 MeV while they enter the tube. When a muon slows to a stop, the excited scintillator emits light. This light is detected by a photomultiplier tube (PMT), eventually producing a logic signal which triggers a timing clock.

After a muon stops, it decays into an electron, a neutrino and an anti-neutrino. Since an electron is much lighter than a muon, $m_\mu/m_e \approx 210$, the electron tends to be very energetic and produces scintillator light all along its pathlength. Some of the muon's total energy is also shared by the neutrino and anti-neutrino but they entirely escape detection. This second burst of scintillator light is also seen by the PMT which is used to trigger the timing clock. The muon lifetime is measured by the the distribution of time intervals between successive clock triggers for a set of muon decays.

Teachspin Apparatus

The electronics module contains all the electronics needed to run the experiment. Connections on the front panel helped examine the PMT signal and monitor that signal as it moves along the readout chain. First, the PMT pulses are amplified. Then they are compared against an adjustable threshold. Pulses that are above the threshold voltage are sent to timing circuitry implemented in a FPGA (field programmable gate array) chip. As soon as the first flash of the scintillator is received, the timing system starts. Now, if a second flash occurs within 20 microseconds of the first flash, the readout electronics measures the time interval between the two flashes. This data is then passed onto the lifetime display software.

If there is no second flash within 20 microseconds of the first one, the software simply records the pulse as a charged particle that has passed through the detector. Furthermore, the Communication circuitry transfers the data to a PC or laptop through either a serial or a USB port.

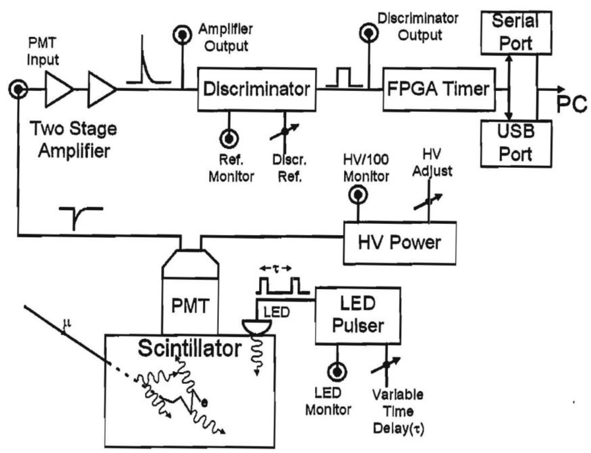

FIG. 3. *Block diagram of the readout electronics. The amplifier and discriminator outputs are available on the front panel of the electronics box. The HV supply is inside the detector tube.*

FPGA Programming

Field Programmable Gate Arrays (FPGAs) are semiconductor ICs where a large majority of the electrical functionality inside the device can be modified for specific use. They are based around a matrix of configurable logic blocks (CLBs) connected via programmable interconnects. For the FPGA to function, the user provides a design in a hardware description language (HDL) or as a schematic design. In this experiment, we have used Altera Cyclone V FPGA board and VHDL as the HDL language. We have used intel Quartus softaware to complile and run the code. For the hardware, we used USB Blaster to successfully impliment the VHDL code on the FPGA board. We carried out basic combinational and sequential circuits. We implemented OR gate, AND gate and D flip-flop circuits on FPGA. The codes for all these are attached in the appendix. The diagram of pin planner of Quartus prime software can be found in figure 4.

OBSERVATION

The high voltage supply required for the functioning of the PMt was supplied by connecting the detector to a high voltage supply. The **"Geco"** software was used to adjust the voltage value. For this experiment, the HV supply for PMT tube was kept at constant 700 V and data was taken accordingly.

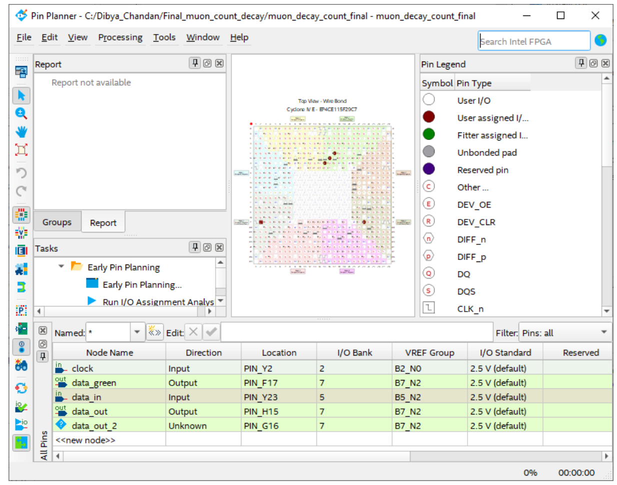

FIG. 4. *Pin planner of Quartus Prime software*

Discriminator voltage vs muon count

It is important to distinguish the Muon detection counts from the muon decay counts. When a muon hits the detector or decays, the scintillation detector increases by 1. On the other hand, muon decay counts are only those detected muons which have hit the scintillation detector and decayed there. The time period of this decayed muon is noted by the software.

TABLE I. *Variation of number of muon counts with discriminator voltage*

S. No.	Discriminator Voltage (mV)	No. of decayed muons pulses detected / 5 min
1	26	21247
2	48	125
3	75	45
4	100	38
5	125	35
6	150	34
7	175	33
8	199	27
9	224	11
10	250	2

The number of muon decay counts that the detector detects highly depends on the discriminator voltage. The pulse counts are very high when discriminator voltage is less than 25mV. When a photon of sufficient energy hits the detector and signals from noise are also picked, we get the high pulse counts. For example, the gamma rays getting detected could add to the noise.

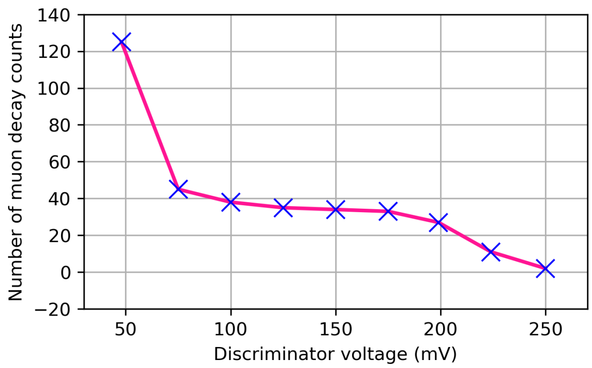

FIG. 5. *Plot showing the variation of number of muon decays between discriminator voltage 50 mV and 250 mV.*

On the contrary, when the discriminator voltage is increased beyond 250 mV, the pulse counts received are too low. This is because very few muons have energy greater than 250 mV. In order to observe significant number of counts, the experiment would take longer time.

As evident from the tables and graphs, the discriminator voltages between 100 mV up to 175 mV are optimum for the experiment. Table I shows how the number of muon counts detected by the scintillation detector and the Teachspin apparatus varies with different discriminator voltages. Figure 5 shows the plot for table I from 50 mV to 250 mV.

Mean life Calculation:

In this report, the histograms are plot from the collected datasets and exponential fits are made. As shown in figures 6 to 11, the histogram for the detected muon decays are plotted. The corresponding error bars are also shown in the plot.

The initial frequency bins for some datasets were removed from consideration because of the presence of instrumental and technical error which led to non-physical values of μ^+/μ^- ratio.

TABLE II. *Obtained half lives of the datasets*

Figure No.	Half life (μs)	Associated error (μs)
6	2.1261	0.0189
7	2.0558	0.0303
8	2.1749	0.0619
9	2.1463	0.1000
10	2.0907	0.0119
11	2.1380	0.0535
Avg	2.1220	0.0553

The half life of the muon decay for each dataset was obtained from the exponential fits. The average of 6 observations gives $\tau_{obs} = 2.1220 \pm 0.0553$ microseconds as shown in table II. This value is used to evaluate muon charge ratio and Fermi coupling constant.

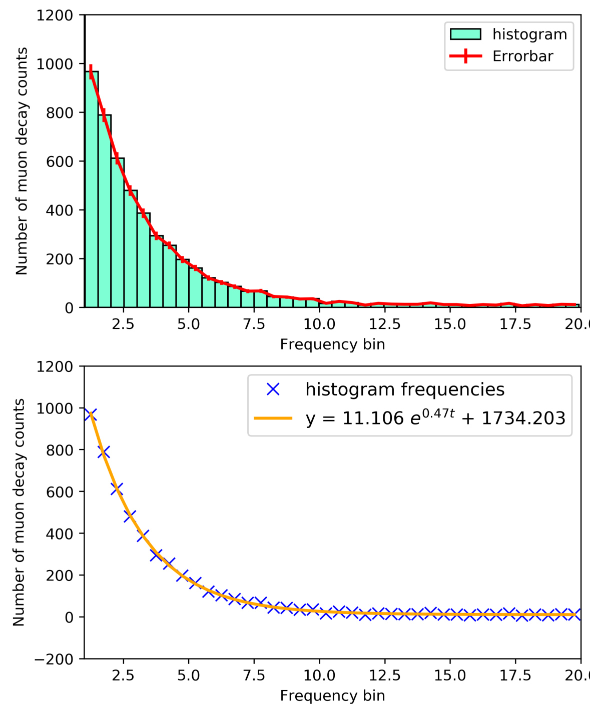

FIG. 6. *Histogram and exponential fit for dataset 1*

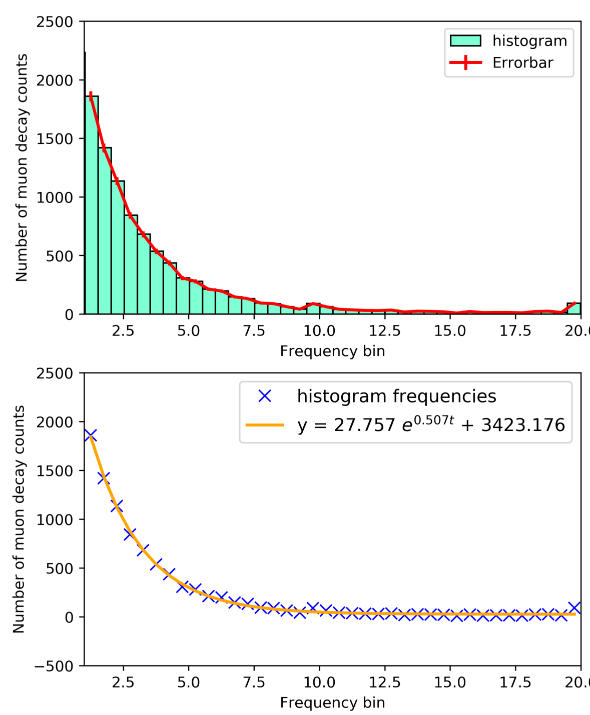

FIG. 7. *Histogram and exponential fit for dataset 2*

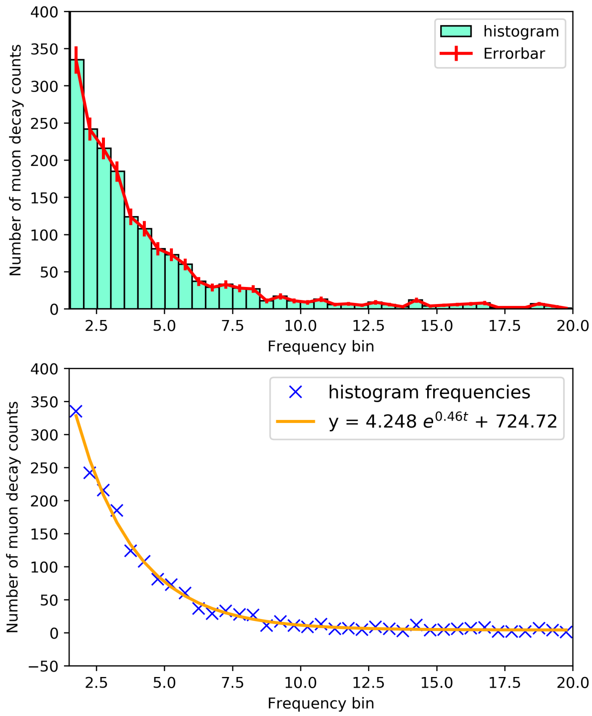

FIG. 8. *Histogram and exponential fit for dataset 3*

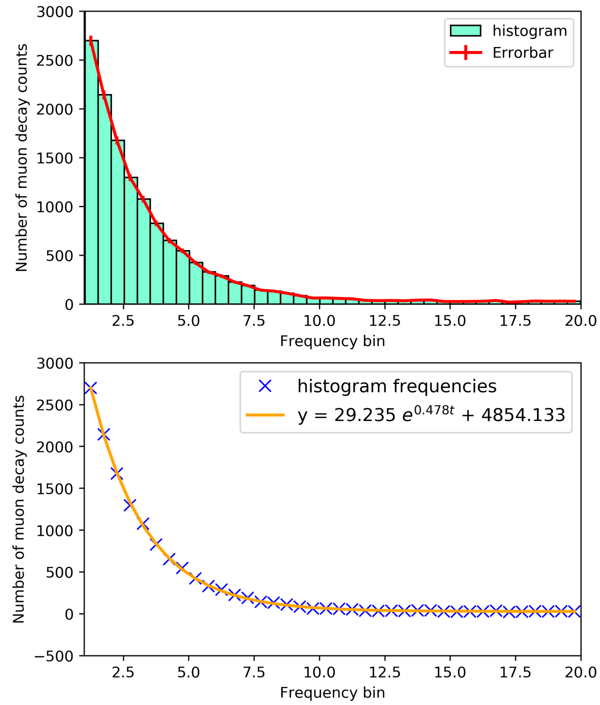

FIG. 10. *Histogram and exponential fit for dataset 5*

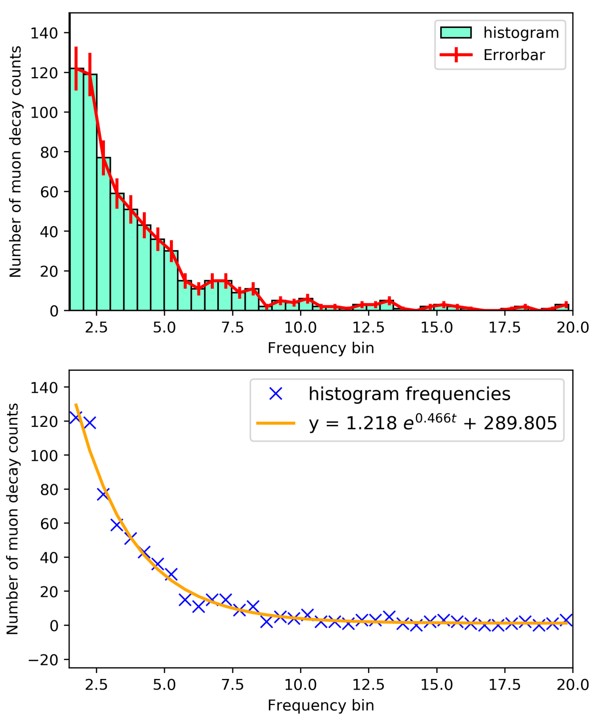

FIG. 9. *Histogram and exponential fit for dataset 4*

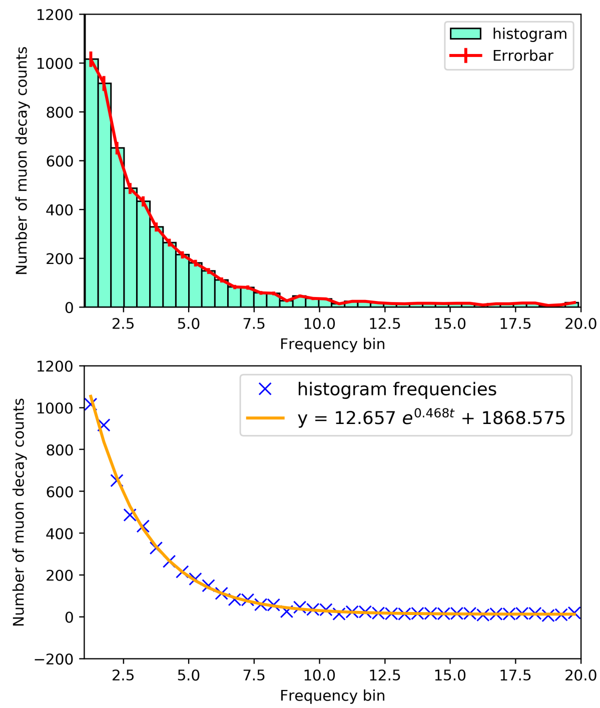

FIG. 11. *Histogram and exponential fit for dataset 6*

CALCULATION

Muon Charge Ratio Calculation:

Muon charge ratio $\rho = \dfrac{N^+}{N^-}$ is defined as the ratio of the number of positively-charged muons to the negatively-charge muons.

The magnitude and momentum dependence of ρ are determined by their production and interaction cross sections Usually positive meson production is favoured because cosmic rays and nuclei with which they interact are mostly positively charged, hence we expect ρ to be more than 1.

Negative muons can interact with protons to give neutron and muon neutrino. Since negative muons have two ways to disappear, their life time in matter is effectively less than that of free muons. Life time of negative muons in carbon: $\tau_c = 2.043 \pm 0.003 \mu s$. (Reiter, 1960 [5]).

The effective decay constant or mean life can be written in terms of muon charge ratio.

$$\bar{\lambda} = \frac{1}{\tau_{obs}} = \frac{N^+\lambda^+ + N^-\lambda^-}{N^+ + N^-} = \frac{\rho\lambda^+ + \lambda^-}{1+\rho} \tag{1}$$

$$\implies \qquad \rho = -\frac{\tau^+}{\tau^-}\left(\frac{\tau^- - \tau_{obs}}{\tau^+ - \tau_{obs}}\right) \tag{2}$$

Hence, we put the values of the respective times $\tau_{obs} = 2.122\ \mu$s (from the experiment), $\tau^+ = 2.197\mu$s (mean lifetime of free muons), $\tau^- = \tau_e = 2.043\mu$s (lifetime of μ^- in carbon) to get a estimate of the ration of positive to negative muons.

We obtain the final ratio as

$$\rho = 1.133$$

Hence our estimation shows that there are more positive muons than negative muons in the incoming muon flux from sky.

Fermi's coupling Constant:

As mentioned above in the theory, the value of the Fermi coupling constant G_F is easily determined from the relation:

$$\tau = \frac{192\pi^3\hbar^7}{G_F^2 m_\mu^5 c^4}$$

Inverting the relation and putting $\tau = \tau_{obs}$ we get:

$$G_F = 91.065 \text{ eV fm}^{-3}$$

Execution using FPGA

We implemented three gates using the FPGA whose properties are mentioned below:

AND Gate: Tt is a basic digital logic gate that implements logical conjunction of two signals. It has been applied using the VHDL code presented in appendix A.

XOR Gate: It is a digital logic gate that gives a true (1 or HIGH) output when the number of true inputs is odd. An XOR gate implements an exclusive OR from mathematical logic. It has been applied using the VHDL code presented in appendix B.

D flip-flop: It is a digital electronic circuit used to delay the change of state of its output signal (Q) until the next rising edge of a clock timing input signal occurs. It has been applied using the VHDL code presented in appendix C.

ERROR ANALYSIS

Error in mean lifetime from table II is 0.0553 μs.

Error in ρ : this can be written as

$$\begin{aligned}\Delta\rho &= \frac{\Delta\tau}{\tau}\rho \\ &= \frac{0.0553}{2.1220} \times 1.133 = 0.0295\end{aligned}$$

Error in G_F : this can be written as

$$\begin{aligned}\Delta G_F &= \frac{\Delta\tau}{\tau}G_F \\ &= \frac{0.0553}{2.1220} \times 91.065 = 2.373 \text{ eV fm}^{-3}\end{aligned}$$

RESULTS

The values obtained for the muon mean lifetime is close to the theoretical values of mean lifetime of muon and Fermi coupling constant.

1. The mean life of muon τ_{obs} was observed to be $= 2.1220 \pm 0.0553\mu s$.
2. The muon charge ratio was calculated to be around 1.133 ± 0.0295.
3. The Fermi coupling constant is calculated to be $G_F = 91.065 \pm 2.373$ eV fm^{-3}.

Here, the muon charge ratio is in close agreement with the theoretical and physical constraints. Literature value of Fermi coupling constant is ≈ 90 eV fm^{-3} which is also near to the calculated value.

The FPGA part of the experiment were executed for AND gate, OR gate, XOR gate and D-flip-flop. However, the muon decay simulation using a sample signal was tried to run. This part of the experiment will be continued and shall be included in the next report.

CONCLUSION

Muon is one of the elementary particles and a very clean probe for the study of weak interaction. The muon lifetime experiment is considered to be one of the standard experiments today. The results agree with the experimentally reported values in the literature. After caculating the mean lifetime of the muons from this experiment, the ratio of positively-charged to negatively-charged muon has also been calculated. Further, the Fermi coupling constant has been calculated from the calculated lifetime of muon, τ.

The FPGA module showed expected results for the basic gates. Simulating a muon detection and its decay using a sample signal will be included in the next report for this project.

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[4] Particle Data Group - Wikipedia

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[6] DE2-115 FPGA User manual

[7] Cyclone IV GX FPGA Development Kit

Appendix A: Code for AND Gate

```
----------------------------------------------
-- AND Gate VHDL Code
----------------------------------------------

library ieee;
use ieee.std_logic_1164.all;

----------------------------------------------

entity AND_gate is
port(
    x: in std_logic;
    y: in std_logic;
    F: out std_logic
);
end AND_gate;

----------------------------------------------

architecture behav of AND_gate is
begin

    process(x, y)
    begin
        -- compare to truth table
        if (x='1' and y='1') then
            F <= '1';
        else
            F <= '0';
        end if;
    end process;

end behav;

----------------------------------------------
```

Appendix B: Code for XOR Gate

```
----------------------------------------------
-- XOR Gate VHDL Code
----------------------------------------------

library ieee;
use ieee.std_logic_1164.all;

----------------------------------------------

entity XOR_gate is
port(
    x: in std_logic;
    y: in std_logic;
    F: out std_logic
);
end XOR_gate;

----------------------------------------------

architecture behv1 of XOR_gate is
begin

    process(x, y)
    begin
        -- compare to truth table
        if (x/=y) then
            F <= '1';
        else
            F <= '0';
        end if;
    end process;

end behv1;

----------------------------------------------
```

Appendix C: Codes for D flip-flop

```
----------------------------------------------
-- D Flip-Flop with Timer
----------------------------------------------
-- Here the D-flip-flop has implemented with
-- a timer which increases the clock rising
-- time and makes the D flip-flop active for
-- every 2 seconds using an inbuid 50MHz clock.
----------------------------------------------

library ieee ;
use ieee.std_logic_1164.all;
use work.all;

---------------------------------------------

entity D_flip_flop is
port(
    data_in: in std_logic;
clock: in std_logic;
data_out: out std_logic
);
end D_flip_flop;

----------------------------------------------

architecture behv of D_flip_flop is

signal count:integer range 0 to 50000000:=0;
signal a:std_logic:='0';
begin

    process(clock,data_in)
    begin
     if(data_in='1') then
            if (clock='1' and clock'event) then
                count <= count+1;
if(count =50000000)then
        a<=not a;
count<=0;
data_out<=a;
end if;
end if;
end if;
    end process;

end behv;

----------------------------------------------
```