

Chunyu Xue 518021910698

## 1 Problem 1

**Describe the details of Caesars Cipher (50 points):**

(a) Message space  $M$ .

**Answer:** The message space of Caesars Cipher is:

$M = \{m \mid m \text{ is any string consists of 26 English letters with the length of } n\}$

(b) Key-generation algorithm  $KeyGen$ .

**Answer:** The key-generation algorithm of Caesars Cipher is:

$KeyGen(\lambda) = \{k \mid k \in \mathbb{N} \text{ and } k \text{ is randomly chosen in the range of } [0, 25] \text{ according to some probabilistic distribution}\}$

(c) Encryption algorithm  $Enc$ .

**Answer:** The encryption algorithm of Caesars Cipher is:

$$Enc(k, m) = \{c \mid c[i] = (m[i] + k) \bmod 26, i \in [1, n]\},$$

which means  $c$  is the string that operates  $k$  (  $k \in \mathbb{N}$  and  $k \in [0, 25]$  ) shifts from original message  $m$ .

(d) Decryption algorithm  $Dec$ .

**Answer:** The decryption algorithm of Caesars Cipher is:

$$Dec(k, c) = \{m \mid m[i] = (c[i] - k) \bmod 26, i \in [1, n]\},$$

which means  $m$  is the string that operates  $k$  (  $k \in \mathbb{N}$  and  $k \in [0, 25]$  ) shifts (the direction is opposite to the  $Enc$  algorithm) from ciphertext  $c$ .

(e) Key space  $K$ .

**Answer:** The key space of Caesars Cipher is:

$$K = \{k \mid k = 0, 1, 2, \dots, 25\},$$

which means that  $|K| = 26$ .

(f) Ciphertext space  $C$ .

**Answer:** The ciphertext space of Caesars Cipher is:

$C = \{c \mid c \text{ is the string that operates } k \text{ ( } k \in \mathbb{N} \text{ and } k \in [0, 25] \text{ ) shifts from original message } m, \text{ which is also any string consists of 26 English letters with the length of } n\}$

## 2 Problem 2

**Describe the details of Simple Substitution Cipher (50 points):**

(a) Message space  $M$ .

**Answer:** The message space of Simple Substitution Cipher is:

$M = \{m \mid m \text{ is any string consists of 26 English letters with the length of } n\}$

(b) Key-generation algorithm  $KeyGen$ .

**Answer:** The key-generation algorithm of Simple Substitution Cipher is:

$KeyGen(\lambda) = \{k \mid k[i] \in \mathbb{N} \text{ and } k \text{ is a random permutation of } 1, 2, \dots, 26 \text{ according to some probabilistic distribution}\}$

(c) Encryption algorithm  $Enc$ .

**Answer:** The encryption algorithm of Simple Substitution Cipher is:

$$Enc(k, m) = \{c \mid c[i] = k[m[i]], i \in [1, n]\},$$

which means  $c$  is the string that operates hash mapping from original message  $m$  according to  $k$ .

(d) Decryption algorithm  $Dec$ .

**Answer:** The decryption algorithm of Simple Substitution Cipher is:

$$Dec(k, c) = \{m \mid m[i] = k^{-1}[c[i]], i \in [1, n]\},$$

which means  $m$  is the string that operates anti-hash mapping from ciphertext  $c$  according to  $k$ .

(e) Key space  $K$ .

**Answer:** The key space of Simple Substitution Cipher is:

$$K = \{k \mid k = permutation(1, 26)\},$$

which means that  $k$  is a random permutation of  $1, 2, \dots, 26$  and  $|K| = 26!$ .

(f) Ciphertext space  $C$ .

**Answer:** The ciphertext space of Simple Substitution Cipher is:

$C = \{c \mid c \text{ is the string that operates hash mapping from original message } m \text{ according to } k, \text{ which is also any string consists of 26 English letters with the length of } n\}$