

# **COM-500**

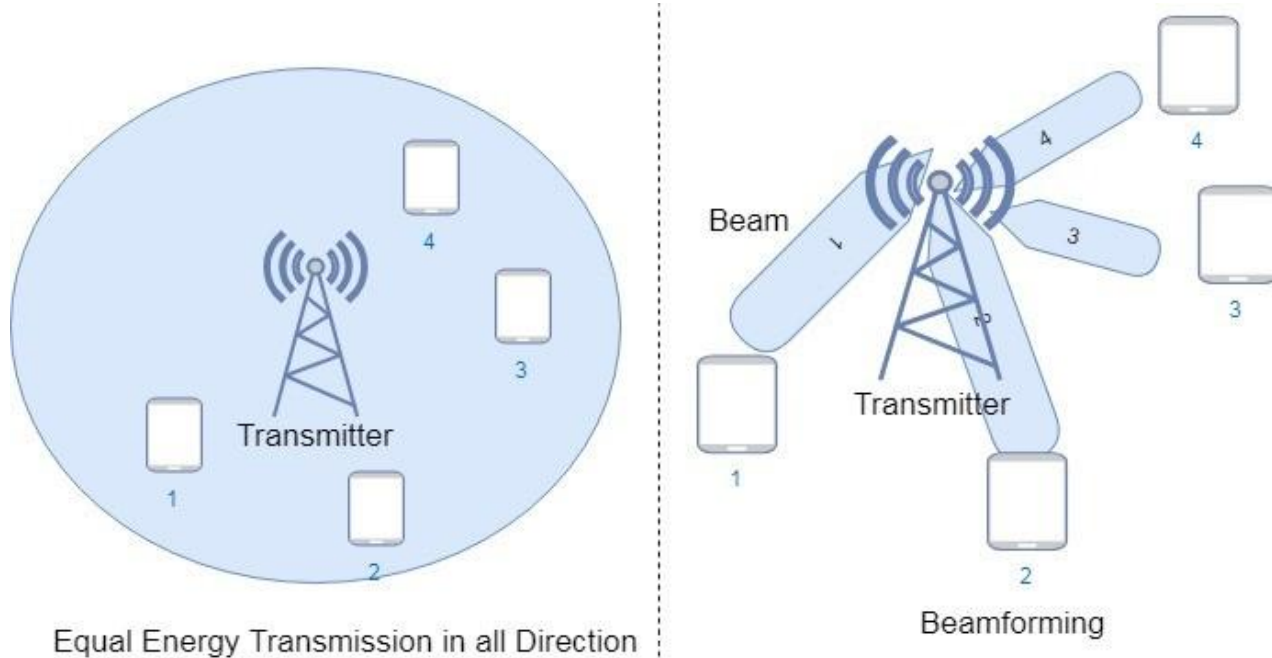
## **Throughput optimization for 5G networks**

Saged Bounekhel  
Franck Khayat  
Salim Najib

June 2023

# Introduction - Beamforming in 5G

**Motivation : Short Transmission Distance, Increased Throughput and Energy Efficiency**



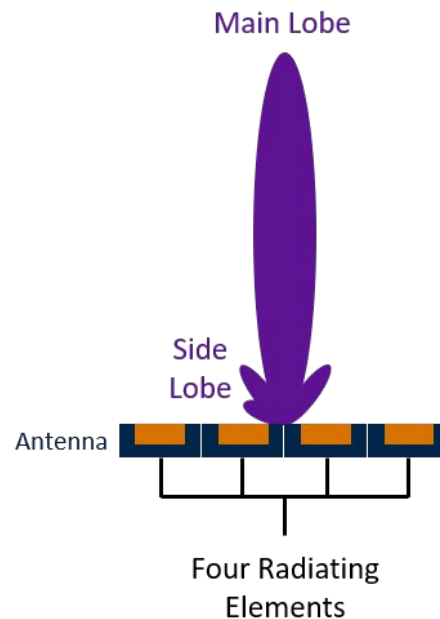
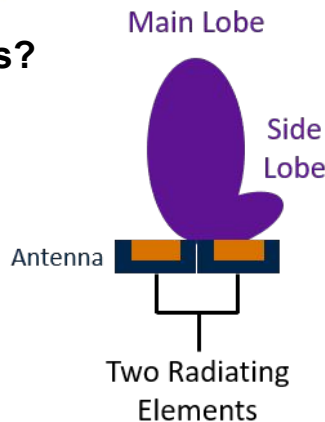
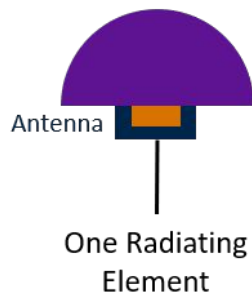
# EPFL Introduction - Beamforming in 5G

**Motivation : Short Transmission Distance, Increased Throughput and Energy Efficiency**

Noiseless Signal :

$$\begin{aligned} y(t, r) &= s(t) \left( \sum_{i=1}^L \gamma_i e^{j\phi_i} e^{-j2\pi \langle r, p_i \rangle} \right) \\ &= s(t) \left( \sum_{i=1}^L w_i^* e^{-j2\pi \langle r, p_i \rangle} \right) \\ &= s(t) b^*(r) \end{aligned}$$

**How to design beamforming weights?**



# Matched beamforming

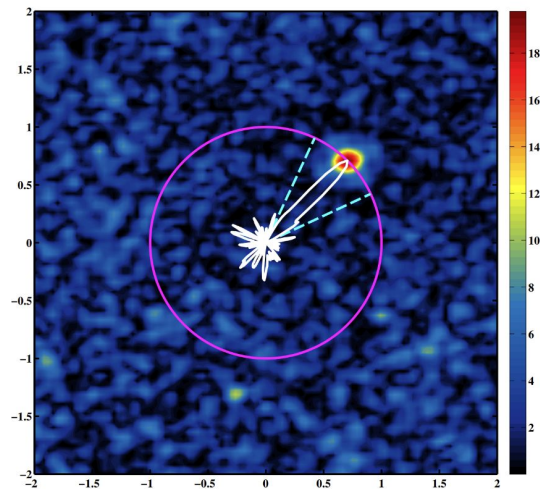
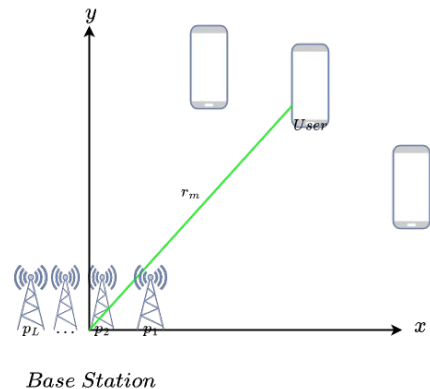
Array Beam Shape :  $b(r) = \sum_{i=1}^L w_i e^{j2\pi \langle r, p_i \rangle}$

Emitted Signal :  $x_i(t) = b_i s(t) = \gamma_i e^{j\phi_i} s(t), \forall i = 1, 2, \dots, L.$

Noiseless Signal :  $y(t, r) = s(t) \left( \sum_{i=1}^L \gamma_i e^{j\phi_i} e^{-j2\pi \langle r, p_i \rangle} \right)$   
 $= s(t) \left( \sum_{i=1}^L w_i^* e^{-j2\pi \langle r, p_i \rangle} \right)$   
 $= s(t) b^*(r)$

Matched weights :  $w_i = e^{-j2\pi \langle r_0, p_i \rangle}$

$r_0$  being the steering direction  
estimated by MUSIC.





Notion of a continuum of antennas in  $\mathbb{R}^2$

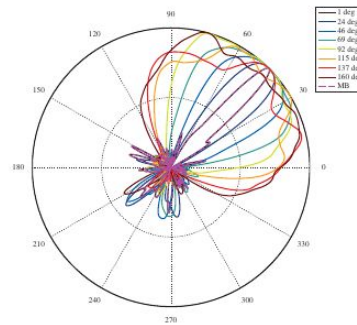
More flexible beam-shape

Sensor function :  $x(t, p) = \int_{\mathbb{S}^1} s(t) e^{-j2\pi \langle r, p \rangle} dr \in \mathbb{C}$

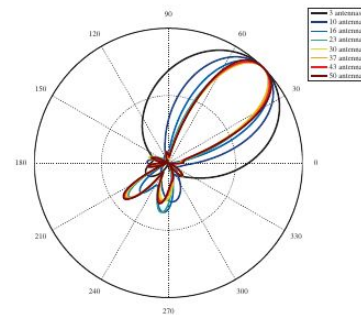
Noiseless transmitted signal :  $z(t) = \langle x(t, \cdot), w \rangle_{\mathcal{L}^2(\mathbb{R}^2)}$

$$\begin{aligned}
 &= \int_{\mathbb{R}^2} x(t, p) w^*(p) dp \\
 &= \int_{\mathbb{R}^2} dp w^*(p) \int_{\mathbb{S}^1} s(t) e^{-j2\pi \langle r, p \rangle} dr \\
 &= \int_{\mathbb{S}^1} dr s(t) \underbrace{\int_{\mathbb{R}^2} dp w^*(p) e^{-j2\pi \langle r, p \rangle}}_{=b^*(r)} \\
 &= s(t) \int_{\mathbb{S}^1} b^*(r) dr
 \end{aligned}$$

Observed signal :  $y(t, r) = s(t) b^*(r)$

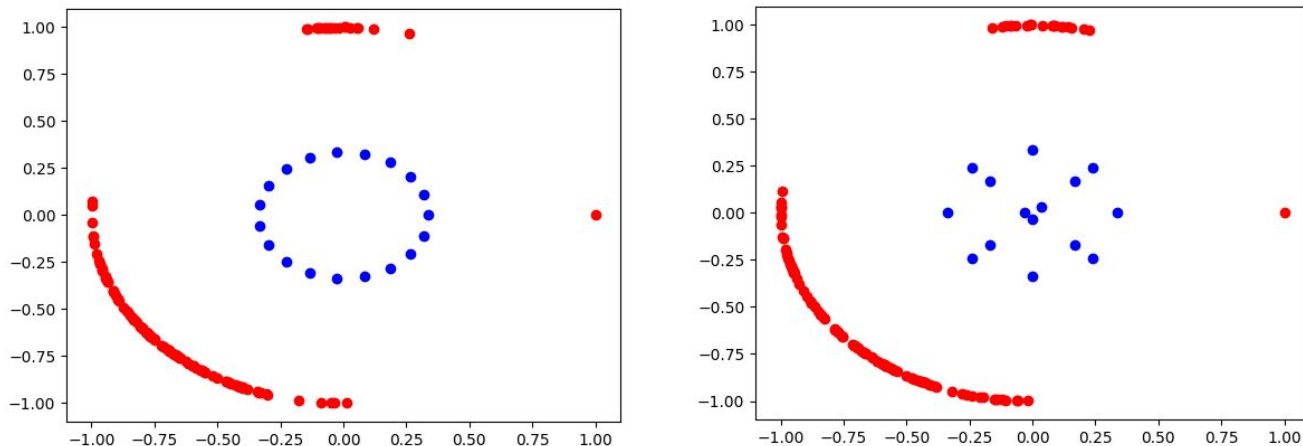


(a) Filtering a range of directions with flexibeam for various angles and 96 antennas.



(b) Filtering a range of directions with flexibeam for  $\Theta = 40^\circ$  with varying number of antennas.

# Throughput estimations - Setup

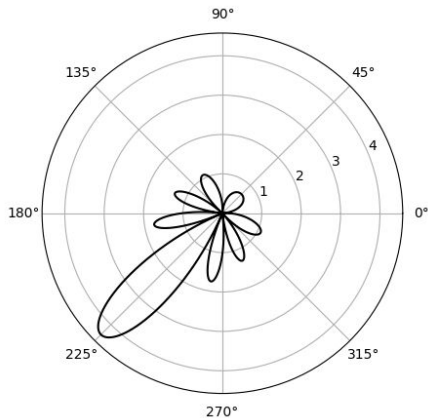


Antenna and transmitter positions, data 1 (left) and 2 (right). Antennas are denoted as blue dots and user positions as red dots.

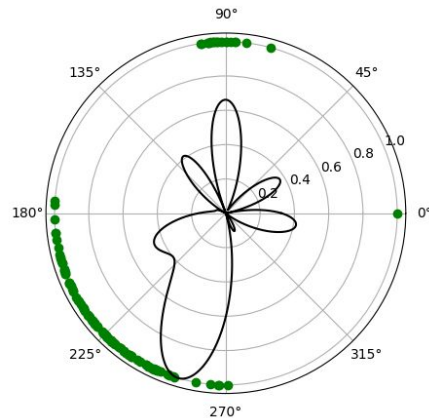
$$\text{Bitrate}(r) = B \log_2 \left( 1 + C_0 \frac{|b(r)|^2}{\sigma^2} \right)$$

With  $B$  being the bandwidth,  $C_0$  a scaling constant and  $\sigma$  the white noise variance

# Throughput estimations



Beamshape generated by matched beamforming  
Dataset 1



Beamshape evaluated by flexibeam  
Dataset 1

Green dots  
denote user  
positions

|                     | Dataset 1 | Dataset 2  |
|---------------------|-----------|------------|
| Matched beamforming | 7.27 Mb/s | 6.87 Mb/s  |
| Flexibeam           | 8.77 Mb/s | 10.86 Mb/s |

Received signal at a station:  $v_s(t) = A_s s(t) + n(t) \in \mathbb{C}^L$

with steering matrix:  $A_s \in \mathbb{C}^{L \times Q}$

Autocorrelation of received signal:  $R_{v_s} = \mathbb{E} (v_s(t) v_s(t)^H) = A_s \Sigma A_s^H + \Sigma_n$

Cross-correlation:  $R_{y_s} := R_s = \mathbb{E} (y_s(t) y_s(t)^*) = \sum_{q=1}^Q \sigma_q^2 |w_s^H a_{s,q}|^2 + \sigma_n^2 \|w_s\|_2^2$

Cross-correlation is a sufficient statistic of the users' density!

LASSO Problem Formulation for user density estimation  $\hat{f}$

$$\arg \min_{\hat{f}, \hat{\sigma}_n^2} \left\| \hat{R} - W^H \left( A \text{diag}(\hat{f}) A^H - \hat{\sigma}_n^2 I_{SL} \right) W \right\|_F^2 + \lambda \|\hat{f}\|_1$$

Implemented using





Received signal at a station:  $v_s(t) = A_s s(t) + n(t) \in \mathbb{C}^L$

with steering matrix:  $A_s \in \mathbb{C}^{L \times Q}$

Autocorrelation of received signal:  $R_{v_s} = \mathbb{E} (v_s(t) v_s(t)^H) = A_s \Sigma A_s^H + \Sigma_n$

Cross-correlation:  $R_{s_1, s_2} = \mathbb{E} (y_{s_1}(t) y_{s_2}(t)^*) = \sum_{q=1}^Q \sigma_q^2 w_{s_1}^H a_{s_1, q} w_{s_2}^H a_{s_2, q} \in \mathbb{C}$

Cross-correlation is a sufficient statistic of the users' density!

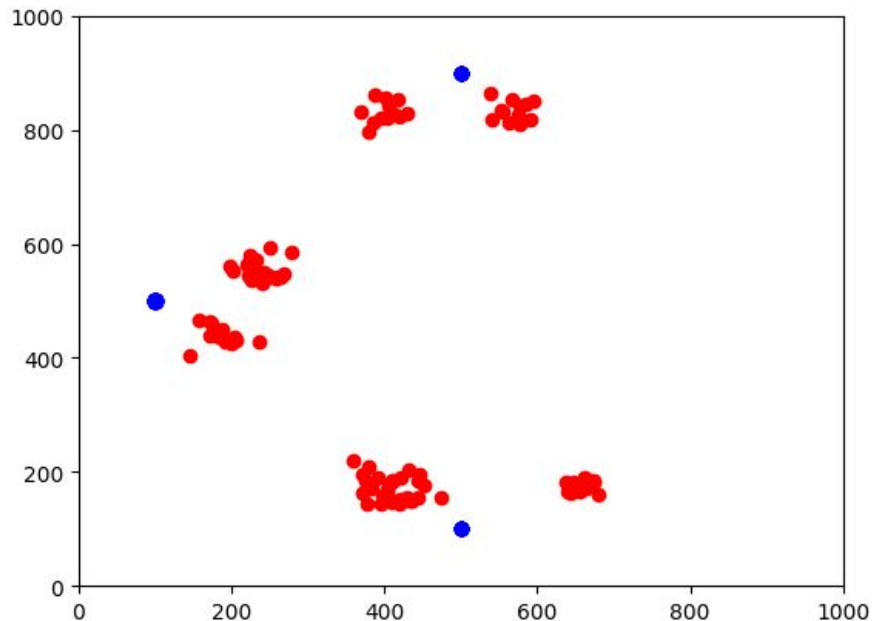
LASSO Problem Formulation for user density estimation  $\hat{f}$

$$\arg \min_{\hat{f}, \hat{\sigma}_n^2} \left\| \hat{R} - W^H \left( A \text{diag}(\hat{f}) A^H - \hat{\sigma}_n^2 I_{SL} \right) W \right\|_F^2 + \lambda \|\hat{f}\|_1$$

Implemented using

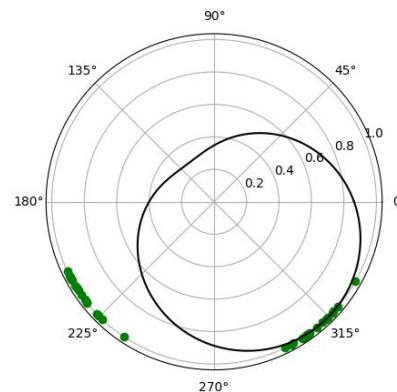
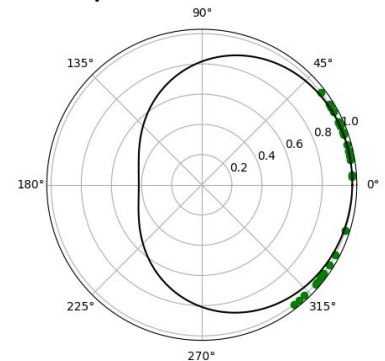


# Testing collaborative beamforming with flexibeam

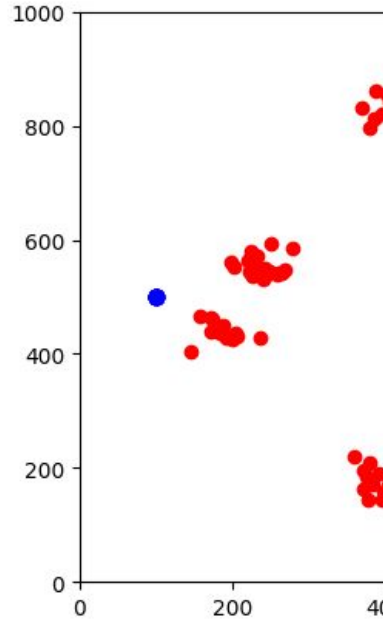


Test setup: 3 stations of 20 antennas, 100 users  
in 6 GMMs - 3000 iterations of PGD

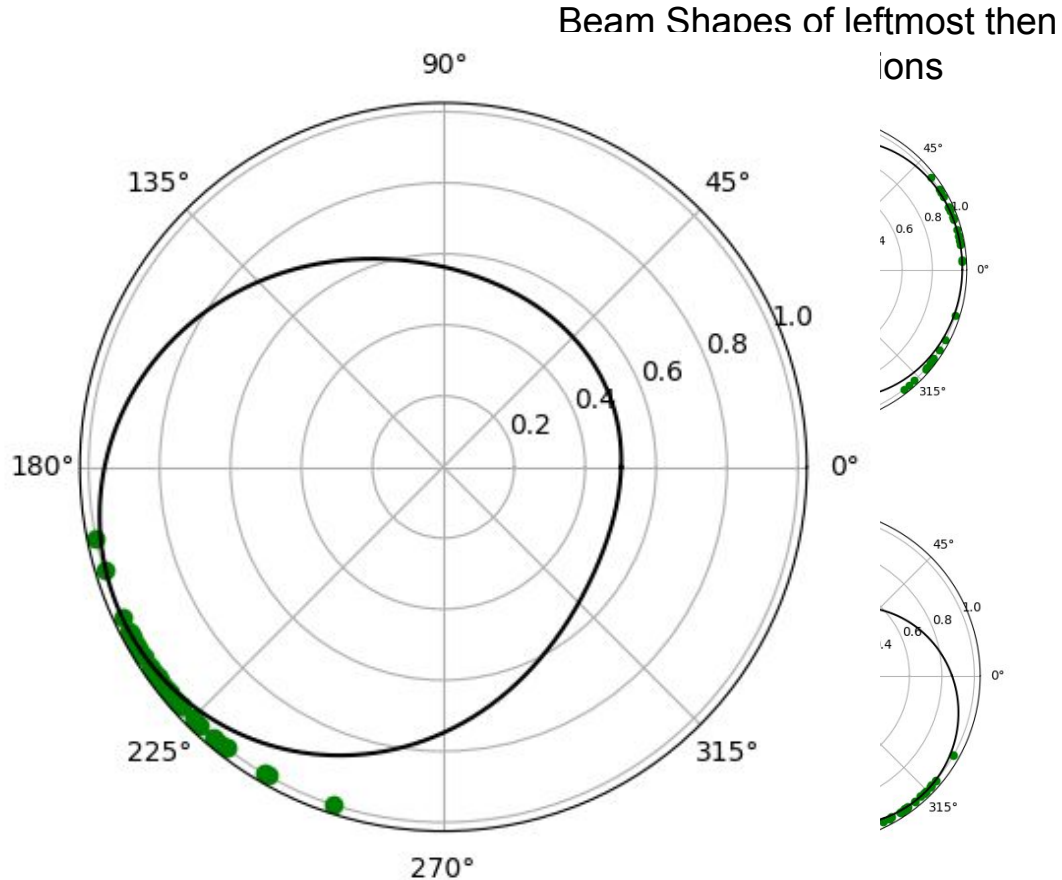
Beamshapes of leftmost then  
topmost stations



# Testing collaborative beamforming with flexibeam



Test setup: 3 stations  
in 6 GMMs - 3



# Conclusions and further works

- Flexibeam induces beamshapes with multiple side-lobes, targeting multiple user clusters at once rather than selecting a single one.
- As expected from the theory, flexibeam leads to higher throughput than matched beamforming.
- Collaborative beamforming is theoretically promising, but is numerically expensive and slow. Optimization algorithms better suited for penalized LASSO problems may improve speed and accuracy.

# Questions?

## References

- [1] P. Hurley and M. Simeoni, “Beamforming towards regions of interest for multi-site mobile networks,” A. Lapidoth and S. M. Moser, Eds. Zurich: ETH Zurich, 2016, Conference Paper.
- [2] —, “Flexibeam: Analytic spatial filtering by beamforming,” in 2016 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2016, pp. 2877–2880.
- [3] O. Açal, P. Hurley, G. Cherubini, and S. Kazemi, “Collaborative randomized beamforming for phased array radio interferometers,” in 2015 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2015, pp. 5654–5658.
- [4] A.-J. Veen and S. Wijnholds, Signal Processing Tools for Radio Astronomy, 05 2013, pp. 421–463.
- [5] H. Krim and M. Viberg, “Two decades of array signal processing research: the parametric approach,” IEEE Signal Processing Magazine, vol. 13, no. 4, pp. 67–94, 1996.
- [6] C. E. Shannon, “A mathematical theory of communication,” Bell Syst. Tech. J., vol. 27, pp. 623–656, 19
- [7] “Pycsou github repository,” <https://github.com/matthieumeo/pycsou>.
- [8] A. Jarret, J. Fageot, and M. Simeoni, “A fast and scalable polyatomic frank-wolfe algorithm for the LASSO,” IEEE Signal Processing Letters, vol. 29, pp. 637–641, 2022. [Online]. Available: <https://doi.org/10.1109%2FIspl.2022.3149377>