Overview drawdown descriptions

This document gives an overview of the empirical equations given in the literature to describe the drawdown height related to the primary water motion of a ship. The following parameters are used in the equations:

- Δh: Maximum water level depression related to the primary water motion [m]
- V_s: Ship speed [m/s]
- V_{lim}: Natural limit speed of the ship [m/s]
- U_r: Maximum return current velocity along the ship [m/s]
- D_s: Draught of the ship [m]
- B_s: Beam of the ship [m]
- L: Length of the ship [m]
- C_m: Midship coefficient of the ship
- A_s: Underwater cross-section of the ship amidships defined as $D_s B_s C_m$ [m^2]
- d_s: Distance from the sailing line [m]
- A_c: Wet cross-sectional area of the undisturbed channel [m^2]
- W_s: Undisturbed channel width at the water surface [m]
- $\bar{h}$: Cross-sectionally averaged water depth defined as A_c/W_s [m]
- α: Slope of the bank [rad]
- W_b: Channel width at the bed level defined as $W_s - 2h \cot \alpha$ [m]
- C_B: Block coefficient of the ship
- g: Gravitational acceleration [m/s^2]

Schijf (1949)

The first attempt to formulate the drawdown or primary water motion a ship creates was based on Bernoulli's theorem and the equation of continuity (Schijf, 1949; Janssen and Schijf, 1953). If the drawdown is set equal to the head difference, Bernoulli's theorem can be stated as follows:

$$\Delta h = \frac{(V_s + U_r)^2}{2g} - \frac{V_s^2}{2g} \tag{1}$$

Using a Lagrangian approach, where the observer moves with the ship, the continuity equation requires the following:

$$V_s A_c = (V_s + U_r)(A_c - A_s - W_s \Delta h) \tag{2}$$

By rewriting Equation 2 for the return current (U_r) and inserting this in Equation 1, an expression for the water level drawdown (Δh) is observed, which can be solved iteratively:

$$\Delta h = \frac{V_s^2}{2g} \left(\left(\frac{A_c}{A_c - A_s - W_s \Delta h} \right)^2 - 1 \right) \tag{3}$$

Van Koningsveld et al. (2023) describes that this formulation is based on a number of assumptions: (1) the flow can be considered one-dimensional, (2) the sinkage of the ship is equal to the water level depression, (3) the ship uses no trim, (4) the channel is straight, prismatic and infinitely long, (5) the shape of the ship is prismatic, (6) the vessel speed is constant, (7) the ships sails along the channel axis, (8) the return current is uniform in the whole channel, (9) the water level depression is uniform over the channel width, (10) no energy losses and (11) there are no influences of ship-initiated waves.

Schijf (1949) including a correction factor

Equation 1 was modified by introducing a correction factor after tests conducted by Delft Hydraulics (1953) revealed certain deviations from the theory:

$$\Delta h = \alpha_s \frac{(V_s + U_r)^2}{2g} - \frac{V_s^2}{2g} \tag{4}$$

Where the correction factor is defined as:

$$\alpha_s = 1.4 - 0.4 \frac{V_s}{V_{lim}} \tag{5}$$

Similarly to Equation 3, an expression for the water level drawdown (Δh) can be observed by rewriting Equation 2 for the return current (U_r) and inserting this in Equation 4.

$$\Delta h = \frac{V_s^2}{2g} \left(\alpha_S \left(\frac{A_c}{A_c - A_s - W_s \Delta h} \right)^2 - 1 \right) \tag{6}$$

The correction factor depends on the ratio between the speed of the ship (V_s) and the natural limit speed of the ship (V_{lim}). Schijf (1949) argues that the natural limit speed of a ship is reached when the return current (U_r) is supercritical. A flow is supercritical when the Froude number is equal to one. Van Koningsveld et al. (2023) shows that the limit speed can be iteratively calculated with the following equation:

$$V_{lim} = \sqrt{g\bar{h} \left(3 \left(\frac{V_{lim}}{\sqrt{g\bar{h}}} \right)^{\frac{2}{3}} - 2 \left(1 - \frac{A_s}{A_c} \right) \right)} \tag{7}$$

Gates and Herbich (1966)

Gates and Herbich (1977) developed an equation for drawdown at the Natural Research Council of Canada and the David Taylor Model Basin (Bhowmik et al., 1981b). This is, in fact, the same equation as Schijf (1949), however, the velocity is expressed in knots, the drawdown in feet and the gravitational acceleration is integrated into the constant.

$$\Delta h = \frac{V_s^2}{22.6} \left(\left(\frac{A_c}{A_c - A_s - W_s \Delta h} \right)^2 - 1 \right) \tag{8}$$

Hochstein (1967)

Hochstein (1967) developed the following relation for drawdown where K is a containment factor which is a function of the ratio of the ship length to beam and the blockage ratio. Almström and Larson (2020) propose that a factor $K = 0.7$ represents the average ship.

$$\Delta h = V_s^2 (C_1 - 1) \frac{C_2}{2g} \tag{9}$$

Where:

$$C_1 = \left(\frac{A_c}{A_c - A_s} \right)^{2.5} \tag{10}$$

$$C_2 = \begin{cases} 0.3 \exp\left(\frac{1.8 V_s}{K\sqrt{g\bar{h}}} \right), & \text{if } \frac{V_s}{K\sqrt{g\bar{h}}} \leq 0.65 \\ 1, & \text{if } \frac{V_s}{K\sqrt{g\bar{h}}} > 0.65 \end{cases} \tag{11}$$

Gelencser (1977)

Another equation was developed by Gelencser (1977) which includes the length of the ship (L_s) and the distance of the ship from the shoreline (d_s) in its definition. This description was derived from prototype and model tests by fitting the calculated drawdown to the measured drawdown (Bhowmik et al., 1981b).

$$\Delta h = 2 \times 10^{-6} \left(\left(\frac{V_s A_s L^2}{d_s \sqrt{A_c}} \right)^{1/3} \right)^{2.8} \tag{12}$$

Dand and White (1978)

Dand and White (1978) developed an expression based on scale ship experiments (Bhowmik et al., 1981b):

$$\Delta h = 8.8 \left(\frac{A_c}{A_s}\right)^{-1.4} \left(\frac{V_s^2}{2g}\right) \tag{13}$$

Bhowmik (1981)

Bhowmik et al. (1981a) relation for drawdown includes the length of the ship and the distance from the shore:

$$\Delta h = 1.03 \frac{V_s^2}{2g} \left(\frac{A_s}{A_c}\right)^{0.81} \left(\frac{L_s}{d_s}\right)^{0.31} \tag{14}$$

Maynord (1996)

Maynord (1996) developed an elaborate definition, including the limit speed and the distance from the shore:

$$\Delta h = C \times \left(\frac{\left(V_s + \left(V_s \frac{A_c}{A_c - A_s} - V_s\right)\left(1.9 - 1.29 \frac{V_s}{V_{lim}}\right)\right)^2}{2g} - \frac{V_s^2}{2g} \right) \times \sqrt{0.75 \left(\frac{A_c}{A_s}\right)^{0.18}} \exp\left(3 \ln\left(\frac{1}{0.75 \left(\frac{A_c}{A_s}\right)^{0.18}}\right)\right) \tag{15}$$

Where C depends on the location of the ship in the fairway, and V_{lim} is the limit speed, which requires to be solved iteratively:

$$C = \begin{cases} 1.65 - 1.3 \frac{d_s}{W_s}, & \text{if } \frac{d_s}{W_s} \leq 0.5 \\ 1.35 - 0.7 \frac{d_s}{W_s}, & \text{if } \frac{d_s}{W_s} > 0.5 \end{cases} \tag{16}$$

$$V_{lim} = \sqrt{2g\bar{h} \left(\frac{A_s}{A_c} + 1.5 \left(\frac{V_{lim}^2}{g\bar{h}}\right)^{1/3} - 1 \right)} \tag{17}$$

Kriebel (2003)

The expression of Kriebel et al. (2003) includes the block coefficient of the vessel (C_B), and does not consider the river and vessel width:

$$\Delta h = D_s(0.0026 C_B - 0.001) \times \exp\left(\left(\frac{-215.8 D_s}{L_s} + 26.4 \right) \frac{V_s}{\sqrt{g L_s}} \times \exp\left(\frac{2.35(1 - C_B) D_s}{\bar{h}} \right) \right) \tag{18}$$

CIRIA (2007)

The analytical description by CIRIA (2007) is very similar to that of Schijf (1949) with the correction factor, described in Section , however, it includes an expression for a trapezoidal river cross-section, where α is the slope of the river banks.

$$\Delta h = \frac{V_s^2}{2g} \left(\alpha_s \left(\frac{A_c}{A_c^*}\right)^2 - 1 \right) \tag{19}$$

$$A_c^* = W_b(h - \Delta h) + \cot\alpha (h - \Delta h)^2 - A_s \tag{20}$$

The correction value is calculated by Equation 5. Furthermore, it includes three methods to calculate the limit speed. The minimum value should be used for further calculations.

$$V_{lim,A} = \sqrt{g \frac{A_c}{W_s} \left(3 \left(\frac{V_{lim}}{\sqrt{g \frac{A_c}{W_s}}} \right)^{\frac{2}{3}} - 2 \left(1 - \frac{A_s}{A_c} \right) \right)} \tag{21}$$

$$V_{lim,B} = \sqrt{\frac{g L_s}{2\pi}} \tag{22}$$

$$V_{lim,C} = \sqrt{g \bar{h}} \tag{23}$$

$$V_{lim} = \min \left(V_{lim,A}; V_{lim,B}; V_{lim,C} \right) \tag{24}$$

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