

# RENDEZVOUS PROXIMITY OPERATIONS USING MODEL PREDICTIVE CONTROL WITH DYNAMIC EQUATIONS DERIVED USING SINDY AND DUAL QUATERNIONS

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The abstract should briefly state the purpose of the manuscript, the problem to be addressed, the approach taken, and the nature of results or conclusions that can be expected. It should stand independently and tell enough about the manuscript to permit the reader to decide whether the subject is of specific interest. The abstract shall be typed single space, justified, centered, and with a column width of 4.5 inches. The abstract is not preceded by a heading of “Abstract” and its length may not extend beyond the first page.

## INTRODUCTION

Space rendezvous missions have been extensively analyzed implementing various forms of constrained path planning and optimization techniques; many of which require high fidelity system identification of the chaser satellite. The combination of improvements in space-certified hardware and modern control techniques have expanded the capability of spacecraft control and the scope of on-orbit services. These advances have begun to democratize industry-changing missions like satellite refueling, inspection, and repair, along with debris removal or avoidance.<sup>?,?</sup> The increased need and application of these mission types has inspired further research in methods to lower costs, increase fuel savings, and find solutions to rendezvous problems with increasingly complex constraints, all while optimizing for computational efficiency.

One of those well-studied rendezvous methods utilizes model-predictive control (MPC), which has been shown to use less fuel due to minimum-fuel trajectory optimization while finding the best approach path within constraints of sensor visibility and safety. Richards and How developed and evaluated a new MPC implementation that optimizes using a novel mixed-integer linear programming method.<sup>1</sup> Fuel saving improvements over traditional MPC methods and the heritage glideslope approach. Singh and Bortolami presented an MPC solution to control one of the Space Shuttle’s approach phases; they optimized for fuel and constrained the sensor line-of-sight and thruster firing directions to avoid plume impingement.<sup>2</sup> They analyzed seven real-world cases of the space shuttle’s standard orbit raising maneuver on its way to the ISS. Cairano and Park shows another example of how an MPC can be used for RPO maneuvers.<sup>?</sup> Their MPC implementation was robust to thrust errors, air drag (low Earth orbit), and solar pressure (geostationary orbits). They optimized considering time-to-dock and fuel consumption while constraining thrust magnitude, line-of-sight, and

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approach velocity. Kannan and Sajadi-Alamdari apply MPC to spacecraft rendezvous maneuvers while considering fuel-efficiency and collision avoidance in their cost function.<sup>2</sup> Park and Zagaris dive deeper into collision avoidance during rendezvous operations. Both linear and non-linear MPC methods are used to optimize fuel while avoiding obstacles, limit thrust magnitudes, and operating within safe entry cones. The linear technique uses hyperplanes to convexify obstacles while the nonlinear solution utilizes ellipsoids. The linear method proved to be much more robust, stable, and computationally practical.

Another control scheme that has been used for rendezvous maneuvers is called adaptive control. Filipe and Tsiotras proposed an adaptive position and attitude (pose) tracking controller for proximity operations that requires no knowledge about the mass and inertia of the chaser satellite.<sup>3</sup> It can also take into account gravitational acceleration, gravity-gradient torque, and other constant disturbance forces and torques. This method can perform system identification of the chaser satellite, given that sufficient conditions are met, and then proceed to approach and then dock the target satellite.

For MPC, adaptive control, and other methods, offline and online system identification techniques are a necessity if high accuracy and precision control is desired, which many times requires costly and expensive volumes of data collection and processing. Within the previously cited adaptive control implementation, Filipe and Tsiotras calculate mass and inertia properties of the system by using known sets of disturbance forces and torques. With this method, the system identification is limited by the measurement quality of the applied forces and torques. There is another system identification technique that was investigated by Kaiser, Kutz, and Brunton that was used within an MPC method called sparse identification of nonlinear dynamics (SINDY).<sup>4</sup> SINDY is a data-driven system identification algorithm which can be used to derive governing equations for a nonlinear dynamic systems. In this work, the authors compare SINDY to dynamic mode decomposition (DMD) and a multilayer neural network (NN). They show that sparse identification is preferred when a low volume of noisy data is available and a fast computation time is required. Also, SINDY sheds light on the underlying nonlinear dynamic equations of the system, instead of just having a black box that sheds little to no physical insight. Another strength of SINDY is that it is fast enough to run on embedded systems.<sup>6</sup> Lastly one of the most useful features of SINDY is that the system identification can be used in with control inputs along with other external forcing. SINDY has a few weaknesses, but they can be mitigated. The first is that a sufficient library of functions must be assumed to identify high-dimensional systems;<sup>7</sup> this can be prevented by increasing the function library size. Another drawback to this technique is that it does not react effectively to abrupt changes in dynamics, but this can be mitigated by using linear methods like DMD while the system dynamics settle.<sup>8</sup>

Robotics and in-orbit servicing go hand-in-hand, and a powerful tool for both of those fields is the concept of dual quaternions. This idea extends the utility found in representing attitude as a quaternion to describe position or translation. This set of numbers is preferred for several reason.

In fact, often controllers or filters that can be used with attitude quaternions can be adapted and used with their dual counterparts.<sup>3</sup>

Filipe and Tsiotras also used a particular set of numbers called dual quaternions, which describe the position and orientation (pose) of a body, and will be explored in this paper. Dual quaternions have been demonstrated to be more computationally efficient, which is especially useful for a processor burdened by the calculations required for MPC.

MPC requires knowledge of the system dynamics, which is not always available to the spacecraft

controller. Data-driven system identification can be used to derive the equations of motion for nonlinear dynamic systems. Brunton, Proctor, and Kutz used data to derive the nonlinear governing equations for a predator-prey model using Sparse Identification of Nonlinear Dynamics (SINDY), including forcing and input functions, which were then implemented in a controller.<sup>5</sup> The same authors also used SINDY to derive the dynamics of oscillators, chaotic Lorenz systems, and fluid vortex shedding behind an obstacle.<sup>6</sup>

The contribution of this paper to the topics herein discuss This paper will use SINDY for system identification within a model predictive controller for space rendezvous proximity operations (RPO). The improved efficiency from using dual quaternions over regular quaternions will be explored.

## **THIS IS A SAMPLE OF A GENERAL SECTION HEADING**

### **This Is a Sample of a Secondary (Sub-Section) Heading**

*Equations.*

*Abbreviations.*

*Figures.*

*Graphic Formats.*

*References and Citations.*

## **MANUSCRIPT SUBMISSION**

### **Journal Submission**

## **CONCLUSION**

## **ACKNOWLEDGMENT**

## **NOTATION**

## **APPENDIX: TITLE HERE**

### **Miscellaneous Physical Dimensions**

## **REFERENCES**

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