Optimal Guidance and Thruster Control in Orbital Approach and Rendezvous for Docking using Model Predictive Control

Conference Paper · July 2010

DOI: 10.2514/6.2010-7754

Leena Singh
Draper Laboratory

Simone B Bortolami
BAE Systems, Inc

Optimal Guidance and Thruster Control in Orbital Approach and Rendezvous for Docking using Model Predictive Control

Leena Singh*, Simone Bortolami†, Lance A. Page‡

The Charles Stark Draper Laboratory, Cambridge, MA 02139

In this paper, we present an integrated approach for proximal guidance and control of a docking vehicle during its terminal rendezvous and approach flight phases. The paper particularly focusses on one rendezvous orbit transfer sequence in the Space Shuttle's standard approach protocol of orbit-raising maneuvers during its approach to ISS: from MC4 (mid-course point 4) to MC5 directly beneath the Space Station. In addition to the relative position and velocity requirements in this transfer, optimal attitude pointing requirements are imposed on the shuttle, as this transfer is in close proximity and the shuttle must orient so that its Trajectory Control Sensor (TCS) can maintain *line of sight* to the ISS docking port to acquire ISS-relative navigation (relative position, velocity and attitude). The performance of the optimal relative translation and orientation guidance and control is shown in this paper utilizing the distinct, known mass properties and state dispersions from 7 recent shuttle missions (STS114-STS121).

We use model-predictive control (MPC) to define and solve the optimal transfer guidance problem. The approach divides the problem into separate translational and associated attitude trajectory design phases that operate sequentially and replan at each guidance update period, consistent with MPC designs. To solve the attitude guidance problem, we introduce a method that analyzes a series of constrained optimal control problems at various candidate attitudes and torques to best achieve the desired translational trajectory synthesized in the previous step; the (optimal) attitude control problem is produced from a robust regression analysis of the optimal trajectory solutions.

A linear programming solver selects and optimally schedules the vehicle jets to deliver the desired body force and torque commands subject to range-dependent plume impingement constraints imposed for safety of the ISS at ranges closer than 1000 ft.

I. Introduction

Orbital rendezvous with a satellite in stationary low-earth orbit such as with the International Space Station (ISS) has been extensively analyzed.[1–4] Beyond the dynamic considerations of the problem, however, are path and operational control constraints derived from satellite safety considerations. Various forms of constrained path planning and optimization have been analyzed using modern optimization and search

*Aerospace and Strategic GN&C Division, Draper Laboratory, AIAA Senior Member

†Chief Science Officer, AgustaWestland Helicopters. AIAA Senior Member. This work was performed while Dr. Bortolami was at Draper Laboratory.

‡Mission Systems Division, Draper Laboratory, AIAA Senior Member

tools in recent years.[5,6] However, as the capability and availability of targetting aids on the target (ISS) and visiting vehicles for cooperative proximity operations and docking have increased, the nature of the approach problem and the operational constraints during it have changed. With the incorporation of the laser-based Trajectory Control Sensor (TCS) on the shuttle, its capability to acquire, target and guide to the docking ports on the ISS from further away have increased the safety of the approach and docking maneuver. Nevertheless, it introduced new and different trajectory and attitude control challenges to incorporate new maneuvers and constraints required for a safe transport. To aid in these challenges, while the shuttle continues to be astronaut pilotted during these approach (and departure) phases, it flies with a laptop-based situational awareness unit called Rendezvous and Proximity Operations Program (RPOP)[7] that collects the raw TCS sensor measurements and computes and presents a recommended trajectory and attitude maneuver profile given the shuttle's current relative state to the ISS. The pilots are able to view this candidate trajectory and then may or may not choose to follow it.

In this paper, we present a new onboard optimal guidance and targetting approach using Model Predictive Control which explictly incorporates the trajectory constraints associated with this transfer. A novel contribution of this approach is its ability to explicitly handle the trajectory state, control and mission safety constraints. Another contribution of the paper is the unique formulation of the coupled atitude control problem to meet TCS pointing constraints as well as optimally deliver the desired trajectory force.

One aspect of the shuttle rendezvous problem that has not really changed over time has been the approach and capture protocol defined as a sequence of orbit-raising maneuvers by the visiting vehicle to monotonically approach the orbit of the ISS and then safely rendezvous to dock along its R-bar . One orbit-raising transfer in this sequence prior to the final approach-to-docking is a transfer from a point below and behind the ISS, termed Mid-Course point4 (MC4), to a set-point 600 ft directly beneath ISS at MC5 with an associated orientation and velocity; this transfer is the focus of this paper. Figure 1 shows this approach segment.

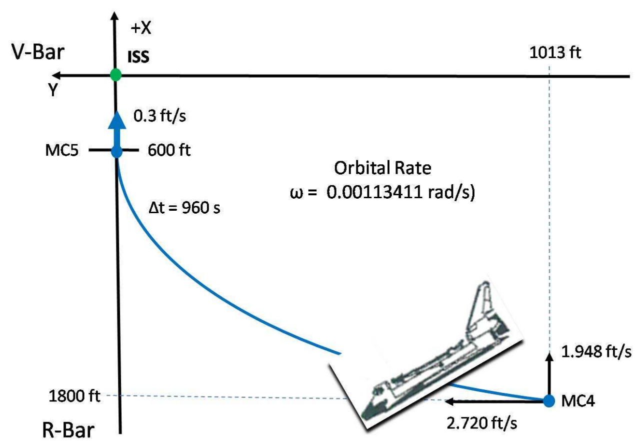


Figure 1. Visiting Vehicle Orbit transfer from MC4 to MC5

As the Orbiter approaches the ISS, several different sensor units are available to provide relative navigation measurements including: ground tracking data, radar and lasers. At ranges under ˜1 km, the Trajectory Control Sensor (TCS) located in the Orbiter's payload bay tracks reflectors on the ISS. RPOP uses this information together with the shuttles ownship IMUs, to produce very accurate relative position and velocity estimates with an onboard Extended Kalman Filter.

Consequently, in addition to the translation and attitude requirements at the transfer points MC4 to MC5, the approach guidance algorithm must adhere to additional constraints throughout the flight. During this transfer, the Orbiter navigates using the TCS and can close its guidance and control loops with this relative state information. Therefore, the shuttle must acquire and maintain a suitable orientation such that the TCS cone maintains *line of sight* (*LOS*) to the ISS docking port through to the end of the docking maneuver, see Figure 2. Lastly, to accommodate station safety concerns, NASA imposes plume impingement constraints: within approximately 1000 ft from the ISS, during this R-bar maneuver, the shuttle may not fire those of its primary thrusters whose would plumes fire towards the station. In this maneuver, given the pointing requirements, it is tantamount to disallowing the shuttle's up-firing (+Z) jets. Adhering to this constraint is thus called the LOW-Z control mode in the shuttle guidance lexicon.

Due to the mode-dependent guidance objectives and the existence of these hard guidance constraints, we present a Model Predictive Control (MPC) based approach to the rendezvous problem. MPC is a compelling onboard control or guidance methodology (appled here to outer-loop control) for problems with time-varying objectives and which impose hard state or control constraints.[8–11] Compounded with the ability to formulate the optimal control problem in such a way that a modern computation system (such as most modern-day laptops) can solve in bounded time,[8,9,12] it presents a powerful tool for trajectory control problems such as this. In this paper, a new MPC-based method computes real-time, low delta-V, low-dispersion translational and attitude rendezvous trajectory controls, based on relative navigation states. Results for one shuttle mission (STS118) are presented in some detail, and results for 6 additional missions are summarized.

The paper is organized as follows: Section II formally defines the overall approach guidance problem for this flight phase. Section III presents the MPC-based translational and Section IV, the optimal attitude guidance algorithms for a fuel-efficient and constraint-satisfying transfer. Section V presents the linear programming-based optimal jet selection logic for the complete (44 jet) shuttle thruster array. Finally, Section VI presents simulation results evaluated using initial conditions and the known mass and inertia properties from shuttle missions STS114-STS121.

II. Guidance Problem Definition: Models, Objectives and Constraints

This paper addresses the problem of computing the optimal thruster burns to transfer a visiting vehicle from its true *dispersed* or off-nominal initial conditions about the MC-4 point, to MC-5, a point on the R-bar. The [R-bar, V-bar, Cross-bar] reference frame conventions are defined in Figure 2. The transfer between MC4 and MC5 is mostly a planar one as the vehicle starts this mission parked in the proximity of MC4 which itself is defined to be in the ISS's orbital plane behind and below ISS at $[-1800, -900, 0]$ ft. MC5, the target point for this transfer, is located on the Station's R-bar, directly below the ISS docking port at $[-600, 0, 0]$ ft, and the shuttle must acquire MC5 with a net ISS-relative velocity of 0.3 fps up the R-bar to coast up to dock. There are variances or dispersions about MC4 due to transfer dispersions during prior maneuvers. The radar-based Trajectory Control Sensor (TCS) on the shuttle must be continuously oriented towards the docking port (defined as $(0, 0, 0)$ for this analysis) within the TCS's acquisition window $(-20,^\circ +10)^\circ$. Finally, at ranges of under 1000 ft from the station, the vehicle is not allowed to use jets that strongly plume up towards the station – in shuttle parlance, we change from a NORM-Z jet selection mode to a LOW-Z jet firing mode.

We define the trajectory and attitude control problem to find the sequence of jet controls that minimizes the amount of delta-V expended to transfer from the initial conditions about MC4 to the terminal MC5 desired position and velocity state subject to range-dependent operating and control constraints. These controls sequences are computed online, in real-time during the transfer. Constraints imposed during the transfer include the following: (a) the TCS line-of-sight pointing constraints must always be obeyed, (b) the only admissible forces and torques are those that can be generated by superposing the full (in NORM-Z

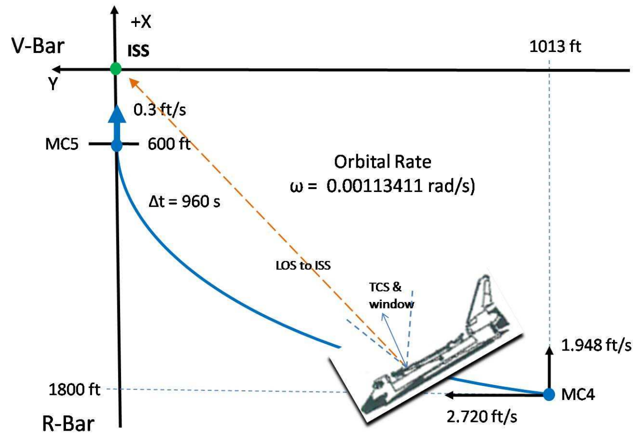


Figure 2. Path and permissible attitude of Shuttle in ISS-LVLH frame with TCS constraints

mode or normal mode) or reduced (in LOW-Z mode) set of jets on the shuttle.

Furthermore, each shuttle jet is an on-off jet and produces a constant average thrust when active. The jets are pulse width modulated, and can only be activated in an integer number of pulses - hence the total force output by any jet is quantized. The 44 jets (38 Primary Reaction Control System and 6 Verniers) associated with the shuttle each produce a known force and a torque characterized in the shuttle body frame. A table of the complete jet force/torque impulse vectors is maintained in a thrust map specific to each shuttle mission and is used to compute the duration of the burns necessary to produce the commanded forces from the appropriate jet.

A. Outline of Guidance and Control Approach

The transfer guidance and control approach presented here solves a sequence of sub-problems to eventually synthesize the optimal shuttle jet firing solutions to effect the MC4-MC5 transfer appropriately. These will be outlined here and developed in subsequent sections. The onboard guidance and control loops run regularly at fixed update rates throughout the transfer.

1. At each guidance update time t_j, we first define a finite horizon optimal control problem in the ISS-relative LVLH space to transfer from the current to the desired (MC5) state. An overbound on the approximate total transfer time is assumed known and the objective in this step is to compute the sequence of controls firing at fixed sparse intervals within this prediction interval. This stage of the problem thus computes the optimal sequence of *Cartesian* forces, $f^{opt}(t_j : t_N) = \mathbf{f}_x^*$ in the ISS-LVLH frame, to transfer the position to MC5.

2. The second objective is to determine the shuttle orientation to (a) effect the commanded LVLH forces in the most fuel-efficient way i.e., by best choice of shuttle body orientation and (b) to observe the TCS pointing requirement when the range to the ISS is within 1000 ft.

3. Finally, we solve the optimal jet selection problem to schedule and identify the individual jet burns at the given control interval, given their impulse characteristics. This is solved optimally from the admissible jet force map using linear programming methods.

Recognizing that in this transfer, roll and yaw should remain small and close to zero, and that the attitude maneuvers will be primarily pitch maneuvers to orient the spacecraft to effect a fuel-optimizing f_j^{opt} and maintain the TCS pointing constraints, we partition the attitude dynamics to separate the pitch dynamics from the roll-yaw dynamics. Attitude control then consists of identifying the necessary pitch angles and associated pitch torques that will properly meet the pointing and commanded LVLH force ($F^{opt}(t_j : t_N)$) realization constraints and then regulating roll and yaw states to stabilize roll (see[13]).

Note that the constraints here are two-fold. First, the TCS pointing constraint requires that the LOS from the instrument on the back of the shuttle be oriented towards the Space Station within a $(-20, +10)^\circ$ window. Secondly, the applied forces and torques are formed from a superposition of jet firings using the (known) jet selection matrix. The admissible jets available for use change from the full set of 44 jets when the shuttle range to ISS is more than 300 m (or 1000 ft) (NORM-Z jets) to a reduced set (derived by eliminating the upward firing jets) when the shuttle is within the safety range.

B. Summary of Necessary Dynamic models

The critical dynamics equations and key parameters used here-in are summarized next. The shuttle relative-translational dynamics states are computed relative to the Space Station LVLH-frame. The key orbital parameters used for the Space Station in this work are in Table 1.

Semi-major axis	6.7142574E+6 m
Eccentricity	7.252E-4
Inclination	51.72133 deg
Ascending node longitude (RAAN)	18.13 deg
Argument of Periapsis	43 deg

Table 1. ISS Orbit Parameters used in this study

Because the orbit of the Space Station is so very nearly circular, it is convenient and quite accurate to use the linearized Hills or Clohessy-Wiltshire equations to represent the relative dynamics of the shuttle. For a more general treatment, the linear-time-varying gauss equations can just as easily be used to capture the relative dynamics.[14,15] This work primarily used the latter but results were virtually identical to those found using the C-W equations and the ease and simplicity of formulation of the C-W will be leveraged in this paper. The relative dynamic model for the visiting vehicle in the rotating LVLH frame of the Space Station may thus be written as:

$$\dot{x} = A^{ISS}x + B^{ISS}u \tag{1}$$

where A^{ISS}, B^{ISS} are the dynamic and forcing matrices for the CW equations as defined by Battin,[1] and x represents the target (ISS) relative translational states of the chaser (shuttle) vehicle.

Additionally, the chaser's nonlinear rotational dynamics are quite generally written as:

$$J\dot{\omega} = \omega \times J\omega + \tau \tag{2}$$

Essentially, the MC4 to MC5 transfer with the TCS constraints is a pitch maneuver in the attitude space. Consequently, the above attitude dynamics are linearized about the associated nominal pitch rate $\dot{\theta}$ to yield equations in terms of Euler angles (α, θ, ξ) and developed in the citation.[13] These dynamics are partitioned into pitch plane dynamics which generally see larger angles, and roll-yaw dynamics which are generally small angles. The attitude dynamics equations are used to produce the desired pointing controls and eventually synthesize the jet firing sequences required to effect the transfer.

Finally, there are state constraints that must be observed during the transfer.

$$\theta \in \theta^{los} + [TCS_{low}, TCS_{high}] \tag{3}$$

where the procedure to compute θ^{los} will be formally introduced in Section IV but basically represents the pitch angle required to point the TCS directly to the origin of the ISS reference frame.

Finally, we must introduce the shuttle force and torque maps. For each shuttle mission, a jet map is available that represents the impulsive vectors in the shuttle body frame of the force m_l and torque κ_l produced by each jet individually firing for one on-interval. We introduce a unitized force map, M_r and a scaled torque map, T_r formed by normalizing each jet force vector (scaling by the force magnitude) and then scaling the torque vectors by the same amount to produce a normalized jet force map:

$$\mathcal{M}_r = \begin{bmatrix} \hat{F}_1 & \hat{F}_2 & \dots & \hat{F}_{N_{jets}} \end{bmatrix} \tag{4}$$

$$\mathcal{T}_r = \begin{bmatrix} \bar{\tau}_1 & \bar{\tau}_2 & \dots & \bar{\tau}_{N_{jets}} \end{bmatrix} \tag{5}$$

$$\mathcal{S} = \begin{bmatrix} s_1 & s_2 & \dots & s_{N_{jets}} \end{bmatrix} \tag{6}$$

A force command of $2s_1$ on jet 1 would therefore recompute as $2s_1[\hat{F}_1; \bar{\tau}_1] = 2[m_1; \kappa_1]$ and be applied with twice minimum on-times of jet 1.

III. Finite Horizon Optimal Control for Trajectory Guidance

First we compute the forces in the LVLH frame that will transfer the shuttle from its current position and velocity state to the MC-5 point with the desired terminal velocity. We fix a prediction horizon, T_H by the expected arrival time, and choose a discrete number, N, of (uniformly spaced) time-points over this entire horizon at which forces may be impulsively applied such that at the end of this prediction interval, the spacecraft is at the desired terminal state (MC5). This arrival problem is articulated as the following, familiar finite horizon optimal control problem:

$$J_i = \min_{u_i, u_{i+1} \dots u_N} (\textstyle\sum_{j=i}^{N} u_j^T R u_j + (X_{N+1} - X_{des})^T S (X_{N+1} - X_{des})) \tag{7}$$

The continuous dynamics of Equation 1 are transformed into the equivalent discrete time system:

$$X_{k+1} = \Phi_{cw}^{ISS} X_k + \Gamma_{cw}^{ISS} u_k \tag{8}$$

where X consists of the ISS-frame relative position and velocity, and X_{des} is the desired value of X at MC5. Note that the optimizer penalizes the controls and terminal state error only. Using a sufficiently high terminal error penalty weight S, we avoid constraining terminal error and instead rely upon the high terminal penalty to produce sufficient terminal accuracy.

Recognizing that the system response can be predicted forward in time using the system models and formulating the state and input grammians,[12] the plant response at any future time can be written as:

$$\begin{aligned} X_{k+M+1} = \; & (\Phi_{cw}^{ISS})^{M+1} X_k + [(\Phi_{cw}^{ISS})^M \Gamma_{cw}^{ISS}, (\Phi_{cw}^{ISS})^{M-1} \Gamma_{cw}^{ISS}, \dots, \Phi_{cw}^{ISS}\Gamma_{cw}^{ISS}, \Gamma_{cw}^{ISS}] \cdot \\ & [u_k, u_{k+1}, \dots, u_{k+M-1}, u_{k+M}]^T & (9) \\ = \; & F_{k+M} X_k + G_{k+M} U_k & (10) \end{aligned}$$

where now, the state at time index (k+M+1) is expressed in terms of the current state X_k and a mapping of the vector U_k of *all* the future controls between time k and time $k + M$. Note that care must be taken in producing the grammians above; the control computed at time t_k is applied impulsively at time t_k (or almost impulsively modulo the PWM action), rather than being applied with a *zero order hold* for the duration off the guidance update period.

Finding the sequence of error regulating optimal LVLH forces in Eq. 7 amounts to finding the optimal controls $u_k, u_{k+1} \dots u_N$. We use the methods developed in[10,11] to require that the minimizing controls to Eq. 7 conform to the optimal feedback-feedforward control structure

$$u_j^* = K_j X_j + FF_j(X_{des}) \tag{11}$$

and solve the bounded Riccati equation as developed in the references above, and where $j \in [i, N]$. This produces the following optimal time-varying structure for the feedback and feedforward control matrices:

$$\begin{aligned} K_j &= -R^{-1} G_j^T G_{j+1}^{-1} M_{j+1} F_j && (12)\\ FF_j &= R^{-1} G_j^T G_{j+1}^{-1} (F_j^T G_{j+1}^{-1})^{(N-j)} X_{des} && (13)\\ \text{where:} \quad G_{j+1} &= I + M_{j+1} B R^{-1} G_j^T && (14)\\ M_{j+1} &= (F_j^T G_{j+1}^{-1})^{N-j} S F_j^{N-j} && (15) \end{aligned}$$

With this formalism, the feedback and feedforward control gain pairs (K_j, FF_j) can be computed for each of the $j \in [i, N]$ points in time that were designated as impulse action points. Note that F_j, G_j represent the appropriate grammians computed above.

The above series of control gains from current time t_k to the predicted arrival time are re-computed at the outer-loop guidance rate, or every time the trajectory is updated. The prediction horizon and the number of steps within that horizon is reduced at every update interval, consistent with the reducing time-to-go, to preserve the terminal state and time conditions. The first in the sequence of LVLH frame controls is the control to be applied at the current time t_k:

$$u_k^* = K_k X_k + FF_k$$

This control will later be transformed into the body frame and eventually into a jet firing sequence. But note that although only the first control vector is ever utilized, the entire sequence of controls is produced at every update cycle.

Meanwhile, while u_k^* represents the force to be applied at time t_k, the stack of all $u_{j=k:N}^*$ comprises the optimal control sequence computed at time t_k and is represented as f_k^{opt}:

$$f_k^{opt} = \begin{bmatrix} u_k^*, & u_{k+1}^*, & u_{k+2}^*, & \dots u_N^* \end{bmatrix} \tag{16}$$

f_k^{opt} had been introduced earlier to represent a certain optimal forcing sequence.

IV. Attitude Command Generation for TCS and thrusting efficiency

Given the desired relative-LVLH forces that will produce the trajectory to effect the transfer to MC5, we next determine the associated attitudes that will realize both the TCS pointing constraint and ensure that the commanded inertial force can be optimally realized with the shuttle's thruster array (minimizing total delta-V). The correspondence between a shuttle body-frame force and one in the LVLH frame (such as u_j^*) is as follows:

$$f^{lvlh} = R_b^{lvlh}(\phi, \theta, \xi) \cdot \mathcal{F}_b = R_b^{lvlh}(\phi, \theta, \xi) \cdot (\sum_{l=1}^{N_{jets}} f_l^{body}) \tag{17}$$

where: f^{lvlh} is a force in the LVLH frame, R_b^{lvlh} is the rotation matrix from the body to the LVLH frame, F_b is the resolved force in the body frame, N_{jets} is the number of admissible shuttle jets being utlized, and f_l^{body} is the total force produced by the l^{th} jet in the body frame. Hence to realize an command u_j^*, we need to solve two problems: (1) to find the trajectory of desireable (optimal) shuttle attitude angles $(\phi_i, \theta_i, \xi_i)$ and (2) compute the array of jet forces f_l^{body} from the known force unit vectors stored in the jet thrust matrix, M_r.

Given the optimal control gain structure computed above, we can now use the control sequence at the scheduled burn times to synthesize the associated Cartesian solution trajectory given the plant model - let this Cartesian trajectory planned at time t_j be: L_j^{nom} where $L_j \in R^{6\times M}, M = N - j + 1$. (If the plan is correct, $L_{j=1:N}^{nom}(:, N)$ converges to $MC5$).

Initial roll and yaw angles are small and the attitude controller will attempt to keep them small over the course of the transfer - for planning purposes, we assume that these will be zero. Along this candidate trajectory we identify the shuttle pitch angles that would keep the TCS oriented towards the Space Station (at the origin of this relative frame). This is referred to as the *nominal pitch profile*, θ_j^{nom}. If the initial condition at MC4, at the start of the run, is outside the admissible TCS pointing window, we blend this nominal pitch trajectory with the initial angles to produce a smooth transfer to the desired TCS-pointing profile; this modified blended trajectory then becomes the *nominal pitch profile.*

An attitude guidance objective is then to ensure that vehicle pitch angle stays within a [+10,° -20]° bound of this nominal pitch profile to ensure that the TCS constraints are always met. The nominal pitch profile is not particularly necessary here - it was only needed to produce the bounds on the admissible pitch angles during this transfer.

Now, the attitude guidance problem must evaluate the optimal pitch angle, within the admissible range just computed that would minimize the total delta-V subject to realizing the translational acceleration sequence f_j^{opt}. To do so, we set up the following local optimal control problem that attempts to penalize pitch angle changes (effectively, minimize pitch rates) and torques along the cartesian trajectory in the following way:

$$J_\theta(i) = \min_{\tau_{j=i:N}} \sum_{j=i}^{N} \begin{bmatrix} \theta_j - \theta_j^{nom} \\ \tau_j \end{bmatrix}^T \begin{pmatrix} Q_{\theta,j} & S_{\theta,j} \\ S_{\theta,j}^T & R_{\theta,j} \end{pmatrix} \begin{bmatrix} \theta_j - \theta_j^{nom} \\ \tau_j \end{bmatrix} \tag{18}$$

$$\text{Subject to:} \quad \begin{bmatrix} \theta_{k+1} \\ \dot{\theta}_{k+1} \end{bmatrix} = \Phi_\theta \begin{bmatrix} \theta_k \\ \dot{\theta}_k \end{bmatrix} + \Gamma_\theta \tau_k \tag{19}$$

$$\text{and:} \quad \theta_k \in \theta_k^{nom} + [-20, +10]^\circ, \quad \forall \ \ k \in [i, N] \tag{20}$$

Note that this is just the generalized quadratic programming form of the linear quadratic regulator problem in state $[\theta, \dot{\theta}]^T$, control τ and weighting matrix $T_j = (Q_{\theta,j}, S_{\theta,j}, R_{\theta,j})$.

Now, while one could arbitrarily choose the weights $(Q, S, R)_{\theta,j}$ to solve a constrained linear quadratic regulator problem of the form above, it would not meet our ultimate goal to find the desired pitch angles and associated command torques that *optimize the jet selection problem to achieve* f_j^{opt}. This tells us that matrices $(Q, S, R)_{\theta,j}$ must be specified in a way to correlate to the desired force profile.

So to define problem above, we declare the following.

Lemma 1. *The optimal solution to the above constrained linear-quadratic pitch control problem must also optimally satisfy the jet-selection problem to realize the commanded translation trajectory* L_j^{opt}.

Thus the optimal solution to the attitude control problem will setup the optimum shuttle pointing configuration to solve the jet selection problem. Therefore, we define and solve the following sub-problem for the suitable Q, R, S matrices, conditioned on θ_j^{nom}.

For a discrete set of small pitch angles in $[\theta_{low}, \theta_{high}]$ about the *Nominal Pitch Profile* at the current time, θ_j^{nom}, and a candidate range of pitch axis torques $[\tau_{low}, \tau_{high}]$, we define the following series of linear programming (LP) problems:

$$\forall \quad \delta\theta_m \in \quad [\theta_{low}, \theta_{high}] \tag{21}$$

$$\text{and} \quad \forall \quad \tau_n \in \quad [\tau_{low}, \tau_{high}] \tag{22}$$

$$J^*(m,n,j) = \min_{\eta_{1:N_{jets}}} \textstyle\sum_{l=1}^{N_{jets}} \eta_l \tag{23}$$

$$\text{Subject to:} \quad \eta_l >= 0 \tag{24}$$

$$\text{and:} \quad R_b^{lvlh}(0, \theta_j^{nom} + \delta\theta_m, 0)\mathcal{M}_r[\eta_1, \eta_2 \ldots, \eta_{N_{jets}}]^T = u_j^* \tag{25}$$

$$\text{and:} \quad \mathcal{T}_r[\eta_1, \eta_2 \ldots, \eta_{N_{jets}}]^T = \boldsymbol{\tau}_n \tag{26}$$

where $\mathcal{M}_r$ and $\mathcal{T}_r$ were defined as the normalized jet force and scaled torque matrices respectively, and $\boldsymbol{\tau}_n = [0 \ \tau_n \ 0]^T$ represents the torque vector.

Equations 23-26 collectively define a straightforward linear programming problem. With Equations 21-22, the above become a series of very straightforward LP problems.

Upon solving the above LPs, we now know the value of the *optimum cost* (=sum of the magnitude of the force applied by each jet utilized) $J^*(m,n,j)$ for each point along a predicted trajectory L_j^{opt}, that will, at each pitch angle variation about θ_j^{nom}, realize the desired force u_j^* and achieve the candidate torque using the array of jets.

Next, the variation of this optimal cost with $\delta\theta_m$ and τ_n is approximated as a quadratic function $H(\delta\theta, \tau)$, using regression analysis:

$$H_j(\delta\theta_m, \tau_n) = \quad [1, \delta\theta_m, \tau_n, \delta\theta_m^2, \tau_n^2, \delta\theta_m \cdot \tau_n] \cdot C_{lp}(j) \approx J^*(m,n,j) \tag{27}$$

This function captures the convex cost function manifold whose minimum we wish to evaluate and sets up the conditions to satisfy Lemma 1 to pose the optimization problem we need to solve, i.e. that the minimizing attitude control problem also minimizes the fuel spent to realize the trajectory L_j^{opt}.

For each time step j, we used the Mathworks robust regression function `robustfit()` to compute the vector of regression coefficients $C_{lp}(j)$. The pitch guidance cost function which directly captures our overall goal of minimizing the cumulative thrusting force magnitudes, assuming that the optimal jet selection is performed for the desired trajectory, while capturing the effect that the pitch angle has on thruster selection is:

$$J_\theta(i) = \min_{\tau_{j=i:N}} \textstyle\sum_{j=i}^{N} H_j(\theta_j - \theta_j^{nom}, \tau_j) \tag{28}$$

$$= \min_{\tau_{j=i:N}} \textstyle\sum_{j=i}^{N} \begin{bmatrix} \theta_j - \theta_j^{nom} \\ \tau_j \end{bmatrix}^T \begin{pmatrix} Q_{\theta,j} & S_{\theta,j} \\ S_{\theta,j}^T & R_{\theta,j} \end{pmatrix} \begin{bmatrix} \theta_j - \theta_j^{nom} \\ \tau_j \end{bmatrix} + \begin{bmatrix} D_{1,j} & D_{2,j} \end{bmatrix} \begin{bmatrix} \theta_j - \theta_j^{nom} \\ \tau_j \end{bmatrix} \tag{29}$$

$$\text{Subject to:} \quad \begin{bmatrix} \theta_{k+1} \\ \dot{\theta}_{k+1} \end{bmatrix} = \Phi_\theta \begin{bmatrix} \theta_k \\ \dot{\theta}_k \end{bmatrix} + \Gamma_\theta \tau_k \tag{30}$$

$$\text{and:} \quad \theta_k \in \theta_k^{nom} + [-20, +10]^\circ \quad \forall \ k \in [i, N] \tag{31}$$

The weighting matrices $Q_\theta, S_\theta, R_\theta$ are formed from the correlation coefficients as follows:

$$Q_{\theta,j} = C_{lp,j}(4), S_{\theta,j} = C_{lp,j}(6)/2, R_{\theta,j} = C_{lp,j}(5), D1_{\theta,j} = C_{lp,j}(2), D2_{\theta,j} = C_{lp,j}(3) \tag{32}$$

While this is similar in structure to the linear quadratic regulator form Equation 18, the additional linear terms, particularly the D_1 term, can attract the pitch trajectory away from θ^{nom} when doing so reduces the cumulative thrusting forces required. We have found that the computations for the parameters of $H_j(\delta\theta_j, \tau_j), j \in \{1, N\}$ only need to be done once for a given mission, as the shuttle translation and attitude trajectory generally remain quite close to the initial solution synthesized at the first step. It is straightforward to use the dynamic constraint in Equation 30 to substitute for θ_j and $\dot{\theta}_j$ in Eq. 29 and 31 to define and solve in terms of the independent variables $\tau_{j=:N}$. Solving this optimization problem produces the optimal pitch axis torque τ_i which should effect the optimal pitch angle at the next computation step.

Note that this only produces the pitch torque command. The vehicle roll-yaw torques which we have decoupled from the pitch control are computed from a roll-yaw regulator form of a controller: $\tau_{lat}(j) = -K_{lat}[\phi_j, \xi_j, \dot{\phi}_j, \dot{\xi}_j]^T$ which augments the $\tau_{\theta,j}$ derived above to produce the torque vector $\boldsymbol{\tau}_j$. See the reference by Walker and Hall[13] for a treatment and discussion of the issues of the controller for the lateral attitude dynamics which we quite simply effect as a 2-dimensional linear regulator.

V. Optimal Jet Selection - a Simplex Problem

Now, we have all the ingredients necessary to solve the jet selection problem and we use the convenient and popular linear programming simplex method to identify the firing jets and their firing durations. The force-torque map $[M_r, T_r]$ captures the force and torque vectors produced by each jet represented in the shuttle body-frame. We had the optimal LVLH force u_k^* desired to be applied at the current time, t_k. We know the current attitude of the vehicle. We also now have the current torque command vector expressed in the shuttle body-frame that will put the vehicle into a desirable orientation for the next outer-loop guidance command cycle.

Now, we solve the jet selection LP problem already introduced previously as part of a problem definition step, this time used to compute the jet pulses that will be applied at the present moment only. So, now, we solve the LP problem:

$$J = \min_{\eta_{1:N_{jets}}} \sum_{l=1}^{N_{jets}} \eta_l \tag{33}$$

$$\text{Where:} \quad \eta_l >= 0 \tag{34}$$

$$R_b^{lvlh}(\phi_j, \theta_j, \xi_j) M_r \cdot [\eta_1, \eta_2 \ldots, \eta_{N_{jets}}]^T = u_i^* \tag{35}$$

$$T_r * [\eta_1, \eta_2 \ldots, \eta_{N_{jets}}]^T = \tau_i \tag{36}$$

The N_{jets} vector η contains the scaling to be applied to each of the N_{jets} jets. Note that N_{jets} and indeed even the mapping matrices M_r, T_r reflect the admissible set of jets depending on the guidance mode. In LOW-Z mode at ranges of under 300 m (1000 ft), the up-firing jets are removed from the map and N_{jets} in this treatment accordingly decreases.

The jets themselves are finite height impulses with a prescribed minimum on-time, (see Equation 6 for the introduction to the scaling). Consequently, the scaling parameter for the m^{th} jet is itself scaled by the known magnitude of the jet pulse, s_m - this tells us how long that particular jet would have to fire to approximate the desired total force required from that jet. Since the jet application is itself quantized, the pulse width must be adjusted to ensure that an integral on-time firing is commanded. This finite height force on a firing jet, firing for some T time-elements is applied to the vehicle.

VI. Performance Monitoring and Results in Shuttle Experiments

We present simulation results realized by using the above guidance and control algorithms to produce desired trajectory and attitude solutions between two near-ISS proximal points with the space shuttle. Depending on the initial conditions at MC-4, i.e. the shuttle's true dispersed coordinates when it completes the transition to MC-4, the results will vary. There are variances or dispersions about MC4 due to transfer dispersions during prior maneuvers these dispersions encountered in each LVLH axis vary on the order of: $[(+380, -10), (180, -50), (\pm 50)]$ ft as per the test data on which our analysis was performed. Of course, its mass and inertia properties also vary from shuttle mission to shuttle mission. Depending on these initial parameters then, the results vary in the amount of time taken for the transfer and the amount of fuel expended. We will present simulation results for shuttle missions STS114 to STS121.

All initial conditions are within some bounded off-nominal positions about MC4 around $(-550, -275, 0)$ m (along ISS-R-bar , ISS-V-bar , ISS-Cross-bar) and velocities dispersed about: $(+0.5, 0.8, 0)$ m/s and some initial attitude.

We are also given the complete jet force-torque map for the shuttle (not reproduced in this paper). Likewise, the mass and and inertia properties for each of the missions are known, but will not be repeated herein.

Simulation results generated for initial conditions at MC4 in one particular shuttle mission, STS118, given its published mass properties, are presented here in detail.

To recapitulate, the objective as described in the methods above is to effect the transfer to MC5 on the ISS R-bar , with a small residual upward coast velocity (0.3 fps or 0.083 m/s), to ensure that the *LOS* to the origin is contained within the TCS window and finally to ensure that no upward thrusting jets are used within 300 m of the station.

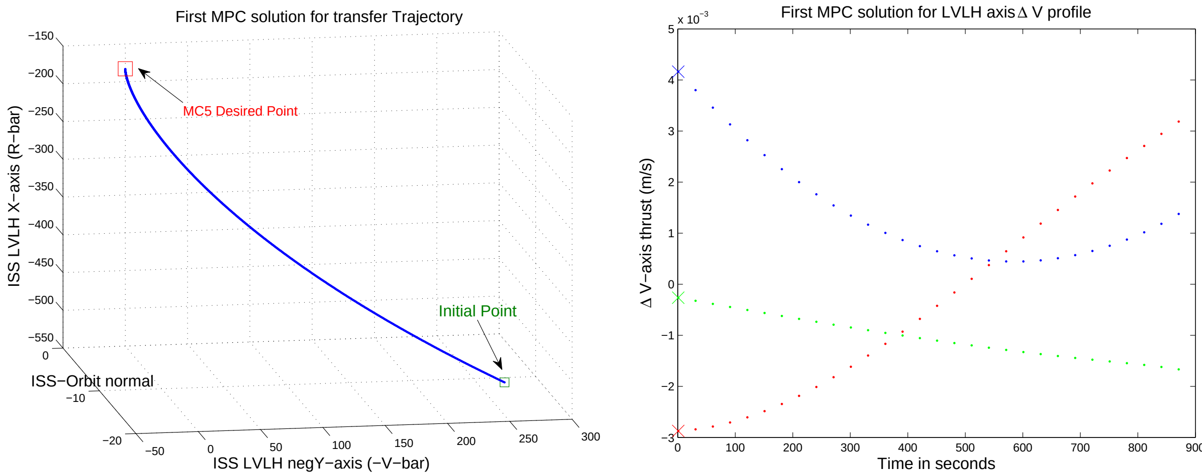
Based on an initial time-to-go estimate from the initial conditions, we set up an MPC problem with a prediction horizon of 900 seconds, consisting of 30 equally spaced control points where a control may be computed and modeled as *impulsively* applied during this MC4-MC5 transfer. Figure 3 shows the Cartesian acceleration (delta-Velocity) vectors computed at the first MPC computation step and the trajectory associated with that control sequence. These LVLH delta Velocities required and on the basis of which the jet selection will be performed are marked with crosses seen at time 0 in Figure 3-B. The delta LVLH Velocity plan for the future is shown as dots.

In order to realize the force and torque sequence computed previously (torque profile not shown), the LP jet selection logic engages. The jet burns that it computes to realize the time 0 accelerations are shown in Figure 5-A.

The solution is replanned at regular intervals with the control horizon shrinking to 29 control steps spaced 30 seconds apart at the next update point. The new solution is computed based on the new true state of the vehicle which will be dispersed from the planned trajectory shown above due to model errors and the fact that the engines cannot deliver the entire acceleration in a single impulse and therefore spread the burn out over a small window. The force profile is recomputed based on the new initial conditions along the planned trajectory. The execution is not identical to the planned trajectory, hence the new solution is slightly different from the initial plan. The inertial forces that will be the basis for the next round of jet selection are shown with the second pair of crosses in Figure 4-B. The plan for the future is shown with dots that are only slightly perturbed from the original plan in Figure 3. Note that the new trajectory profile shown in Figure 4-A is also very similar to the previously planned trajectory.

The new jet selection solution is shown in Figure 5-B.

The completed trajectory executed in position and velocity space associated with the STS118 MC4 initial



(a) Cartesian Trajectory Plan (b) LVLH Force Sequence Plan

Figure 3. The 30-step Cartesian forcing sequence computed at the start of the mission. The associated Cartesian trajectory is shown alongside.

conditions are shown in Figure 6. A stacked sequence showing the jet burn sequences are also included, in Figure 7.

In the final analysis, using this optimal guidance and optimal thrust control/scheduling approach, the shuttle would expend a total of 0.1 m/s of Delta-V or 7.46 lbm of fuel to arrive at MC5 on the R-bar 898.6 seconds after commencing the transfer from MC4 in this particular scenario.

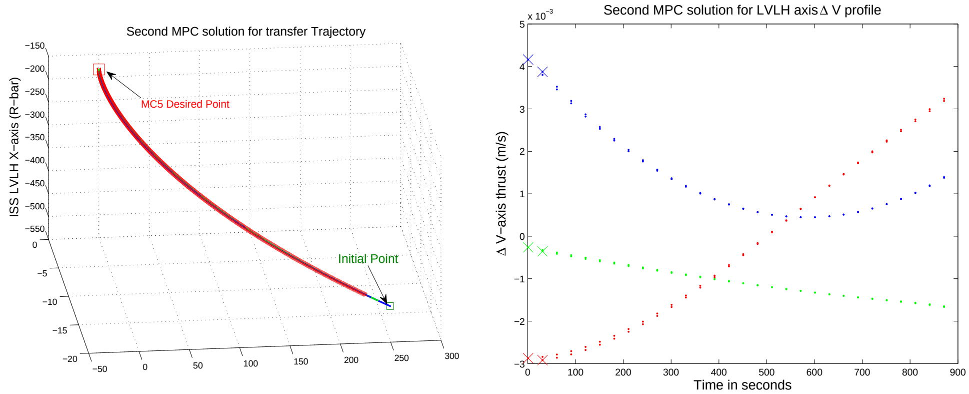
Finally, we present the collection of trajectories achieved using the methods of the paper applied to the dispersed initial conditions for 7 shuttle missions, STS114-121. These are all simulation results only - only the initial conditions and shuttle mass properties for that particular mission represent real data. Figure 8 shows the position, velocity, and pitch traces for these transfers, and Table 2 reports the transfer times, delta-V expended and the terminal position and velocity accuracy at the end of the maneuver for the transfers.

Mission	Time(s)	Δ-V(m/s)	Pos Err (m)	Vel. Err(mm/s)
114	868	0.11	$[-88.7, 0, -70]e-3$	$[0.0, 1.8, 1.3]E-3$
115	871	0.13	$[-0.1, 0.003, -0.05]$	$[1.1, -4.2, -1.1]E-3$
116	891	0.2	$[-0.66, -0.23, -0.05]$	$[0, 6.5, -2.1]E-3$
117	781	0.3	$[-11.4, 10.7, -31.1]e-3$	$[7, -1.8, 4.9]E-3$
118	898.6	0.1	$[-28.4, 103, 81]e-3$	$[-0.3, -2.5, 0.8]E-3$
120	855.4	0.14	$[1.42, -0.01, -0.001]$	$[11.1, 4.4, 2.3]E-3$
121	926	0.23	$[-.225, -.153, .077]$	$[-0.2, -4.1, -3.8]E-3$

Table 2. Measures of Performance for Simulated MC4-MC5 Transfers

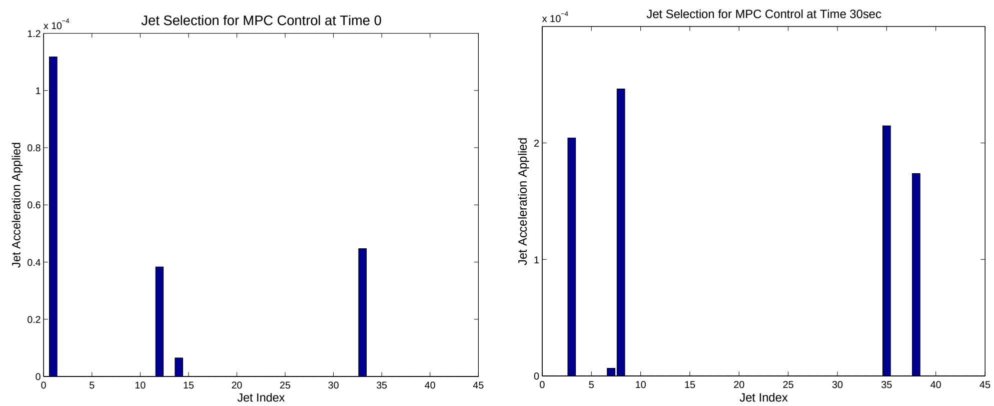
VII. Conclusions

In this paper, we presented our onboard receding horizon optimal control-based approach to the problem of constrained, fuel-optimizing, low-dispersion rendezvous guidance and control for a highly redundant



(a) Cartesian Trajectory Plan (b) LVLH Force Sequence Plan

Figure 4. Cartesian forcing sequence re-computed after 30-seconds after the next computation step. The associated Cartesian trajectory is shown alongside.

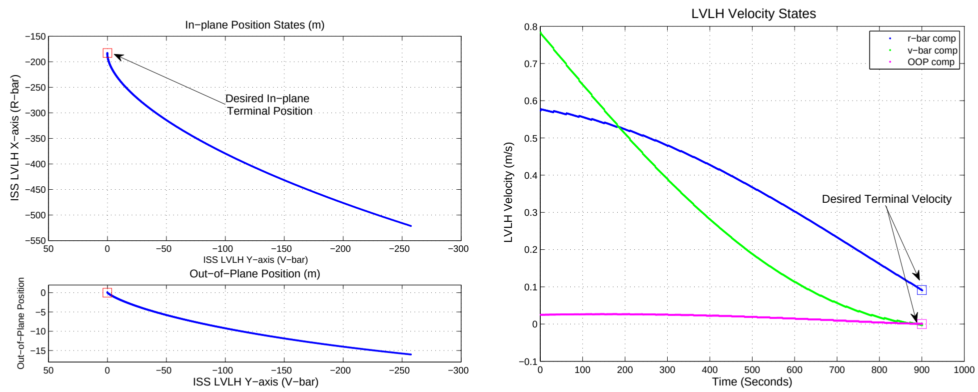


(a) Jet Selection Schedule at MPC Step-0 (b) Jet Selection Schedule at MPC step-2

Figure 5. LP-based computations of optimal jet selection at control step 1 (time=0) and then control step 2 (time = 30)

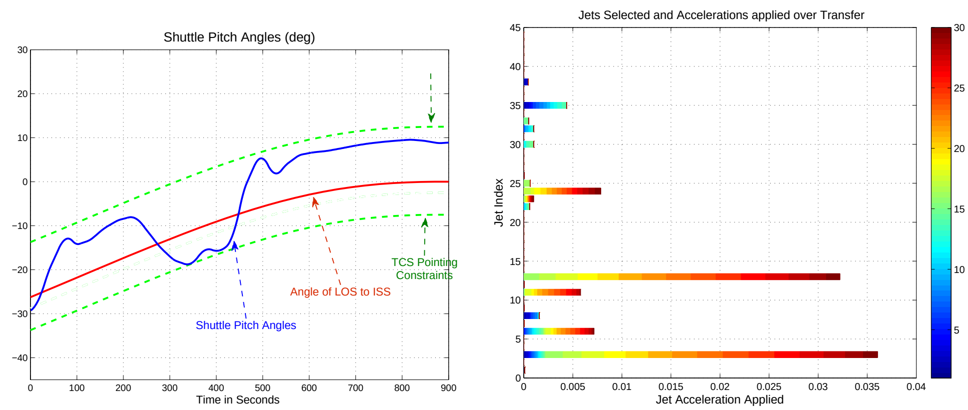
reaction-jet based vehicle such as the space shuttle operating in close proximity of a target vehicle such as the Space Station. We defined an approach for computing the force, trajectory, attitude and control sequence in an optimal fashion to transfer between two key MidCourse points in the rendezvous stages - MC4 to MC5. The transfer is complicated by the fact that attitudes must be commanded not only to optimize the trajectory control but also to meet relative pointing constraints. The approach consists of (a) a Finite-Horizon optimal control step for synthesizing a finite number of impulsive force applications within an approach time-window to produce a candidate trajectory, (b) identifying suitable attitudes and torques along that trajectory that can optimally meet shuttle pointing constraints with a good orientation for realizing the desired translational forces, and then (c) solving for the optimal jet selection logic using the full array of admissible jets depending on the guidance mode at the time.

A brief set of results obtained using initial conditions from one particular shuttle mission were also presented. Overall, we presented the delta-V and fuel expended and the times of arrival produced by this approach.



(a) Final MC4-MC5 Trajectory Executed (b) MC4-MC5 Velocity Traces

Figure 6. LVLH-frame Translational position and velocity trajectory states realized during the transfer



(a) Final MC4-MC5 Pitch Angle Trajectory (b) Total Jet Firing Sequence

Figure 7. A shows Pitch angles achieved relative to the TCS angle to ISS; TCS desired to remain between (-5, 10) degrees of this angle. (B) shows All jets pulses (1-44) as they were used during the transfer. Color coding shows the particular firing time (between time index 1 - 30) with blue earlier in the sequence and red later in the sequence.

References

[1]R.H.Battin, *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA Education Series, Reston, VA, 1999.

[2]Bate, R., Mueller, D., and White, J., *Fundamentals of Astrodynamics*, Dover, New York, 1971.

[3]V.A. Chobotov, E., *Orbital Mechanics, 2nd Ed.*, AIAA Education Series, Ed: J.S.Przemieniecki, Reston, VA, 1996.

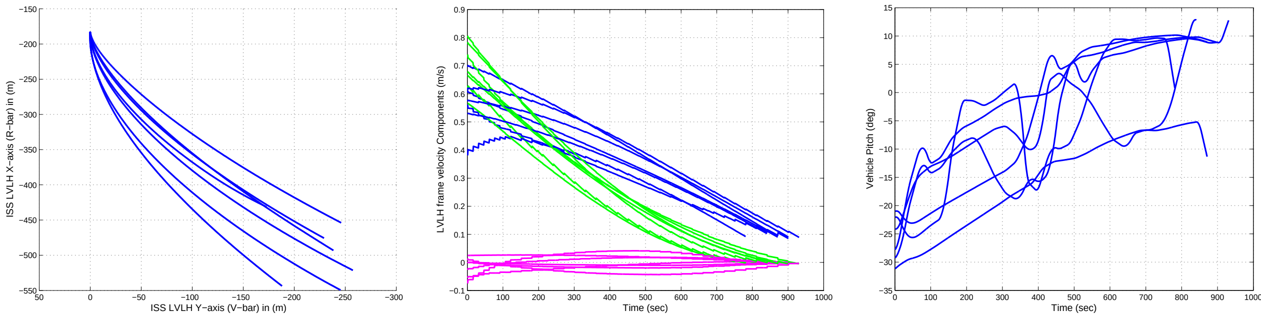
[4]Taur, D.-R., Coverstone-Carroll, V., and J.E.Prussing, "Optimal Impulsive Time-Fixed Orbital Rendezvous and Interception with Path Constraints," *Journal of Guidance, Control and Dynamics*, 1, Jan. 1995.

[5]Breger, L. and How, J. P., "Safe Trajectories for Autonomous Rendezvous of Spacecraft," *AIAA Guidance, Navigation, and Control Conference*, Aug. 2007.

[6]Schouwenaars, A., How, J., Feron, E., and Richards, A., "Plume avoidance maneuver planning using mixed integer linear programming," *AIAA Guidance, Navigation and Control Conference*, Aug. 2001.

[7]F.D.Clark, Spehar, P., J.P.Brazzel, and Hinkel, H., *Laser-Based Relative Navigation and Guidance for Space Shuttle Proximity Operations*, No. AAS 03-014, American Astronautical Society, 2003.

[8]C.E.Garcia, D.M.Prett, and M.Morari, "Model Predictive Control: Theory and Practice - A Survey," *Automatica*, Vol. 25, 1989.



(a) Dispersed MC4-MC5 Trajectories Executed

(b) Dispersed MC4-MC5 Velocity Traces

(c) Dispersed MC4-MC5 Pitch Traces

Figure 8. Translational position, velocity and Attitude states for 7 distinct, dispersed initial conditions

[9]D.Q.Mayne, J.B.Rawlings, C.V.Rao, and P.O.M.Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, Vol. 36, 2000, pp. 789–814.

[10]Maciejowski, J., *Predictive Control with Constraints*, Prentice-Hall. Harlow, England, 2002.

[11]W.H.Kwon and S.Han, *Receding Horizon Control*, Springer, 2005.

[12]L.Singh and J.Fuller, "Trajectory Generation for a UAV in Urban Terrain, using Nonlinear MPC," *American Control Conference*, June 2001.

[13]S.R.Walker, J.A.LoPresti, M.B.Schrock, and R.A.Hall, "Space Shuttle Rbar Pitch Maneuver for Thermal Protection System Inspection," *AIAA Guidance, Navigation and Control Conference*, Aug. 2005.

[14]Gim, D. and Alfriend, K. T., "The State Transition Matrix of Relative Motion for the Perturbed Non-Circular Reference Orbit," *AAS/AIAA Space Flight Mechanics Meeting*, No. AAS 01-222, Feb. 2001.

[15]Alfriend, K. T., Schaub, H., and Gim, D., "Gravitational Perturbations, Nonlinearity and Circular Orbit Assumption Effects on Formation Flying Control Strategies," *AAS/AIAA Space Flight Mechanics Meeting*, No. AAS 00-012, 2000.