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RENDEZVOUS PROXIMITY OPERATIONS USING MODEL PREDICTIVE CONTROL WITH SPARSE DYNAMICS IDENTIFICATION

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The abstract should briefly state the purpose of the manuscript, the problem to be addressed, the approach taken, and the nature of results or conclusions that can be expected. It should stand independently and tell enough about the manuscript to permit the reader to decide whether the subject is of specific interest. The abstract shall be typed single space, justified, centered, and with a column width of 4.5 inches. The abstract is not preceded by a heading of “Abstract” and its length may not extend beyond the first page.

INTRODUCTION

Space rendezvous missions have been extensively analyzed implementing various forms of constrained path planning and optimization techniques; many of which require high fidelity system identification of the chaser satellite. The combination of improvements in space-certified hardware and modern control techniques have expanded the capability of spacecraft control and the scope of on-orbit services. These advances have begun to democratize industry-changing missions like satellite refueling, inspection, and repair, along with debris removal or avoidance.[?,?] The increased need and application of these mission types has inspired further research in methods to lower costs, increase fuel savings, and find solutions to rendezvous problems with increasingly complex constraints, all while optimizing for computational efficiency.

One of those well-studied rendezvous methods utilizes model-predictive control (MPC), which has been shown to use less fuel due to minimum-fuel trajectory optimization while finding the best approach path within constraints of sensor visibility and safety. Richards and How developed and evaluated a new MPC implementation that optimizes using a novel mixed-integer linear programming method.[1] Fuel saving improvements over traditional MPC methods and the heritage glideslope approach. Singh and Bortolami presented an MPC solution to control one of the Space Shuttle’s approach phases; they optimized for fuel and constrained the sensor line-of-sight and thruster firing directions to avoid plume impingement.[2] They analyzed seven real-world cases of the space shuttle’s standard orbit raising maneuver on its way to the ISS. Cairano and Park shows another example

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of how an MPC can be used for RPO maneuvers.[?] Their MPC implementation was robust to thrust errors, air drag (low Earth orbit), and solar pressure (geostationary orbits). They optimized considering time-to-dock and fuel consumption while constraining thrust magnitude, line-of-sight, and approach velocity. Kannan and Sajadi-Alamdari apply MPC to spacecraft rendezvous maneuvers while considering fuel-efficiency and collision avoidance in their cost function.[?] Park and Zagaris dive deeper into collision avoidance during rendezvous operations. Both linear and non-linear MPC methods are used to optimize fuel while avoiding obstacles, limit thrust magnitudes, and operating within safe entry cones. The linear technique uses hyperplanes to convexify obstacles while the nonlinear solution utilizes ellipsoids. The linear method proved to be much more robust, stable, and computationally practical.

Another control scheme that has been used for rendezvous maneuvers is called adaptive control. Filipe and Tsiotras proposed an adaptive position and attitude (pose) tracking controller for proximity operations that requires no knowledge about the mass and inertia of the chaser satellite.[3] It can also take into account gravitational acceleration, gravity-gradient torque, and other constant disturbance forces and torques. This method can perform system identification of the chaser satellite, given that sufficient conditions are met, and then proceed to approach and then dock the target satellite.

For MPC, adaptive control, and other methods, offline and online system identification techniques are a necessity if high accuracy and precision control is desired, which many times requires costly and expensive volumes of data collection and processing. Within the previously cited adaptive control implementation, Filipe and Tsiotras calculate mass and inertia properties of the system by using known sets of disturbance forces and torques. With this method, the system identification is limited by the measurement quality of the applied forces and torques. There is another system identification technique that was investigated by Kaiser, Kutz, and Brunton that was used within an MPC method called sparse identification of nonlinear dynamics (SINDY).[?] SINDY is a data-driven system identification algorithm which can be used to derive governing equations for a nonlinear dynamic systems. In this work, the authors compare SINDY to dynamic mode decomposition (DMD) and a multilayer neural network (NN). They show that sparse identification is preferred when a low volume of noisy data is available and a fast computation time is required. Also, SINDY sheds light on the underlying nonlinear dynamic equations of the system, instead of just having a black box that sheds little to no physical insight. Another strength of SINDY is that it is fast enough to run on embedded systems.[6] Lastly one of the most useful features of SINDY is that the system identification can be used with control inputs along with other external forcing. SINDY has a few weaknesses, but they can be mitigated. The first is that a sufficient library of functions must be assumed to identify high-dimensional systems;[?] this can be prevented by increasing the function library size. Another drawback to this technique is that it does not react effectively to abrupt changes in dynamics, but this concern can be alleviated by using linear methods like DMD while the system dynamics settle.[?]

Robotics and spacecraft rendezvous operations go hand-in-hand, and a powerful tool known as dual quaternions, used mostly with robots, can be brought over to bridge the gap to the on-orbit servicing realm.[?] This set of numbers extends the utility found in representing attitude as a quaternion to describe position and translation. Dual quaternions are preferred over quaternion-vector representations for several reasons. Control laws can be written in a more compact form, pose transformations requires less computations than quaternion-vector method, natural coupling between rotational and translational motion is inherent, there is a strong parallel between rotational-only kinematics and rotation-translation dual quaternion kinematics, control stability is proven in one step, often controllers or filters developed for attitude quaternions can be adapted for use with dual quaternions, kinematics can be obtained easily because of multiplicative properties, constraint equations come in a simple form, and problems involving kinematic chains are more easily solved.[?,?,3] Using this number set does come with what some may see as a drawback. For instance, two additional constraint equations are necessary, derived motion relationships can be complicated, physical significance of variables are not initially apparent, and there is a quadratic cost on required control commands.[?,?,?]

The contribution of this paper to the topics herein discussed is to apply MPC to RPO while gathering system dynamics knowledge solely using SINDY. A trade study will be conducted varying orbit parameters, mass properties, measurement noise, and controller constraints. All analysis will be done with dual quaternions to provide another example of their industry-proven strengths.

DUAL QUATERNION CONCEPT EXAMPLE

All dynamics and controls equations discussed in this work are expressed using dual quaternions; before diving in, a quick introduction of this number set is given via an example. More thorough mathematical properties of quaternions, dual numbers, dual quaternions, and necessary operators are given in reference 3.

Within the aerospace industry, single quaternions are used to express the attitude between two reference frames of interest. Combining quaternions with the concept of a dual number ϵ, where $\epsilon^2 = 0$, but $\epsilon \neq 0$, allows the encoding of both rotational and translational information relating two reference frames. As an example, take the diagram in figure 1 showing the relative positions between chaser and target spacecraft orbiting a gravitational body. Expressing dual quaternions with a hat symbol, one would express the pose of the chaser with respect to its target as

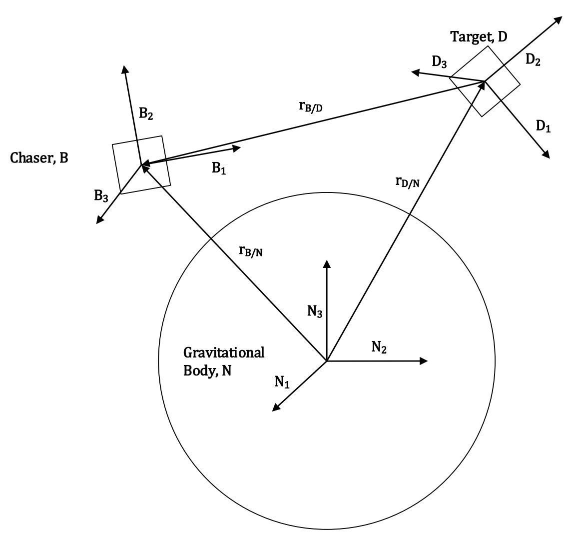

Figure 1. Relative positions between chaser and target spacecraft, orbiting a gravitational body.

$$\hat{q}_{B/D} = q_{B/D} + \epsilon \frac{1}{2} q_{B/D} r^B_{B/D}, \tag{1}$$

where $q_{B/D}$ is the attitude unit quaternion and $r^B_{B/D}$ is the relative position vector of the chaser with respect to the target, in a body fixed basis, represented as a quaternion. Quaternions can be expressed using an ordered pair where the first and second coordinates are the vector and scalar parts, respectively. Using that notation, the attitude is

$$q_{B/D} = (\mathbf{n}\sin(\phi/2), \cos(\phi/2)) \tag{2}$$

where $\mathbf{n}$ is the skew unit vector that one rotates frame D about by ϕ to get frame B. The relative position as a quaternion vector-scalar ordered pair is

$$r^B_{B/D} = (\mathbf{r}^B_{B/D}, 0). \tag{3}$$

RELATIVE DYNAMIC EQUATIONS FOR RENDEZVOUS PROXIMITY OPERATIONS

Assuming quasi-static mass and inertial properties of a chaser spacecraft, B, its motion dynamics relative to a target body, D, are governed by

$$\begin{aligned}(\dot{\hat{\omega}}^B_{B/D})^s = (M^B)^{-1} \star (\hat{f}^B - (\hat{\omega}^B_{B/D} + \hat{\omega}^B_{D/N}) \times (M^B \star ((\hat{\omega}^B_{B/D})^s + (\hat{\omega}^B_{D/N})^s)) \\ - M^B \star (\hat{q}^*_{B/D}\dot{\hat{\omega}}^D_{D/N}\hat{q}_{B/D})^s - M^B \star (\hat{\omega}^B_{D/N} \times \hat{\omega}^B_{B/D})^s)\end{aligned} \tag{4}$$

where the dual inertia of the chaser is

$$M^B = \begin{bmatrix} mI_{3x3} & 0_{3x1} & 0_{3x3} & 0_{3x1} \\ 0_{1x3} & 1 & 0_{1x3} & 0 \\ 0_{3x3} & 0_{3x1} & \bar{I}^B & 0_{3x1} \\ 0_{1x3} & 0 & 0_{1x3} & 1 \end{bmatrix}, \tag{5}$$

given chaser mass m and inertia in spacecraft fixed basis

$$\bar{I}^B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}, \tag{6}$$

and the total external dual force applied to the body about its mass center, in the spacecraft frame is

$$\hat{f}^B = \hat{f}^B_g + \hat{f}^B_{\nabla g} + \hat{f}^B_{J_2} + \hat{f}^B_s + \hat{f}^B_\mu + \hat{f}^B_d + \hat{f}^B_c. \tag{7}$$

The components that make up the total external dual force, in the order shown in (7), are gravitational force, gravity-gradient torque, Earth-oblateness force, solar dual force, atmospheric drag, dual disturbance force, and finally control dual force.

USING SINDY FOR SYSTEM IDENTIFICATION WITHIN MPC

Control methods that utilize system models, such as MPC, are powerful because of their potential application to nonlinear systems with constraints. Nevertheless, this strength in the control domain can come hand-in-hand with a computational efficiency and data logging requirement weakness. The investigations in references **?** and **?** show how both of these potential drawbacks can be mitigated with the use of SINDY for system identification. These same methods will be adapted to rendezvous proximity maneuver applications.

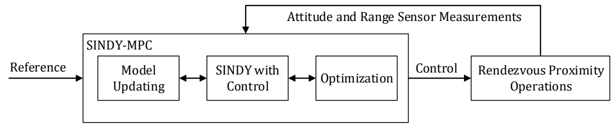

Figure 2. System identification using SINDY within MPC, for RPO.

Figure 2 shows an overview of how SINDY will be applied to rendezvous maneuvers via MPC. The sparse regression algorithm will take in noisy attitude and range sensor data to calculate an approximation of the relative motion dynamics expressed by equation (4). More specifically, mass, inertia, external forces and torques will be determined only with sensor data for use in the MPC optimal control problem.[?]

The diagram shown in figure 3 shows the process of solving for the system model. First, to present the identification problem in solvable way, one must first collect measurement data from the unknown dynamic system represented by

$$\dot{X} = \Xi\Theta(X, U). \tag{8}$$

Then the control developer must select a library of candidate functions that will be used to build $\Theta(X, U)$. The candidate terms can include polynomial, trigonometric, and other nonlinear functions of states X and inputs U. One then substitutes in the measured data into $\dot{X}$ and Θ in equation (8), and we choose Ξ such that we have the minimum number of terms that effectively describe our dynamics. In other words, we solve for

$$\xi_k = \text{arg}\min_{\xi_k} \frac{1}{2}||\dot{X}_k - \xi_k\Theta(X, U)||^2 + \lambda||\xi_k||, \tag{9}$$

where subscript k in $\dot{X}_k$ and ξ_k represents the k^{th} term of $\dot{X}$ and Ξ, and λ is chosen to balance between model complexity and accuracy.

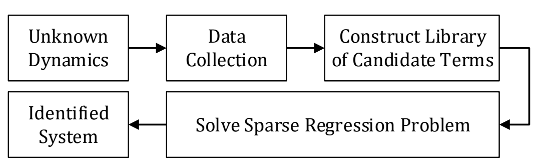

Figure 3. Process of using measured data to identify nonlinear system dynamics.

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Graphic Formats.

References and Citations.

MANUSCRIPT SUBMISSION

Journal Submission

CONCLUSION

ACKNOWLEDGMENT

NOTATION

APPENDIX: TITLE HERE

Miscellaneous Physical Dimensions

REFERENCES

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