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Kelas : C

Tugas Interpolasi

Diket : Data-data berpasangan $(x_i, f(x_i))$

Ditanya : $f(4)$?

Jawab

a) Metode Newton

- Orde 1

$(x_0, f(x_0)) = (3,8)$ | $(x_1), (f(x_1)) = (5,12)$

$f_1(x) = b_0 + b_1 (x - x_0)$

$b_0 = f(x_0) = 8$

$b_1 = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{12-8}{5-3}$

$= 4/2 = 2$ //

$f_1(4) = 8 + 2(4-3)$

$= 10 . (1)$

$= 10$

//

• Orde II

$(x_0, f(x_0)) = (2,5) \mid (x_1, f(x_1)) = (3,8) \mid (5,12)$

$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$

$b_0 = f(x_0) = 5$

$$b_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

$$= \frac{\frac{12-8}{5-3} - \frac{8-5}{3-2}}{5-2}$$

$$= \frac{2-3}{3} = -1/3$$

$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$

$f_2(4) = 5 + 3(4-2) + (-1/3)(4-2)(4-3)$

$= 5 + 6 + (-1/3)(2)(1)$

$= 11 + (-2/3)$

$= \frac{33}{3} + (-2/3) = \frac{33-2}{3} = 31/3$

$= 10,33$

$|\varepsilon_0| = \left| \frac{f_2(x) - f_1(x)}{f_2(x)} \right| \times 100\%$

$= \left| \frac{10,33 - 10}{10,33} \right| \times 100\%$

$= \left| \frac{0,33}{10,33} \right| \times 100\%$

$= 0,03225\%$

b) Metode lagrange

• orde I

$(x_0, f(x)) = (2,5) \mid (x_1, f(x_1)) = (3,8)$

$$f_1(x) = \sum_{i=0}^{1} L_i(x) f(x)$$

$$L_i(x) = \prod_{\substack{j=0 \\ i \neq 0}}^{1} \frac{x - x_j}{x_i - x_j}$$

$$L_0(x) = \frac{x - x_1}{x_0 - x_1}$$

$$L_1(x) = \frac{x - x_0}{x_1 - x_0}$$

$$f(x) = \frac{x - x_1}{x_0 - x_1}(f(x_0)) + \frac{x - x_0}{x_1 - x_0}(f(x_1))$$

$$= \frac{4-5}{3-5}(8) + \frac{4-3}{5-3}(12)$$

$$= \frac{4-5}{-1}(8) + \frac{4-3}{2}(12)$$

$$= \left(\frac{-1}{-1}\right)(4) + (1)(6)$$

$= 4 + 6 = 10$

• orde II

$x_0 = 2, f(x_0) = 5$

$x_1 = 3, f(x_1) = 8$

$x_2 = 5, f(x_2) = 12$

$L_0(x) = \left(\frac{x - x_1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right)$

$L_1(x) = \left(\frac{x - x_0}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right)$

$L_2(x) = \left(\frac{x - x_0}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right)$

$L_0(x) = \left(\frac{x-3}{2-3}\right)\left(\frac{x-5}{2-5}\right)$

$L_1(x) = \left(\frac{x-2}{3-2}\right)\left(\frac{x-5}{3-5}\right)$

$L_2(x) = \left(\frac{x-2}{5-2}\right)\left(\frac{x-3}{5-3}\right)$

$f(x) = L_1(x)(f(x_0)) + L_2(x)(f(x_1)) + L_2(x)(f(x_2))$

$= \left(\frac{x-3}{2-3}\right)\left(\frac{x-5}{2-5}\right)(5) + \left(\frac{x-2}{3-2}\right)\left(\frac{x-5}{3-5}\right)(8)$

$+ \left(\frac{x-2}{5-2}\right)\left(\frac{x-3}{5-3}\right)(12)$

$= \left(\frac{4-3}{2-3}\right)\left(\frac{4-5}{2-5}\right)5 + \left(\frac{4-2}{3-2}\right)\left(\frac{4-5}{3-5}\right)8$

$+ \left(\frac{4-2}{5-2}\right)\left(\frac{4-3}{5-3}\right)12$

$= (-1)(1/3)(5) + 2(1/2)8 + (2/3)(1/2)(12)$

$= (-5/3) + 8 + 4$

$= -5/3 + 12$

$= -\frac{5}{3} + \frac{36}{3} = \frac{31}{3} = 10,33$

$$|\varepsilon_d| = \left| \frac{f_2(x) - f_1(x)}{f_2(x)} \right| \times 100\%$$

$$= \left| \frac{10,33 - 10}{10,33} \right| \times 100\%$$

$$= \left| \frac{0,33}{10,33} \right| \times 100\%$$

$$= 0,3225\ \%$$