ENERGY-EFFICIENT HOMING CONTROL OF UNDERACTUATED MARINE VEHICLES UNDER FAULTY ACTUATOR CONDITIONS

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Marine Robotics School 2023, Goa

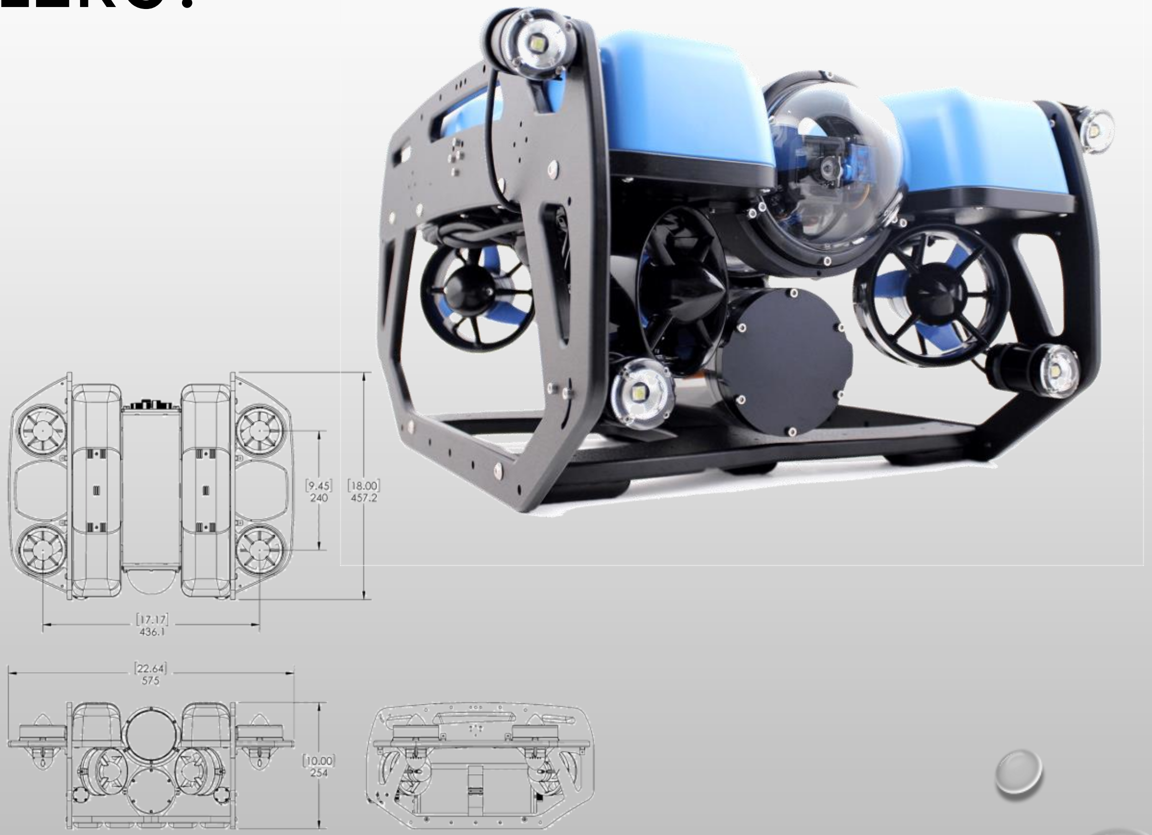
https://github.com/Dictino/mrs2023

WHAT ARE THE STANDARD FAULT TOLERANT CONTROLLERS?

- MANY VEHICLES ARE OVER-ACTUATED
  - IF YOU LOSE ONE MOTOR IT IS POSSIBLE TO COMPENSATE WITH THE REST OR THE MOTORS
- EXAMPLE BLUEROV2 HEAVY
  - 6 DOF (X, Y, Z AND 3 ROTATIONS)
  - 8 ACTUATORS

CONTROL ALLOCATION

$$\tau = Ku$$



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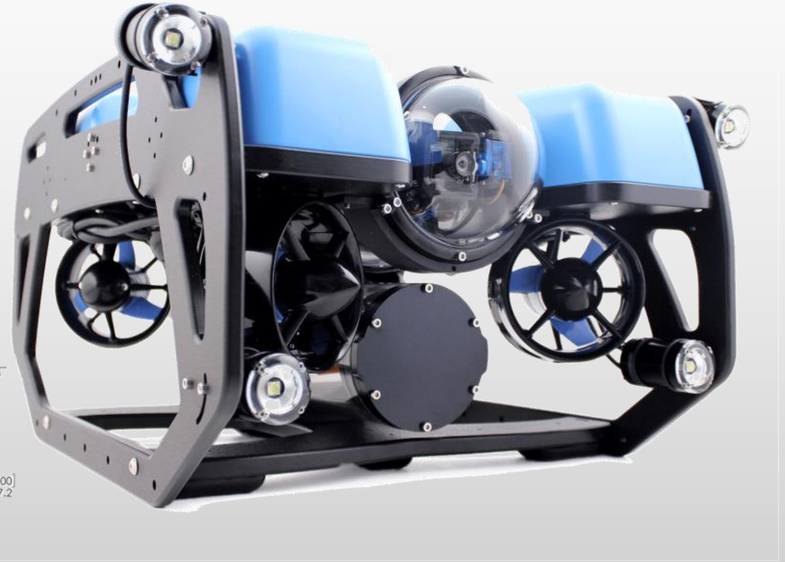

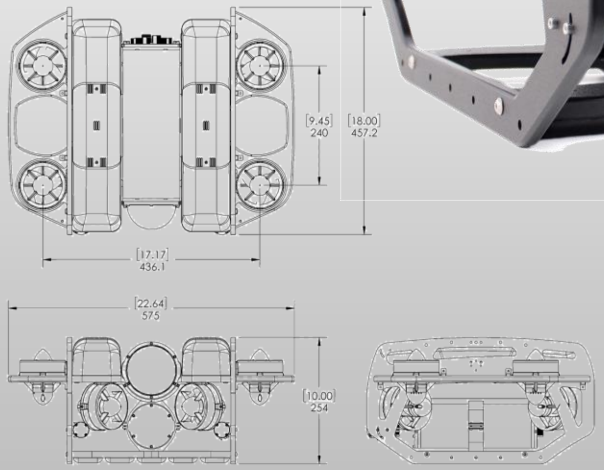

CONTROL ALLOCATION

$$\tau = Ku$$

$$\begin{array}{c} minimize\ u^T W u \\ \text{Subject to } \tau - Ku = 0 \end{array} \longrightarrow u = K_w^{\dagger}\tau$$

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    - YOU CAN BREAK 2 AND NOTHING HAPPENS

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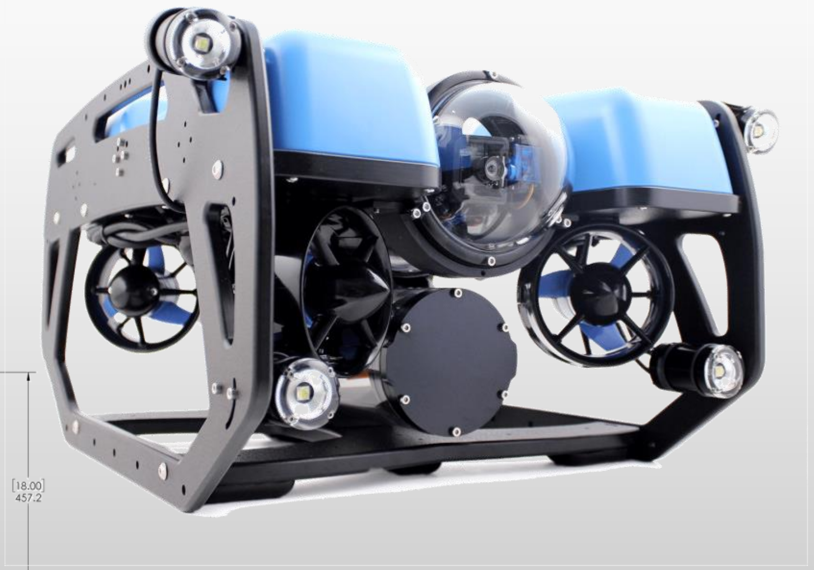

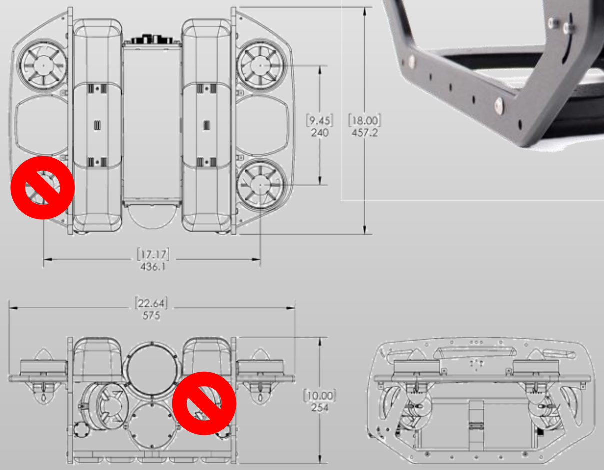

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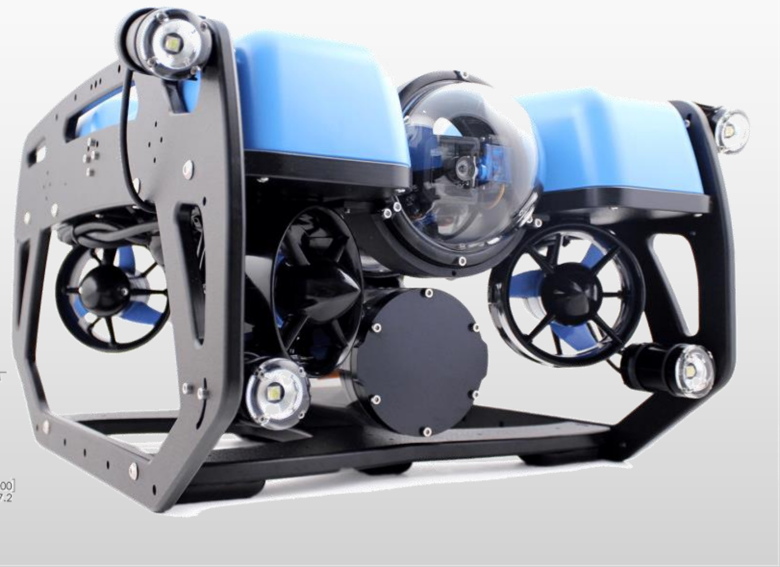
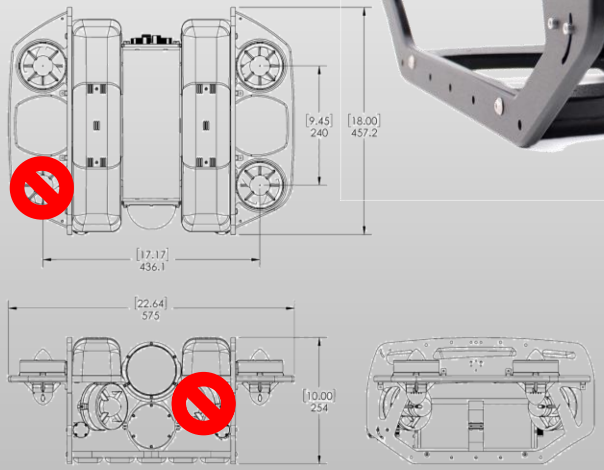
$$\tau = Ku \longrightarrow u = K^{-1}\tau$$





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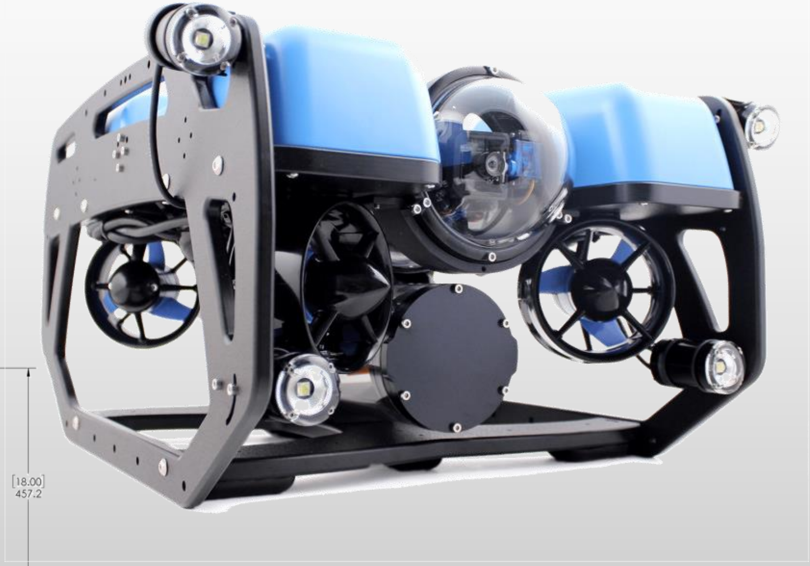
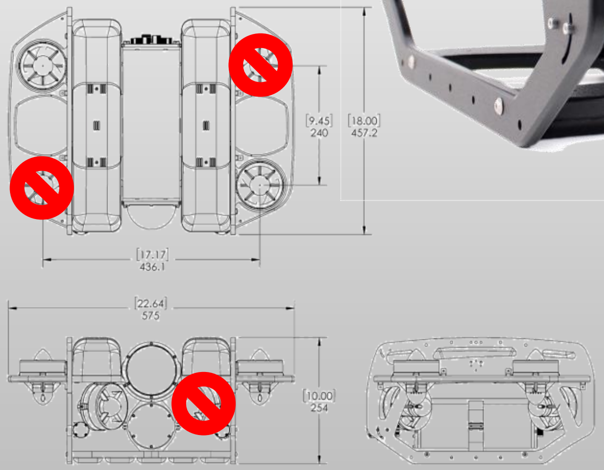
  - 8 ACTUATORS
    - YOU CAN BREAK 2 AND NOTHING HAPPENS
- AND IF YOU LOOSE 3…
  - WE ARE IN TROUBLE (UNDERACTUATED)
    - BUT UNDERACTUATED VEHICLES ARE STILL USEFUL
    - WE LOSE CONTROL ON ROLL/PITCH
    - BUT WE CAN STEER AND MOVE





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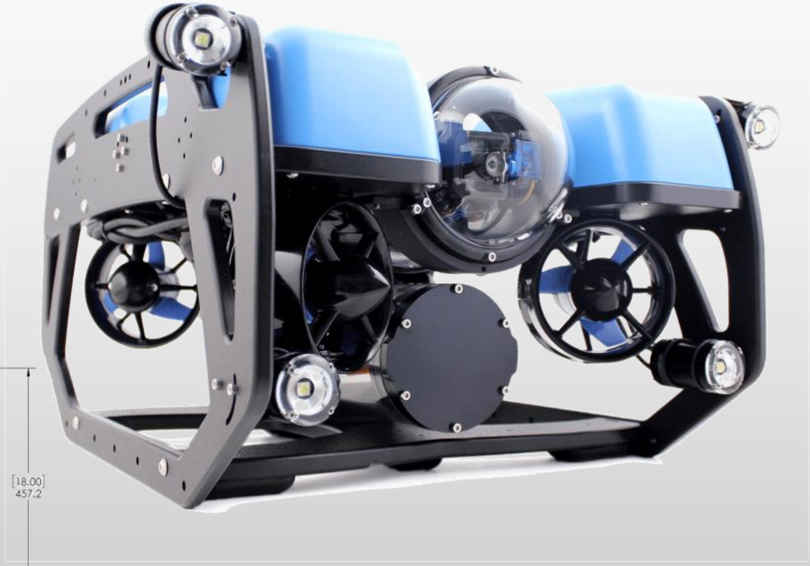
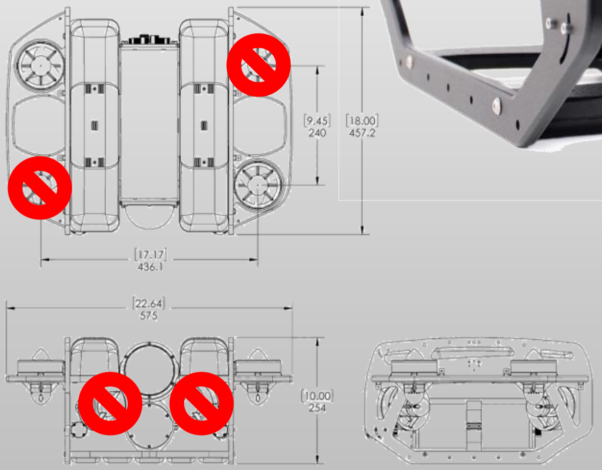
    - YOU CAN BREAK 2 AND NOTHING HAPPENS
- AND IF YOU LOOSE 4!!!!
  - WE ARE IN TROUBLE (UNDERACTUATED)
    - STILL USEFUL!!!
    - WE LOSE CONTROL ON ROLL/PITCH
    - WE CAN'T MOVE LATERALLY BUT STILL TURN AND MOVE FORWARD…





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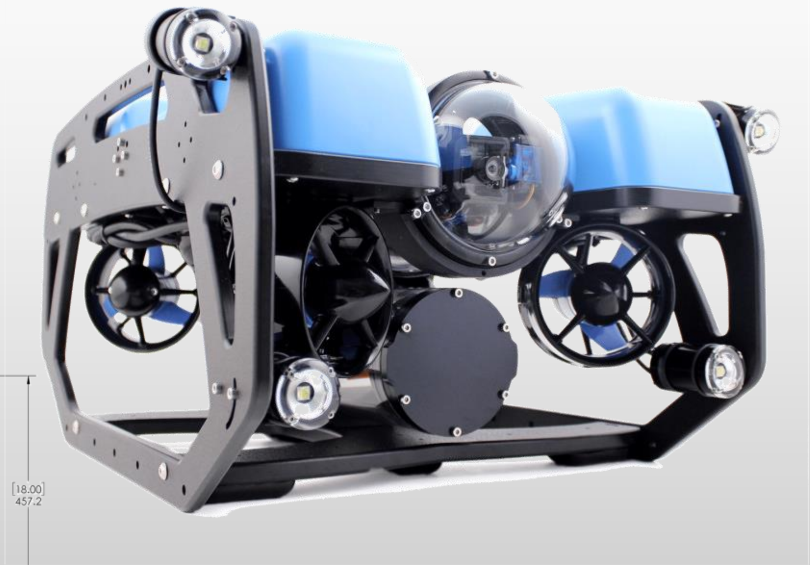
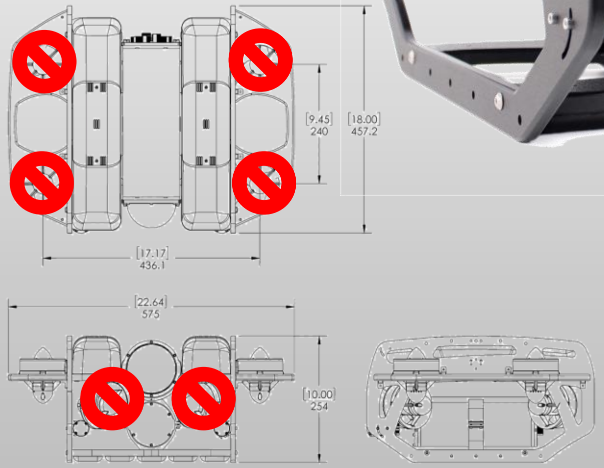
- AND IF YOU LOOSE 6????!!!!
  - IT CAN NOT SUBMERGE
  - BUT WE CAN MOVE ON THE SURFACE AND RECOVER IT





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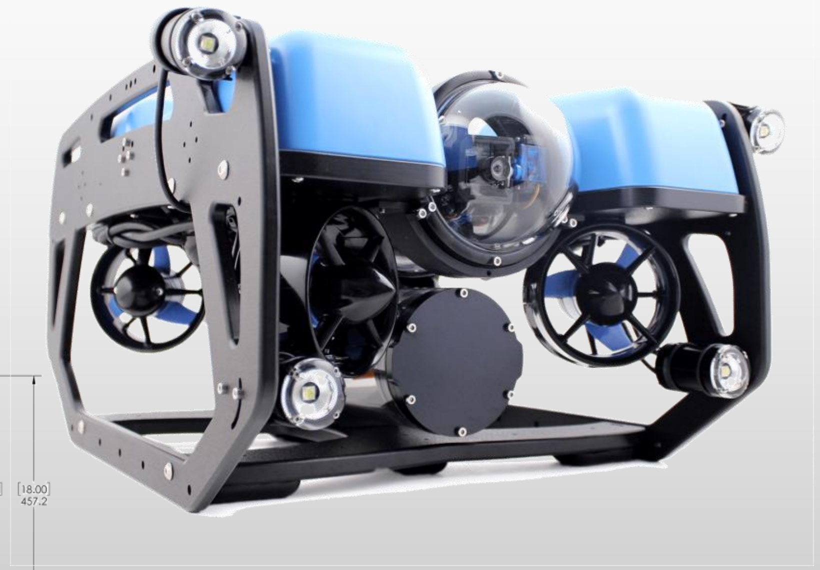
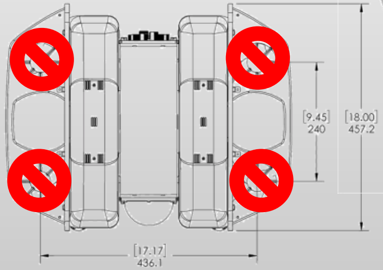
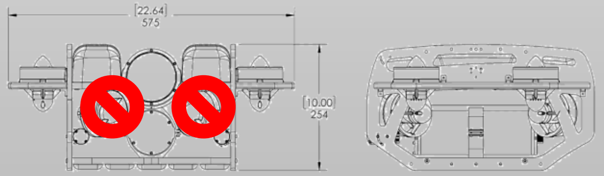






VERY UNLIKELY ☺
AND WE HAVE THE UMBILICAL

WHERE IS THE LIMIT?

MEDUSA VEHICLE

- UNDERACTUATED VEHICLE
  - 4 THRUSTERS
    - 2 FOR DEPTH
    - 2 FOR PLANAR MOTION
  - OPERATES LIKE A “3+1” DOF
    - ROLL AND PITCH ARE STABLE (DON’T NEED ACTUATION)
    - BUT CAN’T ACCELERATE SIDEWAYS
  - POSITIVE BUOYANCY

Surface/submarine vehicle

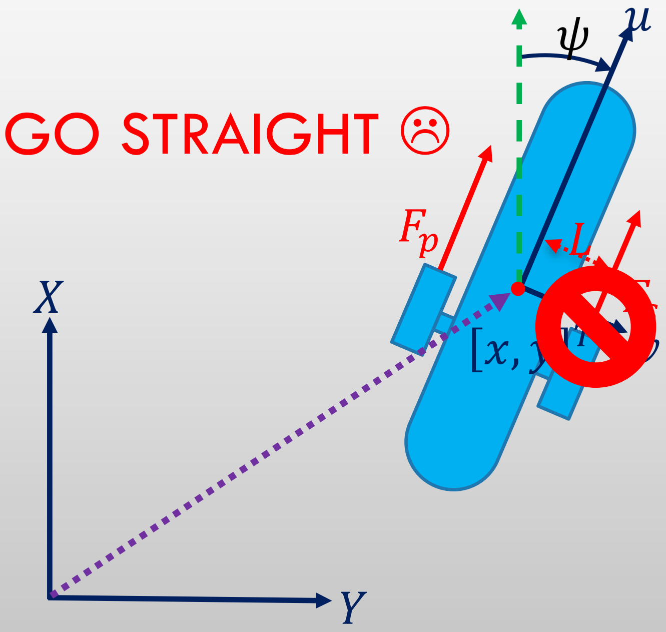
MEDUSA VEHICLE

IF ONE MOTOR FAILS, WE ARE IN TROUBLE, AND THIS IS NOT UNLIKELY!!!

Surface/submarine vehicle

MEDUSA VEHICLE ONLY 1 MOTOR LEFT

WE CAN'T GO STRAIGHT ☹



Planar model

Surface/submarine vehicle

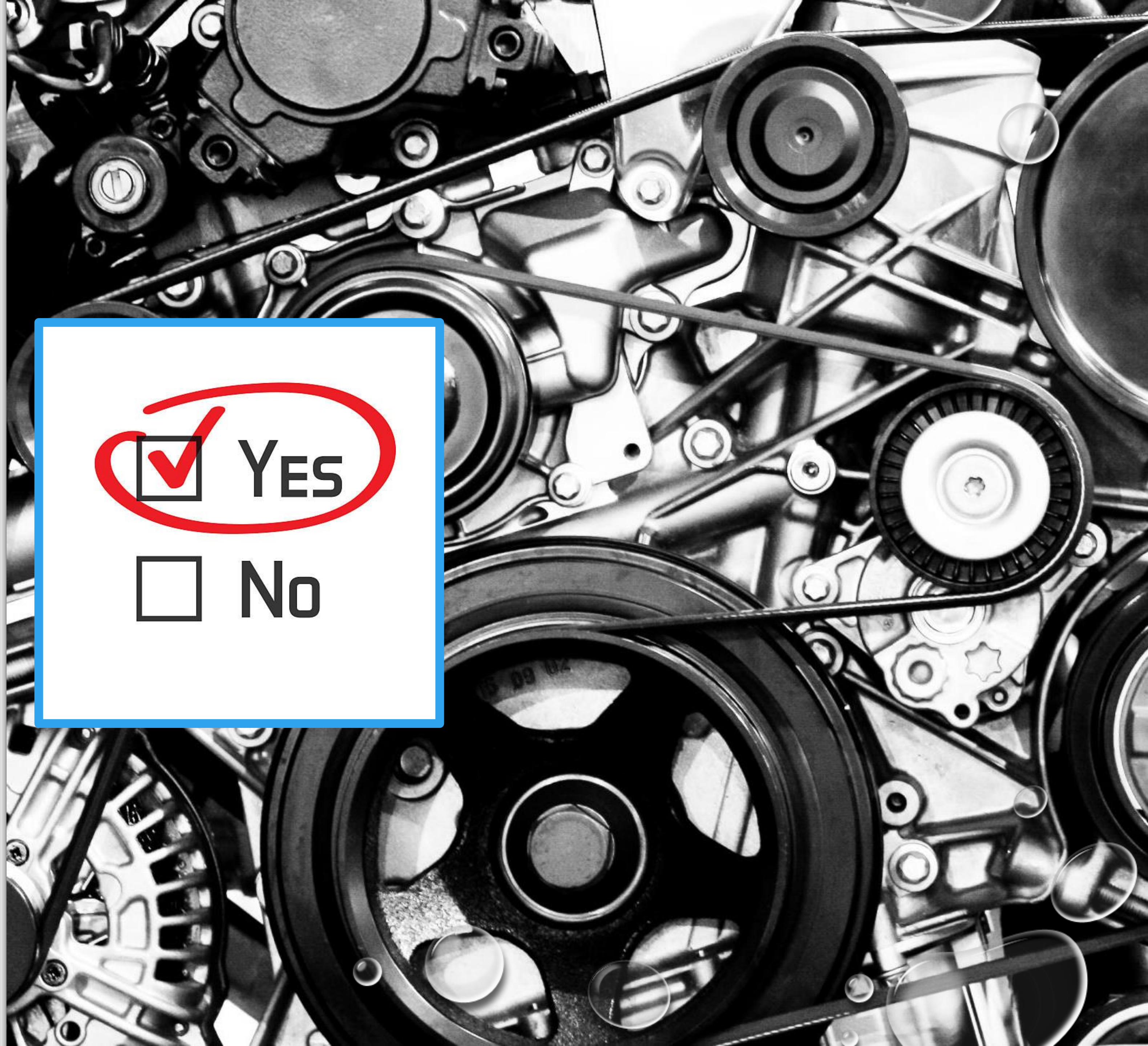
DEPTH IS NOT A PROBLEM (IT FLOATS)

STANDARD RECOVERY SOLUTION!

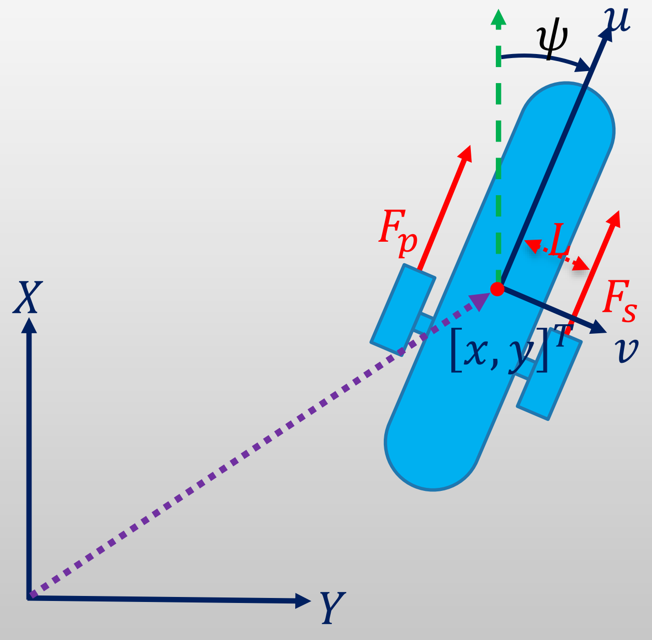
In memorial to Professor JM de la Cruz

IS THERE ANOTHER WAY?

- FAST POLL
  - WHO THINKS IT IS POSSIBLE TO CONTROL THE VEHICLE WITH ONLY ONE MOTOR?



TO BE CONCRETE WE NEED A MODEL



Planar model

NED

(X POINT TO NORTH)

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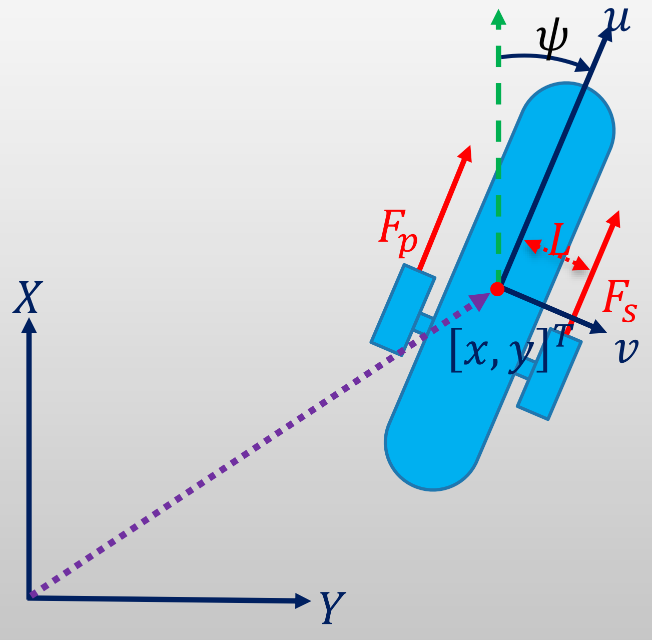

Planar model

NED

(X POINT TO NORTH)

$$\dot{x} = u \cdot \cos(\psi) - v \cdot \sin(\psi)$$
$$\dot{y} = u \cdot \sin(\psi) + v \cdot \cos(\psi)$$
$$\dot{\psi} = r$$

- KINEMATICS
  - ROTATION OF THE VELOCITIES

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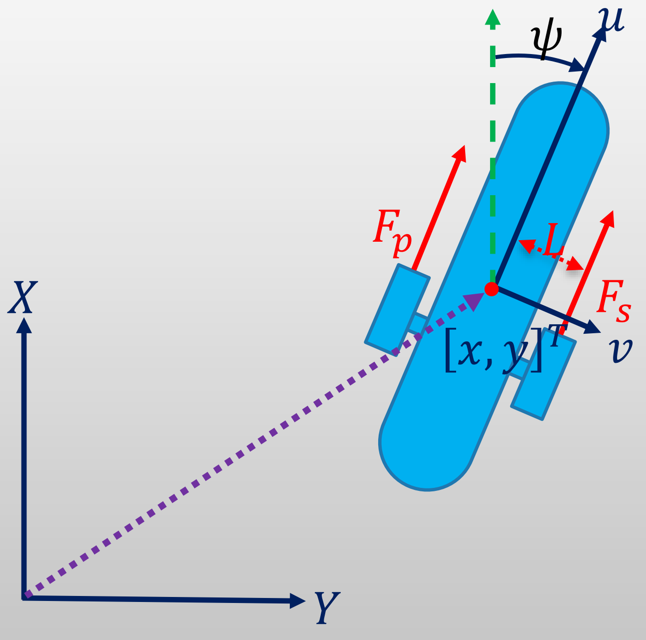

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- KINEMATICS
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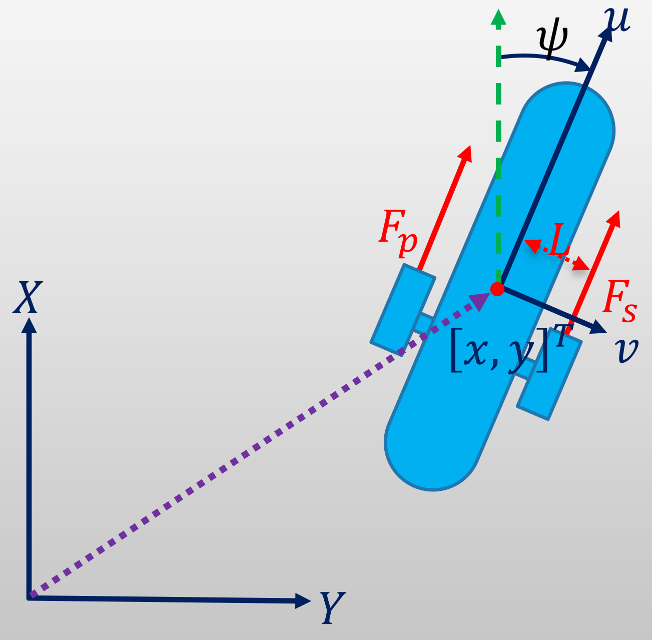
$$m_u \cdot \dot{u} - m_v \cdot v \cdot r + \mathcal{D}_u \cdot u = F = F_p(a_p)$$
$$m_v \cdot \dot{v} + m_u \cdot u \cdot r + \mathcal{D}_v \cdot v = 0$$
$$m_r \cdot \dot{r} - m_{uv} \cdot u \cdot v + \mathcal{D}_r \cdot r = \tau = L \cdot F_p(a_p).$$

- DYNAMICS
  - MASS AND INERTIA
  - ADDED MASSES DUE TO THE WATER
  - CORIOLIS TERMS DUE TO THE ROTATION
  - DRAG

TO BE CONCRETE WE NEED A MODEL



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- DYNAMICS
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DRAG
$$\mathcal{D}_r = -\mathcal{N}_r - \mathcal{N}_{|r|} \cdot |r|$$
$$\mathcal{D}_v = -\mathcal{Y}_v - \mathcal{Y}_{|v|} \cdot |v|$$
$$\mathcal{D}_u = -\mathcal{X}_u - \mathcal{X}_{|u|} \cdot |u|$$

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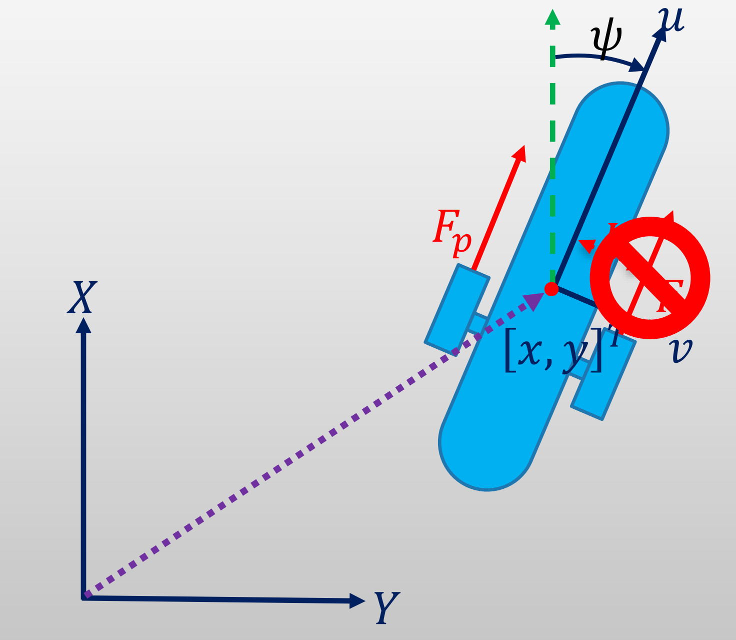

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$$\mathcal{D}_v = -\mathcal{Y}_v - \mathcal{Y}_{|v|} \cdot |v|$$
$$\mathcal{D}_u = -\mathcal{X}_u - \mathcal{X}_{|u|} \cdot |u|$$

$$\cancel{F_s(a_s) = \mathcal{K} \cdot |a_s| \cdot a_s}$$
$$F_p(a_p) = \mathcal{K} \cdot |a_p| \cdot a_p$$

- ACTUATORS MODEL
  - FORCE IS PROPORTIONAL TO THE SQUARE OF THE REVOLUTION

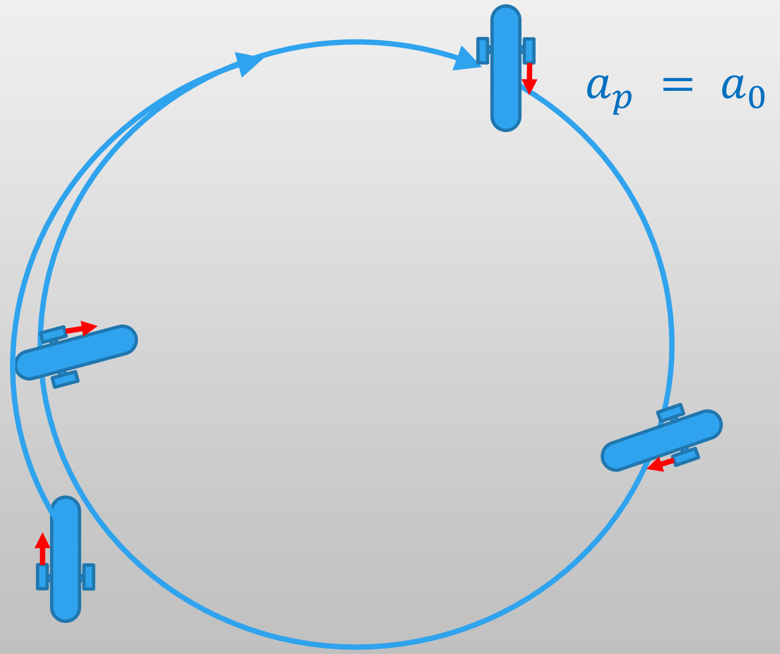
LET'S TRY TO CONTROL IT

- APPLY A CONSTANT FORCE AND…

WHAT DO YOU THINK IT WILL HAPPEN?

LET'S TRY TO CONTROL IT

- APPLY A CONSTANT FORCE AND…

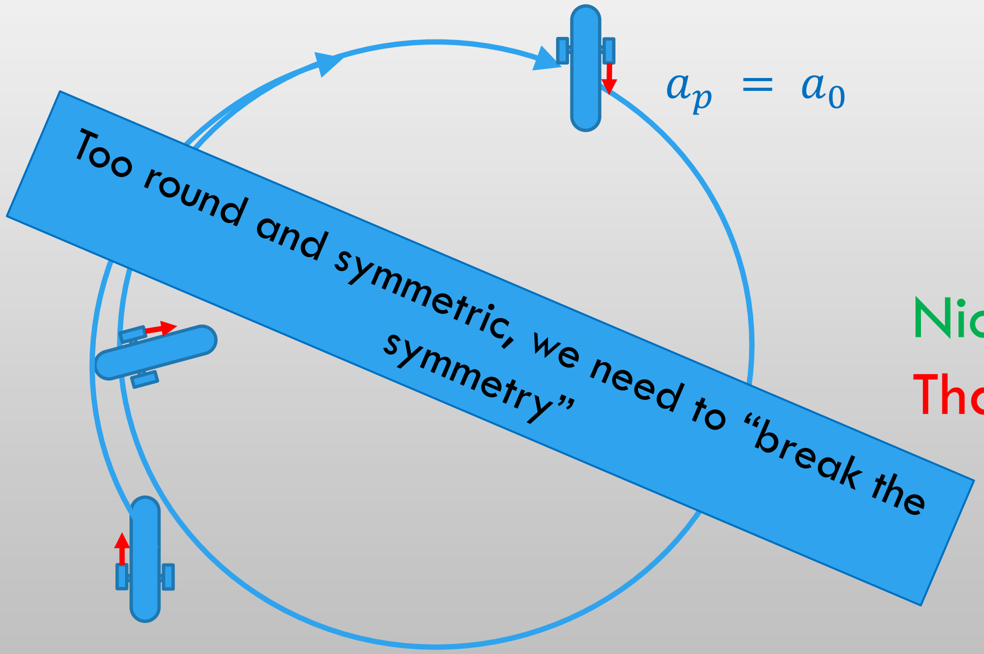


Nice circle…

That goes nowhere

LET'S TRY TO CONTROL IT

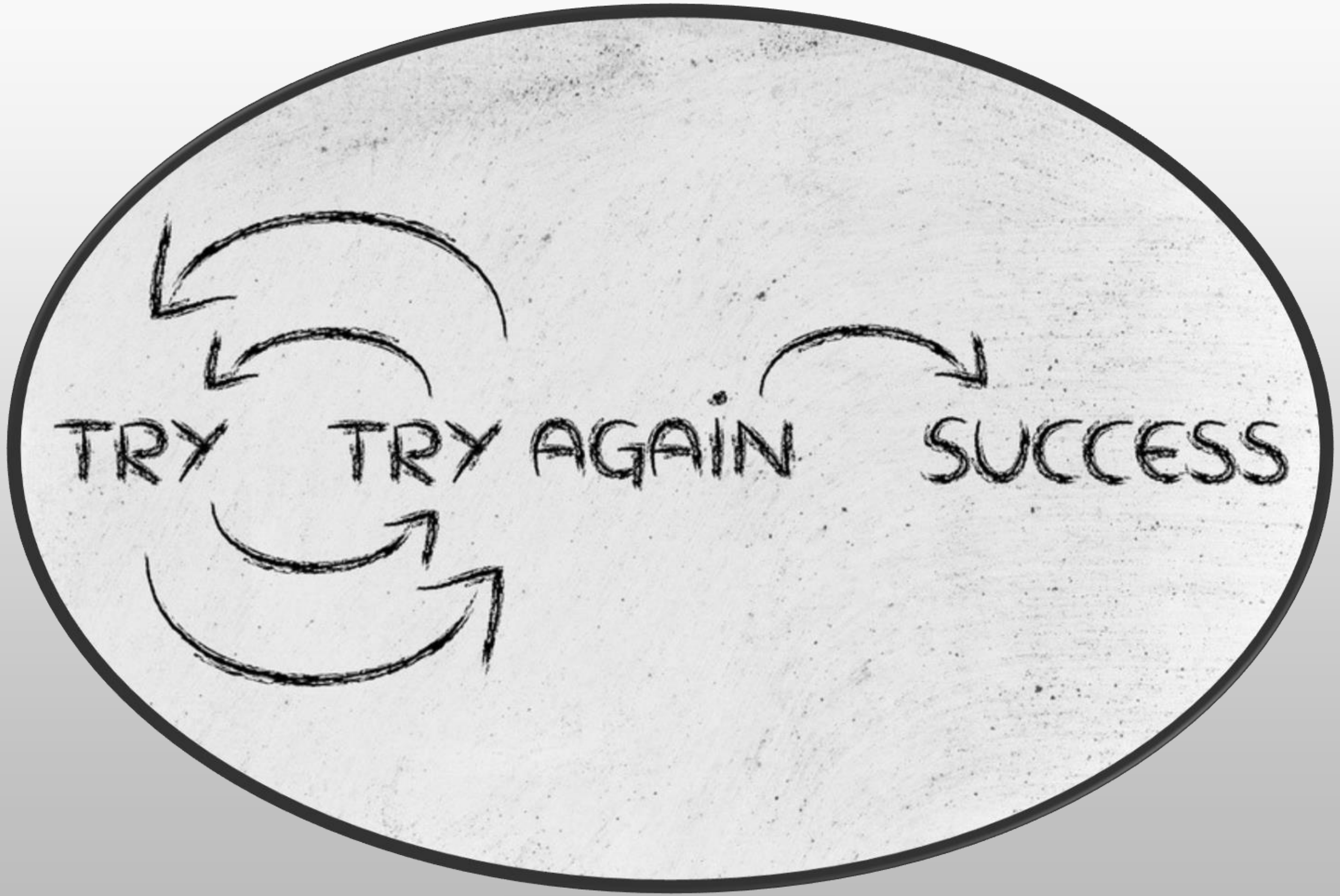
- APPLY A CONSTANT FORCE AND…



Nice circle…

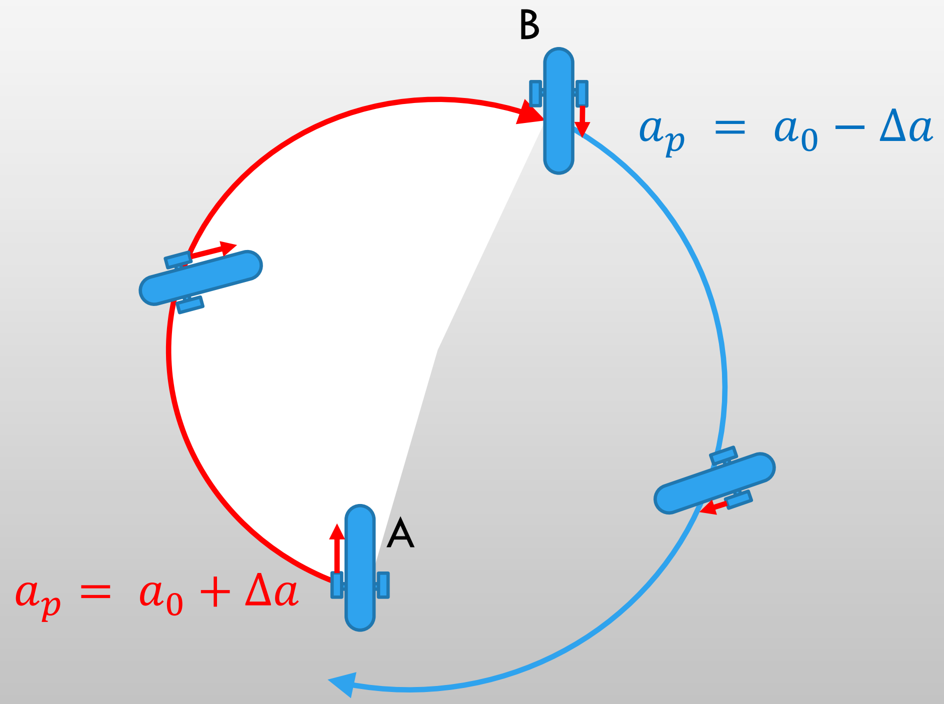
That goes nowhere

DON'T SURRENDER



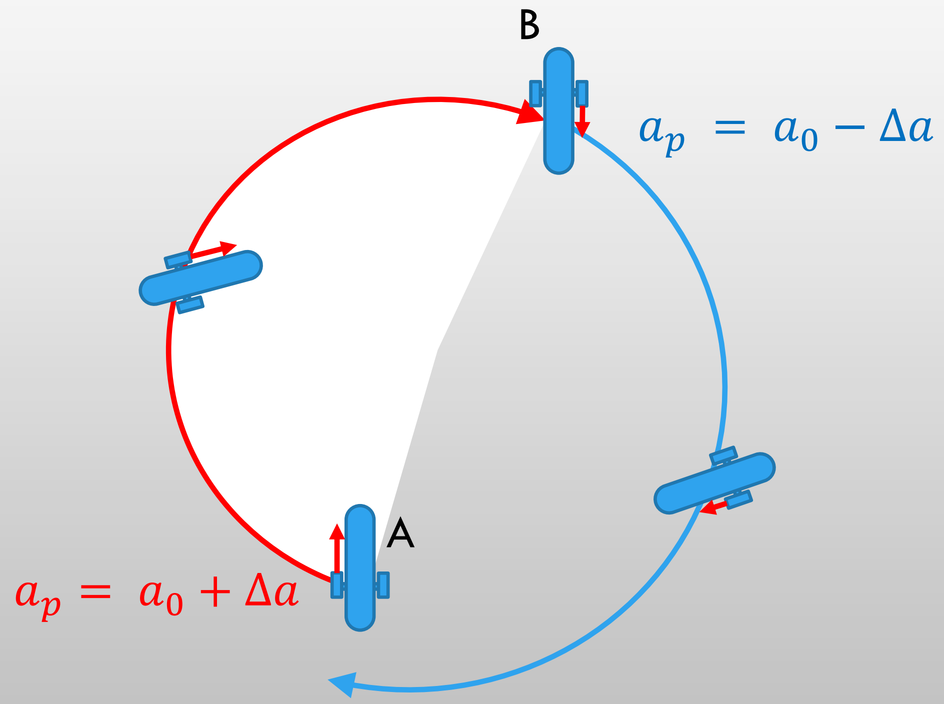
SECOND ROUND

- WHAT IF WE APPLY MORE FORCE WHEN THE VEHICLE POINTS TO THE EAST AND LESS FORCE WHEN IT POINTS TO THE WEST?



SECOND ROUND

- WHAT IF WE APPLY MORE FORCE WHEN THE VEHICLE POINTS TO THE EAST AND LESS FORCE WHEN IT POINTS TO THE WEST?
  - THE CHOICE IS A LITTLE BIT ARBITRARY, BUT WE WILL CHANGE IT LATER…



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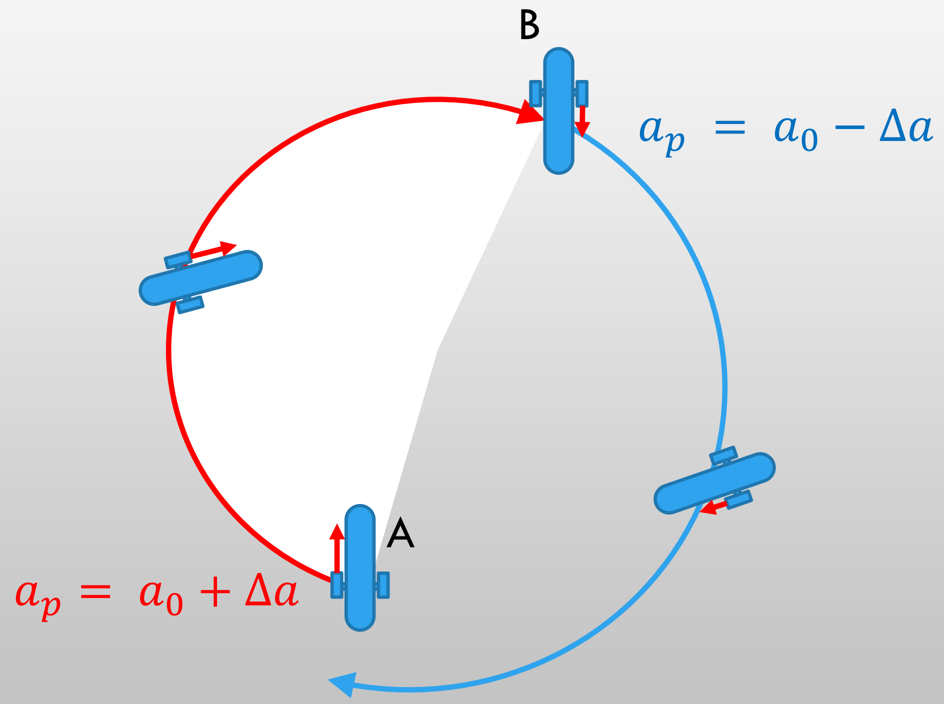
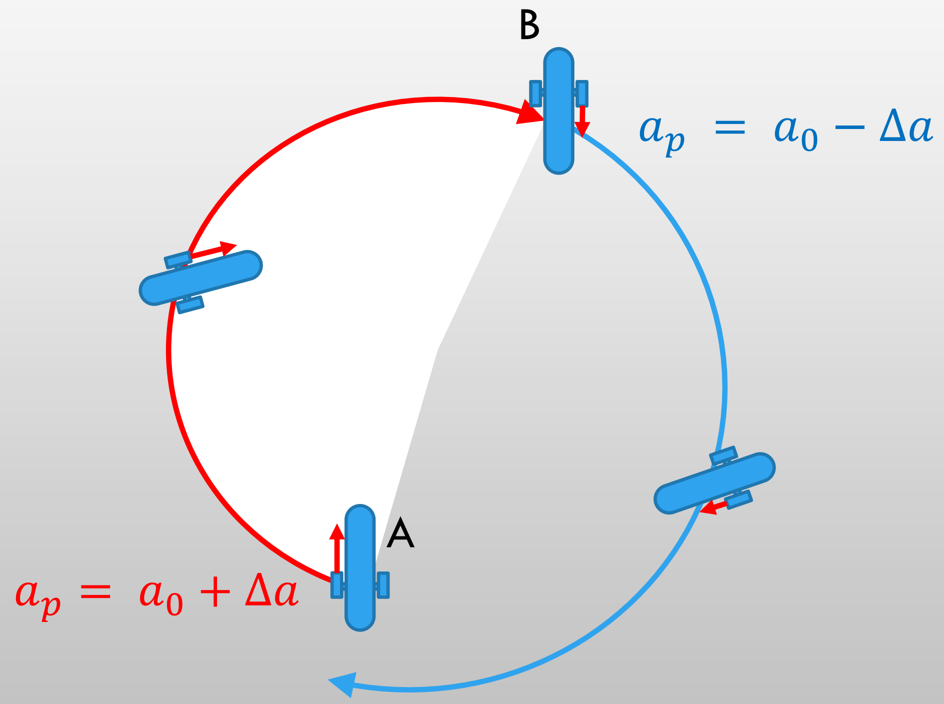


DO YOW WANT TO TRY?
WHAT DO YOU PREDICT IT TO DO?

SECOND ROUND

- WHAT IF WE APPLY MORE FORCE WHEN THE VEHICLE POINTS TO THE EAST AND LESS FORCE WHEN IT POINTS TO THE WEST?
  - THE CHOICE IS A LITTLE BIT ARBITRARY, BUT WE WILL CHANGE IT LATER…
- NOW IT MOVES

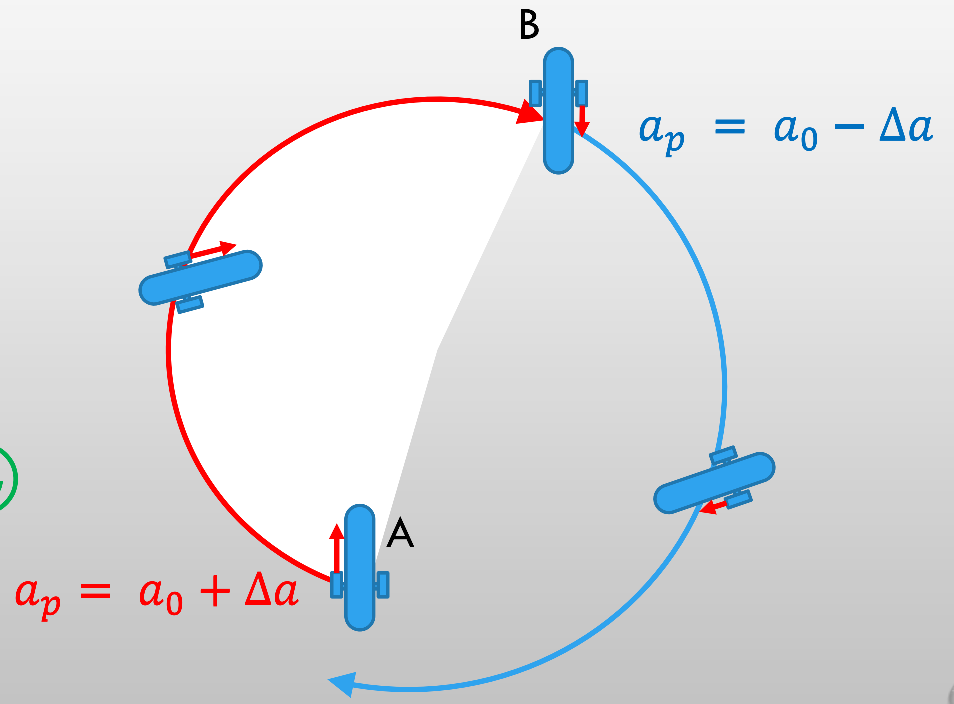
$$a_p = a_0 + \Delta a \cdot \mathrm{sign}(\sin(\psi))$$



QUESTION

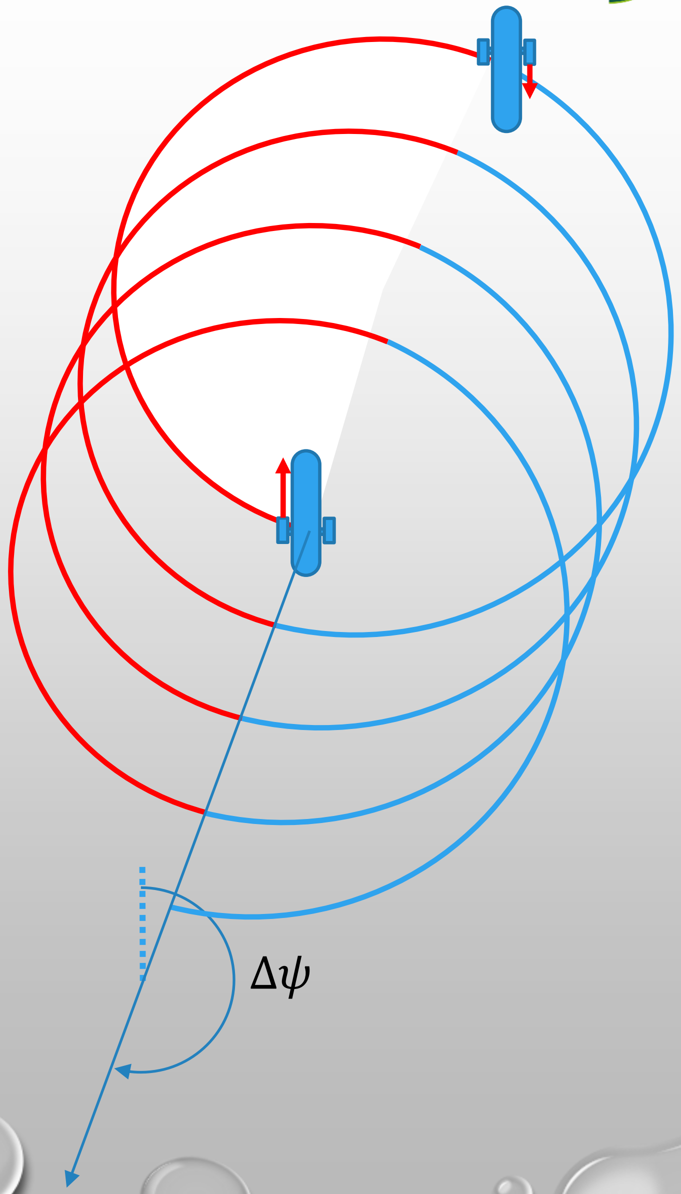
DID YOU PREDICT IN WHICH DIRECTION IT MOVED?

DON'T WORRY, ME NEITHER… ☺



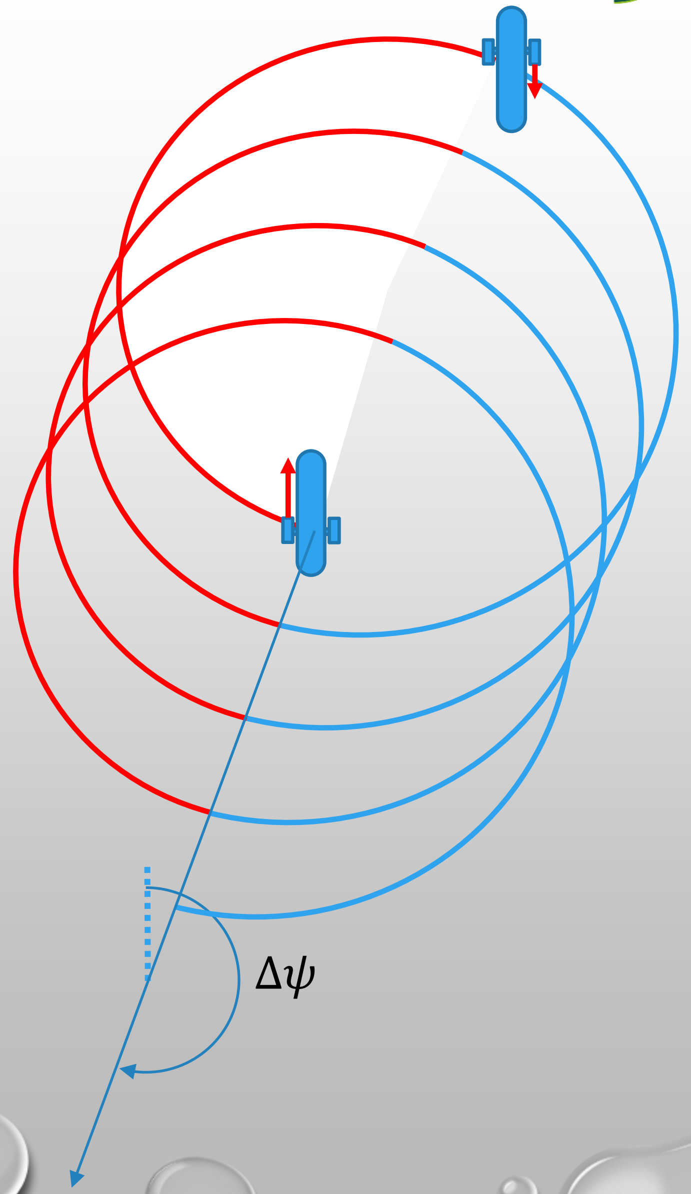
IT'S COMPLICATED

- THE DIRECTION OF THE AVERAGE MOVEMENT DEPENDS ON MANY THINGS IN A COMPLEX WAY…
  - BUT THE IMPORTANT THING IS THAT IT MOVES AND … IT IS EASY TO MEASURE



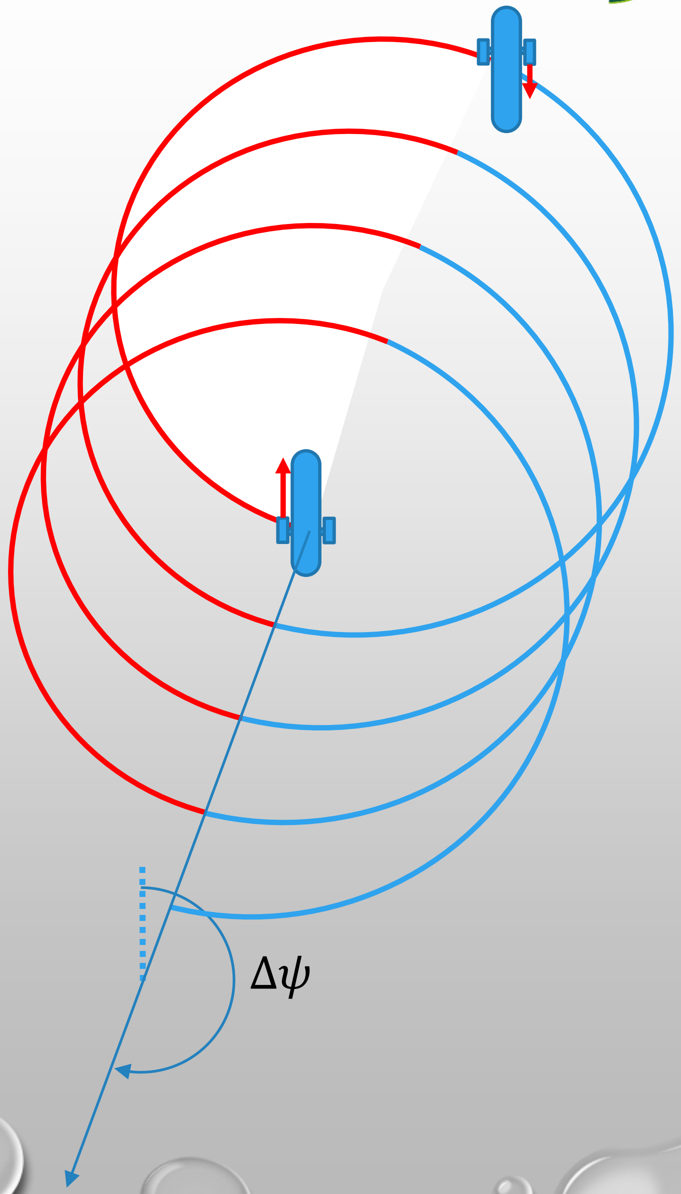
WHERE IT GOES?

- SEEMS NOT VERY USEFUL…IT MOVES IN AN ARBITRARY DIRECTION…
  - BUT REMEMBER EAST-WEST WAS AN ARBITRARY CHOICE



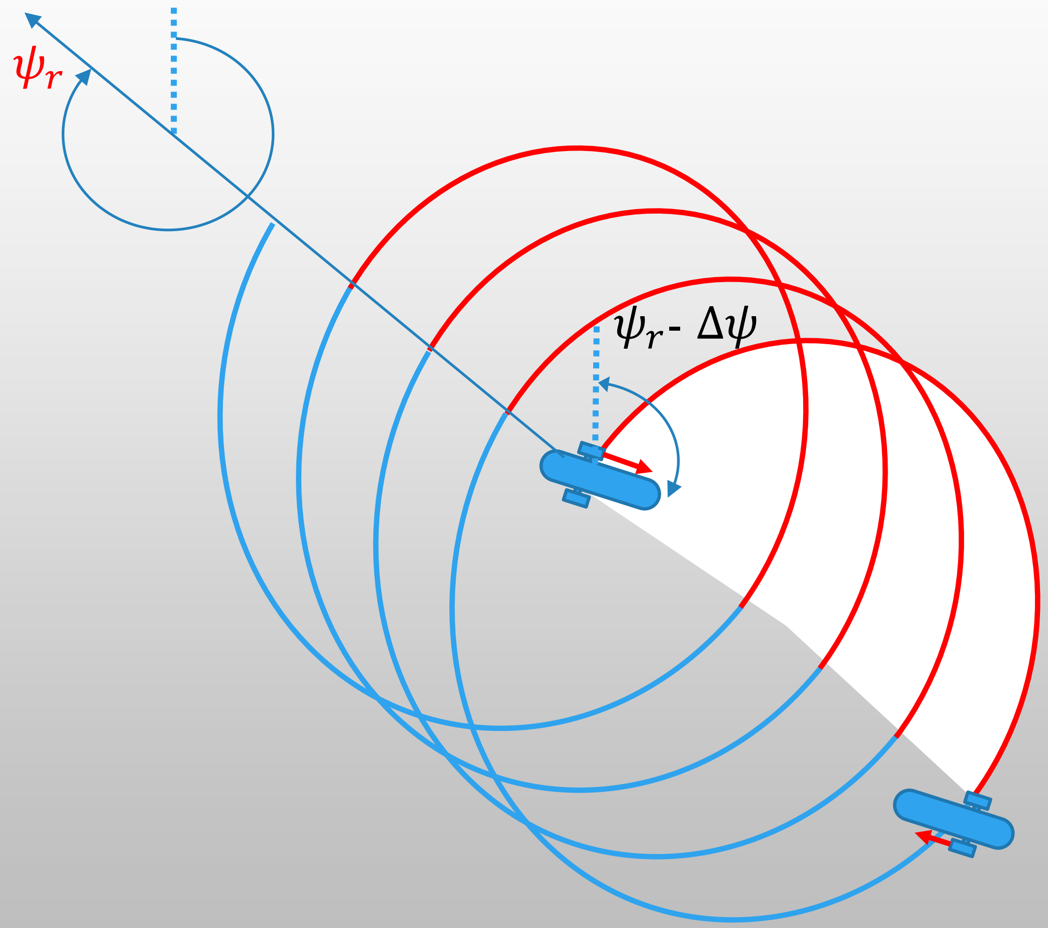
WHERE IT GOES?

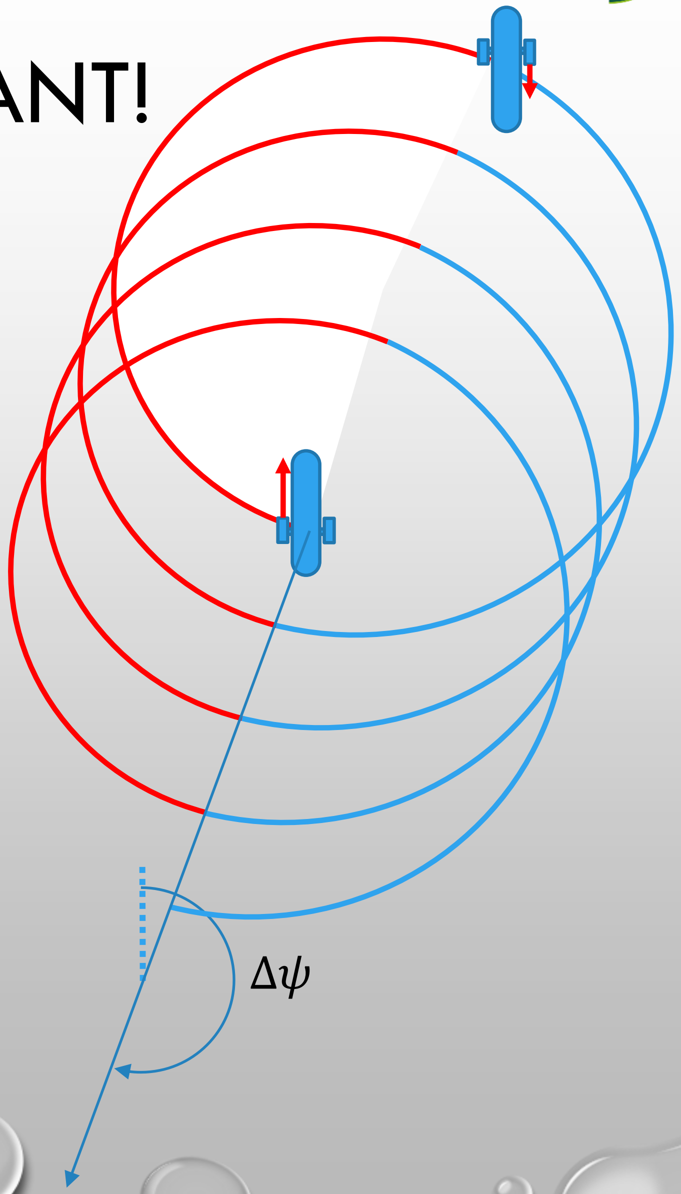
- SEEMS NOT VERY USEFUL…IT MOVES IN AN ARBITRARY DIRECTION…
  - BUT REMEMBER EAST-WEST WAS AN ARBITRARY CHOICE
  - WE CAN CORRECT THE DIRECTION BY ADDING AN OFFSET TO THE ORIENTATION



$$a_p = a_0 + \Delta a \cdot \mathrm{sign}(\sin(\psi - \psi_r + \Delta\psi))$$

IT MOVES WHERE I WANT!





$$a_p = a_0 + \Delta a \cdot \mathrm{sign}(\sin(\psi - \psi_r + \Delta\psi))$$

COULD WE MATHEMATICALLY PROVE IT WOKS?

- YES, THAT'S EASY

ATHEMATICALLY PROVE IT WOKS?

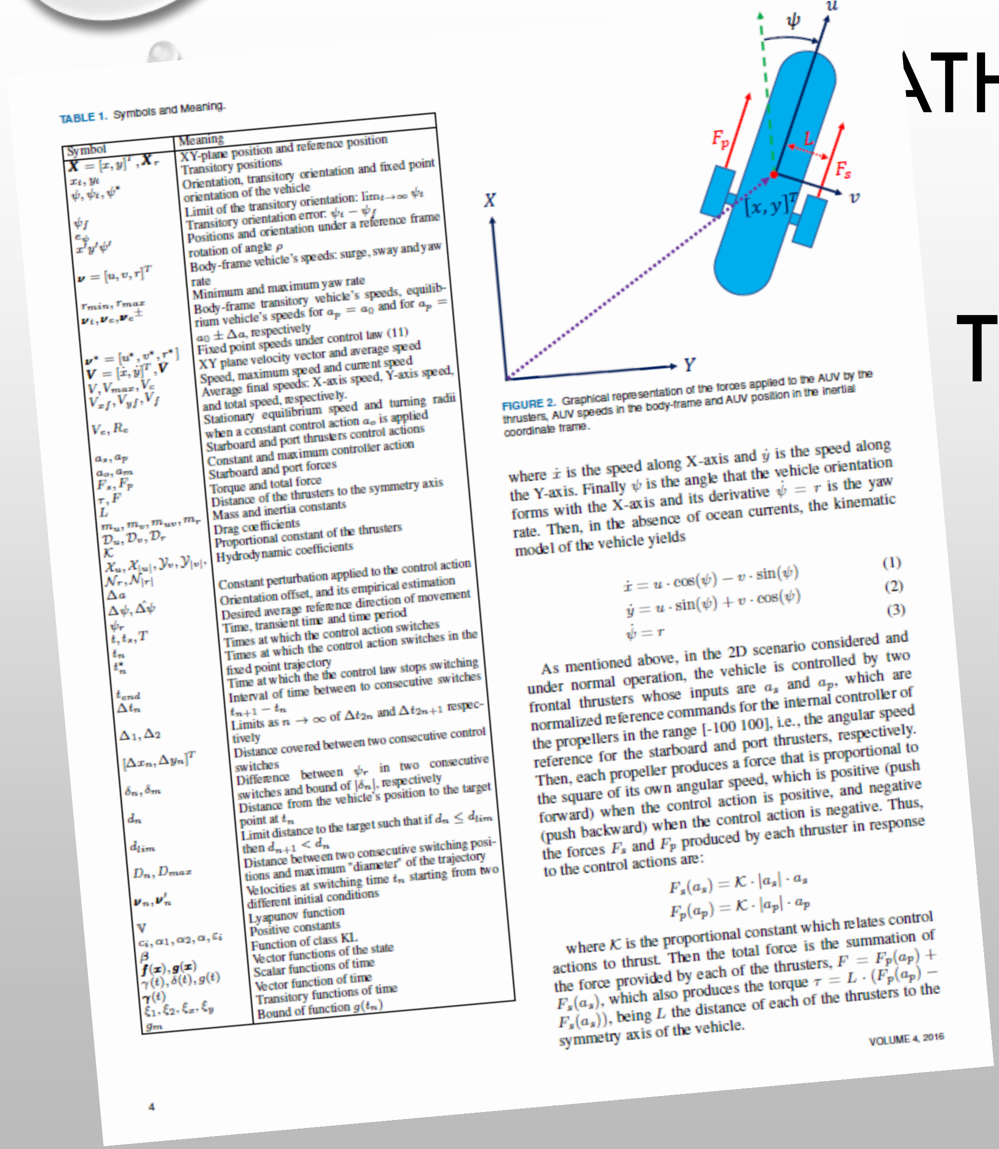


TABLE 1. Symbols and Meaning.

Symbol	Meaning
$\boldsymbol{X} = [x, y]^T, \boldsymbol{X}_r$	XY-plane position and reference position
x_t, y_t	Transitory positions
ψ, ψ_t, ψ^*	Orientation, transitory orientation and fixed point orientation of the vehicle
ψ_f	Limit of the transitory orientation: $\lim_{t\to\infty} \psi_t$
e_ψ	Transitory orientation error: $\psi_t - \psi_f$
$x' y' \psi'$	Positions and orientation under a reference frame rotation of angle ρ
$\boldsymbol{\nu} = [u, v, r]^T$	Body-frame vehicle's speeds: surge, sway and yaw rate
r_{min}, r_{max}	Minimum and maximum yaw rate
$\boldsymbol{\nu}_t, \boldsymbol{\nu}_c, \boldsymbol{\nu}_c^{\pm}$	Body-frame transitory vehicle's speeds, equilibrium vehicle's speeds for $a_p = a_0$ and for $a_p = a_0 \pm \Delta a$, respectively
$\boldsymbol{\nu}^* = [u^*, v^*, r^*]$	Fixed point speeds under control law (11)
$\boldsymbol{V} = [\dot{x}, \dot{y}]^T, \boldsymbol{V}$	XY plane velocity vector and average speed
V, V_{max}, V_c	Speed, maximum speed and current speed
V_{xf}, V_{yf}, V_f	Average final speeds: X-axis speed, Y-axis speed, and total speed, respectively.
V_c, R_c	Stationary equilibrium speed and turning radii when a constant control action a_o is applied
a_s, a_p	Starboard and port thrusters control actions
a_o, a_m	Constant and maximum controller action
F_s, F_p	Starboard and port forces
τ, F	Torque and total force
L	Distance of the thrusters to the symmetry axis
m_u, m_v, m_{uv}, m_r	Mass and inertia constants
$\mathcal{D}_u, \mathcal{D}_v, \mathcal{D}_r$	Drag coefficients
$\mathcal{K}$	Proportional constant of the thrusters
$\mathcal{X}_u, \mathcal{X}_{\|u\|}, \mathcal{Y}_v, \mathcal{Y}_{\|v\|}, \mathcal{N}_r, \mathcal{N}_{\|r\|}$	Hydrodynamic coefficients
Δa	Constant perturbation applied to the control action
$\Delta\psi, \hat{\Delta\psi}$	Orientation offset, and its empirical estimation
ψ_r	Desired average reference direction of movement
t, t_s, T	Time, transient time and time period
t_n	Times at which the control action switches
t_n^*	Times at which the control action switches in the fixed point trajectory
t_{end}	Time at which the the control law stops switching
Δt_n	Interval of time between to consecutive switches $t_{n+1} - t_n$
Δ_1, Δ_2	Limits as $n \to \infty$ of Δt_{2n} and Δt_{2n+1} respectively
$[\Delta x_n, \Delta y_n]^T$	Distance covered between two consecutive control switches
δ_n, δ_m	Difference between ψ_r in two consecutive switches and bound of $\|\delta_n\|$, respectively
d_n	Distance from the vehicle's position to the target point at t_n
d_{lim}	Limit distance to the target such that if $d_n \leq d_{lim}$ then $d_{n+1} < d_n$
D_n, D_{max}	Distance between two consecutive switching positions and maximum "diameter" of the trajectory
$\boldsymbol{\nu}_n, \boldsymbol{\nu}'_n$	Velocities at switching time t_n starting from two different initial conditions
V	Lyapunov function
$c_i, \alpha_1, \alpha_2, \alpha, \varepsilon_i$	Positive constants
β	Function of class KL
$\boldsymbol{f}(\boldsymbol{x}), \boldsymbol{g}(\boldsymbol{x})$	Vector functions of the state
$\gamma(t), \delta(t), g(t)$	Scalar functions of time
$\boldsymbol{\gamma}(t)$	Vector function of time
$\xi_1, \xi_2, \xi_x, \xi_y$	Transitory functions of time
g_m	Bound of function $g(t_n)$

FIGURE 2. Graphical representation of the forces applied to the AUV by the thrusters, AUV speeds in the body-frame and AUV position in the inertial coordinate frame.

where $\dot{x}$ is the speed along X-axis and $\dot{y}$ is the speed along the Y-axis. Finally ψ is the angle that the vehicle orientation forms with the X-axis and its derivative $\dot{\psi} = r$ is the yaw rate. Then, in the absence of ocean currents, the kinematic model of the vehicle yields

$$\dot{x} = u \cdot \cos(\psi) - v \cdot \sin(\psi) \quad (1)$$
$$\dot{y} = u \cdot \sin(\psi) + v \cdot \cos(\psi) \quad (2)$$
$$\dot{\psi} = r \quad (3)$$

As mentioned above, in the 2D scenario considered and under normal operation, the vehicle is controlled by two frontal thrusters whose inputs are a_s and a_p, which are normalized reference commands for the internal controller of the propellers in the range [-100 100], i.e., the angular speed reference for the starboard and port thrusters, respectively. Then, each propeller produces a force that is proportional to the square of its own angular speed, which is positive (push forward) when the control action is positive, and negative (push backward) when the control action is negative. Thus, the forces F_s and F_p produced by each thruster in response to the control actions are:

$$F_s(a_s) = \mathcal{K} \cdot |a_s| \cdot a_s$$
$$F_p(a_p) = \mathcal{K} \cdot |a_p| \cdot a_p$$

where $\mathcal{K}$ is the proportional constant which relates control actions to thrust. Then the total force is the summation of the force provided by each of the thrusters, $F = F_p(a_p) + F_s(a_s)$, which also produces the torque $\tau = L \cdot (F_p(a_p) - F_s(a_s))$, being L the distance of each of the thrusters to the symmetry axis of the vehicle.

4

VOLUME 4, 2016

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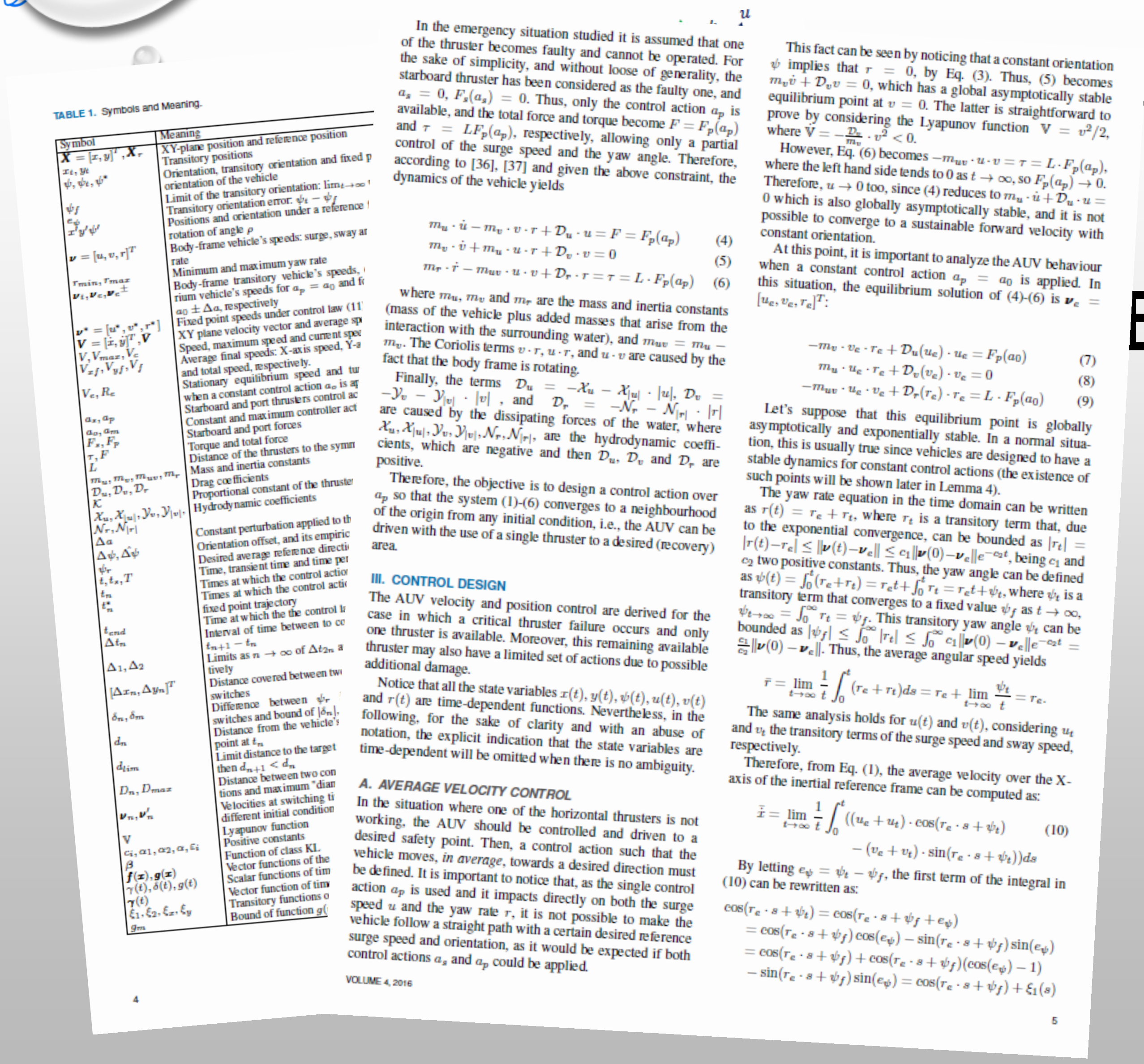
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$\boldsymbol{\nu}_t, \boldsymbol{\nu}_e, \boldsymbol{\nu}_e^{\pm}$	Body-frame transitory vehicle's speeds, ... rium vehicle's speeds for $a_p = a_0$ and f... $a_0 \pm \Delta a$, respectively
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$$m_u \cdot \dot{u} - m_v \cdot v \cdot r + \mathcal{D}_u \cdot u = F = F_p(a_p) \quad (4)$$
$$m_v \cdot \dot{v} + m_u \cdot u \cdot r + \mathcal{D}_v \cdot v = 0 \quad (5)$$
$$m_r \cdot \dot{r} - m_{uv} \cdot u \cdot v + \mathcal{D}_r \cdot r = \tau = L \cdot F_p(a_p) \quad (6)$$

where m_u, m_v and m_r are the mass and inertia constants (mass of the vehicle plus added masses that arise from the interaction with the surrounding water), and $m_{uv} = m_u - m_v$. The Coriolis terms $v \cdot r$, $u \cdot r$, and $u \cdot v$ are caused by the fact that the body frame is rotating.

Finally, the terms $\mathcal{D}_u = -\mathcal{X}_u - \mathcal{X}_{\lvert u\rvert} \cdot \lvert u\rvert$, $\mathcal{D}_v = -\mathcal{Y}_v - \mathcal{Y}_{\lvert v\rvert} \cdot \lvert v\rvert$, and $\mathcal{D}_r = -\mathcal{N}_r - \mathcal{N}_{\lvert r\rvert} \cdot \lvert r\rvert$ are caused by the dissipating forces of the water, where $\mathcal{X}_u, \mathcal{X}_{\lvert u\rvert}, \mathcal{Y}_v, \mathcal{Y}_{\lvert v\rvert}, \mathcal{N}_r, \mathcal{N}_{\lvert r\rvert}$, are the hydrodynamic coefficients, which are negative and then $\mathcal{D}_u$, $\mathcal{D}_v$ and $\mathcal{D}_r$ are positive.

Therefore, the objective is to design a control action over a_p so that the system (1)-(6) converges to a neighbourhood of the origin from any initial condition, i.e., the AUV can be driven with the use of a single thruster to a desired (recovery) area.

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The AUV velocity and position control are derived for the case in which a critical thruster failure occurs and only one thruster is available. Moreover, this remaining available thruster may also have a limited set of actions due to possible additional damage.

Notice that all the state variables $x(t)$, $y(t)$, $\psi(t)$, $u(t)$, $v(t)$ and $r(t)$ are time-dependent functions. Nevertheless, in the following, for the sake of clarity and with an abuse of notation, the explicit indication that the state variables are time-dependent will be omitted when there is no ambiguity.

A. AVERAGE VELOCITY CONTROL

In the situation where one of the horizontal thrusters is not working, the AUV should be controlled and driven to a desired safety point. Then, a control action such that the vehicle moves, *in average*, towards a desired direction must be defined. It is important to notice that, as the single control action a_p is used and it impacts directly on both the surge speed u and the yaw rate r, it is not possible to make the vehicle follow a straight path with a certain desired reference surge speed and orientation, as it would be expected if both control actions a_s and a_p could be applied.

This fact can be seen by noticing that a constant orientation ψ implies that $r = 0$, by Eq. (3). Thus, (5) becomes $m_v\dot{v} + \mathcal{D}_v v = 0$, which has a global asymptotically stable equilibrium point at $v = 0$. The latter is straightforward to prove by considering the Lyapunov function $\mathbb{V} = v^2/2$, where $\dot{\mathbb{V}} = -\frac{\mathcal{D}_v}{m_v} \cdot v^2 < 0$.

However, Eq. (6) becomes $-m_{uv} \cdot u \cdot v = \tau = L \cdot F_p(a_p)$, where the left hand side tends to 0 as $t \to \infty$, so $F_p(a_p) \to 0$. Therefore, $u \to 0$ too, since (4) reduces to $m_u \cdot \dot{u} + \mathcal{D}_u \cdot u = 0$ which is also globally asymptotically stable, and it is not possible to converge to a sustainable forward velocity with constant orientation.

At this point, it is important to analyze the AUV behaviour when a constant control action $a_p = a_0$ is applied. In this situation, the equilibrium solution of (4)-(6) is $\boldsymbol{\nu}_e = [u_e, v_e, r_e]^T$:

$$-m_v \cdot v_e \cdot r_e + \mathcal{D}_u(u_e) \cdot u_e = F_p(a_0) \quad (7)$$
$$m_u \cdot u_e \cdot r_e + \mathcal{D}_v(v_e) \cdot v_e = 0 \quad (8)$$
$$-m_{uv} \cdot u_e \cdot v_e + \mathcal{D}_r(r_e) \cdot r_e = L \cdot F_p(a_0) \quad (9)$$

Let's suppose that this equilibrium point is globally asymptotically and exponentially stable. In a normal situation, this is usually true since vehicles are designed to have a stable dynamics for constant control actions (the existence of such points will be shown later in Lemma 4).

The yaw rate equation in the time domain can be written as $r(t) = r_e + r_t$, where r_t is a transitory term that, due to the exponential convergence, can be bounded as $\lvert r_t\rvert = \lvert r(t) - r_e\rvert \le \lVert\boldsymbol{\nu}(t) - \boldsymbol{\nu}_e\rVert \le c_1\lVert\boldsymbol{\nu}(0) - \boldsymbol{\nu}_e\rVert e^{-c_2 t}$, being c_1 and c_2 two positive constants. Thus, the yaw angle can be defined as $\psi(t) = \int_0^t (r_e + r_t) = r_e t + \int_0^t r_t = r_e t + \psi_t$, where ψ_t is a transitory term that converges to a fixed value ψ_f as $t \to \infty$, $\psi_{t\to\infty} = \int_0^\infty r_t = \psi_f$. This transitory yaw angle ψ_t can be bounded as $\lvert\psi_f\rvert \le \int_0^\infty \lvert r_t\rvert \le \int_0^\infty c_1\lVert\boldsymbol{\nu}(0) - \boldsymbol{\nu}_e\rVert e^{-c_2 t} = \frac{c_1}{c_2}\lVert\boldsymbol{\nu}(0) - \boldsymbol{\nu}_e\rVert$. Thus, the average angular speed yields

$$\bar{r} = \lim_{t\to\infty} \frac{1}{t}\int_0^t (r_e + r_t)ds = r_e + \lim_{t\to\infty} \frac{\psi_t}{t} = r_e.$$

The same analysis holds for $u(t)$ and $v(t)$, considering u_t and v_t the transitory terms of the surge speed and sway speed, respectively.

Therefore, from Eq. (1), the average velocity over the X-axis of the inertial reference frame can be computed as:

$$\bar{\dot{x}} = \lim_{t\to\infty} \frac{1}{t}\int_0^t ((u_e + u_t) \cdot \cos(r_e \cdot s + \psi_t) - (v_e + v_t) \cdot \sin(r_e \cdot s + \psi_t))ds \quad (10)$$

By letting $e_\psi = \psi_t - \psi_f$, the first term of the integral in (10) can be rewritten as:

$$\begin{aligned}\cos(r_e \cdot s + \psi_t) &= \cos(r_e \cdot s + \psi_f + e_\psi)\\ &= \cos(r_e \cdot s + \psi_f)\cos(e_\psi) - \sin(r_e \cdot s + \psi_f)\sin(e_\psi)\\ &= \cos(r_e \cdot s + \psi_f) + \cos(r_e \cdot s + \psi_f)(\cos(e_\psi) - 1)\\ &- \sin(r_e \cdot s + \psi_f)\sin(e_\psi) = \cos(r_e \cdot s + \psi_f) + \xi_1(s)\end{aligned}$$

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where $\xi_1(s)$ tends exponentially to zero as $s \to \infty$ since ψ_t tends exponentially to ψ_f, so $e_\psi \to 0$. Following similar computations, the second term in (10) yields $\sin(r_e \cdot s + \psi_t) = \sin(r_e \cdot s + \psi_f) + \xi_2(s)$, where $\xi_2(s)$ also tends exponentially to zero as $s \to \infty$, and thus, the average velocity over the X-axis of the inertial reference frame becomes:

$$\bar{\dot{x}} = \lim_{t\to\infty} \frac{1}{t} \int_0^t ((u_e + u_t)\cos(r_e \cdot s + \psi_t) - (v_e + v_t)\sin(r_e \cdot s + \psi_t))ds$$
$$= \lim_{t\to\infty} \frac{1}{t} \int_0^t (u_e \cos(r_e s + \psi_f) - v_e \sin(r_e s + \psi_f))ds$$
$$+ \lim_{t\to\infty} \frac{1}{t} \int_0^t (u_t \cos(r_e s + \psi_f) - v_t \sin(r_e s + \psi_f))ds$$
$$+ \lim_{t\to\infty} \frac{1}{t} \int_0^t (u_e + u_t)\xi_1(s) + (v_e + v_t)\xi_2(s)ds$$
$$= \lim_{t\to\infty} \frac{1}{t} \int_0^t (u_e \cos(r_e s + \psi_f) - v_e \sin(r_e s + \psi_f))ds$$
$$= 0$$

Notice that this average speed is zero because the integral of exponentially decreasing functions are bounded, and the trigonometric functions that are periodic with zero average over a period are also bounded. The same analysis holds for $\bar{\dot{y}}$, the average speed over the Y-axis of the inertial reference frame. This result has an intuitive explanation. Notice that when enough time has passed, the vehicle will move with constant speed $V_e = \sqrt{u_e^2 + v_e^2}$ and constant angular velocity r_e. Thus, the trajectory will be a circle of radii $R_e = \frac{V_e}{r_e}$, and the average advance velocity is zero.

In order to obtain an non-zero average advance velocity, the symmetry of the circular trajectory must be broken, so a constant input control action is not suitable. The following control law is proposed:

$$a_p = a_0 + \Delta a \cdot \mathrm{sign}(\sin(\psi)) \quad (11)$$

The concept within (11) is that the control action will be different when the vehicle is oriented to East ($\sin(\psi) > 0$) than when the vehicle is oriented to West ($\sin(\psi) < 0$) breaking the symmetry of the circular trajectory, see Fig. 3. Notice that the sign of the control law (11) changes when $\psi = n\pi$, with $n = 0, 1, 2, \ldots$. The value of ψ for changing the sign of (11) has been chosen for simplicity of explanation, and any other angle could be chosen without loss of generality. In fact, the selected angle will depend on the desired direction of movement, as it will be shown in the forthcoming analysis.

The control law works as follows: Consider that the vehicle turns clockwise ($r > 0$) describing an approximately circular trajectory (for Δa small) and pointing to the interior of the circular path because $u > 0$ and $v < 0$. Therefore, the velocity vector of the vehicle, which is tangent to the trajectory, points to port. When $\psi = 2n\pi$ the vehicle points

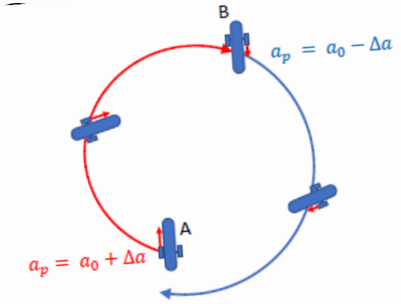

FIGURE 3. Behaviour of control law (11).

exactly to the North, which is the frontier at which the vehicle changes its orientation from West to East (point A of Figure 3), and a_p switches from $a_p = a_0 - \Delta a$ to $a_p = a_0 + \Delta a$, following the red path of Figure 3. Once ψ reaches $(2n+1)\pi$, the vehicle points exactly to the South (point B of Figure 3) and the control law switches again to $a_p = a_0 - \Delta a$, describing the blue path. Since the control action is different in both parts of the path (red and blue) the trajectory does not close, i.e. it is not a circumference. Thus, the vehicle moves in "some" direction, which depends on the angle or condition to switch the control law. This pattern is repeated periodically producing an average movement of the vehicle on a certain course.

In order to prove the above statement, and before starting the stability analysis of the control law proposed, we must resort to some technical results:

Lemma 1. *Consider system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ such that $\boldsymbol{x}_e$ is an exponentially stable equilibrium point of the system and the first derivative of $\boldsymbol{f}$ is bounded and Lipschiz on $\boldsymbol{x}$ in a domain $\|\boldsymbol{x} - \boldsymbol{x}_e\| < c_0$, where c_0 is a positive constant. Let $\boldsymbol{x}_1(t)$ and $\boldsymbol{x}_2(t)$ be two solutions of the differential equation with initial conditions $\boldsymbol{x}_1(t_0)$ and $\boldsymbol{x}_2(t_0)$, respectively. Then $\|\boldsymbol{x}_2(t) - \boldsymbol{x}_1(t)\| < c_1 e^{-c_2(t-t_0)}\|\boldsymbol{x}_2(t_0) - \boldsymbol{x}_1(t_0)\|$ for $t > t_0$, being c_1 and c_2 two positive constants.*

Proof. This is a simple application of Theorem 9.1 from [38] for the unperturbed case ($\boldsymbol{g}(\boldsymbol{x}) = 0$ in equation 9.6 of [38]), when, in addition, the system is autonomous. □

Lemma 2. *Consider system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, a_p)$ continuously differentiable and locally Lipschiz in a domain $\|\boldsymbol{x}\| < c_3 + \varepsilon_1$, where c_3 is a positive constant and ε_1 is a small positive constant. Suppose also that the solutions of the system are globally uniformly ultimately bounded (GUUB) with bound c_3 for $0 \le a_p \le a_m$. Then, $\|\boldsymbol{x}(t)\| \le \boldsymbol{\beta}(\|\boldsymbol{x}(t_0)\|, t - t_0) + c_4 a_m$ for some positive constant c_4 and a function $\boldsymbol{\beta}$ of class KL, i.e. $\boldsymbol{\beta}$ is a non decreasing function of its first argument,*

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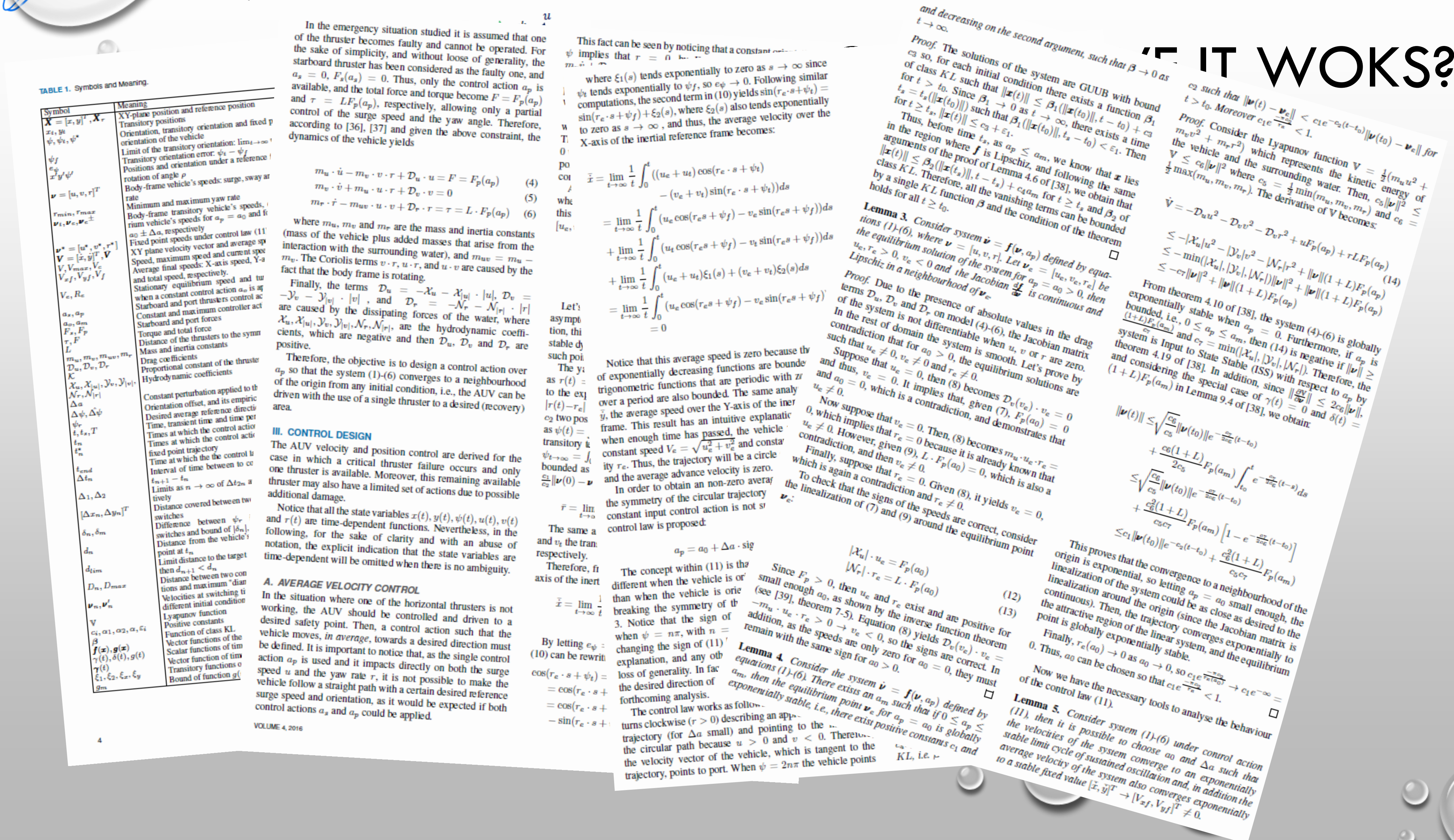

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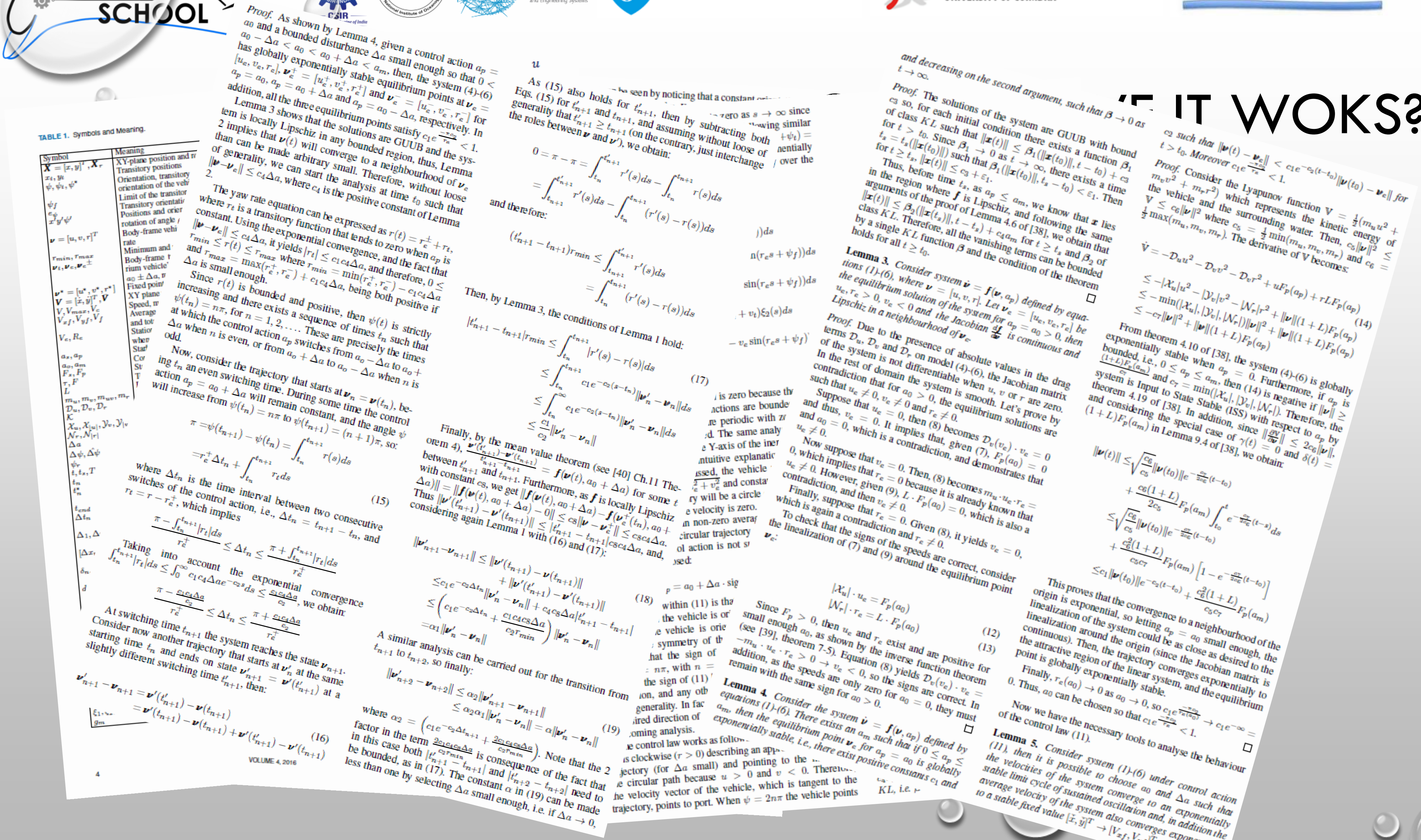
IT WOKS?



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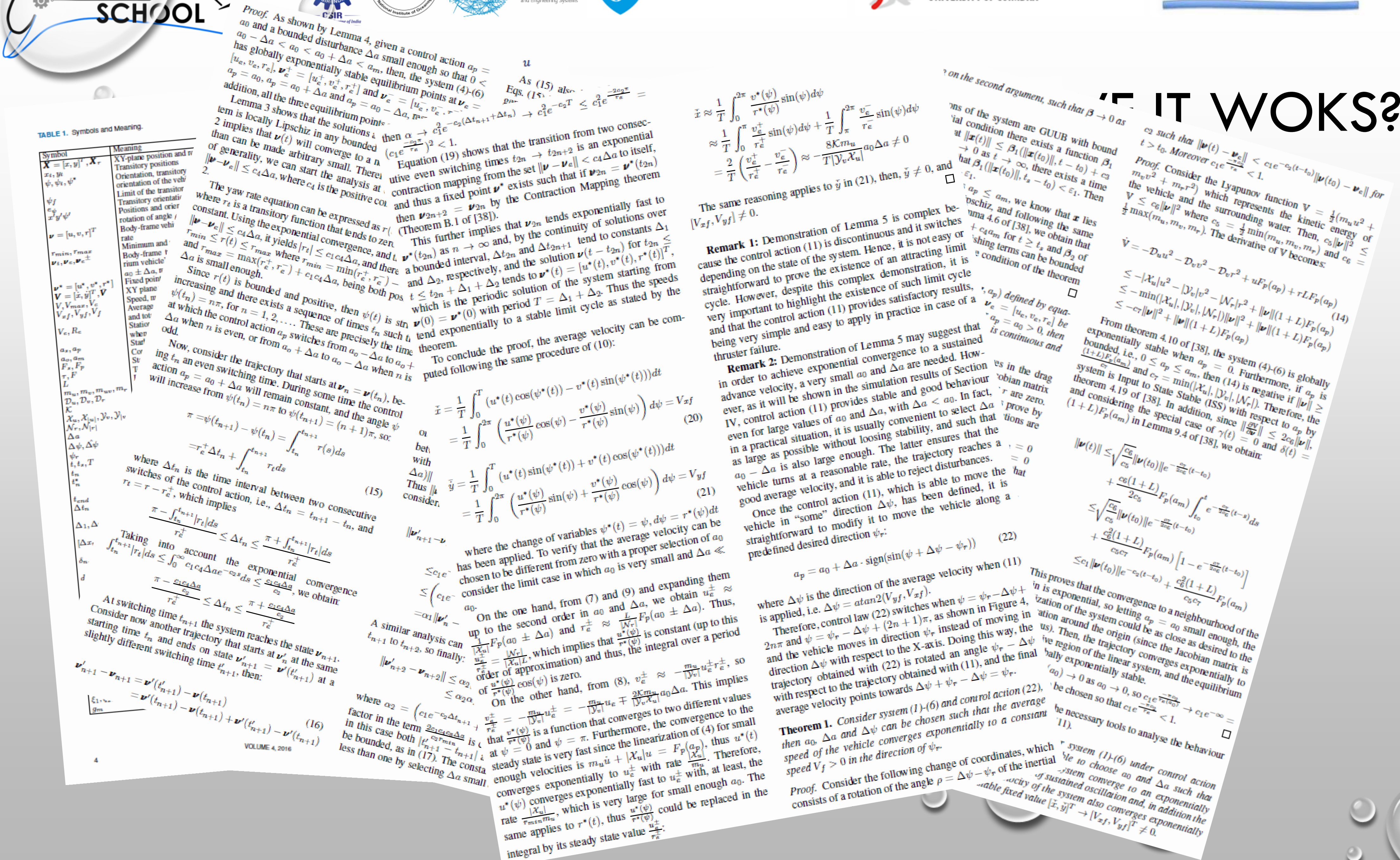

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COULD WE MATHEMATICALLY PROVE IT WOKS?

- IT'S ~~EASY~~ POSSIBLE TO SHOW THAT IT WORKS

  NON LINEAR

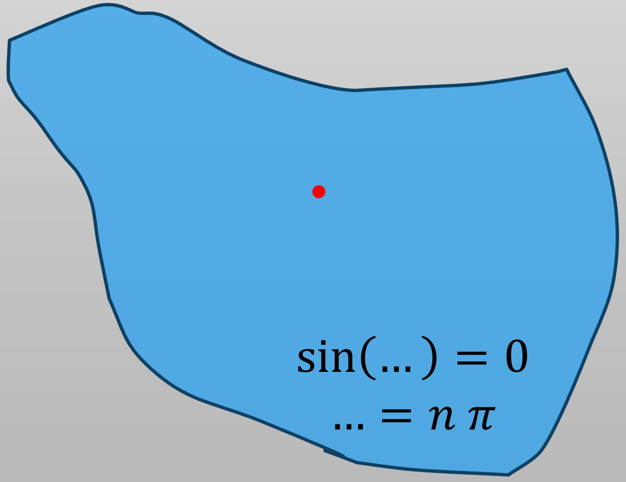
  +WITCHING CONTROLLER (DISCONTINUOUS)

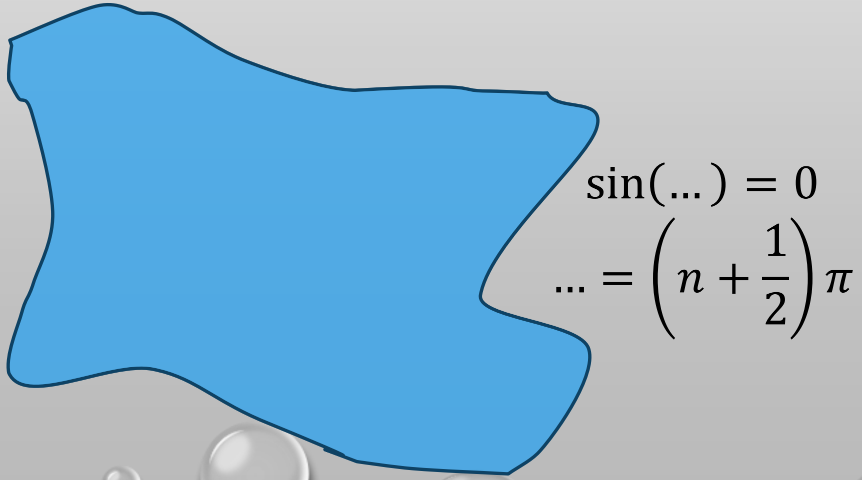
  +NON STANDARD…

  = HARD MATH

IDEAS OF THE PROOF

- *WE NEED TO PROVE THAT A LIMIT CYCLE (SUSTAINED OSCILLATION) EXIST FOR THE VELOCITIES*
  - *IF THE CONTROL ACTION IS CONSTANT* $a_p = a_0 \pm \Delta a$ *THE VEHICLE TENDS TO STABLE BODY FRAME VELOCITIES*
  - *THE CONTROL SWITCHES WHEN IT REACHES A SWITCHING SURFACE (SIN(…)=0)*
  - *IT SWITCHES INFINITELY MANY TIMES→ LOOK WHAT HAPPENS AT THE SWITCHING POINTS*





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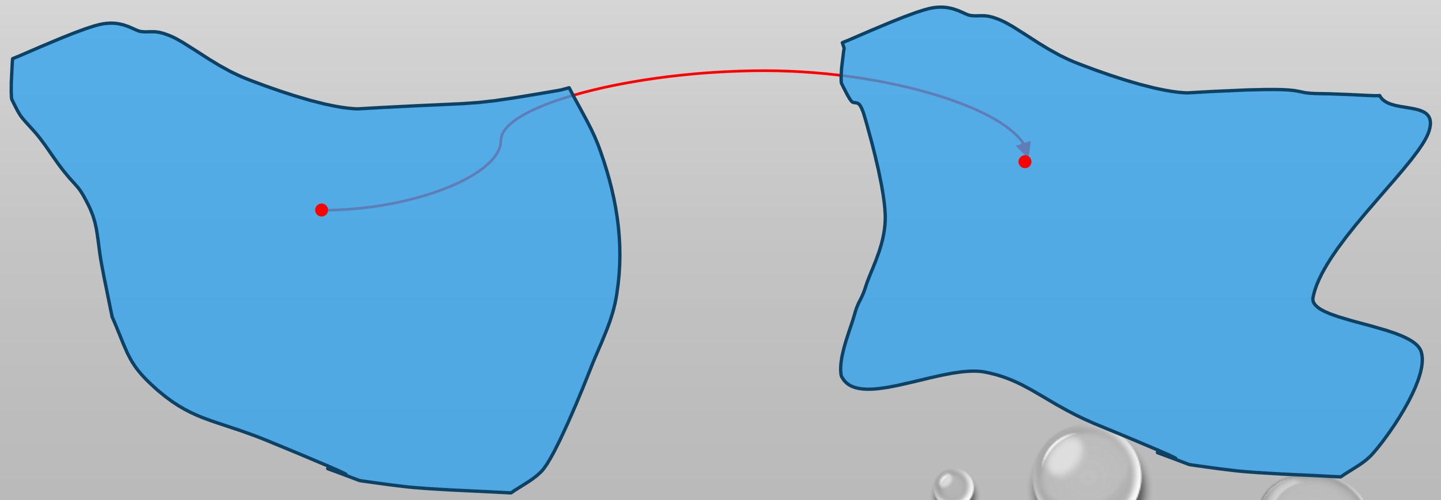

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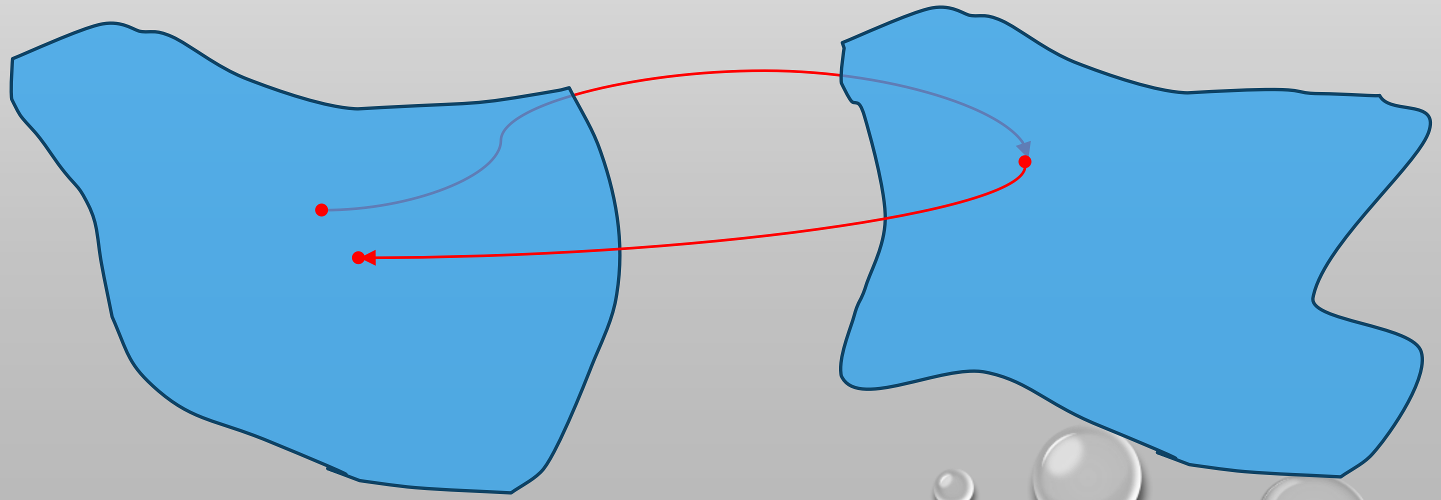
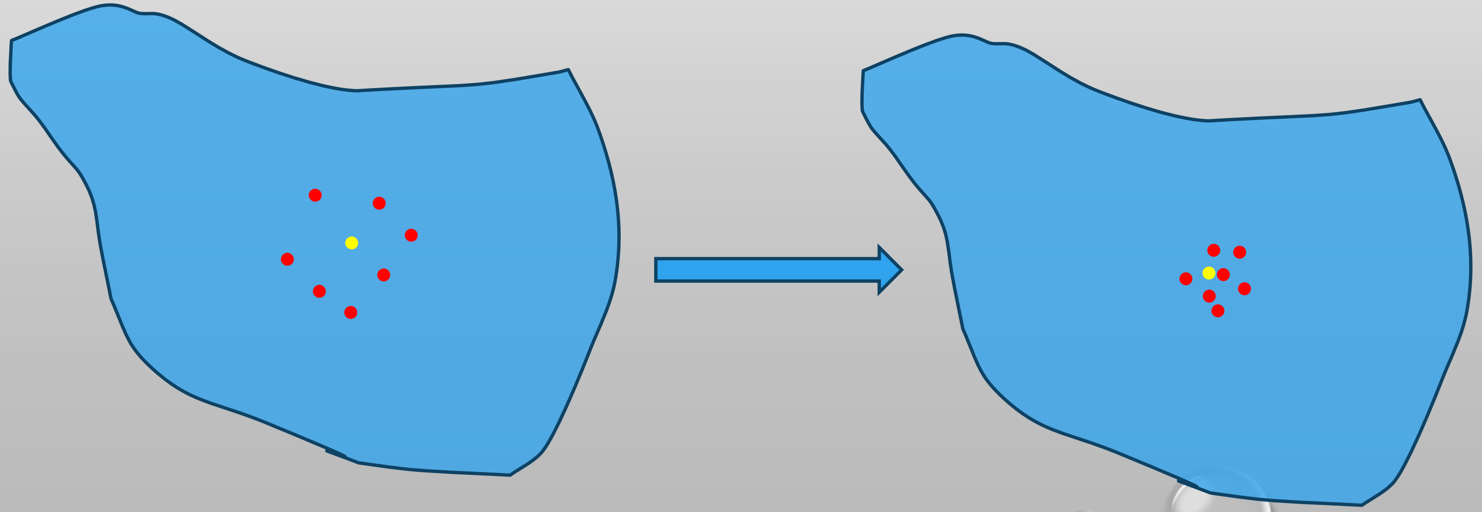


MORE IDEAS

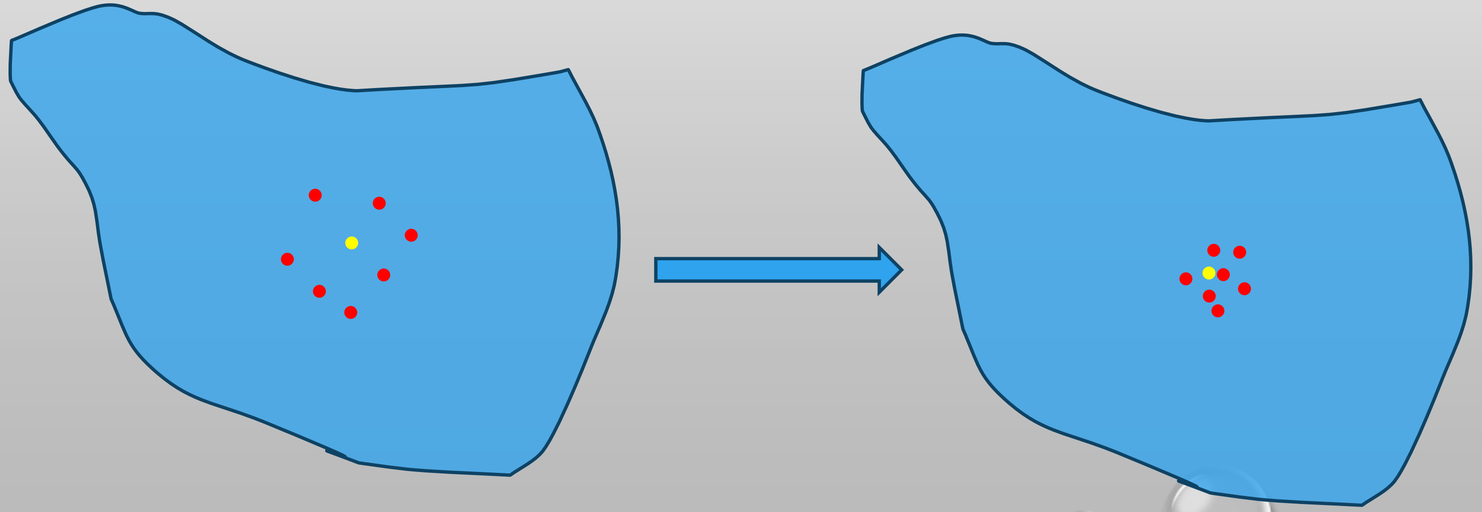
- *WE NEED TO PROVE THAT A LIMIT CYCLE (SUSTAINED OSCILLATION) EXIST FOR THE VELOCITIES*
  - *LOOK WHAT HAPPENS TO CLOSE TRAJECTORIES*
    - *IF THEY APPROACH EACH OTHER → CONTRACTIVE, IT EXIST AN ATTRACTIVE FIXED POINT*



POINCARE MAP

MORE IDEAS

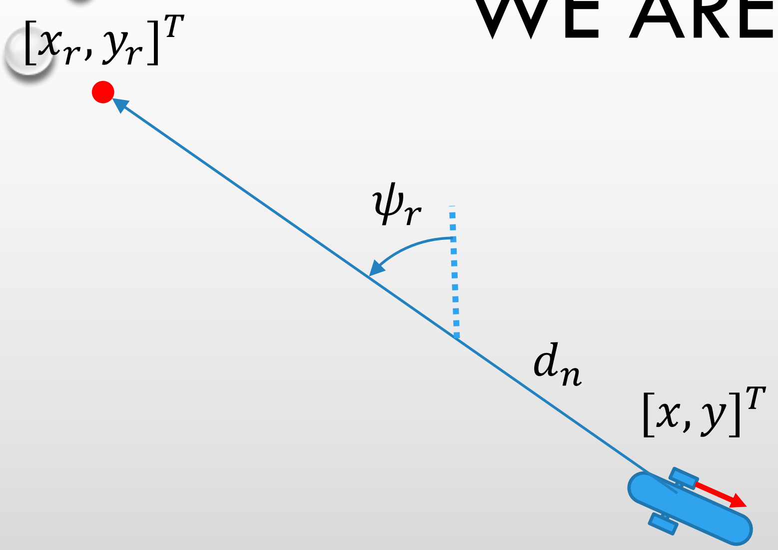
- *WE NEED TO PROVE THAT A LIMIT CYCLE (SUSTAINED OSCILLATION) EXIST FOR THE VELOCITIES*
  - *LOOK WHAT HAPPENS TO CLOSE TRAJECTORIES*
    - *IF THEY APPROACH EACH OTHER → CONTRACTIVE, IT EXIST AN ATTRACTIVE FIXED POINT*
    - *NOW WE NEED TO SHOW THAT THIS FIXED POINT HAS AN AVERAGE VELOCITY ≠0*



POINCARE MAP

BUT WE WANT TO GO HOME!

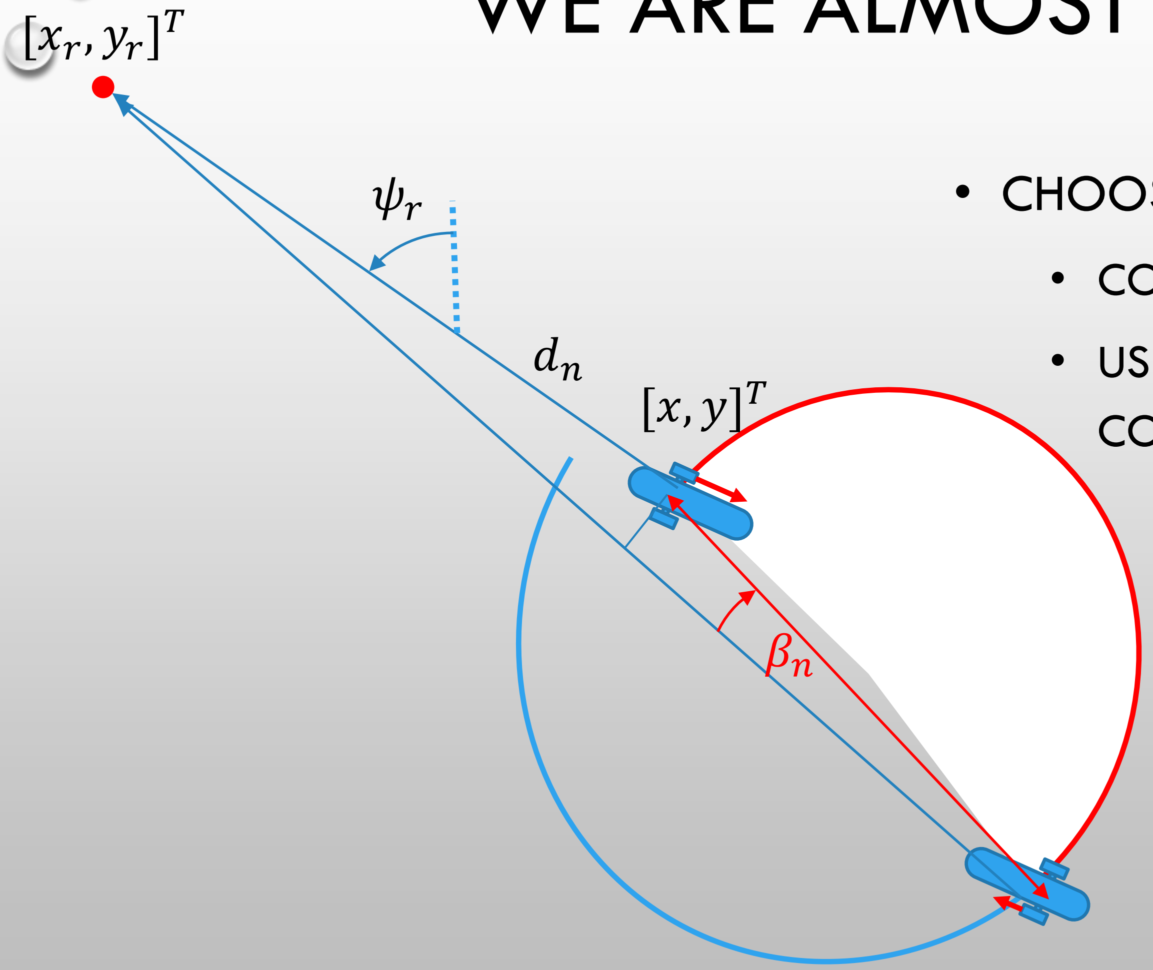
WE ARE ALMOST THERE… LAST STEP



- CHOOSE THE POINT I WANT TO BE (SETPOINT)
  - COMPUTE THE DIRECTION TOWARDS THIT POINT

$$\psi_r = atan2(y_r - y, x_r - x)$$

WE ARE ALMOST THERE… LAST STEP

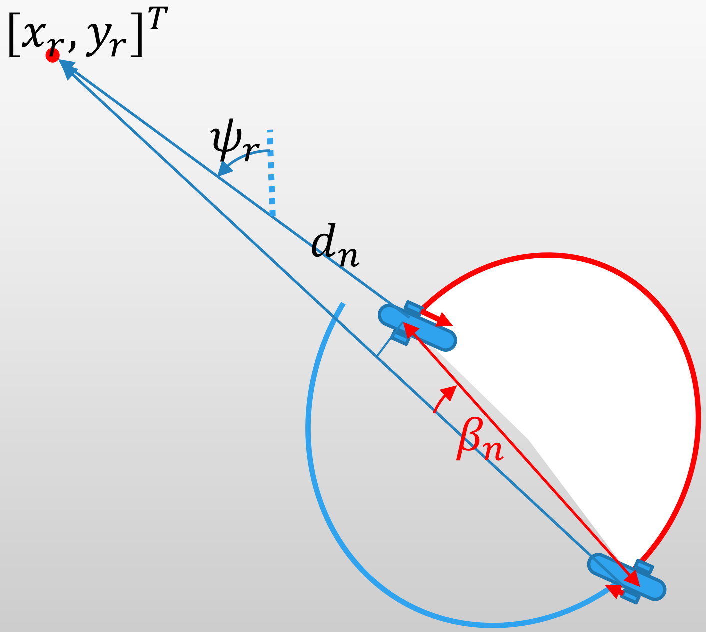


- CHOOSE THE POINT I WANT TO BE (SETPOINT)
  - COMPUTE THE DIRECTION TOWARDS THIT POINT
  - USE IT AS THE DESIRED ORIENTATION IN THE PREVIOUS CONTROLLER

$$\psi_r = atan2(y_r - y, x_r - x)$$
$$a_p = a_0 + \Delta a \cdot \mathrm{sign}(\sin(\psi - \psi_r + \Delta\psi))$$

MORE MATH PROBLEMS…

- WE HAVE A REFERENCE THAT VARIES THROUGH TIME
  - IT INTRODUCES A DISTURBANCE



MORE MATH PROBLEMS…

- WE HAVE A REFERENCE THAT VARIES THROUGH TIME
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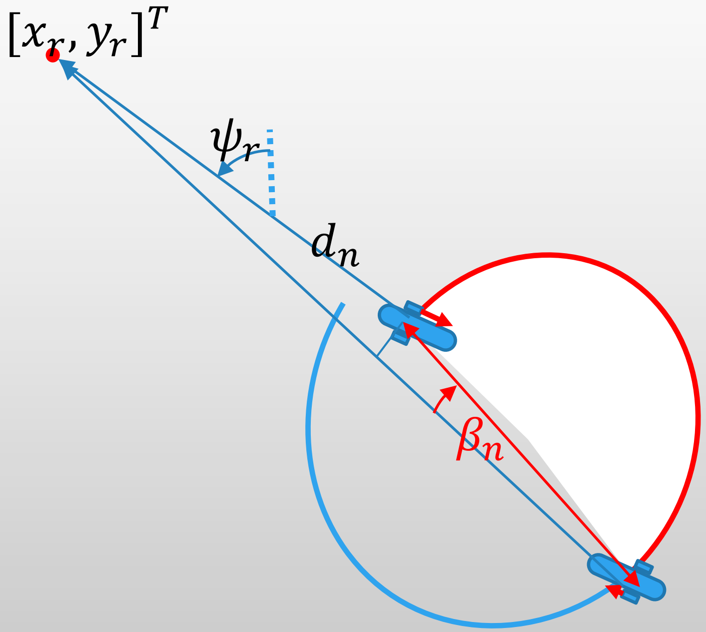
  - IF WE ARE FAR AWAY THE DISTURBANCE IS "SMALL"
    - THE INNER CONTROLLER CAN COMPENSATE IT



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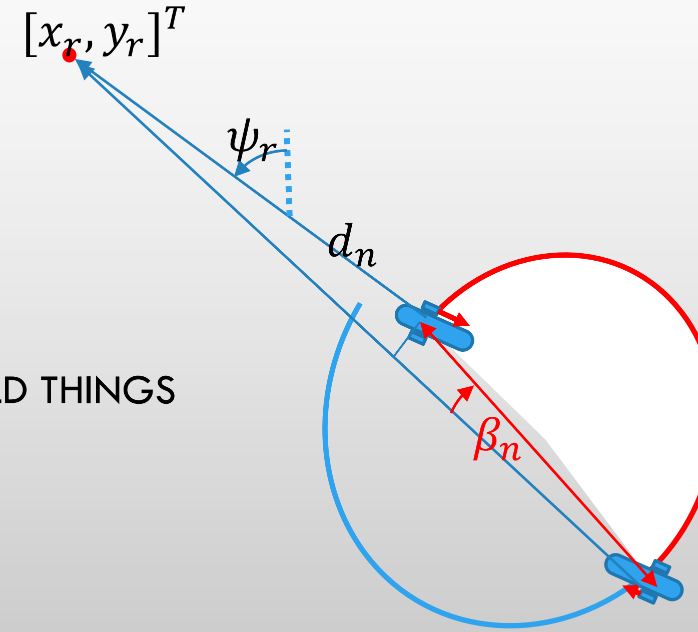
    - THE INNER CONTROLLER CAN COMPENSATE IT
    - WHEN WE ARE CLOSE TO THE REFERENCE THE CONTROLLER COULD DO WILD THINGS
      - OSCILLATE RAPIDLY
      - STOP SWITCHING (ENCIRCLE THE TARGET POINT)



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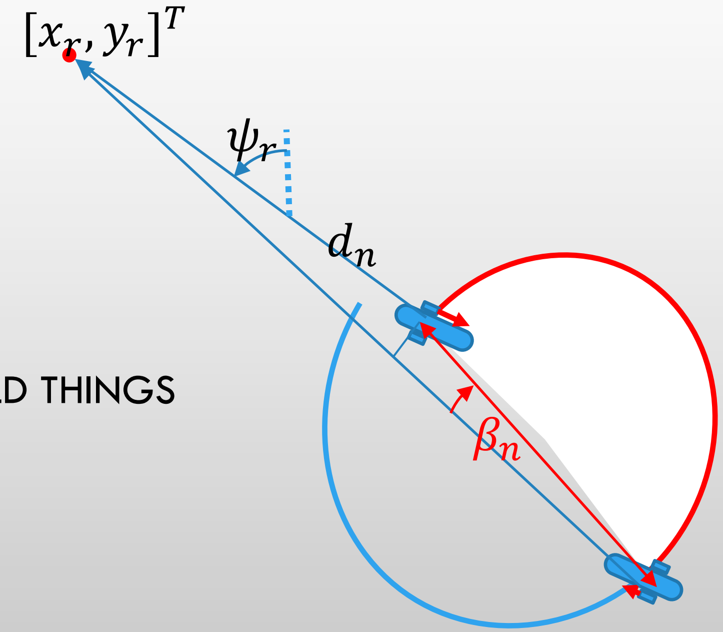
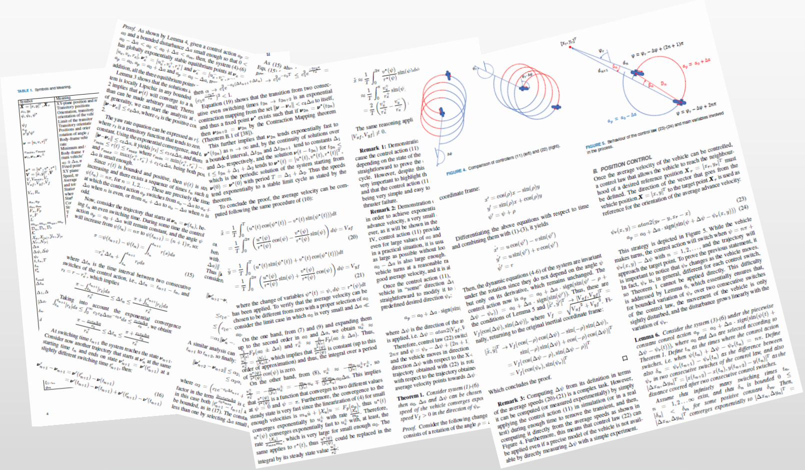
      - STOP SWITCHING (ENCIRCLE THE TARGET POINT)
    - WE DON'T KNOW EXACTLY WHAT WILL HAPPENED BUT IS GFB



- FOR THE INTREPID THE DETAILS ARE ON THE PAPER:
  - D. CHAOS, D. MORENO-SALINAS AND J. ARANDA, "FAULT-TOLERANT CONTROL FOR AUVS USING A SINGLE THRUSTER," IN IEEE ACCESS, VOL. 10, PP. 22123-22139, 2022, DOI: 10.1109/ACCESS.2022.3152190.

REMEMBER, THE IMPORTANT THING IS THE CONTROLLER



$$\psi_r = atan2(y_r - y, x_r - x)$$
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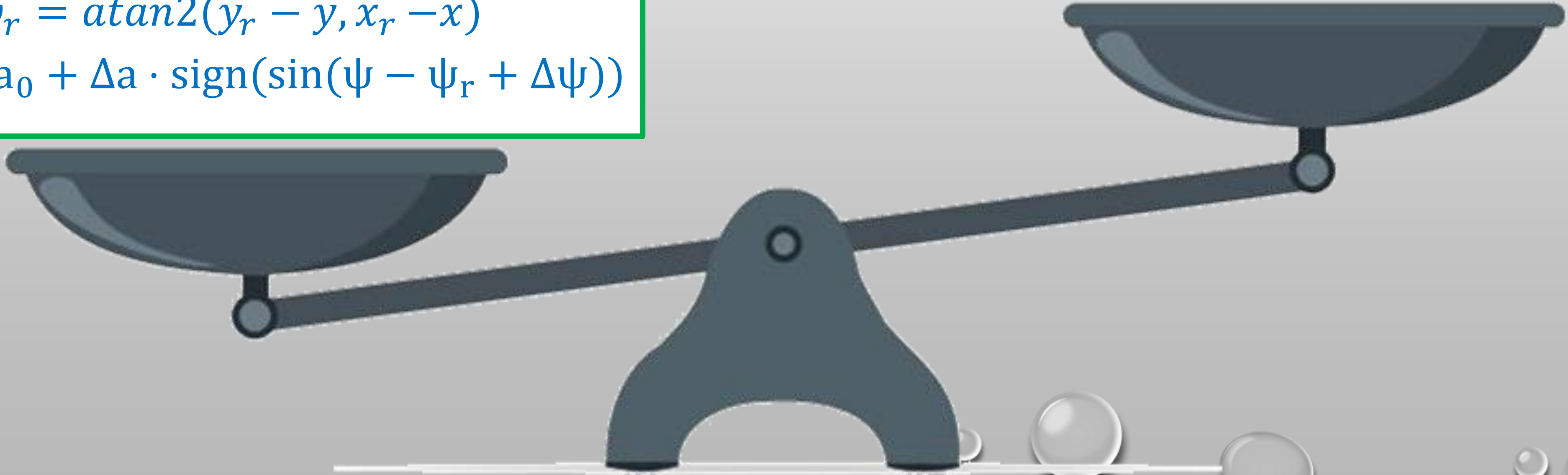
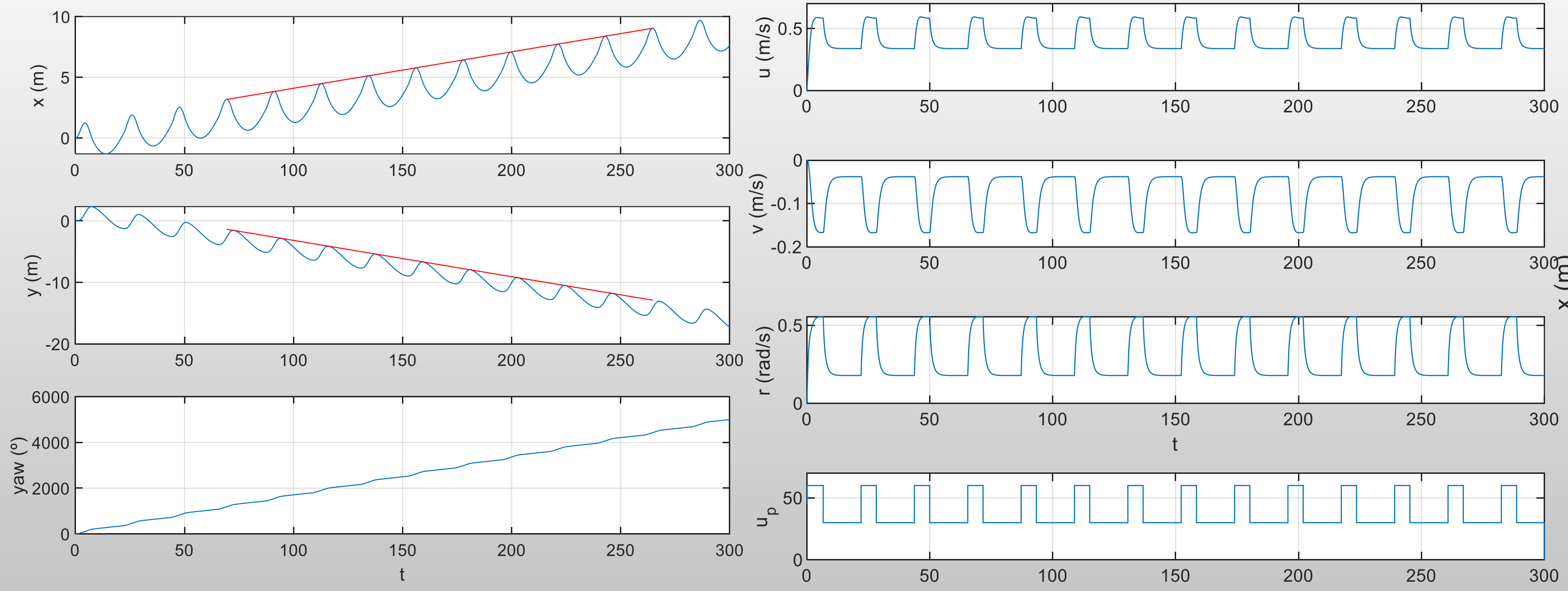
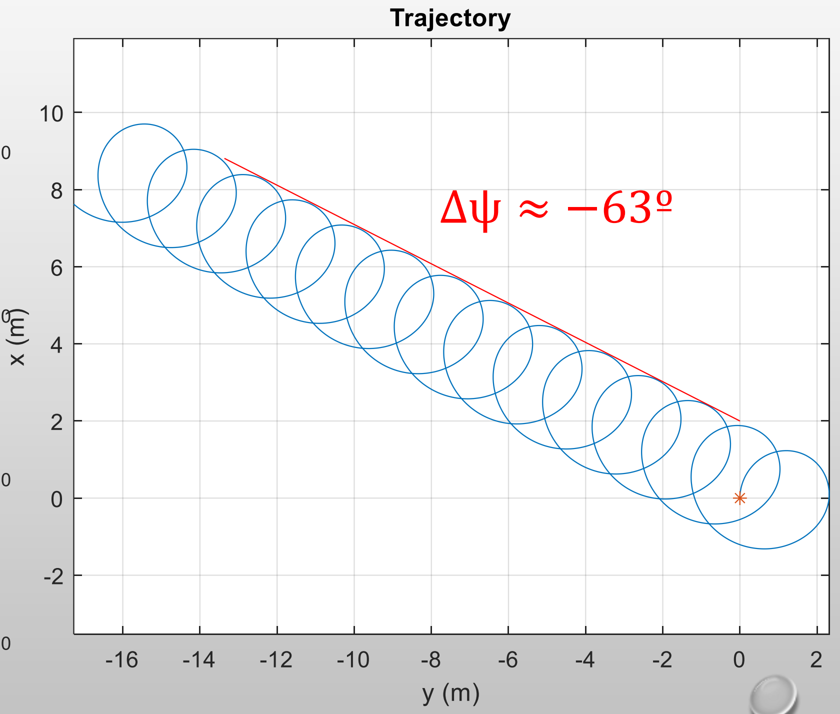


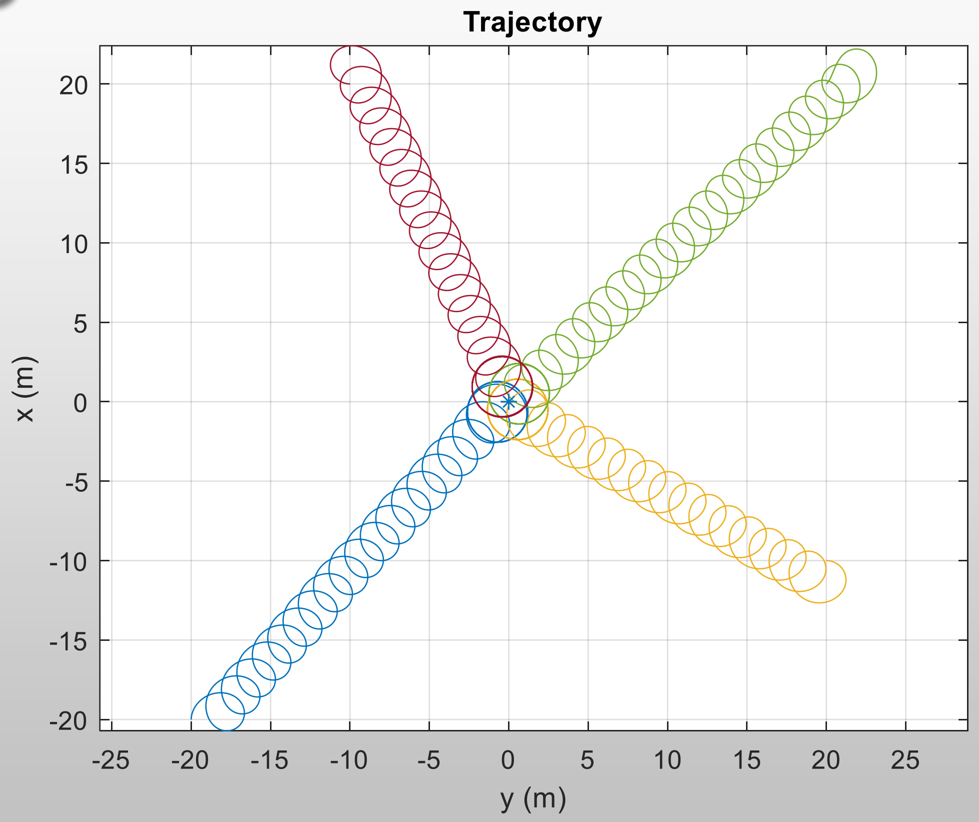
IT WAS EASY TO COMPUTE THE OFFSET





$$a_p = 45 + 15 \cdot \mathrm{sign}(\sin(\psi))$$

ONCE THE CORRECTION IS DONE

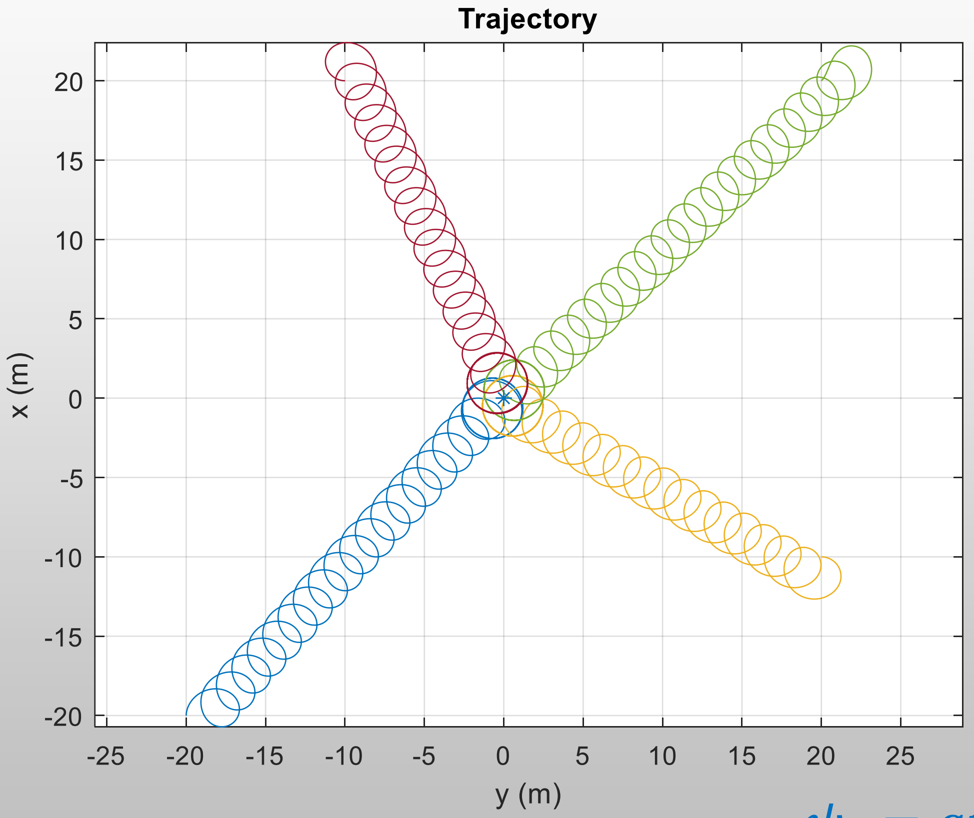


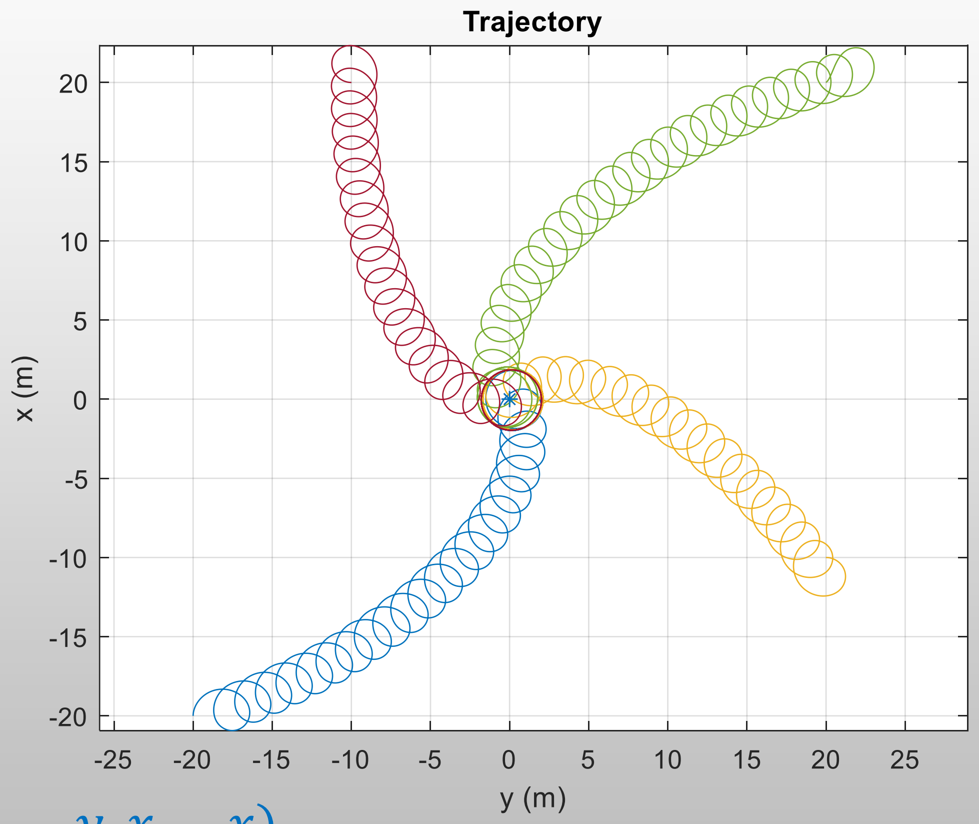
- IT GOES “STRAIGHT” TO THE POINT AND SURROUNDS IT
  - IT STOPS SWITCHING
  - THIS IS WHAT USUALLY HAPPENS
- IT IS ROBUST? WHAT HAPPENS IF THE OFFSET IS WRONG?

$$\psi_r = atan2(y_r - y, x_r - x,)$$

$$a_p = 45 + 15 \cdot \mathrm{sign}(\sin(\psi - \psi_r + 63 \cdot \pi/180))$$

IT STILL WORKS





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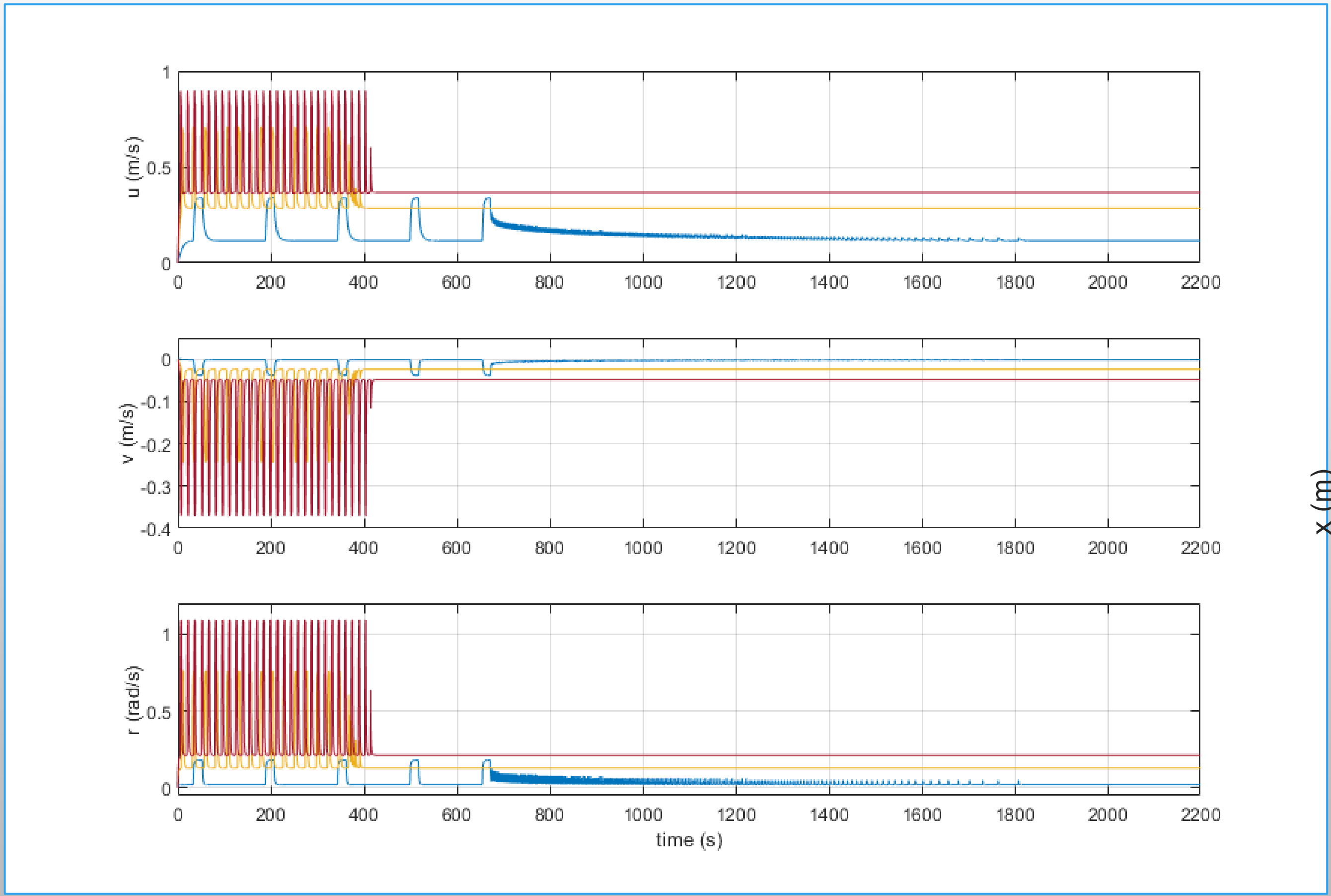
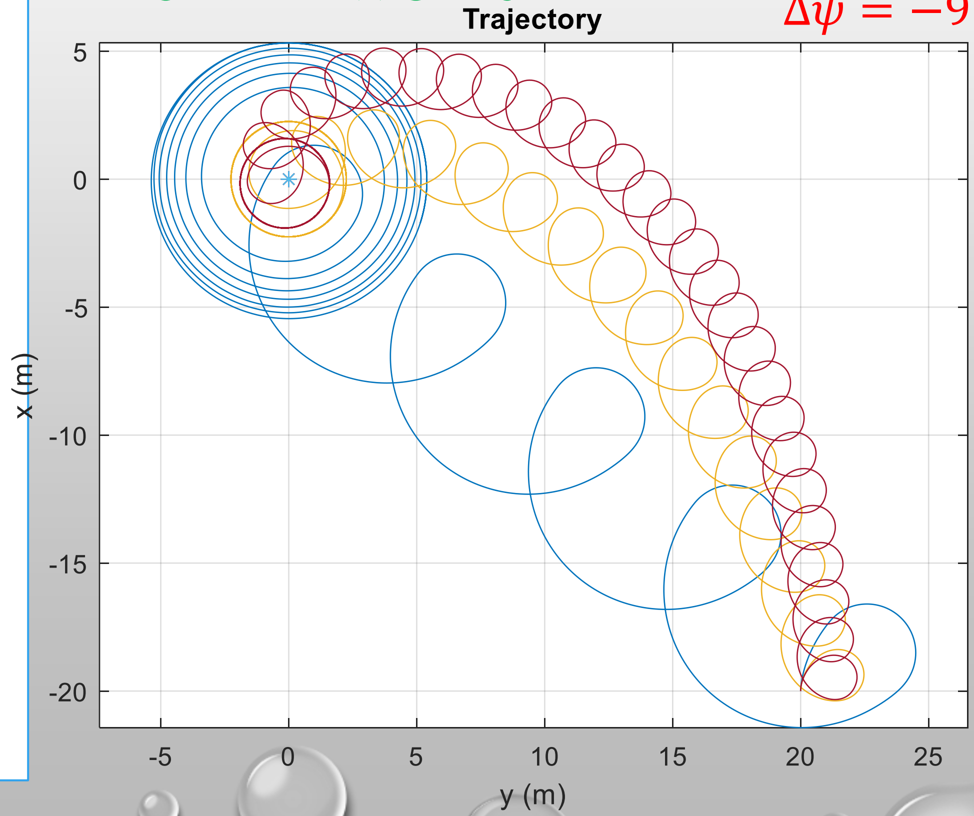
$$a_p = 45 + 15 \cdot \mathrm{sign}(\sin(\psi - \psi_r - \pi/2))$$

WHAT IF THE CONSTANTS ARE NOT WELL TUNNED?



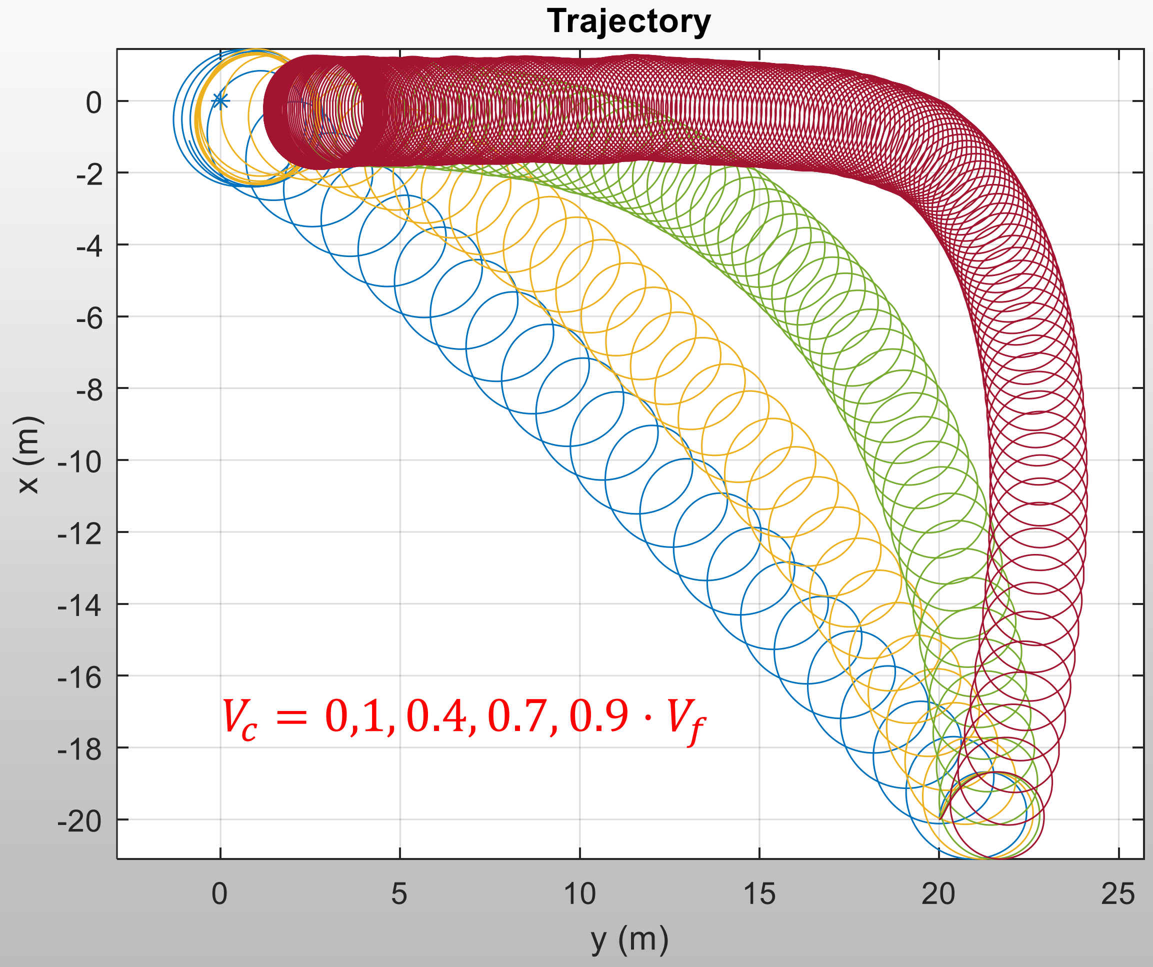
- WORSE RESPONSE
- SLOW CONVERGENCY
- BUT… "IT WORKS"

$$a_0 = 20,40,66$$
$$\Delta a = a_0/2$$
$$\Delta\psi = -90$$



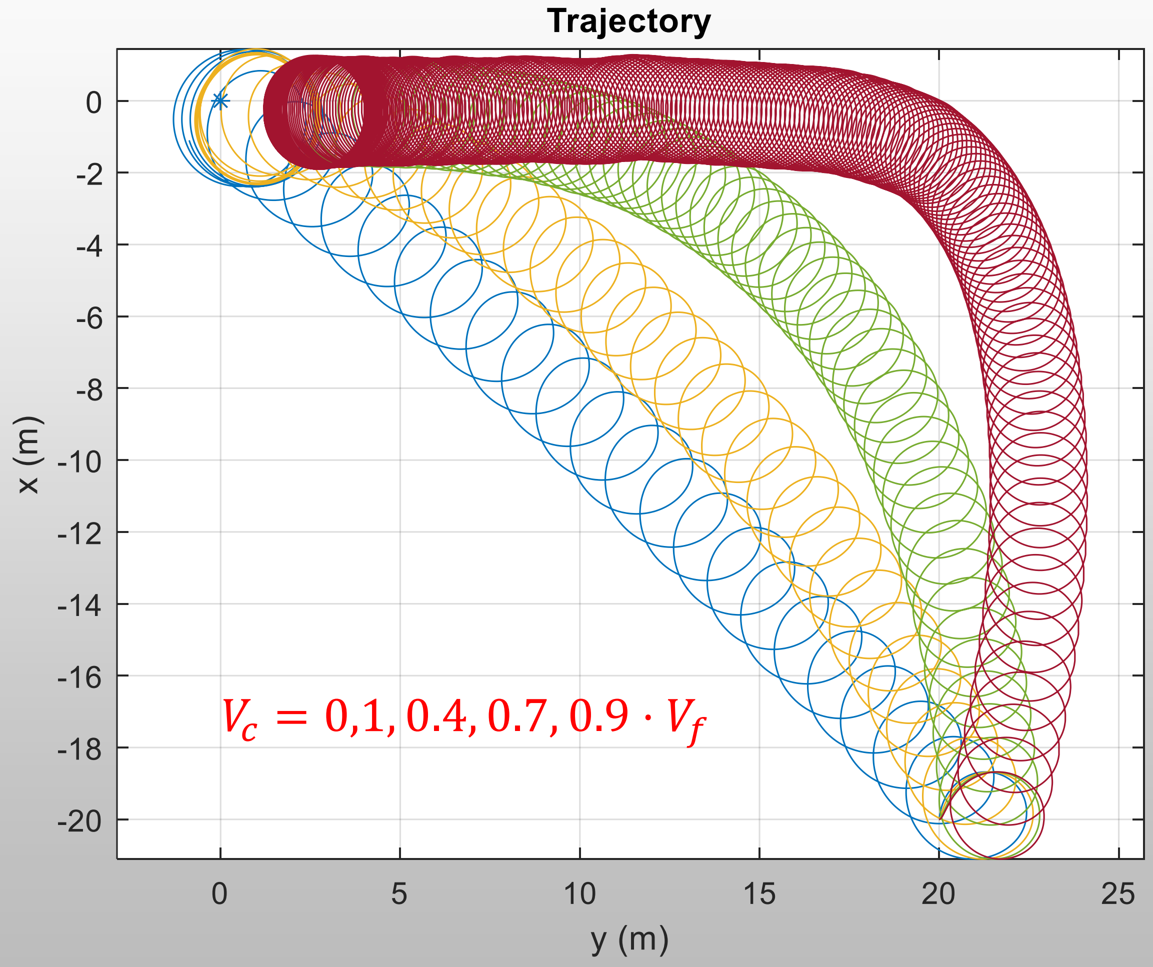
AND IF WE HAVE CURRENTS?

- IF THE SPEED OF THE CURRENT V_c IS LESS THAT THE AVERAGE VELOCITY V_f IT CAN FIGHT THE CURRENT.



AND IF WE HAVE CURRENTS?

- IF THE SPEED OF THE CURRENT V_c IS LESS THAT THE AVERAGE VELOCITY V_f IT CAN FIGHT THE CURRENT.
- PROBLEM, V_f IS TINY (6.6cm/s)

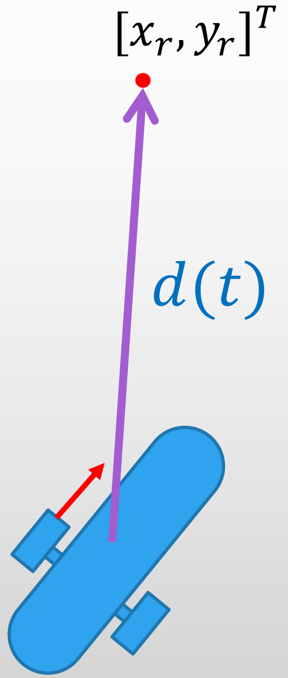


CAN WE DO IT BETTER?

OPTIMAL CONTROL

- NOW WE KNOW THAT IT IS POSSIBLE TO DO SOMETHING
- COULD IT BE DONE BETTER?
  - IT DEPENDS ON THE DEFINITION OF BETTER

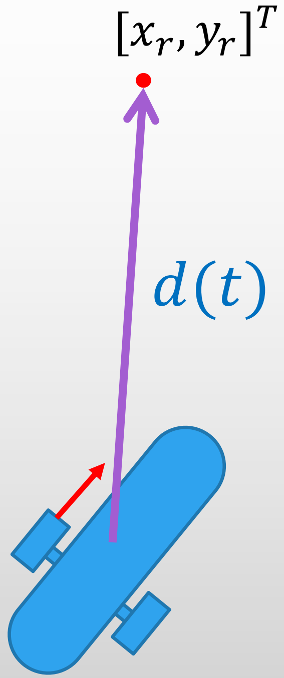
OPTIMAL CONTROL



- NOW WE KNOW THAT IT IS POSSIBLE TO DO SOMETHING
- COULD IT BE DONE BETTER?
  - IT DEPENDS ON THE DEFINITION OF BETTER
  - WE NEED TO DEFINE SOME “COST” LIKE ISE = INTEGRAL SQUARE ERROR

$$J = \int_0^{T_f} d(t)^2 dt$$

OPTIMAL CONTROL



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  - WE NEED TO DEFINE SOME "COST" LIKE ISE = INTEGRAL SQUARE ERROR

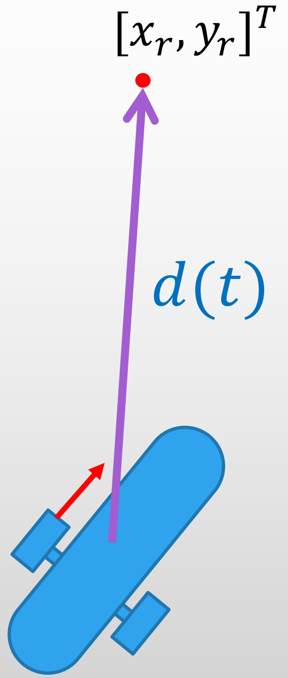
$$J = \int_0^{T_f} d(t)^2 dt$$

  - WE USE A DISCRETE VERSION OF THIS INDEX

$$\boxed{J = \Delta \mathrm{T} \cdot \sum_{i=0}^{N} (x_i - x_r)^2 + (y_i - y_r)^2}$$

OPTIMAL CONTROL

- SO, THE IDEA IS
  - 1) PARAMETRIZE THE CONTROL ACTION WITH PARAMETERS VECTOR θ



OPTIMAL CONTROL

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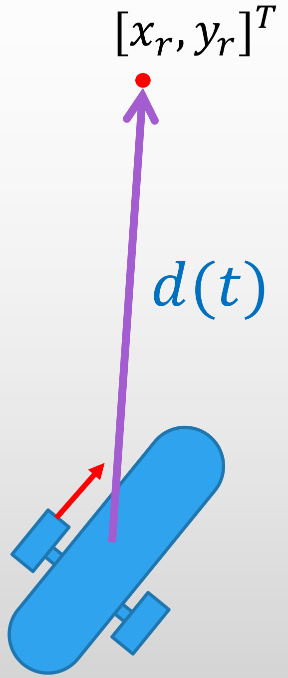
  - 2) TUNE UP θ TO GET THE MINIMUM VALUE OF THE COST J



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$$\theta^* = \arg\min_{\theta} J(\theta)$$

SUBJECT TO

$$J(\theta) = \Delta T \cdot \sum_{i=0}^{n} (x_r - x_i)^2 + (y_r - y_i)^2$$

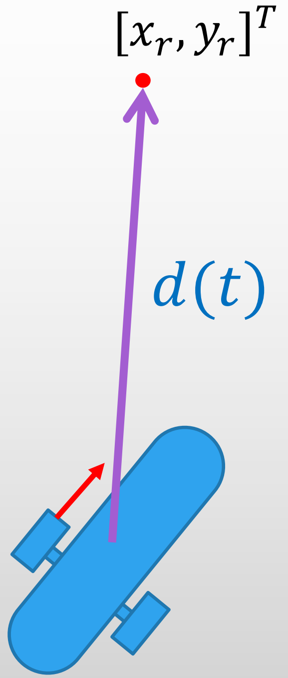
$$t_i = i \cdot \Delta T$$

$$\chi_i = \chi(t_i)$$

$$a_p(t_i) = g(\chi_i, \theta, t_i)$$

$$a_{pmin} \leq a_p(t_i) \leq a_{pmax}$$

$$\chi_{i+1} = \chi_i + \Delta T \cdot f(\chi_i, a_p(t_i))$$



EXAMPLE, OPTIMIZE OUR CONTROL

$$\psi_r = atan2(y_r - y, x_r - x)$$
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$$\theta = [\mathrm{a_0}, \Delta\mathrm{a}, \Delta\psi]^T$$

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OPTIMIZE

$$-60 \leq a_p \leq 60$$

$$\theta = [\mathrm{a_0}, \Delta\mathrm{a}, \Delta\psi]^T$$

EXAMPLE 1, OPTIMIZE OUR CONTROL

$$\psi_r = atan2(y_r - y, x_r - x)$$
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OPTIMIZE

$$-60 \leq a_p \leq 60$$

$$\theta = [\mathrm{a_0}, \Delta\mathrm{a}, \Delta\psi]^T$$

$$\psi_r = atan2(y_r - y, x_r - x)$$
$$\mathrm{a_p} = 40{,}7 + 19{,}9 \cdot \mathrm{sign}(\sin(\psi - \psi_r - 63 \cdot \pi/180))$$

$$\frac{J}{J_{ref}} = 0{,}84$$

$$J_{ref} \rightarrow a = 45, \Delta a = 15, \Delta\psi = -63 \cdot \pi/180$$

EXAMPLE 2, THE BEST POSSIBLE CONTROL FROM A→B

- START AT SOME INITIAL CONDITION (A)

EXAMPLE 2, THE BEST POSSIBLE CONTROL FROM A→B

- START AT SOME INITIAL CONDITION (A)
- PARAMETRIZE THE CONTROLLER SPECIFYING THE CONTROL ACTION AT EACH SAMPLING TIME

$$a_p(t_i) = \theta_i$$

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- JUST OPTIMIZE

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- JUST OPTIMIZE
- THIS KIND OF CONTROL IS COMMONLY USED AS A SLIDING HORIZON STRATEGY
  - APPLY THE FIRST CONTROL ACTION
  - RECOMPUTE THE CONTROL IN THE NEXT TIME STEM
    - LIKE GPS NAVIGATOR RECOMPUTES THE OPTIMAL TRAJECTORY

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JU

JUST

JUST!!!!!!!

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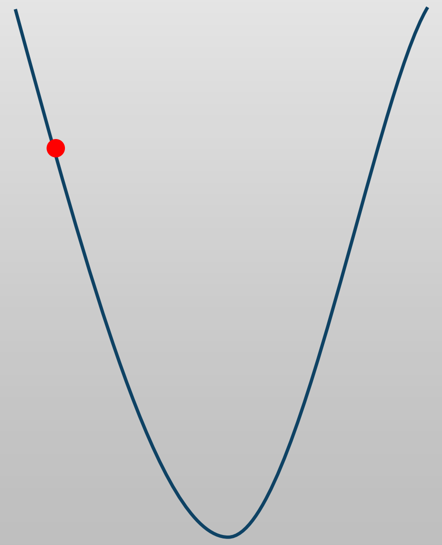
IT IS VERY HARD!!!

- NICE COST FUNCTION ARE CONVEX



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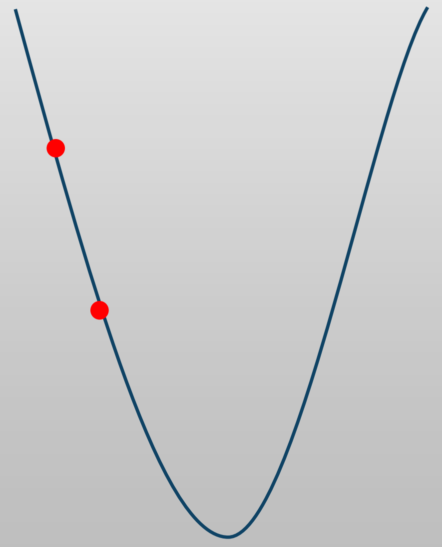

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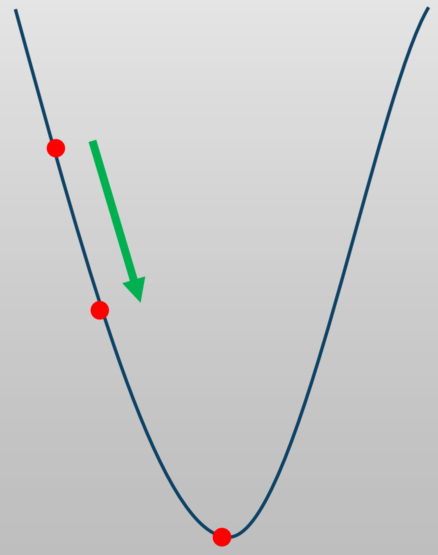
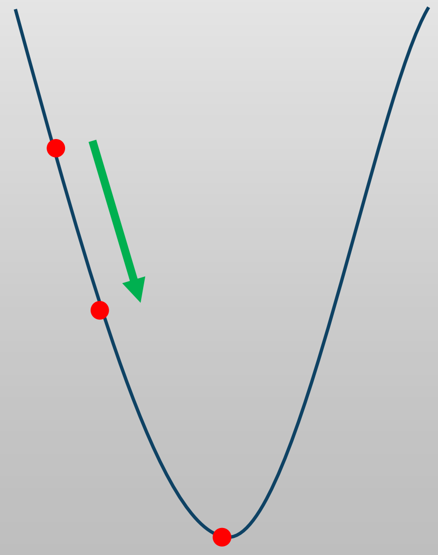
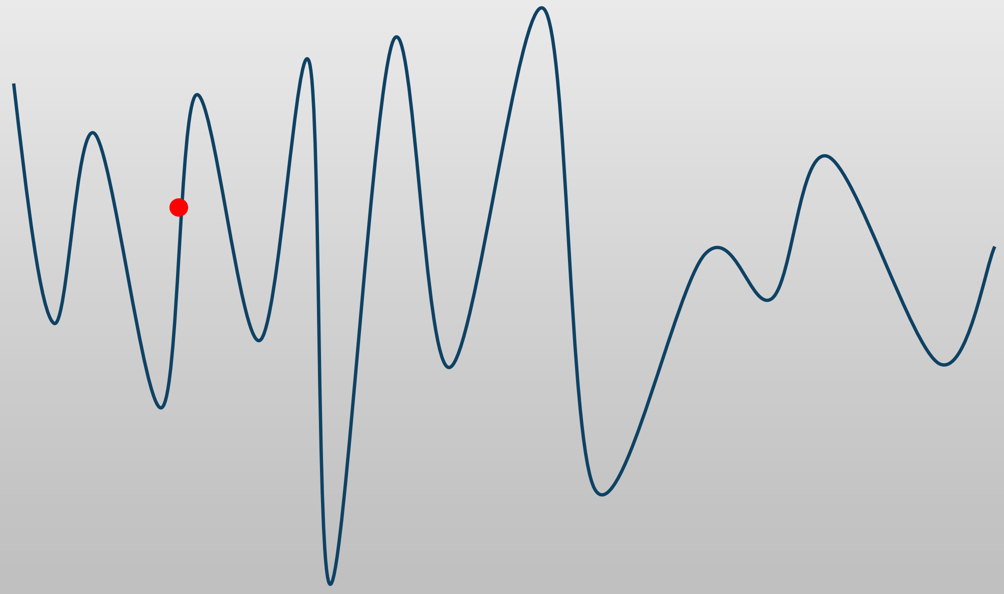


IT IS VERY HARD!!!

- NICE COST FUNCTION ARE CONVEX
  - J THE EXACT OPPOSITE → MANY LOCAL MINIMA





IT IS VERY HARD!!!

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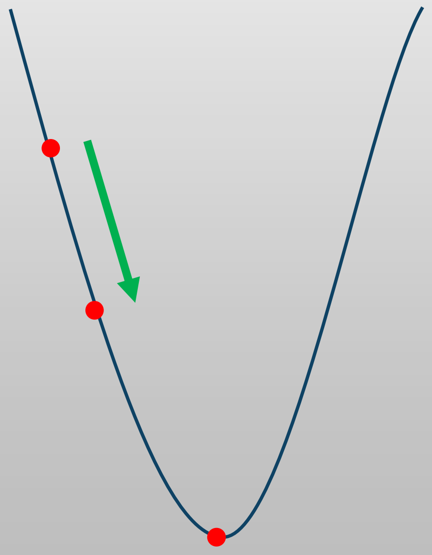

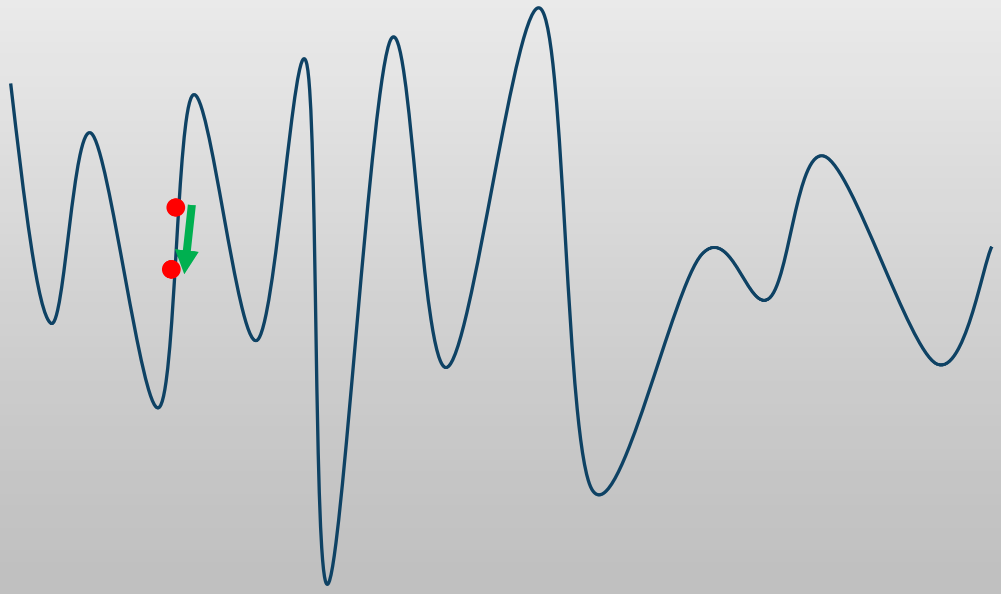

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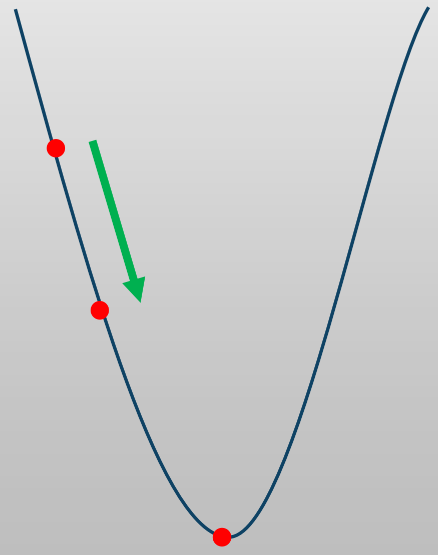

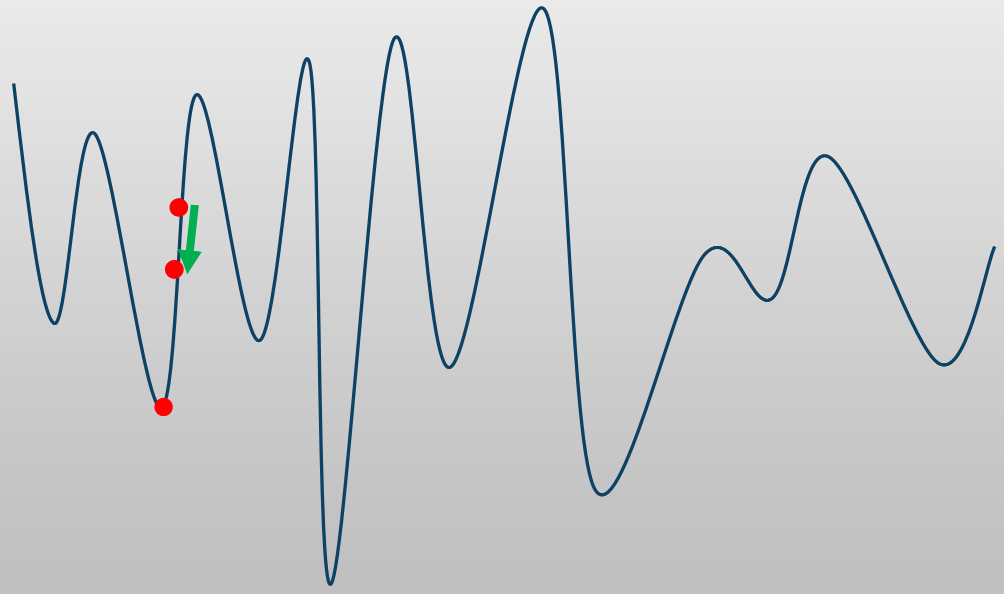

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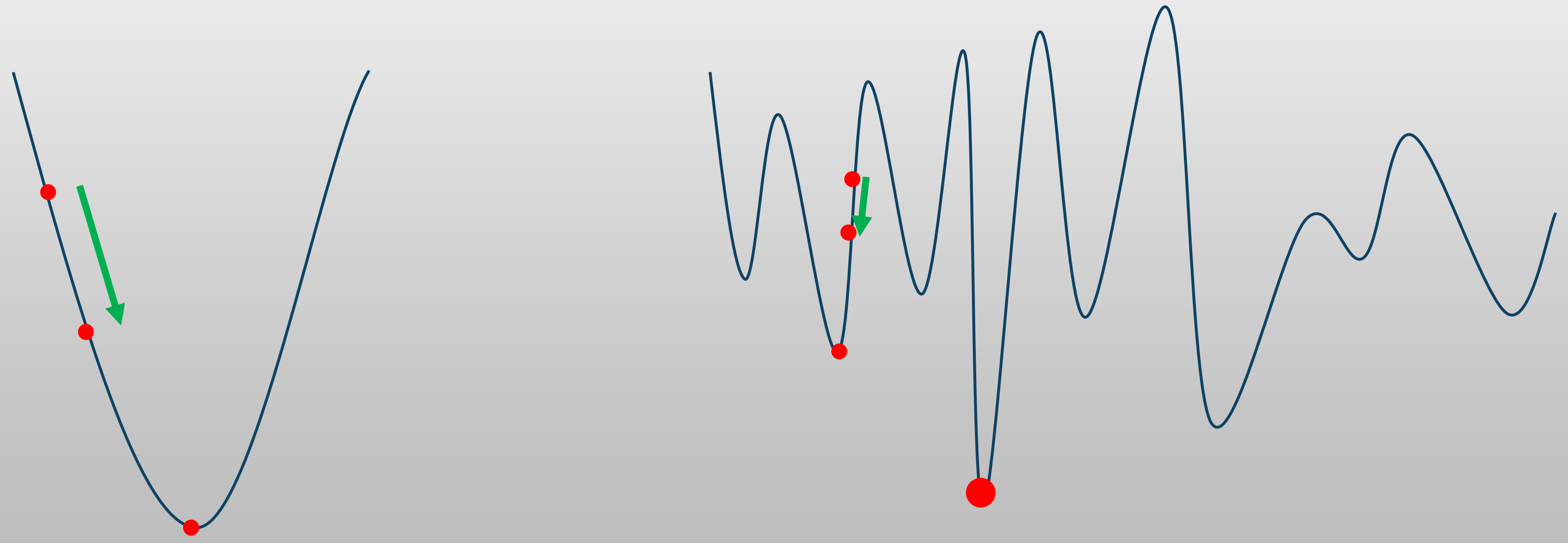
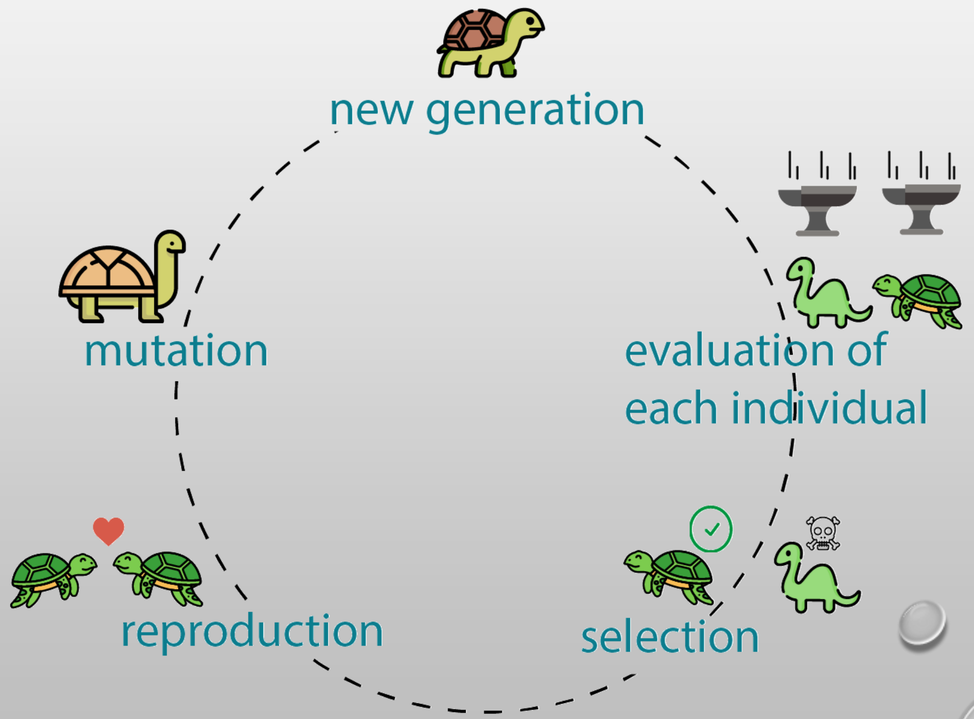


WE USE A HEURISTIC: GENETIC ALGORITHMS

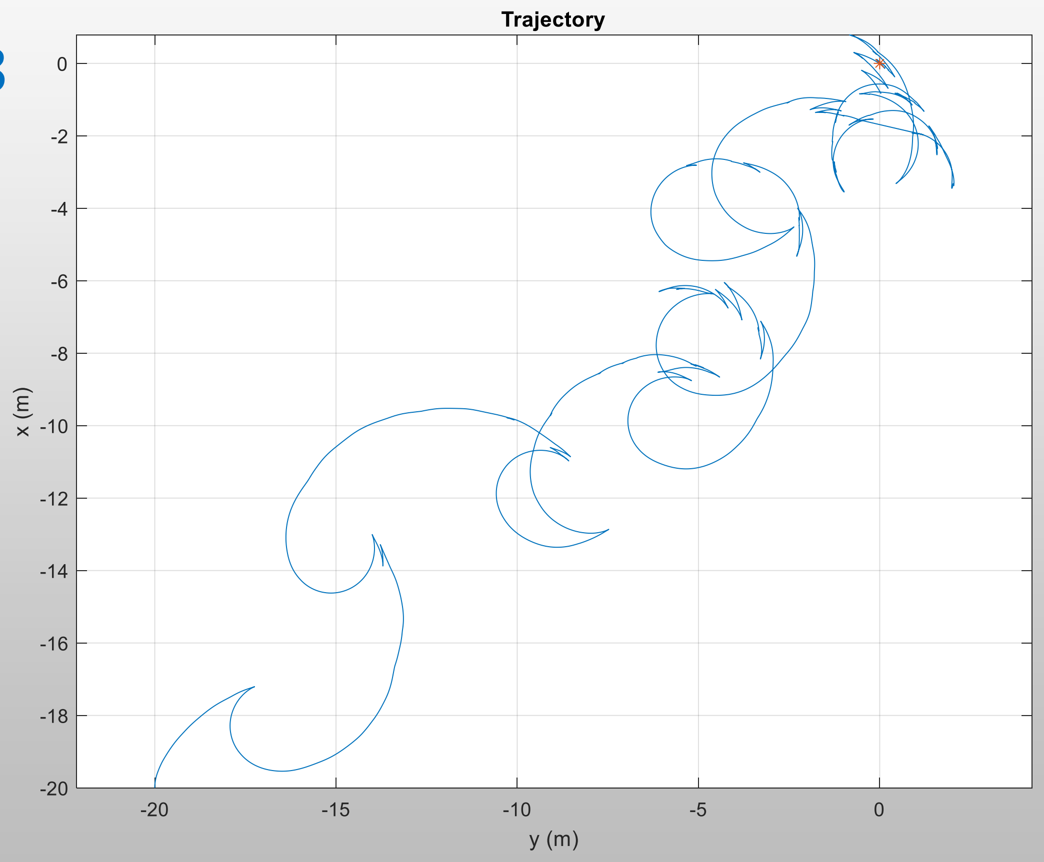
- GENERATE SOME SAMPLES (POPULATION)
  - PARAMETERS ARE THE "GENES"
- SURVIVE WITH A PROBABILITY PROPORTIONAL TO THE SCORE (SELECTION)
  - LESS J MORE SCORE, IE.: SCORE=J_MAX-J
- MIX THEN TOGETHER (SEXUAL REPRODUCTION)
- ALTER THEN RANDOMLY (MUTATION)
- THE BEST ONE ALWAYS SURVIVE (ELITISM)
- REPEAT MANY TIMES



"JUST" DO IT

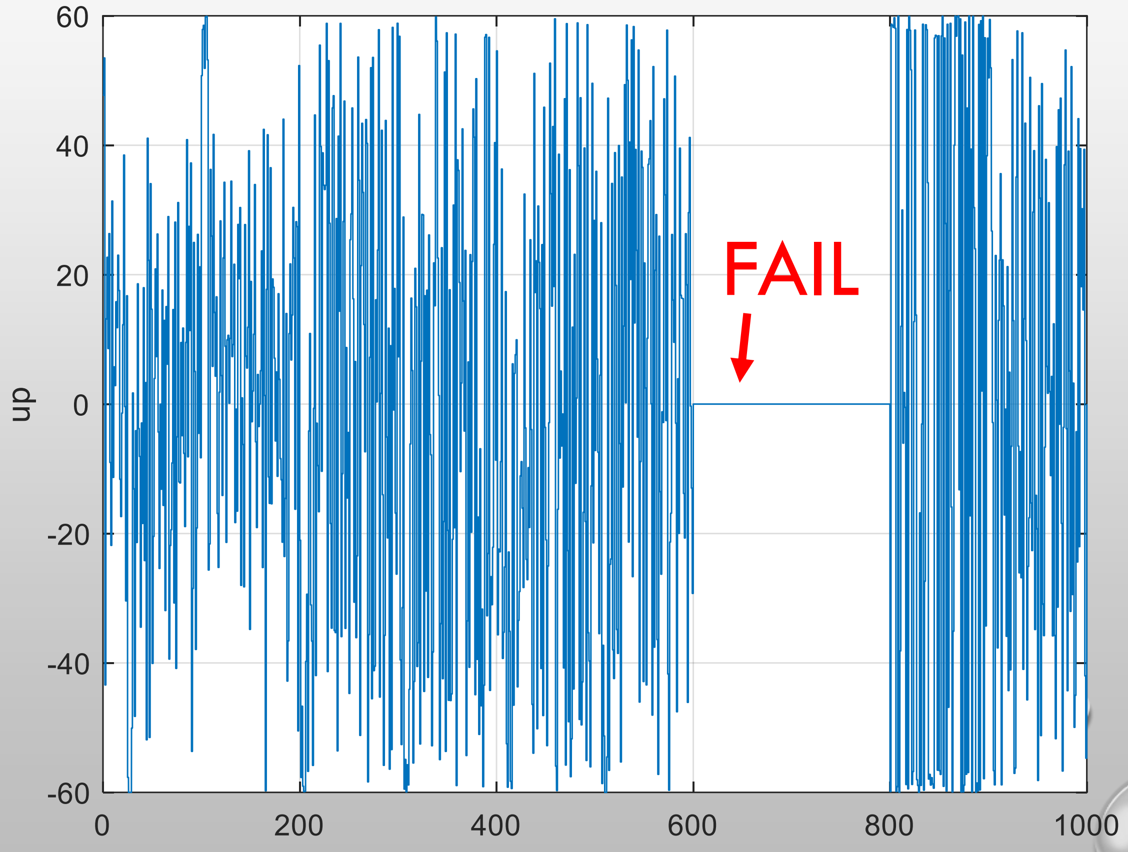
- FIRST "GOOD RESULT" 29N
- BROKEN IN 10 TIME WINDOWS
- INDEPENDENTLY OPTIMIZED
- OPTIMIZATION TOOK MANY HOURS
  - IMPOSSIBLE TO USE IN REAL TIME…

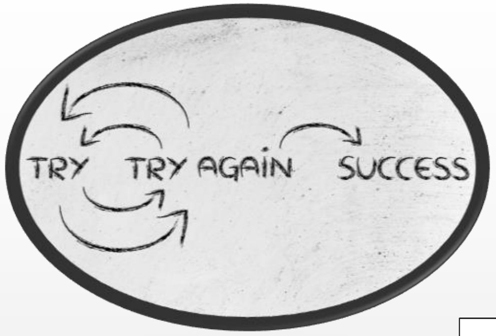
$$\frac{J}{J_{ref}} = 1{,}03$$



"JUST" DO IT

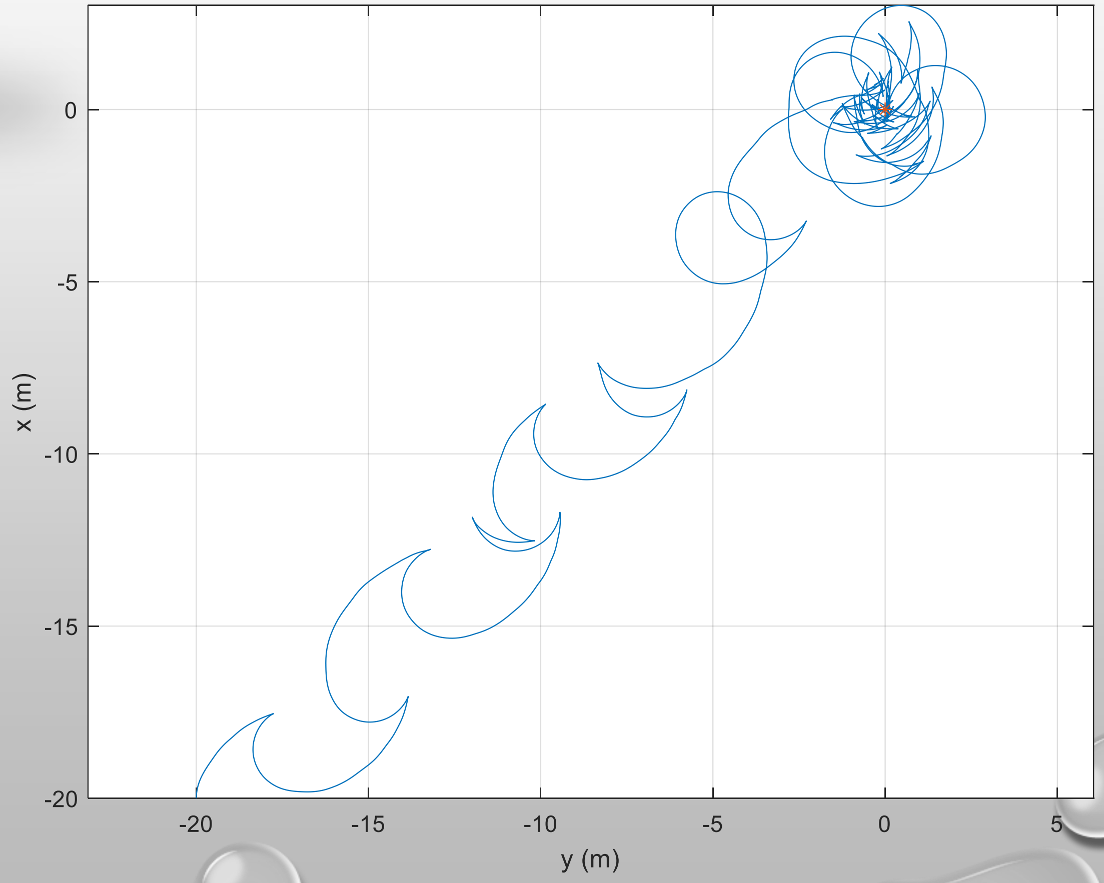
- FIRST "GOOD RESULT" NOV 29 $\dfrac{J}{J_{ref}} = 1{,}03$
- BROKEN IN 10 TIME WINDOWS
- INDEPENDENTLY OPTIMIZED
- OPTIMIZATION TOOK MANY HOURS
  - IMPOSSIBLE TO USE IN REAL TIME…
- CONTROL ACTION IS UGLY

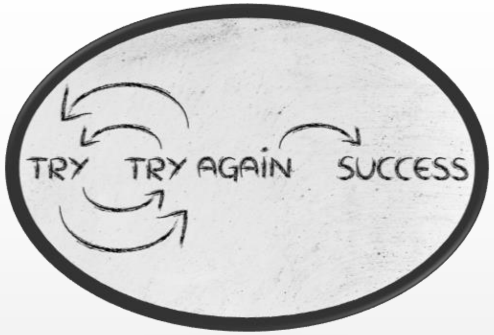




DEC 10

$$\frac{J}{J_{ref}} = 0{,}75$$

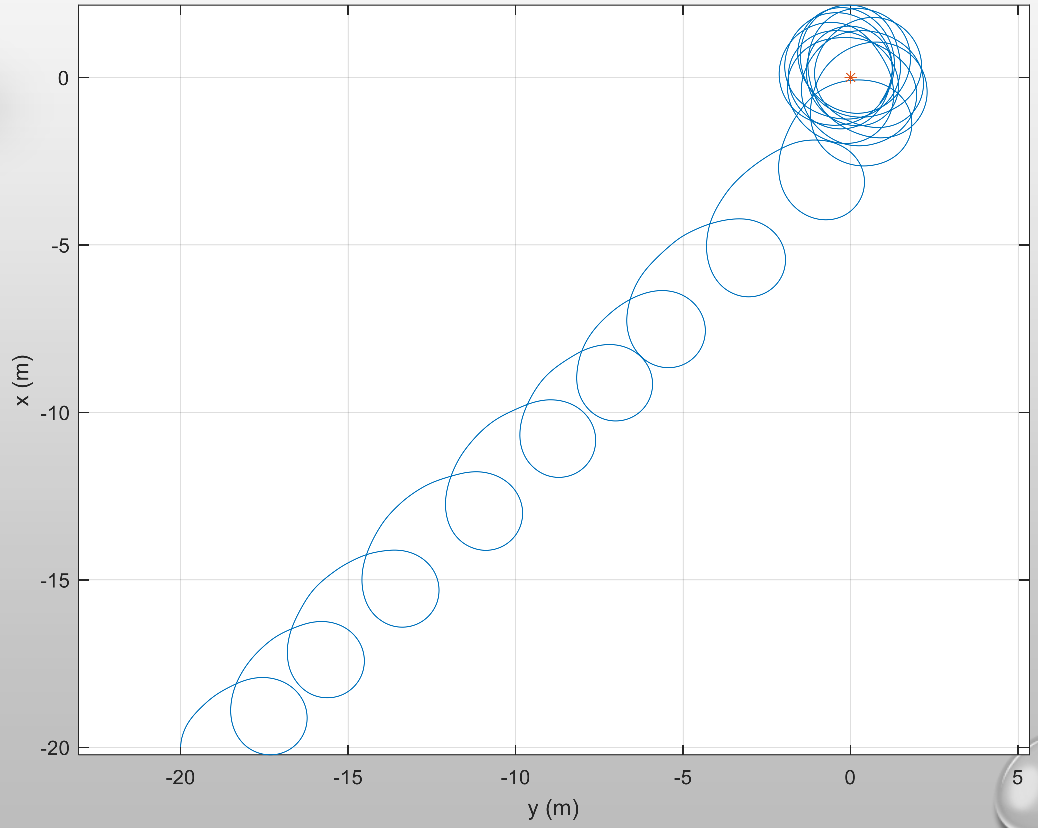


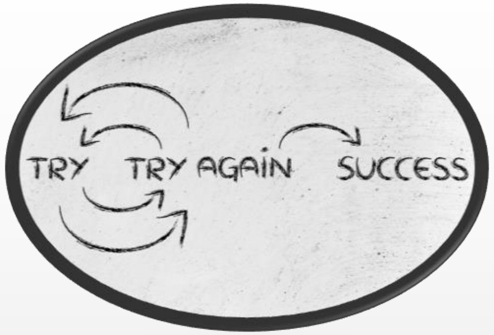


- THE BEST DEC 22
- SPIRAL LIKE
  - NOT QUALITATIVELY FOR THE INITIAL CONTROLLER

$$\frac{J}{J_{ref}} = 0{,}7429$$

- CONTROL ACTION IS UGLY





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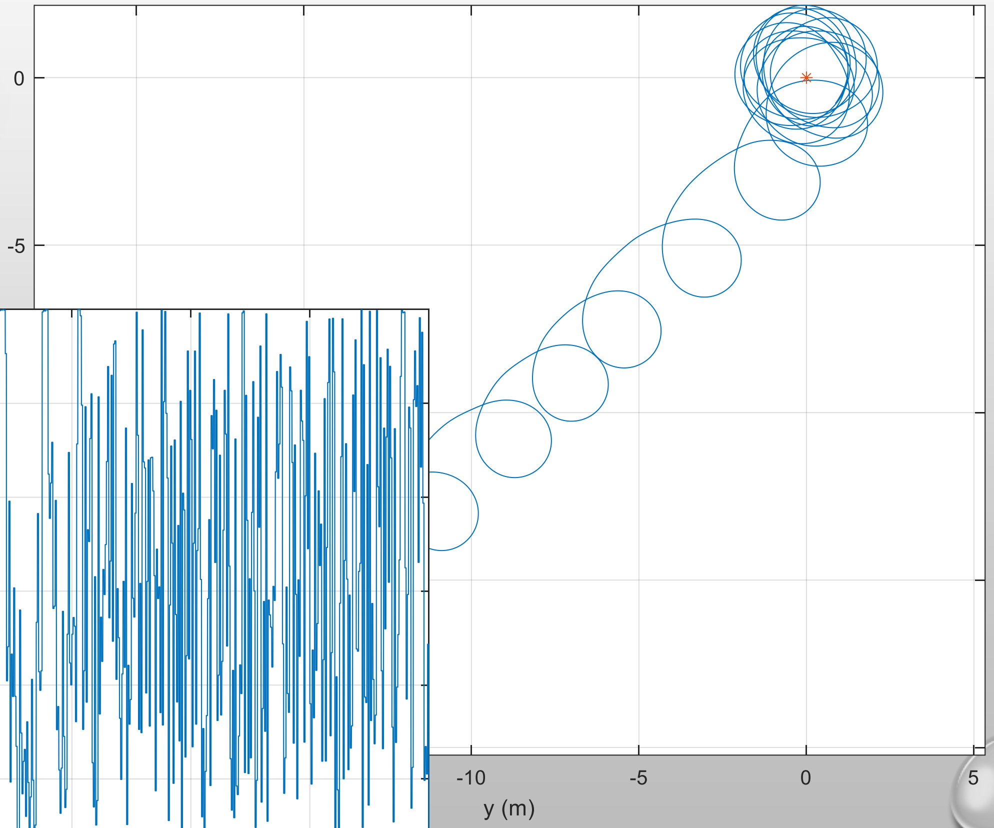

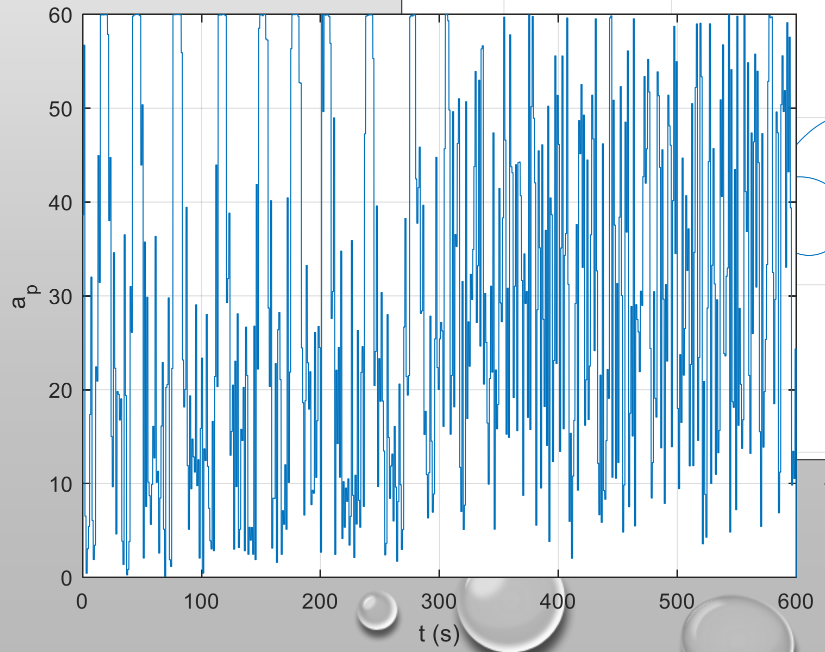

EXAMPLE 3, FOURIER BASED PARAMETRIZATION

- OUR FIRST CONTROL WAS NO BAD AT ALL
- THE CONTROL ACTION IS LESS AGGRESSIVE
- COULD WE HAVE A MORE GENERAL CONTROL LAW?
  - BUT SIMPLE

$$\psi_r = atan2(y_r - y, x_r - x)$$
$$a_p = a_0 + \Delta a \cdot \mathrm{sign}(\sin(\psi - \psi_r + \Delta\psi))$$

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$$\Downarrow$$

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$$\psi_r = atan2(y_r - y, x_r - x)$$
$$a_p = h(\psi - \psi_r)$$

$$h(\alpha) = \frac{b_0}{2} + \sum_{i=1}^{N} b_j \cos(j\alpha) + c_j \sin(j\alpha)$$

$$\theta = [b_0, \dots, b_n, c_1, \dots, c_n]^T$$

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ECOBOTICS.sea

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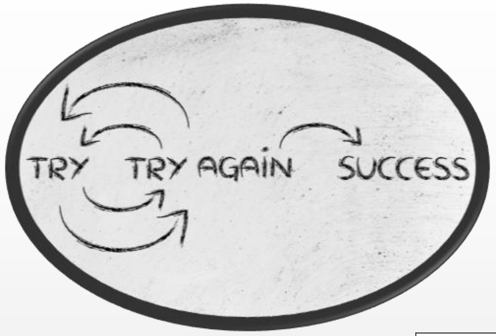
    - PARAMETRIZED BY A FOURIER SERIES
  - PROBLEM: IT IS DIFFICULT TO ENSURE THAT THE CONTROL ACTION IN THE SAFE REGION OF THE ACTUATOR
    - SOLUTION: SATURATE THE CONTROL LAW

$$\psi_r = atan2(y_r - y, x_r - x)$$

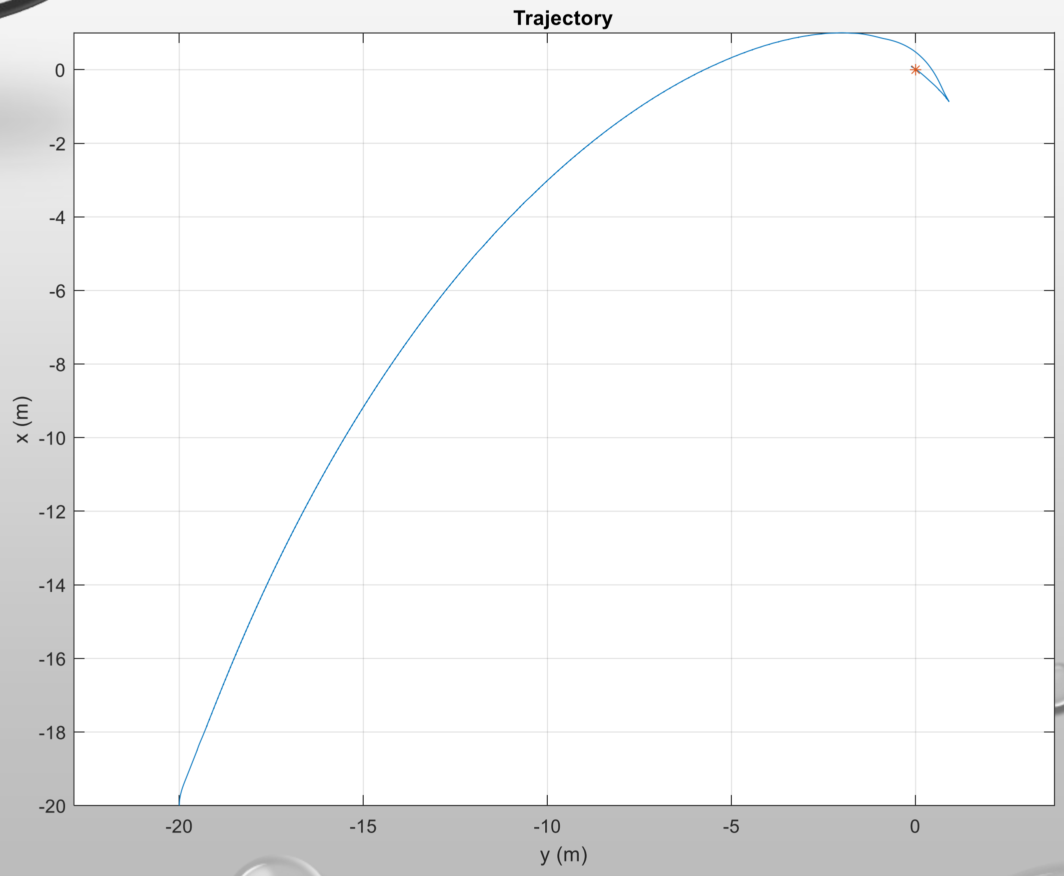
$$\mathrm{a_p} = \max(\mathrm{a_{min}}, \min(\mathrm{h}(\psi - \psi_r), \mathrm{a_{max}}))$$

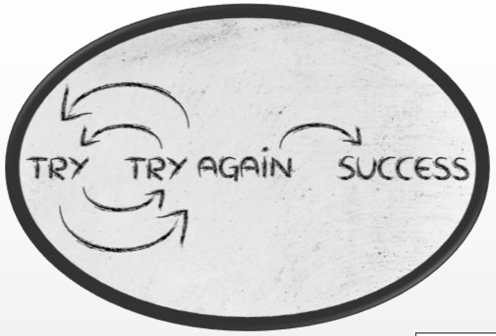
$$\mathrm{h}(\alpha) = \frac{b_0}{2} + \sum_{i=1}^{N} b_j \cos(j\alpha) + c_j \sin(j\alpha)$$

$$\theta = [b_0, \dots, b_n, c_1, \dots, c_n]^T$$


TRY
TRY AGAIN
SUCCESS

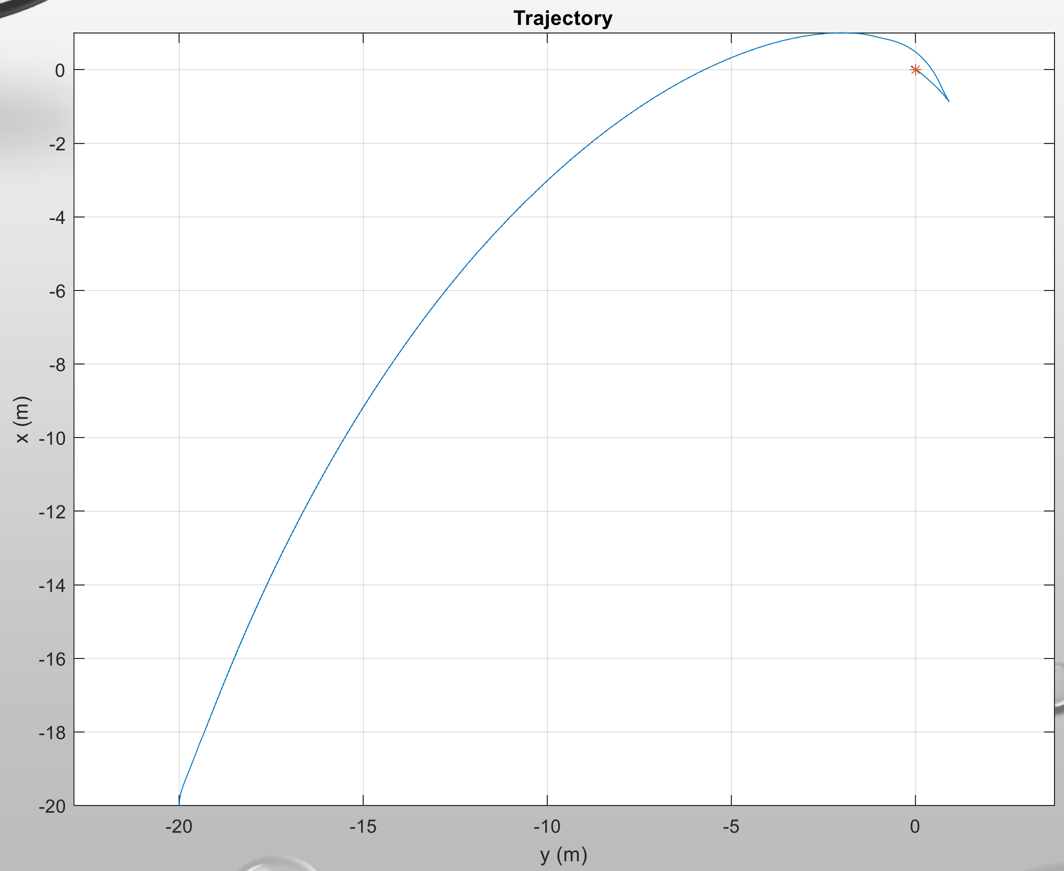
JAN 11☺

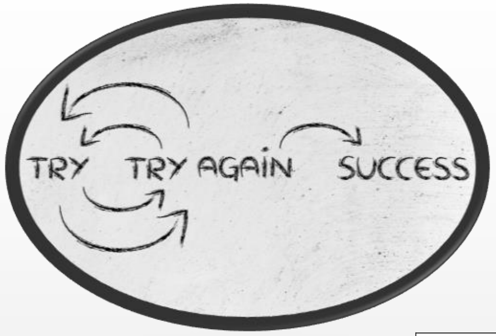

Trajectory
x (m)
y (m)
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-20
-15
-10
-5
0



JAN 11☺

$$\frac{J}{J_{ref}} = 2{,}37$$ ☹





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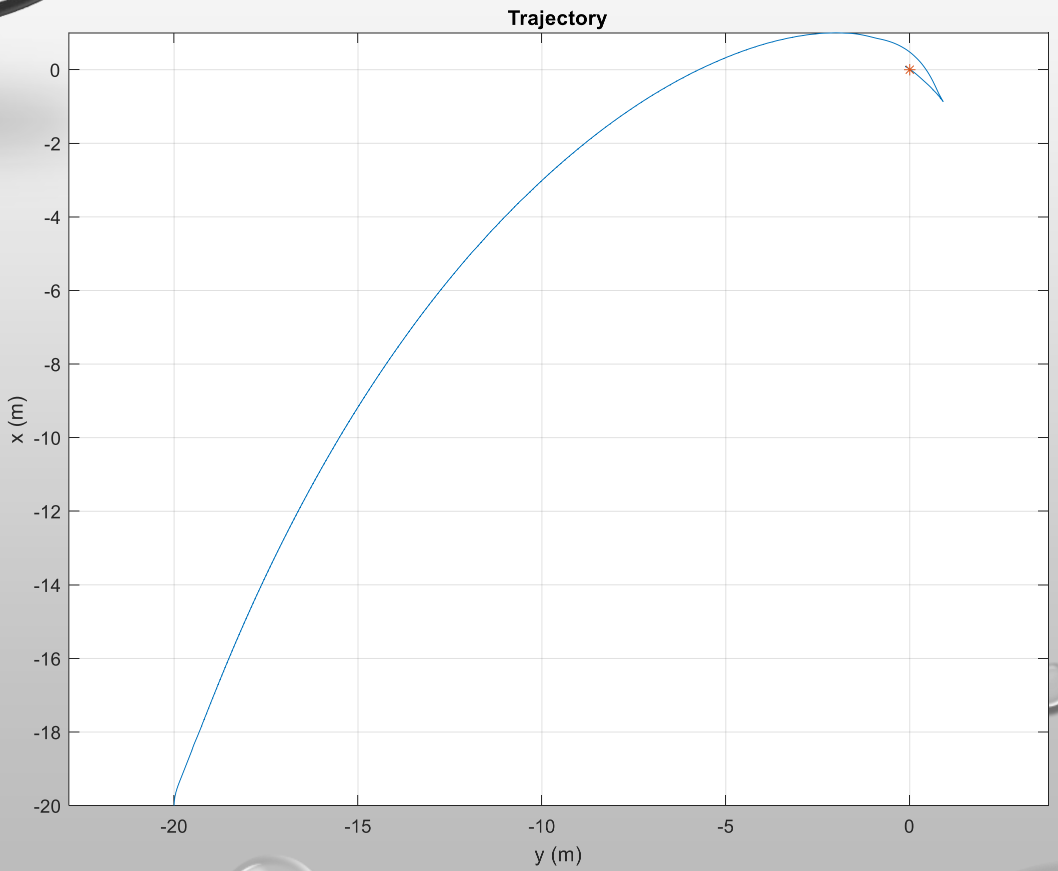
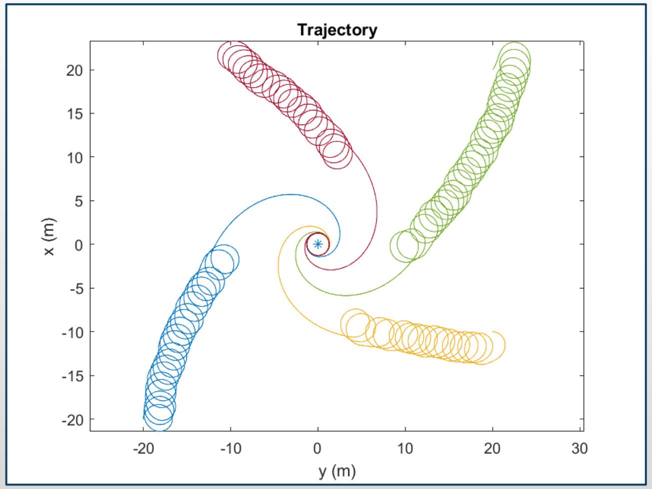
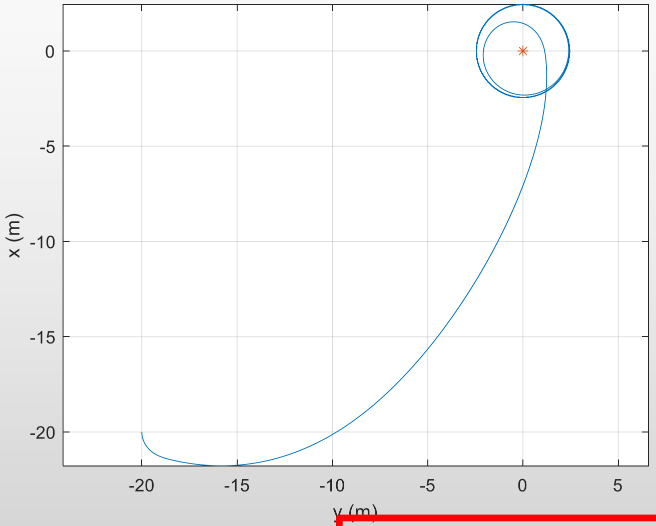
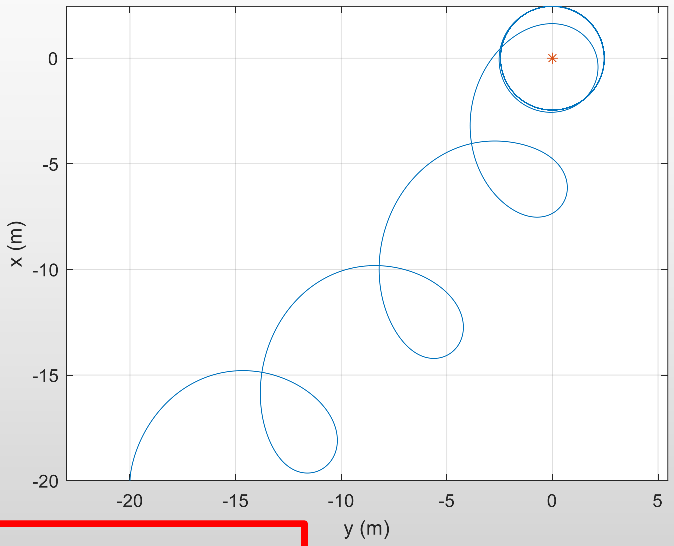
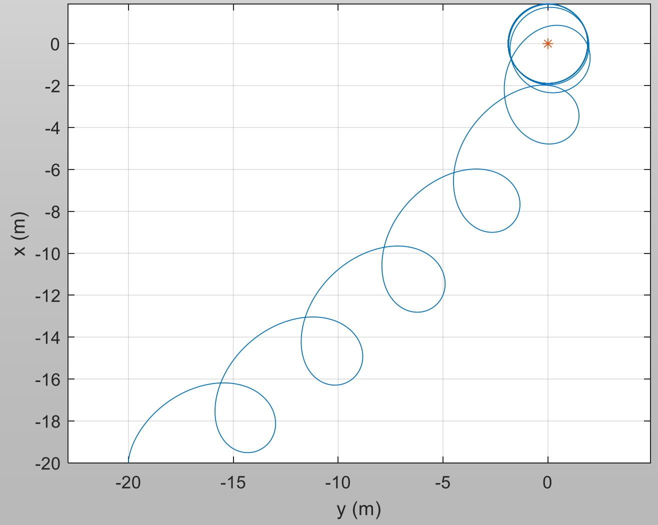


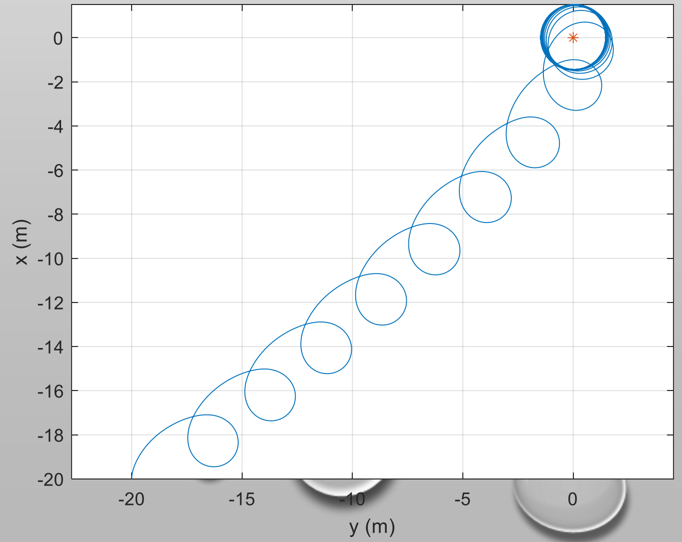
AFTER SOME TRIALS







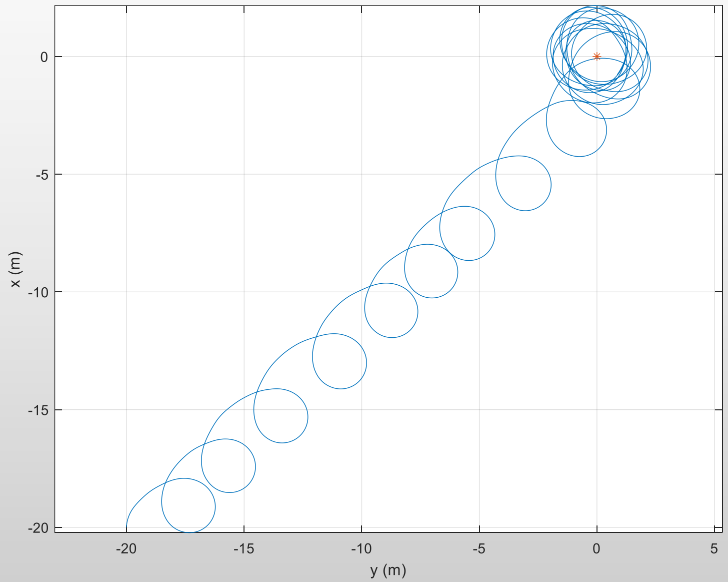




$$\frac{J}{J_{ref}} = 0{,}7422$$

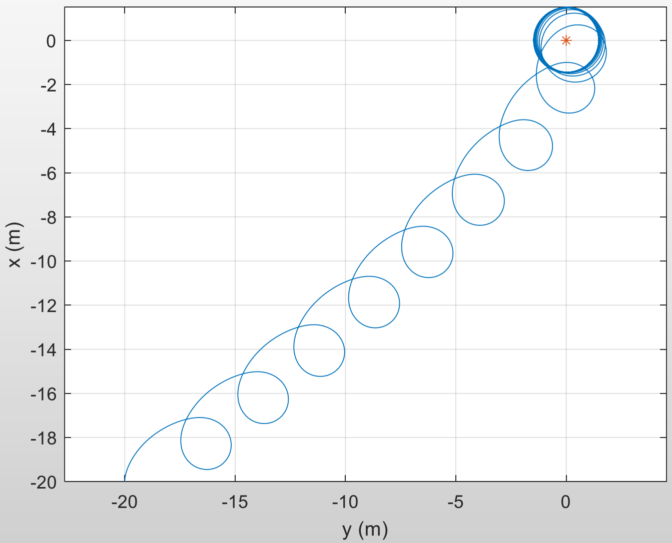
COMPARISON

“BEST POSSIBLE”



$$\frac{J}{J_{ref}} = 0{,}7429$$

FOURIER



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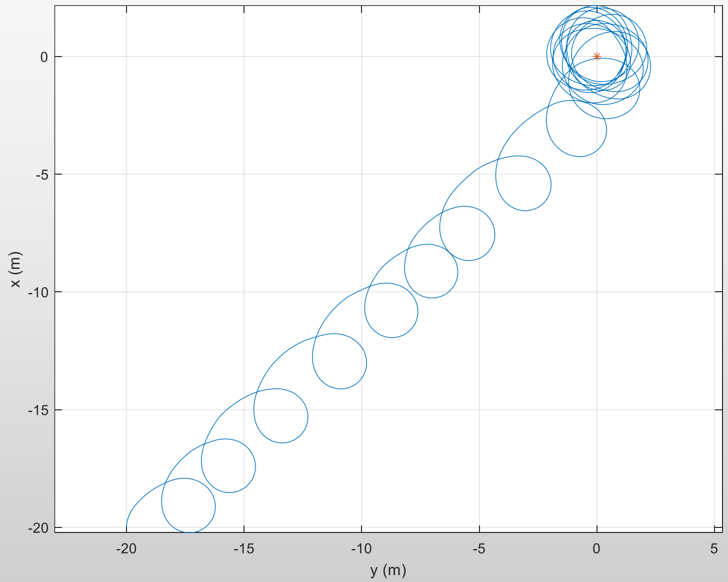

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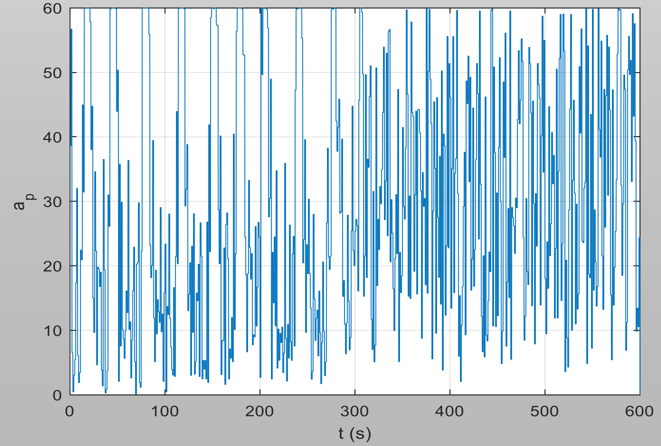

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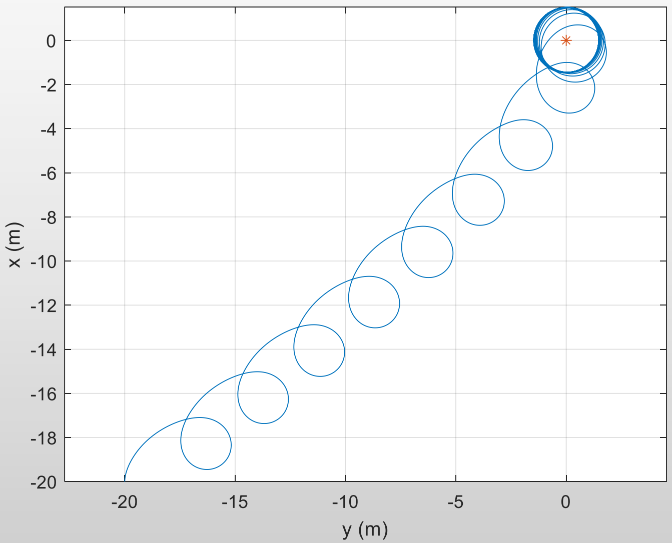

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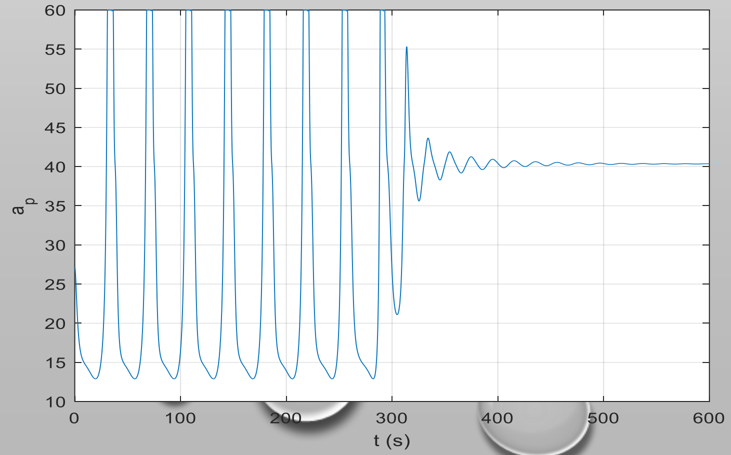

COMPARISON

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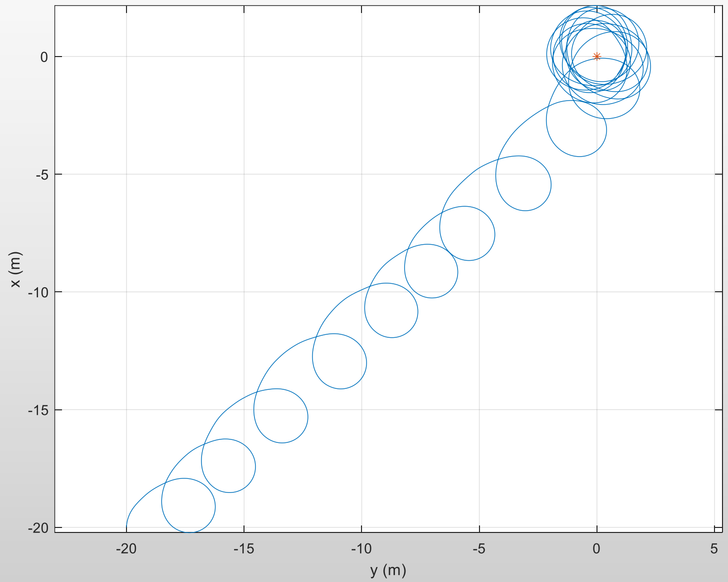

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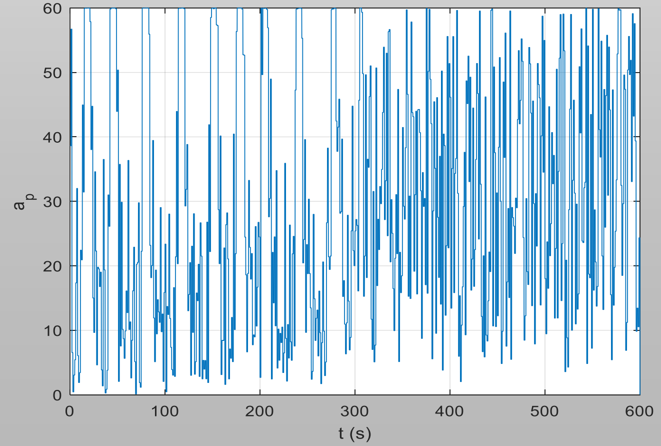
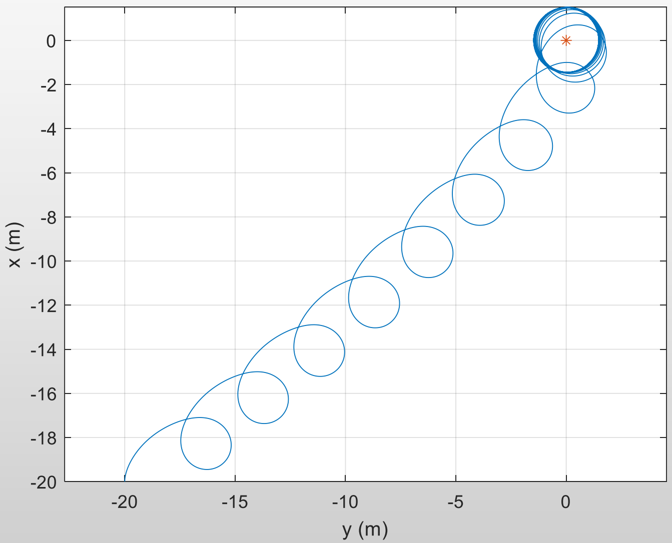


HOURS TO COMPUTE ☹

BAD FOR THE MOTOR ☹

FOURIER



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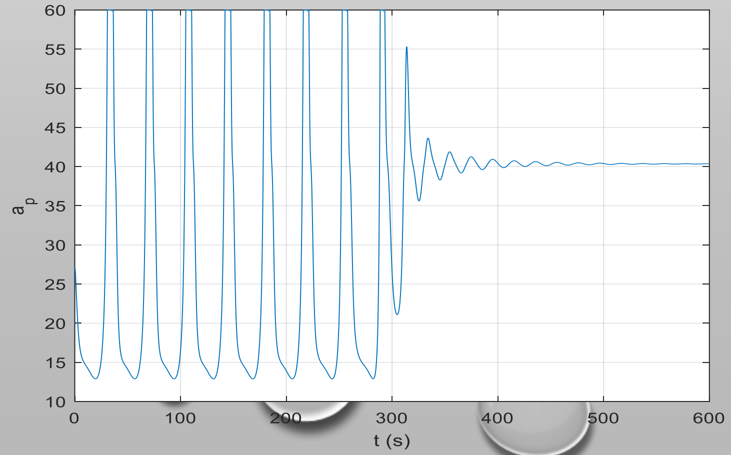
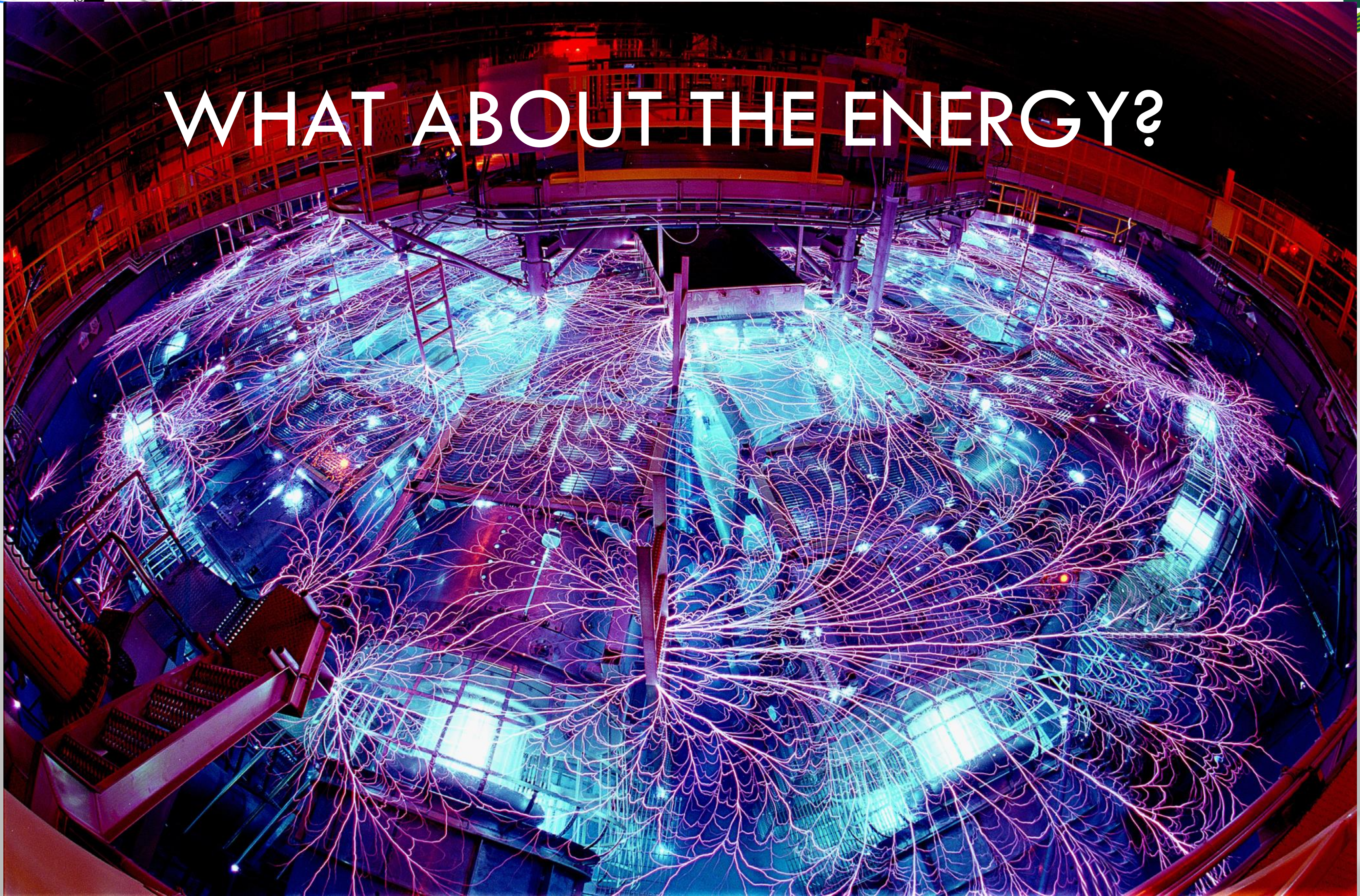


REAL TIME ☺

NICE AND SMOOTH ☺

WHAT ABOUT THE ENERGY?



INCLUDE IT IN THE OPTIMIZATION PROBLEM!!!

- ONCE WE HAVE A GOOD PARAMETRIZATION, WE CAN INCLUDE MORE THINGS
  - ENERGY CONSUMPTION
  - FINAL BOUND FOR ANY INITIAL CONDITION IN AN INITIAL SET
  - GUARANTEE THAT IT WILL REACH A NEIGHBORHOOD OF THE RECOVERY POINT
  - AT A RECOVERY TIME, AND STAY THERE

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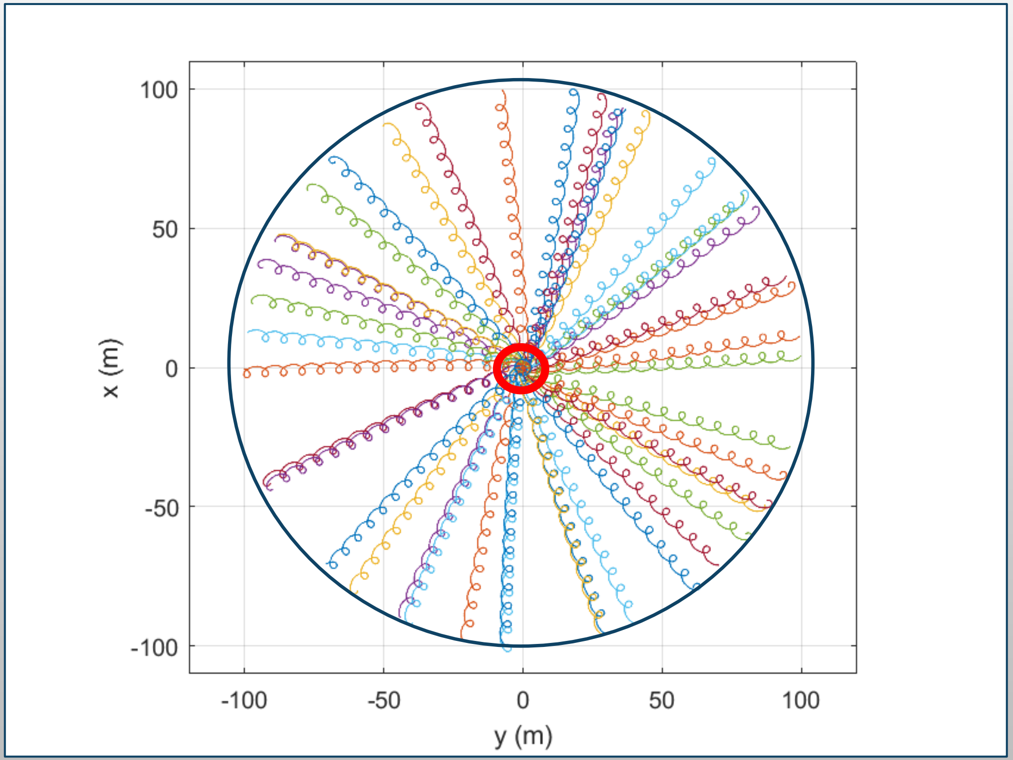
  - GUARANTEE THAT IT WILL REACH A NEIGHBORHOOD OF THE RECOVERY POINT
  - AT A RECOVERY TIME, AND STAY THERE
- NOTHING IS FREE
  - TRADEOFF BETWEEN USING LESS ENERGY AND GOING FAST TO THE RECOVERY POINT

$$J_{Tot}(\boldsymbol{\theta}, \boldsymbol{\chi}_0) = (1-\lambda) \cdot J_{ISE}(\boldsymbol{\theta}, \boldsymbol{\chi}_0) + \lambda \cdot J_E(\boldsymbol{\theta}, \boldsymbol{\chi}_0)$$

$$J_E(\boldsymbol{\theta}, \boldsymbol{\chi}_0) = \Delta T \cdot \sum_{i=0}^{n} P(\boldsymbol{\chi}_i, a_p(t_i), t_i)$$

MIN-MAX PROBLEM (SOLVE FOR THE WORST CASE)



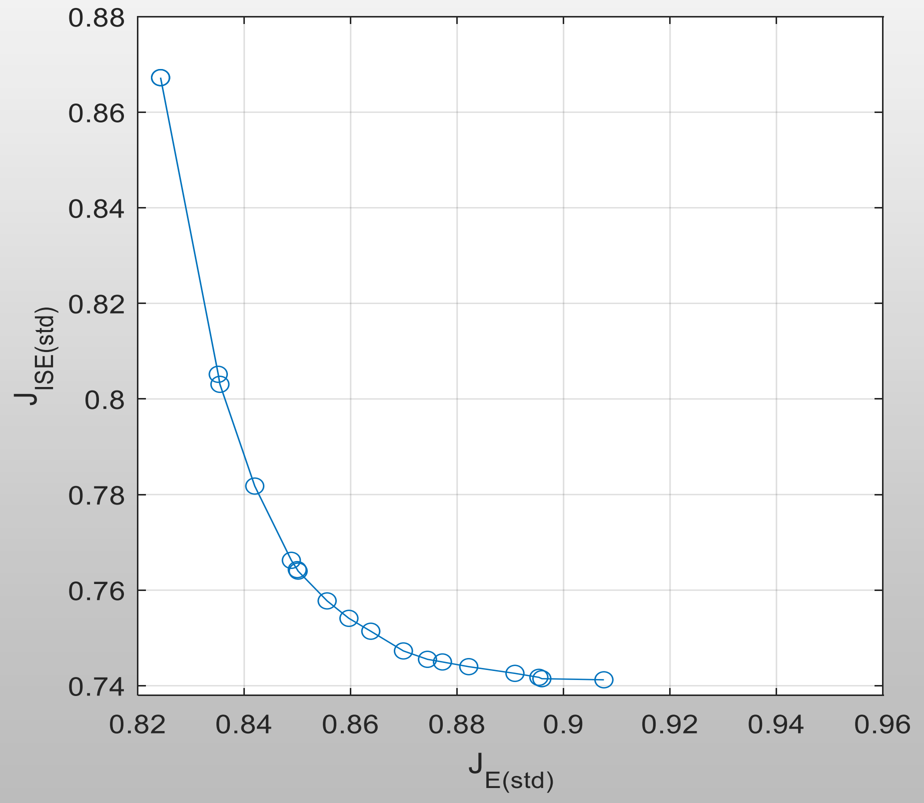
$$\theta^* = \arg\min_{\theta} \max_{\chi_0 \in \Omega} J_{Tot}(\theta, \chi_0)$$

- INITIAL SET $\Omega = \left\{ \begin{array}{c} x^2 + y^2 \leq R_0^2 \\ v = 0 \end{array} \right\}$

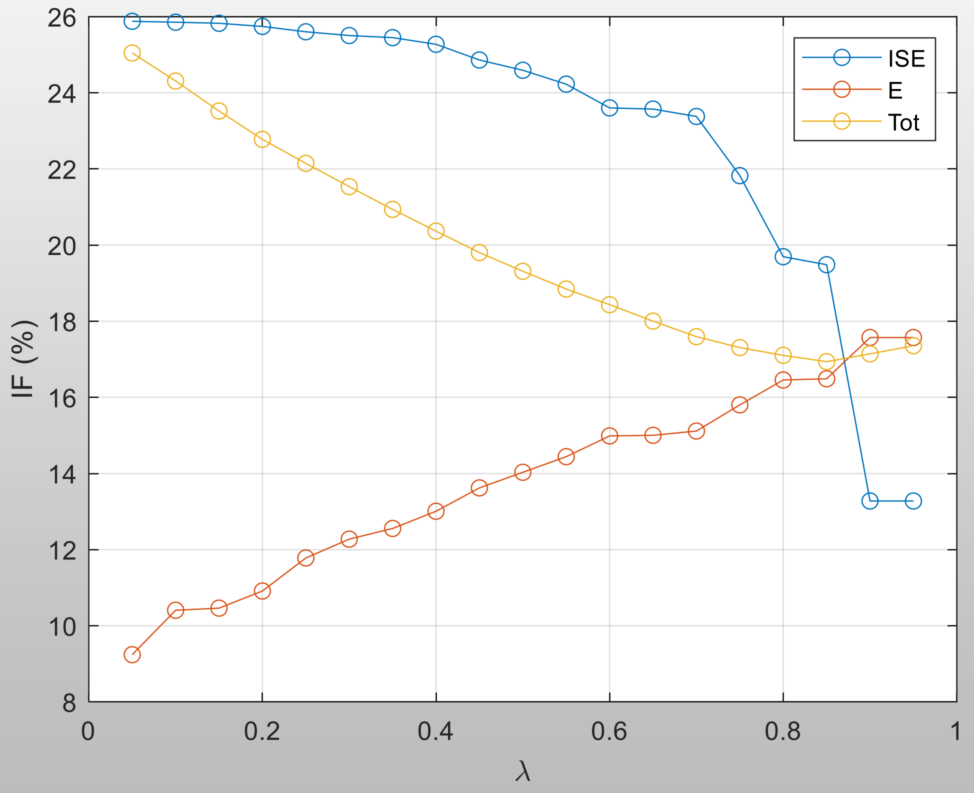
- REACH A NEIGHBORHOOD OF THE RECOVERY POINT AT A RECOVERY TIME, AND STAY THERE

$$(x(t) - x_r)^2 + (y(t) - y_r)^2 < R_f^2 \quad \forall\, t_f \leq t$$

TRADEOFFS

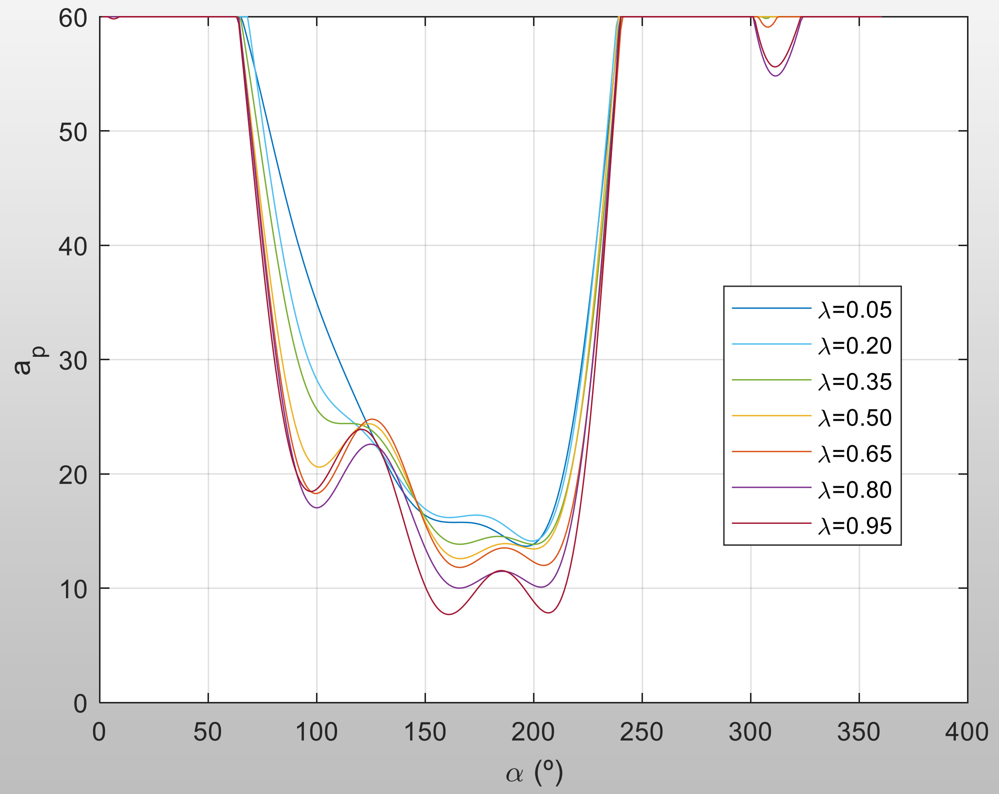


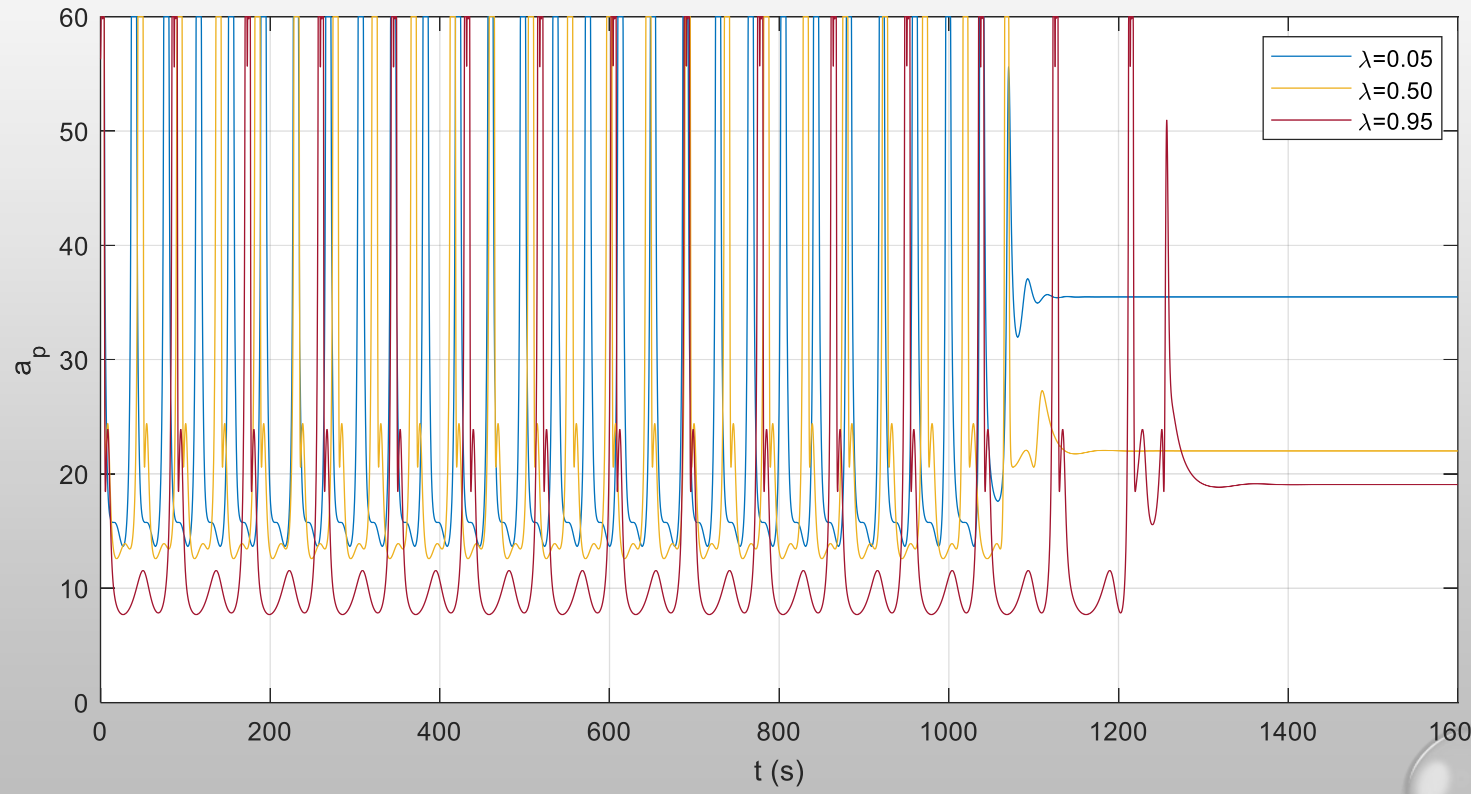
0.88
0.86
0.84
0.82
0.8
0.78
0.76
0.74
0.82
0.84
0.86
0.88
0.9
0.92
0.94
0.96
$J_{ISE(std)}$
$J_{E(std)}$



26
24
22
20
18
16
14
12
10
8
0
0.2
0.4
0.6
0.8
1
IF (%)
λ
ISE
E
Tot

CONTROL ACTION





THANK YOU VERY MUCH FOR YOUR ATTENTION!

QUESTIONS?