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1. $f(y) = \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$

$f'(y) = \frac{1}{2}y^3 + (-\frac{1}{2})y^{-3}$

$f'(y) = \frac{1}{2}y^3 - \frac{1}{2}y^{-3}$

$$S = \int_2^4 \sqrt{1 + (\tfrac{1}{2}(y^3 - y^{-3}))^2}\, dy$$

$$= \int_2^4 \sqrt{1 + \tfrac{1}{4}(y^6 - 2 + y^{-6})}\, dy$$

$$= \int_2^4 \sqrt{y^6 + 2 + y^{-6}}\, dy$$

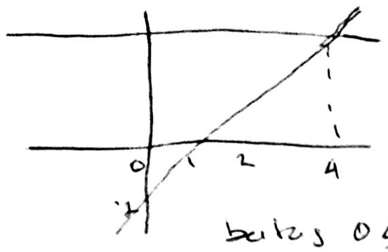
$$= \left[\frac{y^4}{8} - \frac{1}{4y^2}\right]_2^4$$

$$= \left[\frac{4^4}{8} - \frac{1}{4(4)^2}\right] - \left[\frac{2^4}{8} - \frac{1}{4(2)^2}\right]$$

$$= \left[\frac{256}{8} - \frac{1}{64}\right] - \left[\frac{16}{8} - \frac{1}{16}\right]$$

$$= 30{,}04$$

2) $y = 2x - 2 \quad y = 6 \quad y = 0 \quad x = 0$



batas $0 \le x \le 4$

$y = 2x - 2$

$(0,-2)\ (1,0)$

$y = 2x - 2$

$6 = 2x - 2$

$2x = 8$

$x = 4$

$$\bar{x} = \frac{\int_0^4 x(2x\,2-6)\,dx}{\int_0^4 (2x-2-6)\,dx}$$

$$= \frac{\int_0^4 (2x^2 - 8x)\,dx}{\int_0^4 (2x-8)}$$

$$= \frac{\left(\frac{2}{3}x^3 - 4x^2\right)\Big|_0^4}{(x^2 - 8x)\Big|_0^4}$$

$$= \frac{\frac{126}{3} - 64}{16 - 32} = \frac{\frac{128-192}{3}}{-16} = \frac{-64}{3(-16)} = \frac{4}{3}$$

$$\bar{y} = \frac{\frac{1}{2}\int_0^4 (2x-6)^2 - 6^2\,dx}{\int_0^b (2x-2-6)\,dx}$$

$$= \frac{\frac{1}{2}\int_0^4 4x^2 - 24x + 36 - 36\,dx}{-16}$$

$$= \frac{\frac{1}{2}\left(\frac{4}{3}x^3 - 12x^2\right)\Big|_0^4}{-16}$$

$\cancel{= -\frac{1}{32}}$

$$= -\frac{1}{32}\left(\frac{256}{3} - 192\right)$$

$$= -\frac{1}{32}\left(\frac{-320}{3}\right) = \frac{10}{3}$$

$$(x, y) = \left(\frac{4}{3}, \frac{10}{3}\right)$$

3) $y = \sqrt{1-x^2}$, $-1 \le x \le 1$, diputar sumbu x

$$y' = \frac{1}{2}(1-x^2)^{\frac{1}{2}-1}\ \frac{d}{dx}(1-x^2)$$

$$= \frac{\frac{d}{dx}(1) - \frac{d}{dx}(x^2)}{2\sqrt{1-x^2}} = \frac{0-2x}{2\sqrt{1-x^2}} = -\frac{x}{\sqrt{1-x^2}}$$

$$L = \int_{-1}^{1} 2\pi\left(\sqrt{1-x^2}\right)\left(\sqrt{1+\left(-\frac{x}{\sqrt{1-x^2}}\right)^2}\right)dx$$

$$= 2\pi\int_{-1}^{1} \cancel{\sqrt{1-x^2}}\ \frac{\sqrt{1}}{\cancel{\sqrt{1-x^2}}}\ dx.$$

$$= 2\pi\int_{-1}^{1} 1\,dx = 2\pi\left((1) - (-1)\right)$$

$$= 2\pi(2) = 4\pi$$

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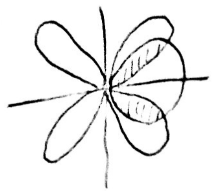
4) $r = \sin 2\theta$ dan $r = \cos\theta$ (irisan)



$r_1 = r_2$

$\sin 2\theta = \cos\theta$

$2\sin\theta\cos\theta = \cos\theta$

$\sin\theta = \frac{1}{2}$

$\theta = \pm \pi/6$

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$A_1 = 2 \cdot \frac{1}{2} \int_{\pi/6}^{\pi/2} r_1^2 \, d\theta$

$= \int_{\pi/6}^{\pi/2} \sin^2 2\theta \, d\theta$

$= \left[\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta\right]_{\pi/6}^{\pi/2}$

$= \left[\left(\frac{1}{2}(\pi/2) - \frac{1}{8}\sin 4(\pi/2)\right) - \left(\frac{1}{2}(\pi/6) - \frac{1}{8}\sin 4(\pi/6)\right)\right]$

$= \dfrac{8\pi + 5\sqrt{3}}{48}$

$A_2 = 2 \cdot \frac{1}{2} \int_0^{\pi/6} r_2^2 \, d\theta$

$= \int_0^{\pi/6} \cos^2\theta \, d\theta$

$= \left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{\pi/6}$

$= \left[\frac{1}{2}(\pi/6) + \frac{1}{4}\sin 2(\pi/6) - 0\right]$

$= \dfrac{2\pi + 3\sqrt{3}}{24}$

$A_{total} = A_1 + A_2$

$= \dfrac{8\pi + 3\sqrt{3}}{48} + \dfrac{2\pi + 3\sqrt{3}}{24}$

$= \dfrac{8\pi + 3\sqrt{3} + 4\pi + 6\sqrt{3}}{48}$

$= \dfrac{12\pi + 9\sqrt{3}}{48} = \dfrac{4\pi + 3\sqrt{3}}{16} = 1{,}4$ //

5. kemiringan $r = 3\sin 3\theta$ di $\theta = \frac{\pi}{6}$

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$$\frac{dr}{d\theta} = \frac{d(3\sin 3\theta)}{d\theta} = 9\cos(3\theta)$$

$$m = \frac{dy}{dx} = \frac{r\cos\theta + \sin\theta \frac{dr}{d\theta}}{-r\sin\theta + \cos\theta \frac{dr}{d\theta}}$$

$$m = \frac{(3\sin 3\theta)\cos\theta + \sin\theta(9\cos(3\theta))}{-(3\sin 3\theta)\sin\theta + \cos\theta(9\cos(3\theta))}$$

$$m = \frac{dy}{dx}\bigg|_{\theta=\frac{\pi}{6}} = \frac{(3\sin 3\frac{\pi}{6})\cos\frac{\pi}{6} + \sin\frac{\pi}{6}(9\cos(3\frac{\pi}{6}))}{-(3\sin 3\frac{\pi}{6})\sin\frac{\pi}{6} + \cos\frac{\pi}{6}(9\cos(3\frac{\pi}{6}))}$$

$$m = \frac{dy}{dx}\bigg|_{\theta=\frac{\pi}{6}} = \frac{0 + \frac{1}{2}\cdot 9 \cdot 1}{-3 \cdot 0 + \frac{1}{2}\sqrt{3}\cdot 9 \cdot 1}$$

$$= \frac{9/2}{\frac{9\sqrt{3}}{2}} = \frac{1}{3}\sqrt{3}$$