Development Economics

Homework 2

Dídac Martí Pinto

CEMFI

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Question 1. Praying for Rain: The Welfare Cost of Seasons

My understanding of the question is that consumption of individual i follows the following process:

$$c_{m,t,i} = z_i e^{g(m)} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i} \quad (1)$$

Where $m \in \{1,2 \dots 12\}$, $t \in \{1,2 \dots 40\}$ represent months and years, and $i \in \{1,2 \dots 1000\}$ represent individuals. Then, $z_i = e^{-\sigma_u^2/2} u_i$ with $ln\, u_i \sim N(0, \sigma_u^2)$ represent households permanent differences in consumption. There is also some common seasonality component for all households $g(m)$ which follows the process described in table 1 provided in the question. And finally $ln\, \varepsilon_{t,i} \sim N(0, \sigma_\varepsilon^2)$ is an idiosyncratic shock that lasts for 12 months. Parameters values are: $\sigma_u^2 = \sigma_\varepsilon^2 = 0.2$.

Individual's i welfare is:

$$W(z_i, \varepsilon_i) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(c_{m,t,i}) \quad (2)$$

Where $(c_{m,t,i}) = \frac{c_{m,t,i}{}^{1-\eta}}{1-\eta}$, and annual "β" is 0.99, so $\beta = 0.99^{1/12}$.

1.1.a. Compute the welfare gains of removing the seasonal component from the stream of consumption separately for each degree of seasonality in Table 1

To compute the welfare gains I try to find for each individual the "g" that satisfies the following equation:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u((1+g) z_i e^{g(m)} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z_i e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) \quad (3)$$

This "g" measures the amount increase in consumption in all months and all years that should be given to an individual facing seasonality to make her as happy as being in a situation without seasonality.

In this case we are assuming that η=1. Therefore, the utility function is approximated by a logarithmic function, i.e. $u(c_{m,t,i}) = lnc_{m,t,i}$.

The results are in the following table:

Table 1. Welfare gains ("g") of removing seasonality (η=1)

Low seasonality	Medium seasonality	High seasonality
g = 0.0042	g = 0.0086	g = 0.0171

Two things are worth commenting from table 1:

First, removing the seasonal component allows all individual to have exactly the same welfare gains, i.e. "g" is the same for all individuals. This is due to the fact that the seasonal component is exactly the same for all individuals.

Second, we can see that the higher is the degree of seasonality the higher are the benefits from removing seasonality. This is because of the choice of the utility function. Notice that it is a CRRA with η=1 and therefore individuals are risk averse, they do not like uncertainty. Any individual with a CRRA utility function and η=1 who has to choose between a certain consumption level, call it c_A, and an uncertain consumption with mean c_A, will prefer the certain consumption level without a stochastic component. And the higher is the uncertainty the less the individual will like the uncertain consumption choice. All this drives the results seen in table 1.

Regarding the real life implications, under all the assumptions we made, we can say that seasonality in consumption has a negative impact in individuals welfare, and that the higher is seasonality the stronger is the negative impact.

1.1.b Compute the welfare gains of removing the non-seasonal consumption risk

Similar to 1.a, in order to compute the welfare gains of removing non-seasonal consumption risk, I try to find for each individual the "g" that satisfies the following equation:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u((1+g) z_i e^{g(m)} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z_i e^{g(m)}) \quad (4)$$

Table 2. Welfare gains ("g") of removing non-seasonal consumption risk (η=1)

	Low seasonality	Medium seasonality	High seasonality
P(10)	0.0077	0.0077	0.0077
P(50)	0.1049	0.1049	0.1049
P(90)	0.2113	0.2113	0.2113
Mean	0.1077	0.1077	0.1077
Std	0.0782	0.0782	0.0782

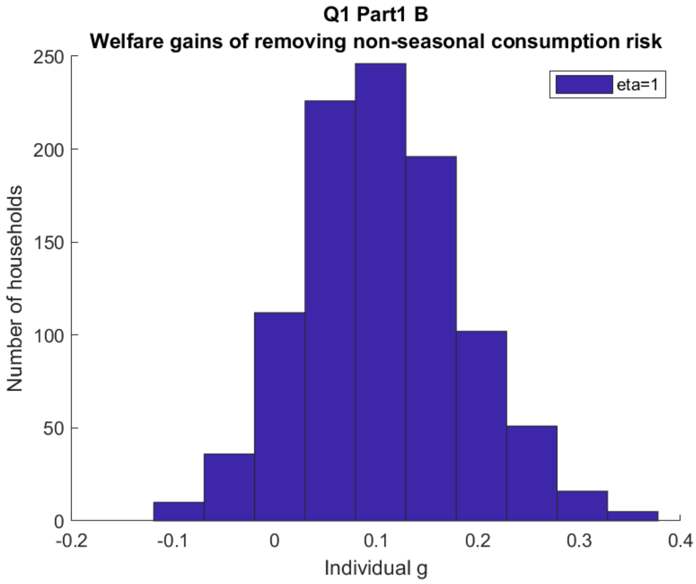
Notice that for all levels of seasonality the welfare gains are exactly the same. It does not matter what degree of seasonality was present before (low, mid or high), when we "turn off" the individual time-varying shocks $\varepsilon_{t,i}$ of consumption the gains are always the same. This is because the shocks $\varepsilon_{t,i}$ do not depend on the degree of seasonality.

I tried an alternative way of measuring the welfare gains of removing non-seasonal consumption risk. I removed also the seasonal component $e^{g(m)}$ from both sides of equation (4) and I measured g. Again, due to the previous reasoning that the previous level of seasonality does not matter, the results were the same. These results are also generated by the MATLAB code but not displayed here.

Then, it is worth noting that the welfare gains are not the same across individuals since the realization of $\varepsilon_{t,i}$ are different across individuals. This is normal; some individuals had better shocks than other individuals, so welfare gains have to be different. In table 2 there is the standard deviation and percentiles 10, 50 (median), and 90 of g's. And in figure 1 we can also see the distribution of welfare gains.

The distribution of welfare gains is similar to a normal distribution, but slightly skewed to the right. Some individuals had "negative welfare gains" or welfare losses from removing non-seasonal consumption risk. Those were probably the lucky individuals who had very good shocks.

Figure 1. Welfare gains of removing non-seasonal consumption risk (η = 1)



1.1.c. Compare and discuss your results in (a) and (b)

Both in parts (a) and (b) the individuals are generally better of removing stochastic components of their consumption paths, reflecting the risk aversion of the utility function. Individuals prefer certainty and smooth consumption paths.

In part (a) the gains are the same across all individuals since the seasonal component is common for all individuals, while in part (b) the welfare gains differ across individuals since $\varepsilon_{t,i}$ is not the same across individuals.

For more specific comments of each section you can look directly at parts (a) and (b).

1.1.d. Redo for η = 2, 4

Part a. for η = 2, 4

The following table is analogous to table 1 for values of η = 2, 4.

Table 3. Welfare gains ("g") of removing seasonality (η = 2, 4)

	Low seasonality	Medium seasonality	High seasonality
η=2	g = 0.0066	g = 0.0185	g = 0.0601
η=4	g = 0.0118	g = 0.0426	g = 0.1867

As I commented in part (a) the welfare gains of removing seasonality are the same for all individuals and are higher the higher is the level of seasonality. What is new here is that we can see that the higher is η the higher are the welfare gains. This is because η is a measure of the individual risk aversion. The higher is η the less individuals like uncertain and non-smooth consumption paths, so the more individual would like to live without seasonality, as it can be seen in table 3.

Part b. for η = 2, 4

The following table is analogous to table 2 for values of η = 2, 4

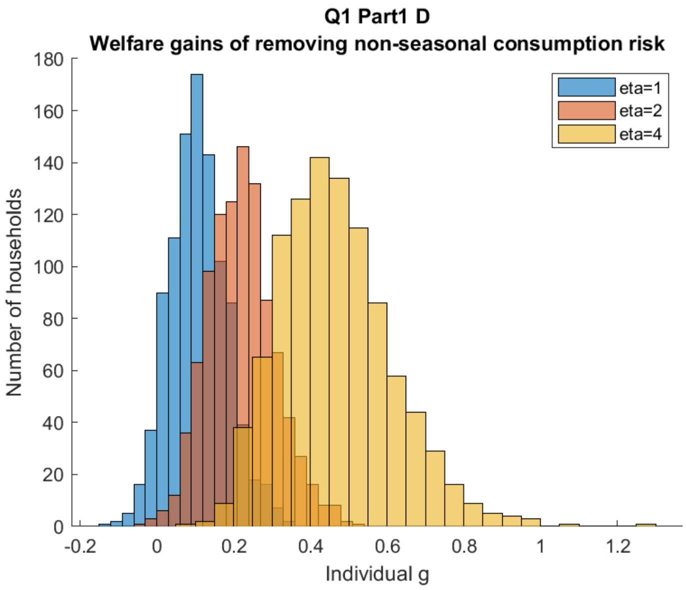
Table 4. Welfare gains ("g") of removing non-seasonal consumption risk (η = 2 and 4)

		Low seasonality	Medium seasonality	High seasonality
η = 2	P(10)	0.1068	0.1068	0.1068
	P(50)	0.2174	0.2174	0.2174
	P(90)	0.3426	0.3426	0.3426
	Mean of g	0.2211	0.2211	0.2211
	Std of g	0.0906	0.0906	0.0906
η = 4	P(10)	0.2881	0.2881	0.2881
	P(50)	0.4520	0.4520	0.4520
	P(90)	0.6884	0.6884	0.6884
	Mean of g	0.4742	0.4742	0.4742
	Std of g	0.1580	0.1580	0.1580

As in part (b) the welfare gains differ across individuals, and are the same for any level of seasonality. What is new with respect to part (b) is that here we can see that the higher is η the higher are the welfare gains of removing non-seasonal consumption risk. This follows from the fact that the higher is η the more risk averse are individuals, as previously commented.

Regarding the differences in welfare gains across individuals they can be seen in figure 2 which is analogous to figure 1 for all levels of η = 1, 2 and 4:

Figure 2. Welfare gains of removing non-seasonal consumption risk (η = 1, 2 and 4)



We can see that the welfare gains follow similar distributions for any value of η, but the higher is η the higher are the welfare gains and the more dispersed and right-skewed they are. Again, this reflects that the higher is η the higher is the risk aversion so the higher are the welfare gains of removing stochastic components of consumption.

Question 1 - Part 2 set-up:

My understanding of the question is that consumption of individual i follows the following process:

$$c_{m,t,i} = z_i e^{g(m)} e^{-\sigma_m^2/2} \varepsilon_{m,i} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i} \quad (5)$$

Where everything is as before, except for $e^{-\sigma_m^2/2}\varepsilon_{m,i}$ which is an individualized stochastic seasonal component with $ln\, \varepsilon_{m,i} \sim N(0, \sigma_m^2)$, and σ_m^2 is given by table 2 in the question. So, there is a common deterministic seasonal component $g(m)$, and an individual seasonal stochastic component $\varepsilon_{m,i}$.

1.2.a. Compute the welfare gains of removing the seasonal component (all combinations of deterministic and stochastic) from the stream of consumption separately for each degree of seasonality in Table 1 and 2

For part 1.2.a I assume that the deterministic seasonal component has always a medium degree of seasonality. And then I allow for the stochastic seasonal component to have different variances (σ_m^2). This already provides enough useful results to get the intuition of the exercise.

Welfare gains of removing only the deterministic seasonal component

These are calculated by finding the "g" that satisfies the following equation for each individual:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u((1+g) z_i e^{g(m)} e^{-\sigma_m^2/2} \varepsilon_{m,i} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z_i e^{-\sigma_m^2/2} \varepsilon_{m,i} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) \quad (6)$$

The results are in the following table:

Table 5. Welfare gains ("g") of removing deterministic seasonality (η=1)

Low σ_m^2	Medium σ_m^2	High σ_m^2
g = 0.0086	g = 0.0086	g = 0.0086

These results are identical to 1.1.a, which is logical since the deterministic levels of seasonality are the same and σ_m^2 does not play any role.

Welfare gains of removing only the stochastic seasonal component

To obtain the welfare gains I estimate the "g" that satisfies the following equation for each individual:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u((1+g) z_i e^{g(m)} e^{-\sigma_m^2/2} \varepsilon_{m,i} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z_i e^{g(m)} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) \quad (7)$$

The results are summarized in the following table:

Table 6. Welfare gains ("g") of removing stochastic seasonality (η=1)

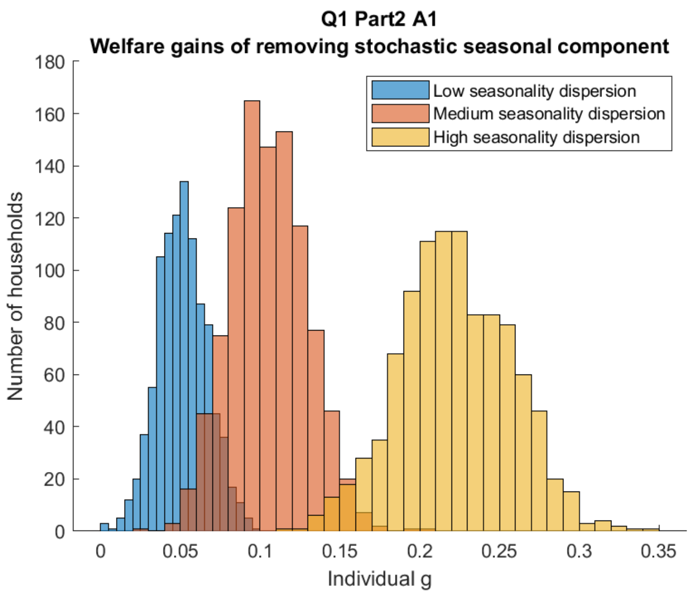
	Low σ_m^2	Medium σ_m^2	High σ_m^2
P(10)	0.0322	0.0752	0.1797
P(50)	0.0512	0.1042	0.2208
P(90)	0.0710	0.1358	0.2681
Mean	0.0513	0.1055	0.2223
Std	0.0154	0.0237	0.0351

It can be seen that the higher is the level of variance of the stochastic component, σ_m^2, the higher are the welfare gains of removing seasonality. This again stems from the fact that individuals are risk averse and prefer smooth consumption paths rather than consumption paths that vary a lot. Precisely, higher σ_m^2 makes consumption paths fluctuate more, and that is something that individuals dislike.

Notice that the effects of the stochastic component are different across individuals, since the realizations of $\varepsilon_{m,i}$ need not to be the same for all individuals. The effects of $\varepsilon_{m,i}$ are similar to those of $\varepsilon_{t,i}$ with the difference that the variance of $\varepsilon_{m,i}$ is not always the same; and that $\varepsilon_{m,i}$ changes each month while $\varepsilon_{t,i}$ changes every year. But both of them differ across individuals.

The differences across individual can be seen more clearly in figure 3. In that figure it is seen that the shape of the distribution of welfare gains is similar for all values of σ_m^2. However, the higher is σ_m^2 the higher are the welfare gains (as already argued), and the higher is the dispersion (as it can be seen in the standard deviation in table 6 too), and the more skewed to the right is the distribution.

Figure 3. Welfare gains of removing stochastic seasonal component (η = 1)



Welfare gains of removing both seasonal components (stochastic and deterministic)

Again, for this part I assume that the deterministic seasonal component has always a medium degree of seasonality, and I allow for the stochastic seasonal component to have different variances (σ_m^2).

In order to measure the welfare gains I estimate for each individual the g that satisfies:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u((1+g) z_i e^{g(m)} e^{-\sigma_m^2/2} \varepsilon_{m,i} e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) = \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z_i e^{-\sigma_\varepsilon^2/2} \varepsilon_{t,i}) \quad (8)$$

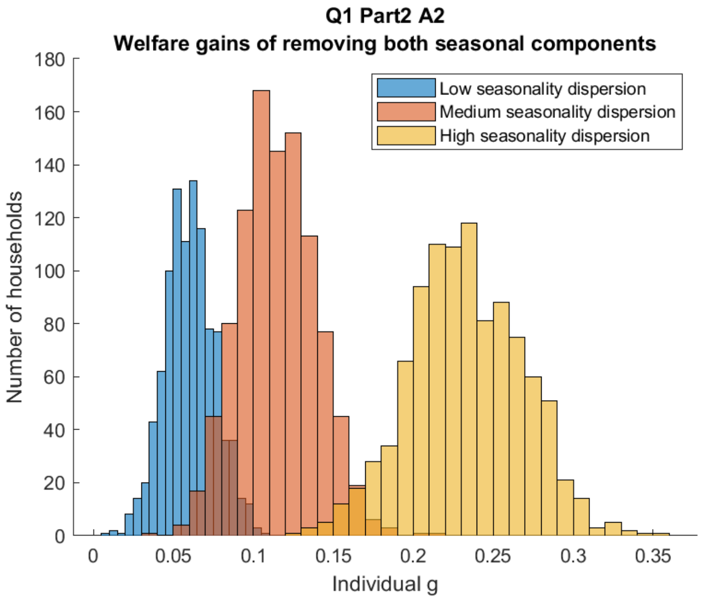
The results are summarized in table 7 and figure 4:

Table 7. Welfare gains ("g") of removing both seasonal components (η=1)

	Low σ_m^2	Medium σ_m^2	High σ_m^2
P(10)	0.0411	0.0845	0.1899
P(50)	0.0602	0.1137	0.2313
P(90)	0.0802	0.1456	0.2791

Mean	0.0603	0.1150	0.2329
Std	0.0155	0.0239	0.0354

Figure 4. Welfare gains of removing both seasonal components (η = 1)



We can see that the results are roughly an addition of the welfare gains of removing the stochastic and the deterministic components separately. Here there are the 2 effects in place:

First, the deterministic component of seasonality, $e^{g(m)}$, which makes all individuals worse off since individuals are risk averse, and the effects are stronger when seasonality is high (even though here we only consider the case in which $e^{g(m)}$ follows the medium degree of seasonality). These effects are the same for all individuals since the deterministic component is the same across all individuals

The second effect comes from the stochastic component of seasonality $e^{-\sigma_m^2/2}\varepsilon_{m,i}$ which also adds more uncertainty to the consumption path, increasing welfare losses. The negative effects are increasing the higher is the variance of this stochastic component since more variance means more uncertainty and higher welfare losses due to the risk aversion.

We do not see the effects separately in the table 5 and figure 3, but they are driving the results.

1.2.b. Compute the welfare gains of removing the non-seasonal consumption risk

Similar to the previous questions, in order to compute the welfare gains of removing non-seasonal consumption risk, I try to find for each individual the "g" that satisfies the following equation:

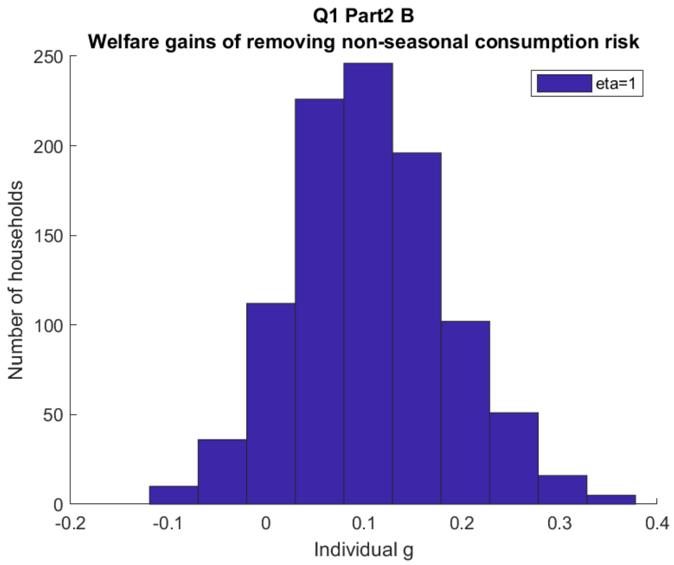
$$\sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u((1+g)z_i e^{g(m)}e^{-\sigma_m^2/2}\varepsilon_{m,i}e^{-\sigma_\varepsilon^2/2}\varepsilon_{t,i}) = \sum_{t=1}^{40}\beta^{12t}\sum_{m=1}^{12}\beta^{m-1}u(z_i e^{g(m)}e^{-\sigma_m^2/2}\varepsilon_{m,i}) \quad (9)$$

The results are summarized in table 8 and figure 4:

Table 8. Welfare gains (“g”) of removing non-seasonal consumption risk (η=1)

	Low σ_m^2	Medium σ_m^2	High σ_m^2
P(10)	0.0077	0.0077	0.0077
P(50)	0.1048	0.1048	0.1048
P(90)	0.2112	0.2112	0.2112
Mean	0.1077	0.1077	0.1077
Std	0.0782	0.0782	0.0782

Figure 4. Welfare gains of removing non-seasonal consumption risk (η = 1)



The results should look familiar, since they are exactly the same as in part 1.1.b. This is because I am using exactly the same $\varepsilon_{t,i}$’s (I do not generate individual $\varepsilon_{t,i}$‘s two times since there is no need to do it, the process is exactly the same).

We can see again that it does not matter the level of seasonality, the welfare gains of removing the non-seasonal consumption risk are always the same. In fact, here the level of seasonality is different than the one in 1.1.b. and still we get the same results. This is because the non-seasonal consumption risk is always the same regardless of the level of seasonality.

1.2.c. Compare and discuss your results in (a) and (b)

Many things can be said (and in fact I already said a lot in the questions themselves), I will make just two comments that I think are worth pointing out:

First, it can be seen that in some cases the welfare gains are different across individuals. This happens when we remove either the stochastic seasonal component ($\varepsilon_{m,i}$) or the non-seasonal stochastic annual component of consumption ($\varepsilon_{t,i}$). This happens because the realizations of these two components are different across individuals; therefore the gains are different for each individual. In this sense the effects of $\varepsilon_{m,i}$ and $\varepsilon_{t,i}$ are similar. On the other hand, we do not see the same when we remove the deterministic seasonal component ($\mathrm{e}^{\mathrm{g(m)}}$), again, this happens because this component is the same for all individuals.

Second, the level of seasonality variance (σ_m^2) is important in part (a) but not in part (b). This is quite obvious since in part (a) we are eliminating the stochastic seasonal component, so the higher it is σ_m^2 the higher are the welfare gains; while in part (b) the stochastic seasonal component is always the same both when we have $\varepsilon_{t,i}$ and when we have not, so it is irrelevant the level of stochastic seasonality variance.

For further comments please refer to the comments in questions 1.2.a and 1.2.b.

1.2.d. Redo for η=2, and 4

Part (a) for η = 2, and 4:

Welfare gains of removing only the deterministic seasonal component

The following table is analogous to table 5 but for different levels of η:

Table 9. Welfare gains ("g") of removing deterministic seasonality (η = 2, 4)

	Low σ_m^2	Medium σ_m^2	High σ_m^2
η=2	g = 0.0185	g = 0.0185	g = 0.0185
η=4	g = 0.0426	g = 0.0426	g = 0.0426

Welfare gains of removing only the stochastic seasonal component

The following table is analogous to table 6 but for different levels of η:

Table 10. Welfare gains ("g") of removing the stochastic seasonal component (η = 2 and 4)

		Low σ_m^2	Medium σ_m^2	High σ_m^2
η = 2	P(10)	0.0779	0.1759	0.4096
	P(50)	0.0999	0.2126	0.4774
	P(90)	0.1220	0.2514	0.5485
	Mean of g	0.1001	0.2130	0.4794
	Std of g	0.0173	0.0293	0.0545
η = 4	P(10)	0.1423	0.3406	0.8571
	P(50)	0.1881	0.4293	1.1053
	P(90)	0.2377	0.5538	1.5776
	Mean of g	0.1907	0.4415	1.1863
	Std of g	0.0414	0.0983	0.3809

Welfare gains of removing both seasonal components (stochastic and deterministic)

The following table is analogous to table 7 but for different levels of η:

Table 11. Welfare gains ("g") of removing both seasonal components (η = 2 and 4)

		Low σ_m^2	Medium σ_m^2	High σ_m^2
η = 2	P(10)	0.0978	0.1976	0.4356
	P(50)	0.1202	0.2350	0.5047
	P(90)	0.1427	0.2745	0.5771
	Mean of g	0.1204	0.2354	0.5067
	Std of g	0.0176	0.0299	0.0555
η = 4	P(10)	0.1909	0.3976	0.9361
	P(50)	0.2387	0.4902	1.1949
	P(90)	0.2904	0.6200	1.6874
	Mean of g	0.2414	0.5029	1.2794
	Std of g	0.0432	0.1024	0.3971

Part (b) for η = 2, and 4:

The following table is analogous to table 8 but for different levels of η:

Table 12. Welfare gains ("g") of removing non-seasonal shocks (η = 2 and 4)

		Low σ_m^2	Medium σ_m^2	High σ_m^2
η = 2	P(10)	0.1066	0.1055	0.1044
	P(50)	0.2159	0.2178	0.2152
	P(90)	0.3384	0.3428	0.3442
	Mean of g	0.2206	0.2210	0.2203
	Std of g	0.0907	0.0906	0.0926
η = 4	P(10)	0.2857	0.2688	0.2183
	P(50)	0.4484	0.4434	0.4136
	P(90)	0.6779	0.6727	0.7204
	Mean of g	0.4716	0.4681	0.4513
	Std of g	0.1621	0.1771	0.2438

Comments part d: The results are similar to those in part (a), the main difference that is worth noticing is that the higher is η the higher are the welfare gains of removing any component of seasonality shocks. All this comes from the fact that the individuals are risk averse and η is a measure of risk aversion.

Question 2. Adding seasonal labor supply

My understanding of the question is that both consumption and labor of individual i follow a process like the one in equation (5). And that individual i welfare is:

$$W(z_i, \varepsilon_i) = \sum_{t=1}^{40} \beta^{12t} [\sum_{m=1}^{12} \beta^{m-1} u(c_{m,t,i}, h_{m,t,i})\,] \;\; (9)$$

Where $(c_{m,t,i}, h_{m,t,i}) = lnc_{m,t,i} + \kappa \frac{h_{m,t,i}{}^{1+\frac{1}{v}}}{1+\frac{1}{v}}$, where =1 and $\kappa = 28.5 \cdot \frac{30}{7} = 122.1429$

2.a. Assume a deterministic seasonal component and a stochastic seasonal component for labor supply both of which are highly positively correlated with their consumption counterparts. Then, compute the welfare gains of removing seasons isolating the effects of consumption and leisure.

Preliminaries/Assumption

1. In order to generate highly positively correlated stochastic components for consumption and labor I assume that $ln\ \varepsilon_{m,i}^{c}$ and $ln\varepsilon_{m,i}^{h}$ follow the following process:

$$\begin{pmatrix} ln\ \varepsilon_{m,i}^{c} \\ ln\ \varepsilon_{m,i}^{h} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & 0.03 \\ 0.03 & \sigma_m^2 \end{pmatrix}\right) \;\; (10)$$

The value of 0.03 is the highest possible I found that ensures that the variance covariance matrix is positive definite. Notice that σ_m^2 is not equal to 0.2 as other people probably assumed, so the covariance in my case cannot be very high.

2. In order to generate highly positively correlated deterministic seasonal components I assume that $g(m)$ has exactly the same values for stochastic and labor.

3. I further assume both σ_m^2 and $g(m)$ are both in the same "low", "middle" or "high" value in the table. e.g when $g(m)$ is in the middle values of the table then for sure σ_m^2 is always in the middle values. This is done in order to reduce the number of cases to study, which is already enough to get the important intuitions.

Solution

1. To measure total effects I try to find the "g" that satisfies for each individual the following:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u\left((1+g) z^c{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^c{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^h{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i} \right)$$

$$= \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z^c{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i}) \;\; (11)$$

2. To measure consumption effects I try to find the "g" that satisfies for each individual the following:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u\left((1+g) z^c{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^c{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^h{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i} \right)$$

$$= \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z^c{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^h{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i}) \quad (12)$$

3. To measure labor effects I try to find the “g” that satisfies for each individual the following:

$$\sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u\left((1+g) z^c{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{g(m)} e^{-\frac{\sigma_m^2}{2}} \varepsilon^h{}_{m,i} e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i} \right)$$

$$= \sum_{t=1}^{40} \beta^{12t} \sum_{m=1}^{12} \beta^{m-1} u(z^c{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^c{}_{t,i}\, , z^h{}_i e^{-\frac{\sigma_\varepsilon^2}{2}} \varepsilon^h{}_{t,i}) \quad (12)$$

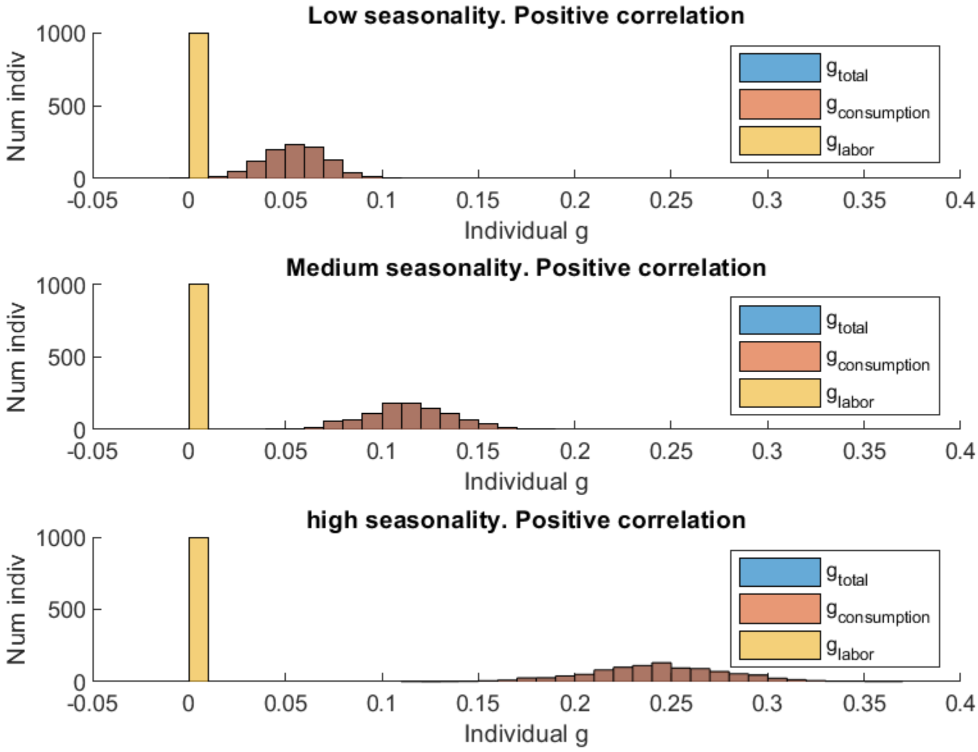
Notice all this procedures come from the slides

Results:

Table 13. Welfare gains (“g”) of removing seasonal risks (positive correlation)

		Low seasonality	Medium seasonality	High seasonality
Total effects	P(10)	0.0354	0.0848	0.1970
	P(50)	0.0553	0.1137	0.2438
	P(90)	0.0748	0.1439	0.2901
	Mean of g	0.0553	0.1150	0.2434
	Std of g	0.0157	0.0230	0.0359
Consum effects	P(10)	0.0354	0.0847	0.1966
	P(50)	0.0553	0.1137	0.2437
	P(90)	0.0748	0.1439	0.2899
	Mean of g	0.0553	0.1150	0.2433
	Std of g	0.0157	0.0230	0.0359
Labor effects	P(10)	1.0e-03*0.0163	1.0e-03*0.0198	1.0e-03*0.0283
	P(50)	1.0e-03*0.0222	1.0e-03*0.0339	1.0e-03*0.0680
	P(90)	1.0e-03*0.0425	1.0e-03*0.0838	1.0e-03*0.2068
	Mean of g	1.0e-03*0.0271	1.0e-03*0.0456	1.0e-03*0.1013
	Std of g	1.0e-03*0.0152	1.0e-03*0.0371	1.0e-03*0.1079

Figure 5. Welfare gains of removing both seasonal risks (positive correlation)



<u>Comment:</u>

I want to mention three things. First, the labor effects are very small in comparison to the consumption and total effects. With different values of kappa the welfare gains that come from labor are higher (and maybe more reasonable too), but let's assume that the results are correct. Notice also that the total effects are roughly the summation of labor and consumption effects.

Second, the distribution of welfare gains varies across individuals since the seasonal stochastic shocks realization are different across individuals. And as it can be seen in figure 5 the consumption effects totally dominate and are almost equal to total effects (which are kind of hidden in the graphs).

Finally, notice that as I especially analyzed in exercise 1, the higher is the seasonality the higher are the welfare gains of removing it; reflecting the risk aversion of the individual.

2.b. Assume a deterministic seasonal component and a stochastic seasonal component for labor supply both of which are highly negatively correlated with their consumption counterparts. Then, compute the welfare gains of removing seasons isolating the effects of consumption and leisure.

<u>Preliminaries/Assumption</u>

1. In order to generate negatively correlated stochastic components for consumption and labor I assume that $ln\ \varepsilon_{m,i}^{c}$ and $ln\varepsilon_{m,i}^{h}$ follow the following process:

$$\begin{pmatrix} \ln \varepsilon^c_{m,i} \\ \ln \varepsilon^h_{m,i} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_m & -0.03 \\ -0.03 & \sigma^2_m \end{pmatrix}\right) \quad (10)$$

2. In order to generate highly negatively correlated deterministic seasonal components I assume that $g^c(m) = -g^h(m)$. Where consumption has exactly the values in the table, and labor the negative sign.
3. I further assume σ^2_m and $g(m)$ are always in the middle value for both consumption and labor.

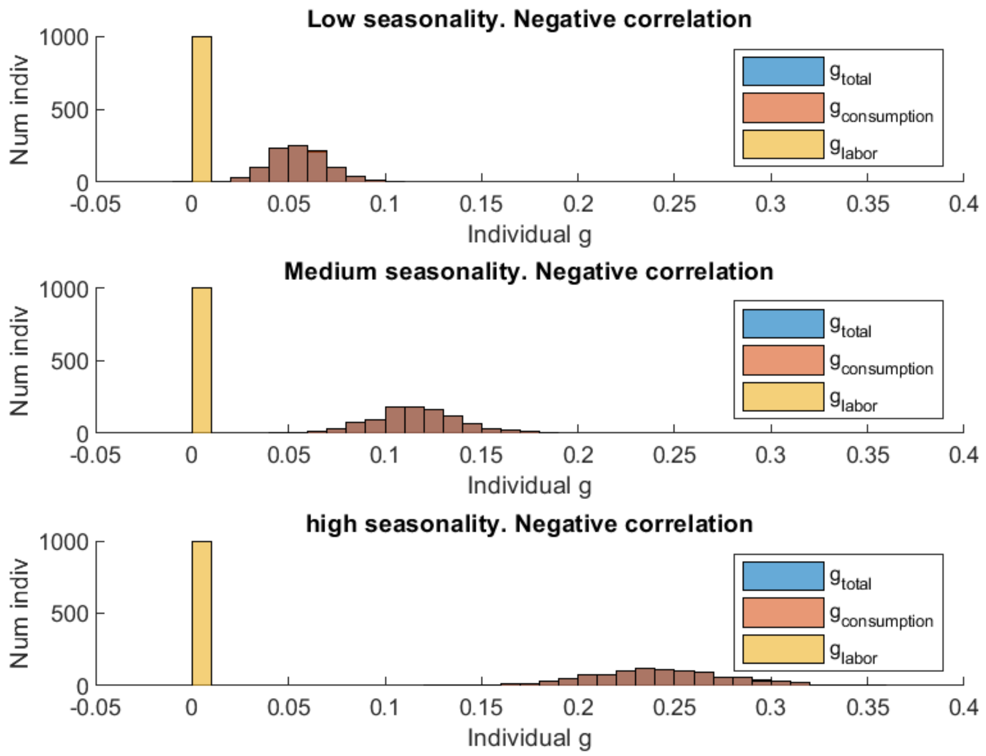
Solution: The procedure is exactly the same as in 2.a. The results, however are different since the process that generates consumption and labor is slightly different.

Results:

Table 13. Welfare gains ("g") of removing seasonal risks (negative correlation)

		Low seasonality	Medium seasonality	High seasonality
Total effects	P(10)	0.0366	0.0864	0.1991
	P(50)	0.0546	0.1156	0.2422
	P(90)	0.0736	0.1432	0.2904
	Mean of g	0.0549	0.1158	0.2439
	Std of g	0.0150	0.0223	0.0355
Consum effects	P(10)	0.0366	0.0864	0.1990
	P(50)	0.0545	0.1155	0.2420
	P(90)	0.0736	0.1431	0.2901
	Mean of g	0.0549	0.1157	0.2438
	Std of g	0.0150	0.0223	0.0355
Labor effects	P(10)	1.0e-03*0.0168	1.0e-03*0.0204	1.0e-03*0.0310
	P(50)	1.0e-03*0.0234	1.0e-03*0.0350	1.0e-03*0.0695
	P(90)	1.0e-03*0.0456	1.0e-03*0.0821	1.0e-03*0.1884
	Mean of g	1.0e-03*0.0284	1.0e-03*0.0453	1.0e-03*0.0972
	Std of g	1.0e-03*0.0157	1.0e-03*0.0337	1.0e-03*0.0874

Figure 6. Welfare gains of removing both seasonal risks (negative correlation)



Comment: The results do not change a lot from those of part (a), so the comments there also apply for this section. The reasons are explained below.

2.c. How do your answers to (a) and (b) change if the non-seasonal stochastic component of consumption and leisure are correlated?

The results of parts (a) and (b) are very similar. No clear pattern can be found. It seems like it does not matter whether correlation between the seasonal components of consumption and work are positively or negatively correlated.

This is because of the fact that the utility function is separable. There is no complementarity between consumption and work, they affect utility independently. Therefore, what matters in determining the level of utility are the levels of consumption and labor themselves, not whether they occur when the other input is high or low.

In parts (a) and (b) all the stochastic components have the same means and variances. Therefore it is normal that the results of part (a) and (b) are the same, since the only thing they differ on are the covariances, and as it has been argued they are irrelevant in determining utility.