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CMPS 351

Assignment 1

In [1]:
```
import numpy as np
from numpy import linalg as la
import matplotlib.pyplot as plt
```

Rosenbrock

In [2]:
```
def rosenbrock(x):
    f = 10 * (x[1] - x[0]**2)**2 + (1 - x[0])**2
    return f
```

Rosenbrock Gradient

In [3]:
```
def rosenbrock_gradient(x):
    g = np.zeros(2)
    g[0] = -40*x[0]*(x[1] - x[0]**2) + 2*x[0] - 2
    g[1] = 20*x[1] - 20*x[0]**2
    return g
```

Backtrack Line Search

In [4]:
```
def backtrack_linesearch(f, gk, pk, xk, alpha = 0.01, beta = 0.6):
    t = 1
    while ( f(xk + t*pk) > f(xk) + alpha*t*gk@pk):
        t *= beta
    return t
```

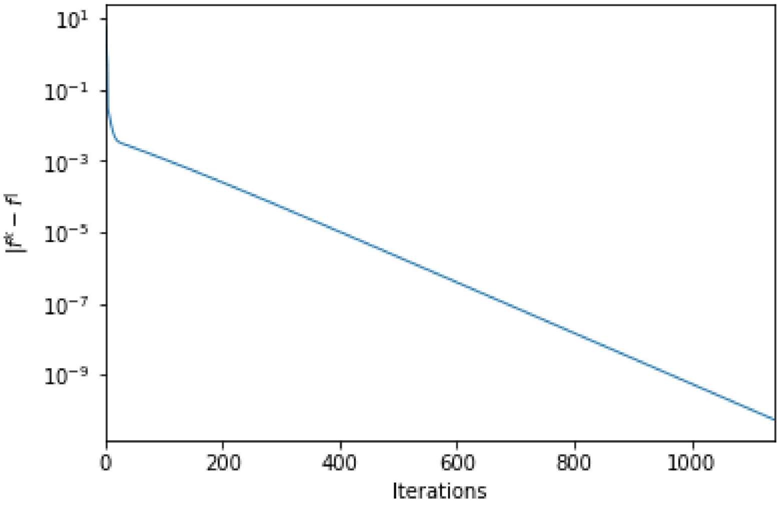
Steepest Descent Backtrack

In [5]:
```
def steepest_descent_backtrack(f, grad, x0, tol = 1e-5):
    x = x0
    history = np.array([x0])
    while (la.norm(grad(x)) > tol):
        p = -grad(x)
        t = backtrack_linesearch(f, grad(x), p, x)
        x += t * p
        history = np.vstack((history, x))
    return x, history
```

Plotting the performance

In [6]:

```
x0 = np.array([-1.2, 1.0])
xstar, history = steepest_descent_backtrack(rosenbrock, rosenbrock_gradient, x
0)

nsteps = history.shape[0]
fhist = np.zeros(nsteps)

for i in range(nsteps):
    fhist[i] = rosenbrock(history[i,:])

plt.figure()
plt.autoscale(enable=True, axis='x', tight=True)
plt.semilogy(np.arange(0, nsteps), fhist, linewidth=1)
plt.xlabel('Iterations')
plt.ylabel(r'$|f^k - f^|$')
plt.show()
```



We see here that initially within the first 50 iterations the error decreases very rapidly, but then the progress becomes very slow. This is due to the contours being very elongated causing the countour lines to be nearly parallel.

Because we are moving in the direction of the steepest descent we find ourselves almost zigzagging in place and making very slow progress towards the minimum.

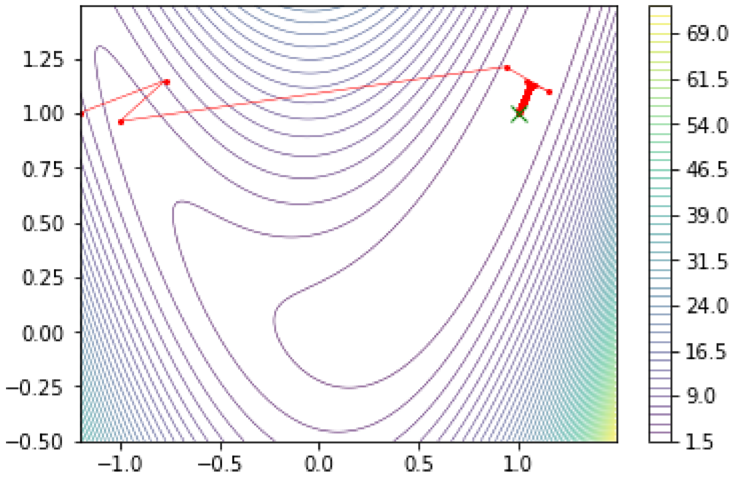
Plotting the contours

In [7]:

```
x0 = np.arange(-1.2, 1.5, 0.01)
x1 = np.arange(-0.5, 1.5, 0.01)

F = np.zeros((x1.shape[0], x0.shape[0]))

for i in range(F.shape[0]):
    for j in range(F.shape[1]):
        x = [x0[j], x1[i]]
        F[i, j] = rosenbrock(x)

plt.figure('Contours')
plt.plot(history[:,0], history[:,1], linewidth=0.5, color='red', marker='o', markersize=2)
plt.plot([1],[1], color='green', marker='x', markersize=8)
plt.contour(x0, x1, F, 50, linewidths=0.5)
#plt.axis('equal')
plt.colorbar()
```

Out[7]:

```
<matplotlib.colorbar.Colorbar at 0x13918e6d860>
```


1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−1.0
−0.5
0.0
0.5
1.0
69.0
61.5
54.0
46.5
39.0
31.5
24.0
16.5
9.0
1.5