Faculty of Arts and Sciences
Department of Computer Science
CMPS 351 – Numerical Optimization
Assignment 5 – Due Feb 25, 2019

Reading material:

- Undergrad review of Lagrange multipliers (handout posted on moodle).
- Chapter 12 from Ref 1

1. Exercise in R^3. Write and solve the first order optimality conditions of the following problem.

$$\begin{array}{ll} \text{minimize} & x_1x_2 + x_2x_3 \\ \text{subject to} & x_1^2 + x_2^2 - 2 = 0 \\ & x_1^2 + x_3^2 - 2 = 0, \end{array}$$

2. Significance of the Lagrange Multipliers. Consider the equality-constrained problem:

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & h_i(x) = 0, \quad i = 1, \dots, p \end{array}$$

Show that the value of a Lagrange multiplier at optimality λ_i^* may be interpreted as the sensitivity of the optimal solution f^* with respect to the right hand side of the i-th constraint. In other words, if the i-the constraint is changed to $h_i(x) = \epsilon$, then: $\Delta f^* \approx -\lambda_i^* \epsilon$.

3. Nonnegativity of the Lagrange Multipliers. Explain with the aid of an appropriate diagram why it is not possible for the Lagrange multiplier of an inequality constraint to be negative at an optimal point.

4. Necessary but not sufficient conditions. Verify that the first order optimality conditions of the following problem are satisfied at the points $(-2, 2)$ and $(2, -2)$ but yet neither is a solution.

$$\begin{array}{ll} \text{minimize} & x_1 - x_2 \\ \text{subject to} & x_1x_2 + 4 = 0 \end{array}$$

5. Healthy Snack. Consider the problem of purchasing afternoon snacks. Health conscious buyers need at least 6 total ounces of chocolate, 10 ounces of sugar, and 8 ounces of cream cheese. There are 2 choices of snacks: brownies and cheescakes whose ingredients are listed below. Brownies cost 50 cents and mini-cheesecakes cost 80 cents.

	Chocolate	Sugar	Cream Cheese
Brownie	3	2	2
Cheesecake	0	4	5

- Formulate the minimum-cost healthy purchase snack problem as a linear optimization problem, assuming a friendly bakery that allows fractional purchases, and solve it using the `linprog` routine from the `scipy.optimize` library.
- Write out explicitly the Lagrangian for this problem, as well as the optimality conditions.
- Compute the values of the Lagrange multipliers from the optimality conditions above. What is their physical interpretation in this problem? Comment on their values.
- Convert the problem to canonical form (i.e., equality constraints with zero-lower bound inequality constraints) and solve it again with the `linprog` routine (you may directly call the inner routine in this case). Do you get the same solution and multipliers?