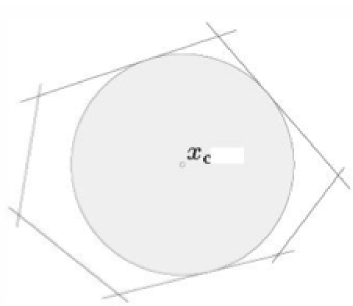
Faculty of Arts and Sciences
Department of Computer Science
CMPS 351 – Numerical Optimization
Assignment 6 – Due Mar 4, 2019

1. Center of Polyhedron. Consider a convex polygon P with m sides defined as $P = \{x \,|\, a_i^t x \leq b_i,\ i = 1 \cdots m\}$. We seek to find the center and radius of the largest inscribable ball in the polygon.

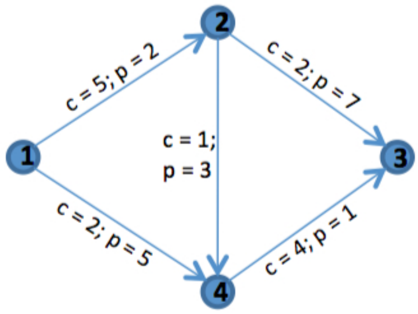
- Formulate the problem as a linear programming problem. Hint. A point x is at a distance R from the line i if $a_i^t x - b_i = -R\|a_i\|$.
- Use the data sample below and solve the problem.
- What is the significance of the Lagrange multipliers in your solution.
- Write out the Lagrangian for this problem as well as the optimality conditions. Verify that the solution satisfies the optimality conditions.

```
A = np.array( [[0, -1], [2, -1], [1, 1], [-1/3, 1], [-1, 0], [-1, -1]] )
b = np.array( [0, 8, 7, 3, 0, -1] )
```



2. Minimum Cost Flow. Many graph problems (shortest-path, multi-commodity flow, minimum-cost flow, etc.) can be formulated as linear programs. In this exercise, we explore the problem of the minimum-cost shipment of an amount of goods from a source node to a destination node in a transportation network, consisting of n nodes and m arcs connecting these nodes. Associated with each arc is a maximum capacity c_i (the maximum amount that can be sent along the arc) and a unit cost p_i (the cost of shipping one unit along the arc). Formulate the problem as an LP which minimizes an appropriate cost function subject to capacity constraints, as well as constraints that the sum of all goods arriving and departing from a node is zero, except for the source and destination (where the sum is q and q respectively, q being the amount of goods shipped). The variables are the amounts sent along the arcs of the network.

- Solve the problem for the graph below where we wish to send 4 units from node 1 to node 3.
- What is the significance of the multipliers of the capacity constraints? What is the significance of the multipliers of the node constraints?



3. Image Reconstruction. A grayscale image is represented as an $M \times N$ matrix of intensities U. Suppose we are given the values of U^0_{ij} for a subset of the pixels of such an image and seek to complete the image by guessing the missing values. The reconstructed image consists of the complete matrix $U \in R^{M\times N}$ where U satisfies the interpolation conditions $U_{ij} = U^0_{ij}$. The reconstruction is found by minimizing an appropriate measure of roughness of the image satisfying these conditions.

- Formulate and solve the image interpolation problem using an L_2 roughness measure

$$\sum_{i=2}^{M}\sum_{j=2}^{N}(U_{ij} - U_{i-1,j})^2 + (U_{ij} - U_{i,j-1})^2$$

  Hint: The simplest formulation will result in a quadratic objective and linear constraints.

- To test your solution, consider the image below. Use the data from the obscured image on the right to see how well you can recover the original image on the left (data posted on moodle).