Calculations of $2\nu\beta\beta$ Decay Nuclear Matrix Elements Using Ab-Initio Methods

Thesis Presentation

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March 31, 2025

Overview

1. Neutrinos and the β-Decay
2. $2\nu\beta\beta$ and $0\nu\beta\beta$
3. Nuclear Matrix Elements (NMEs) and Half-Life
4. Ab-initio method vs Phenomenology
5. IMSRG
6. NMEs Results for ^{22}O & ^{48}Ca with Ab-Initio Methods
7. Concluding Remarks

1. Neutrinos and the β-Decay

β-decay isospin changing nuclear decay governed by the weak nuclear force:

$$\beta^- : \left| n, t_z = -\frac{1}{2} \right\rangle \rightarrow \left| p, t_z = \frac{1}{2} \right\rangle + e^- + (\bar{\nu}_e) \tag{1}$$

$$\beta^+ : \left| p, t_z = \frac{1}{2} \right\rangle \rightarrow \left| n, t_z = -\frac{1}{2} \right\rangle + e^+ + (\nu_e) \tag{2}$$

- Emission of β-particles, either an electron $\beta^- = e^-$ or a positron $\beta^+ = e^+$ along with electron neutrinos ν_e and $\bar{\nu}_e$.
- Historically, neutrino emission was unknown.
- Postulated by Wolfgang Pauli in 1930 to explain *the continuous energy spectra of the β-decay.*
- We focus on β^- decay.

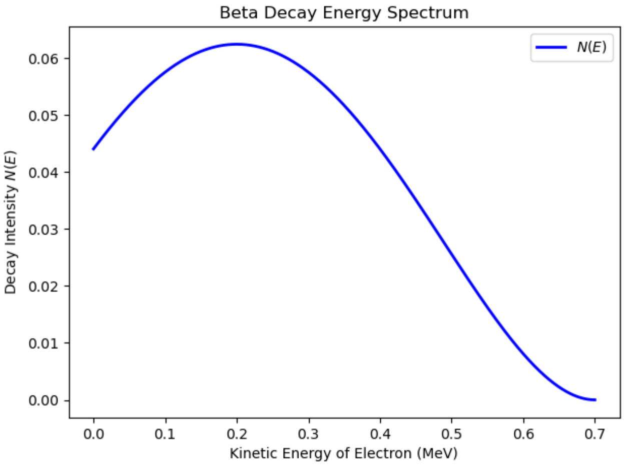
Neutrinos and the β-Decay

- Q-reaction value for nuclear decays and reactions:

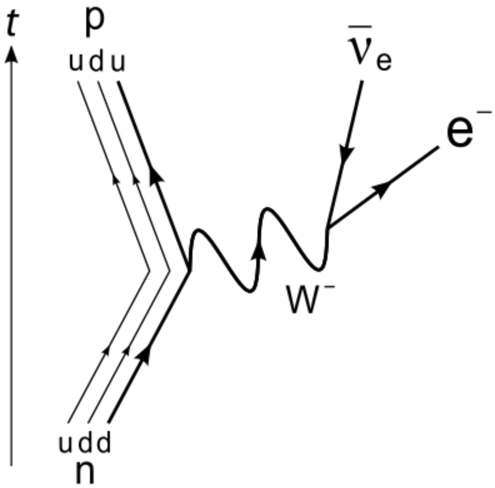
$$Q = T_f + T_i = (m_i - m_f)c^2 \tag{3}$$

- Without neutrinos, expected narrow distribution: $Q \sim T_e \sim -m_e c^2$.
- Observed continuous energy spectrum distribution: $Q \sim T_e + T_{\bar{\nu}_e}$.
- Electron energy shared with another unidentified particle called the neutrino.

Neutrinos and the β-Decay



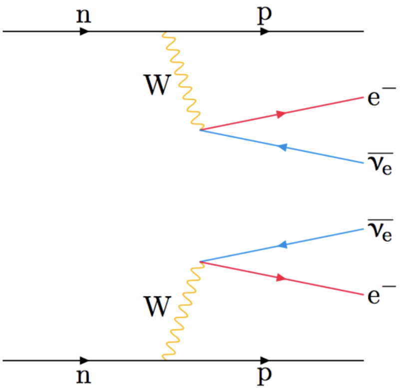
Neutrinos and the β-Decay



2. $2\nu\beta\beta$ and $0\nu\beta\beta$

- First theoretical postulation made by Maria Goeppert-Mayer in 1935.

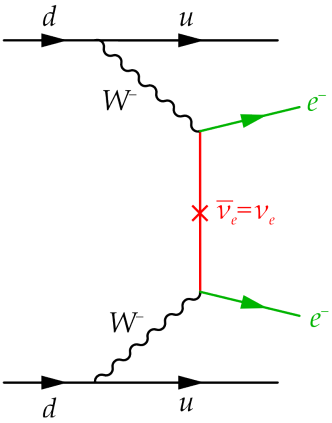
$$2\nu\beta\beta : X(A, Z) \longrightarrow X(A, Z+2) + 2e^- + 2\bar{\nu}_e \tag{4}$$



- Important in the study of $0\nu\beta\beta$ decay.

$2\nu\beta\beta$ and $0\nu\beta\beta$

- Nuclear matrix element correlations for $2\nu\beta\beta$ and $0\nu\beta\beta$.
- Constraining of $0\nu\beta\beta$ NMEs values.
- Probing differences to phenomenological NMEs.
- Majorana or Dirac fermions?



3. Nuclear Matrix Elements (NMEs) and Half-Life

- Half-life equation for $2\nu\beta\beta$ decay given by Fermi's Golden Rule (perturbative weak interaction) is [1]:

$$\frac{1}{T_{1/2}^{2\nu}} = G^{2\nu} g_A^4 |\mathcal{M}^{2\nu}|^2 m_e^2 \tag{5}$$

- We only care about the relation between half-life $T_{1/2}^{2\nu}$ and NMEs $|\mathcal{M}^{2\nu}|$:

$$\frac{1}{T_{1/2}^{2\nu}} \propto |\mathcal{M}^{2\nu}|^2 \tag{6}$$

- Nuclear matrix element for $2\nu\beta\beta$ contributed by Gamow-Teller and Fermi transitions:

$$\mathcal{M}^{2\nu} = \mathcal{M}_{GT}^{2\nu} - \left(\frac{g_V}{g_A}\right)^2 \mathcal{M}_F^{2\nu} \tag{7}$$

[1] Payne, C. G. (2018). Ab initio theory for two-neutrino and neutrinoless double-beta decay (T). University of British Columbia. Retrieved from https://open.library.ubc.ca/collections/ubctheses/24/items/1.0363101

Nuclear Matrix Elements (NMEs) and Half-Life

- General matrix element formulation for selection rules and probabilities.

$$\langle \Psi_f^{J^\pi} | \hat{\mathcal{O}} | \Psi_i^{I^\pi} \rangle \tag{8}$$

- Suppress Fermi contribution [2]:

$$|\mathcal{M}_{GT}^{2\nu}| >> |\mathcal{M}_{F}^{2\nu}| \tag{9}$$

- Fermi transition mismatch of isospins.

[2] Masaru Doi, Tsuneyuki Kotani, Eiichi Takasugi, Double Beta Decay and Majorana Neutrino, Progress of Theoretical Physics Supplement, Volume 83, March 1985, Pages 1–175,

Nuclear Matrix Elements (NMEs) and Half-Life

- Gamow-Teller transition contribution most significant [3] :

$$\mathcal{M}^{2\nu} \approx \mathcal{M}^{2\nu}_{GT} = \sum_k \frac{\langle 0^+_f | \hat{\mathcal{O}}_{GT} | 1^+_k \rangle \langle 1^+_k | \hat{\mathcal{O}}_{GT} | 0^+_i \rangle}{E_k - [(E_i + E_f)/2]} \tag{10}$$

- Gamow-Teller operator $\hat{\mathcal{O}}_{GT} = \sum_a \hat{\tau}^-_a \boldsymbol{\sigma}_a$.
- Parent to daughter nuclei $0^+_i \longrightarrow 1^+_k \longrightarrow 0^+_f$

[3] P. Jokiniemi et al., *Neutrinoless $\beta\beta$-decay nuclear matrix elements from two-neutrino $\beta\beta$-decay data*, Phys. Rev. C, **107**(4), 2023.

4. Ab-initio method vs Phenomenology

- Theoretical nuclear phenomenology focuses on empirically motivated construction of nuclear theory.
- Liquid drop model, nuclear shell model, interacting boson model (IBM), etc
- Phenomenological methods fit nuclear potentials by appropriate mathematical functions to reproduce experiments.
- Lacks power to predict observables with no available data such as $0\nu\beta\beta$.

Ab-initio method vs Phenomenology

- Ab-initio; first principle or starting from the beginning.
- Essential idea: solve $\hat{H}|\Psi\rangle = E|\Psi\rangle$.
- Consider all nucleon-nucleon interactions and build from ground up.
- Apply appropriate approximations (e.g. valence-space and inert-core decoupling from $\hat{H}$ using IMSRG.
- Use of χ EFT to describe effective theory for QCD and complicated many-body interactions in nuclei.
- QCD: quark-gluon degrees of freedom.
- χEFT: nucleon-pion degrees of freedom.
- Nucleon-nucleon interactions for NN and **3N** terms.

Ab-initio method vs Phenomenology

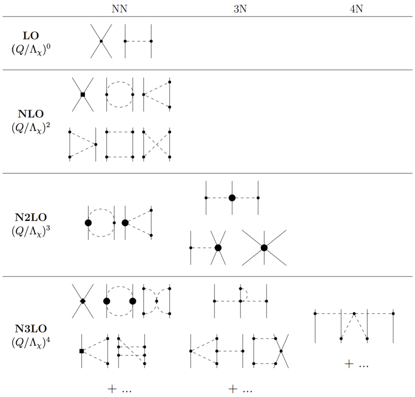


Figure: χEFT interaction diagarams. Full lines are nucleons, dashed lines pions. [4]

[4] Machleidt, R. and Sammarruca, F., *Chiral EFT based nuclear forces: achievements and challenges*, Physica Scripta, vol. 91, no. 8, 2016.

5. IMSRG

- Most quantum systems can be approximated using the quantum harmonic oscillator.
- Approximate nuclear potential interactions with quantum harmonic oscillator Hamiltonian $\hat{H}_{HO} = \frac{\hat{p}^2}{2m} + V_{HO}$ where $V_{HO} = \frac{1}{2}m\omega^2\mathbf{x}^2$.
- Given different $\hbar\omega$ values, we get different complete orthonormal basis sets $\{|\Psi_n\rangle\}$.

$$\hat{H}_{HO}(\omega)|\Psi_n\rangle_\omega = E_n^{HO}(\omega)|\Psi_n\rangle_\omega \tag{11}$$

- Energy splittings defined entirely by $\hbar\omega$:

$$E_n^{HO} = \hbar\omega\left(n + \frac{1}{2}\right) \tag{12}$$

- How to transform a Hamiltonian from the H.O. to the desired basis?

IMSRG

- IMSRG is a quantum many-body method for calculating nuclear interactions.
- Simplification of $\hat{H}$-Hamiltonian via **continuous unitary transformation:**[5]

$$\hat{H}(s) = \hat{U}(s)\hat{H}(0)\hat{U}^\dagger(s) \tag{13}$$

- $\hat{H}(s = 0)$ starting Hamiltonian parametrized by s-flow parameter $s \in [0, \infty)$.

$$\hat{H}(s) = \hat{H}(s)_d + \hat{H}(s)_{od} \tag{14}$$

- As $s \longrightarrow \infty$, $\hat{H}(s) \longrightarrow \hat{H}(s)_d$ and $\hat{H}(s)_{od} \longrightarrow 0$.
- Hamiltonian becomes more "simplified" or diagonal.
- Suppression of undesired terms as $\hat{H}(s)_{od} \longrightarrow 0$.

[5] H. Hergert et al., *In-Medium Similarity Renormalization Group Approach to the Nuclear Many-Body Problem*, in *An Advanced Course in Computational Nuclear Physics*, Springer, 2017.

IMSRG

Find a unitary transformation $\hat{U}(s)$ that renders $\hat{H}(s) \longrightarrow \hat{H}(s)_d$ diagonal.
After some tedious math...

Dynamical Flow Generator

$$\hat{\eta}(s) \equiv \frac{d\hat{U}(s)}{ds}\hat{U}^{\dagger}(s) = -\hat{\eta}^{\dagger}(s) \tag{15}$$

- Solve the following differential equation:

$$\frac{d}{ds}\hat{H}(s) = [\hat{\eta}(s), \hat{H}(s)] \tag{16}$$

- SRG flow equation describes **evolution** of $\hat{H}(s)$ under operation from $\hat{\eta}(s)$.

IMSRG

- Evolution of $\hat{H}(s)$ depends on $\hat{\eta}(s)$ generator.
- What $\hat{\eta}(s)$ generator is appropriate? Recall...

Desired Hamiltonian

$$\lim_{s\to\infty} \hat{H}(s) \to \hat{H}(s)_d, \lim_{s\to\infty} \hat{H}(s)_{od} \to 0 \tag{17}$$

- Wegner-Brockett proposed:

$$\hat{\eta}(s) \equiv [\hat{H}(s)_d, \hat{H}(s)_{od}] \tag{18}$$

- $\hat{\eta}(s)$ vanishes at two points: finite s due to degeneracy and $s \to \infty$.

IMSRG

- Other operators transform along:

$$\hat{\mathcal{O}}(s)_{GT} = \hat{U}(s)\hat{\mathcal{O}}(0)_{GT}\hat{U}^{\dagger}(s) \tag{19}$$

- $2n + l \leq e_{max}$

6. NMEs Results for ^{22}O & ^{48}Ca with Ab-Initio Methods

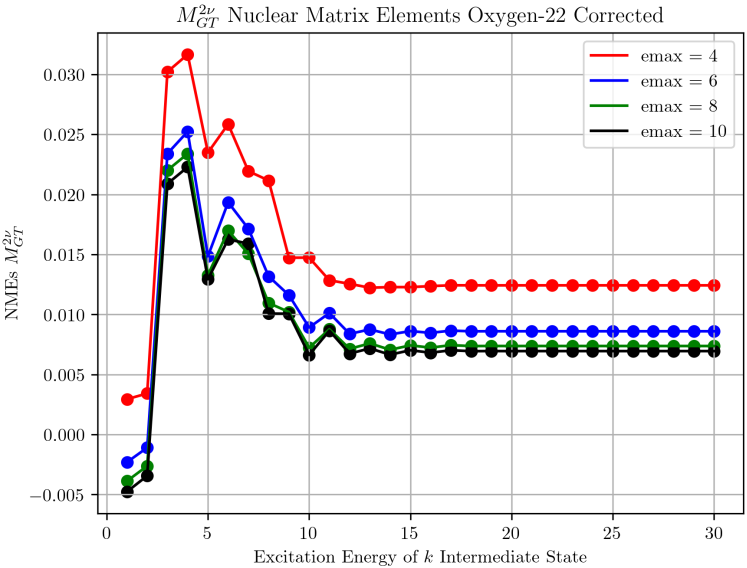
- Oxygen-22 $2\nu\beta\beta$ decay.

$$^{22}O(0^+) \longrightarrow \ ^{22}F(1^+) \longrightarrow \ ^{22}Ne(0^+) \tag{20}$$

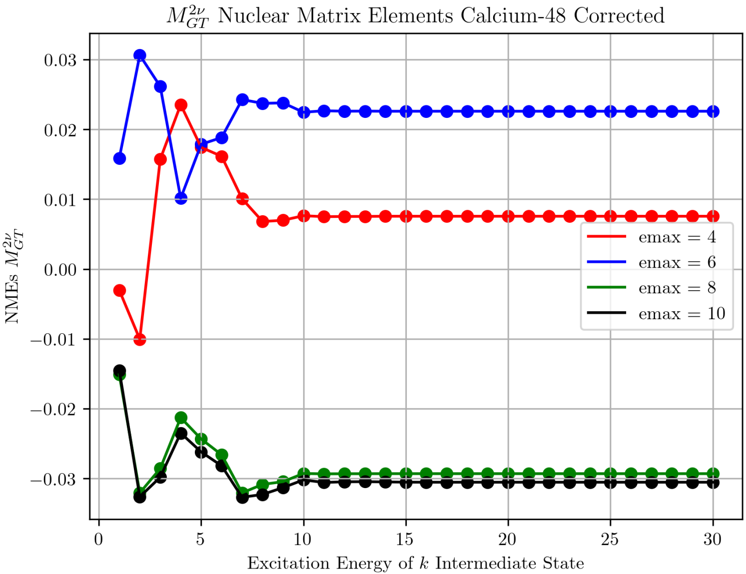
- Calcium-48 $2\nu\beta\beta$ decay.

$$^{48}Ca(0^+) \longrightarrow \ ^{48}Sc(1^+) \longrightarrow \ ^{48}Ti(0^+) \tag{21}$$

NMEs Results for ^{22}O & ^{48}Ca with Ab-Initio Methods



NMEs Results for ^{22}O & ^{48}Ca with Ab-Initio Methods



7. Concluding Remarks

- Important to study $2\nu\beta\beta$ NMEs to correlate and constrain $0\nu\beta\beta$ NMEs.
- First ab-initio calculations of $2\nu\beta\beta$ NMEs.
- Useful tool in probing and understanding neutrino and Beyond the Standard Model physics.

The End