Judging competitions and benchmarks: a candidate election approach SUPPLEMENTAL MATERIAL

Kendall's W

To compute the concordance, we proposed to compute the mean Spearman's ρ of all possible pairs of rankings. However, this algorithm's complexity is exponential $O(2^n)$ with the number of judges. To avoid the problem of complexity, we can compute the concordance using Kendall's W statistics [5] (in practice we use a newer version of Kendall's W accounting for ties [8]). It has been shown in [4] that W is linearly related to $\bar{r_s}$, the mean value of the Spearman's rank correlation coefficients between all $\binom{m}{2}$ possible pairs of rankings between judges. We have published our implementation of Kendall's W and its second version accounting for ties in the Python Package RANKY [7], dedicated to ranking methods and measures.

$$W(M) = \frac{12 \sum_{i=1}^{n} (R_i - \bar{R})^2}{m^2(n^3 - n)}$$

With R_i being the total rank of candidate i on all judges:

$$R_i = \sum_{j=1}^{m} r_{i,j}$$

And $\bar{R}$ being the mean value of all total ranks:

$$\bar{R} = \frac{1}{n} \sum_{i=1}^{n} R_i$$

There is also the correction for ties, implemented in my package ranky, to describe. See Wikipedia for the definition[1].

Relation between Kendall's W and the average Spearman's ρ between all possible pairs of judges:

$$\bar{r_s} = \frac{mW - 1}{m - 1}$$

Kendall's rank correlation coefficient

Intuitively, Kendall's τ measures a correlation linked to the number of "neighbor swaps" needed to transform j into j'. It's definition involves all possible pair

[1] `https://en.wikipedia.org/wiki/Kendall%27s_W`

of observations (j_c, j'_c) and $(j_{c'}, j'_{c'})$. The two pairs are said to be *concordant* if both judges j and j' agrees on their order of the candidates c and c', otherwise they are *discordant*. Kendall's τ is defined as following [6]:

$$\tau = \frac{n_c - n_d}{\binom{n}{2}}$$

where n_c and n_d represent the number of concordant pairs and discordant pairs respectively.

The actual version we use is the τ_b which accounts for ties and have a more complex definition [1]:

$$\tau_b = \frac{n_c - n_d}{\sqrt{(\binom{n}{2} - n_1)(\binom{n}{2} - n_2)}}$$

where $n_1 = \sum_i \frac{t_i(t_i-1)}{2}$ and $n_2 = \sum_{i'} \frac{u_{i'}(u_{i'}-1)}{2}$ with t_i the number of tied values in the i^{th} group of ties for the first quantity and $u_{i'}$ the number of tied values in the j^{th} group of ties for the second quantity.

In practice, Spearman's ρ and Kendall's τ are well correlated [ref?].

Datasets-Algorithms matrices

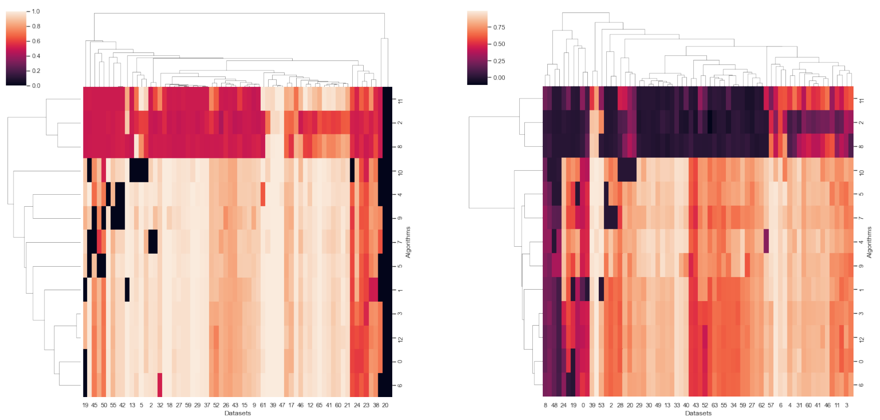

Fig. 1: Heatmap with hierarchical clustering, AutoDL-AUC (left) and AutoDL-ALC (right) score matrix.

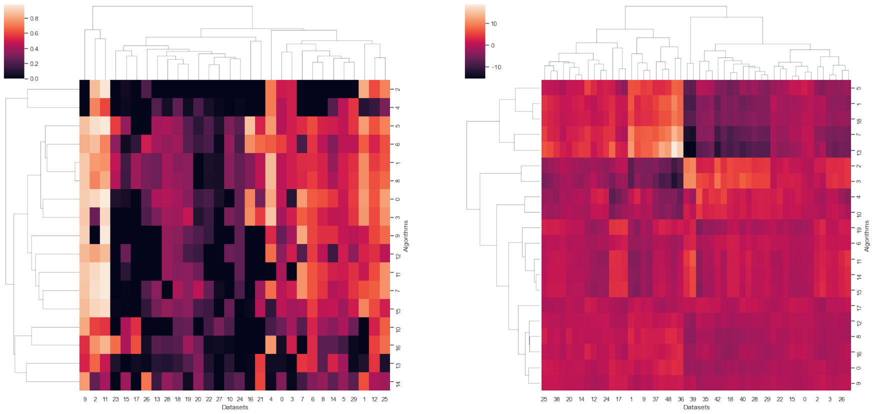

Fig. 2: Heatmap with hierarchical clustering, AutoML (left) and Artificial (right) score matrix.

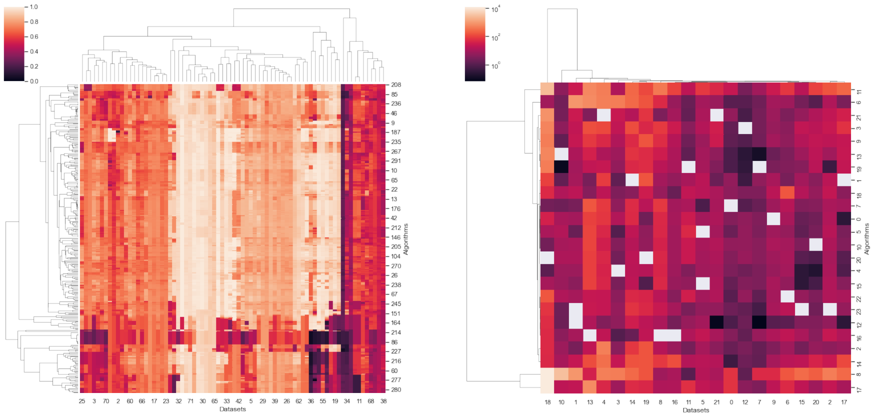

Fig. 3: Heatmap with hierarchical clustering, OpenML (left) and Statlog (right) score matrix.

Interesting plots

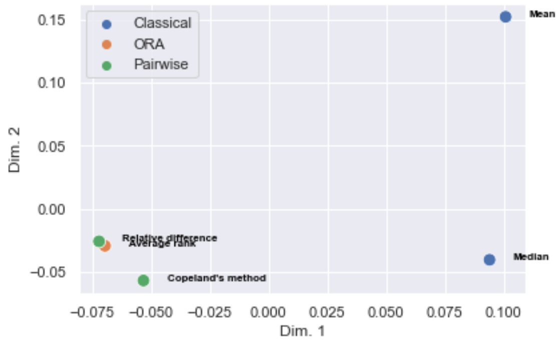

Fig. 4: Multidimensional scale (MDS) plot of the rankings produced by each ranking method. The metric used for the MDS is the Spearman distance, averaged on all benchmarks. This gives an idea of the similarities between the methods.

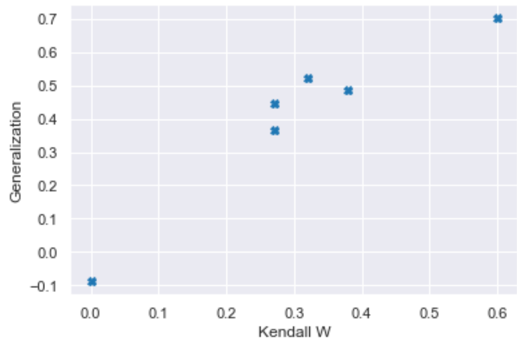

Fig. 5: Concordance of the DA matrices versus the mean generalization score obtained by the ranking functions. Each point is a benchmark.

The results on stability are summarized in Figure 6.

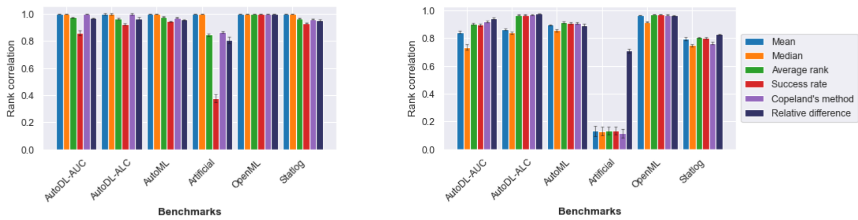

Fig. 6: Stability against candidate axis perturbation (left) and judge axis perturbation (right) of the ranking functions on each benchmark. The error bars represents the standard deviation across the scores obtained on all trials.

Criteria satisfied by success rate method

All the results presented in Figure **??** were found in references (to cite), except for success rate. Let's find out by ourselves.

(1) Success rate doesn't satisfy majority criterion. Counter-example:

	j_1	j_2	j_3
A	1	1	0
B	0.8	0.8	1
C	0.6	0.6	0.6

Table 1: This is a score matrix exposing a counter example. A is ranked first by a majority of judges but its average success rate is the same as B: $\frac{2}{3}$.

(2) Success rate doesn't satisfy Condorcet criterion, as implied by the fact it does not satisfy majority criterion (Majority $\in$ Condorcet).

(3) Success rate meets consistency criterion and (4) participation criterion.

A ranked system is consistent iif it's a scoring function (i.e. positional system) [10]. Success rate is positional as adding a judge improve the score of the candidates according the their position in the ranking of this judge.

(5) Success rate is not LIIA. Counter example:

	j_1	j_2	j_3
A	0.6	0.6	0
B	0.4	0.4	1
C	1	0	0.4

Table 2: This is a score matrix exposing a counter example. If there is only A and B then A wins. If we add C then it is a tie.

(6) Success rate is not IIA, as implied by the fact it is not LIIA (LIIA $\in$ IIA).

(7) Success rate is not clone-proof. Counter-example:

	j_1	j_2	j_3
A	0.6	0.6	0.4
B	0.5	0.4	0.6
C	0.5	0.7	0.5

Table 4: This is a score matrix exposing a counter example. If we repeatedly duplicate the candidate B, A will end up in front of C using the relative difference method.

	A	B	C	Average
A	-	0.6	0.4	0.5
B	0.4	-	0.5	0.45
C	0.6	0.5	-	0.55

Table 3: This is a pairwise success rate table exposing a counter example. If we repeatedly duplicate the candidate B, A will end up in front of C.

Criteria satisfied by relative difference method

(1) Majority: No, counter example found empirically (Majority rate $\neq 1$).
(2) Condorcet: No, counter example found empirically (Condorcet rate $\neq 1$).
(3) Consistency: Yes. The final score given to a candidate can be expressed as the sum (the mean) of the score this candidate obtained on all judges. This means that, for two score matrices X and Y sharing the same candidates but not the same judges, and $[X\ Y]$ representing the judge axis concatenation of X and Y:

$$f(X) + f(Y) = \frac{1}{2} f([X\ Y])$$

(4) Participation: Yes. Once again, the final score given to a candidate can be expressed as the sum (the mean) of the score this candidate obtained on all judges. Therefore, a judge which prefers a candidate $\mathbf{u}$ against another candidate $\mathbf{v}$ can only improve the position of $\mathbf{u}$ relatively to $\mathbf{v}$.
(5) LIIA: No, same counter example as success rate above.
(6) IIA: No, because not LIIA
(7) Clone-proof: No, counter example

Optimal rank aggregation and Kemeny-Young

Optimal rank aggregation (ORA) methods is a family of ranking methods that consists in proposing a distance function $d : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+$ and finding a ranking r which minimizes the following objective function:

$$l(\mathbf{r}) = \sum_{\mathbf{j} \in \mathcal{J}} d(\mathbf{r}, \mathbf{j})$$

Some well-known distance functions that can be used are Kendall's τ distance, Spearman distance, Cayley distance or the Euclidean distance.

The ORA using Kendall's τ as a distance function is known as the *Kemeny-Young* method. It has interesting properties such as being a Condorcet method; however, its computation is NP-Hard. The ORA using the Spearman distance also has interesting properties and is computationally linear as it produces the same ranking as the *average rank* method [3].

In practice we perform the optimization using differential evolution [9]. A good overview of ORA and rank distance functions is given in [2].

They can all be computed in polynomial time in n and m, except Kemeny-Young.

The high complexity of *Kemeny-Young* method prevented us from including it in the experiments.

References

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