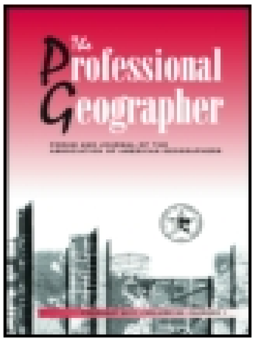

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Polygon Characterization With the Multiplicatively Weighted Voronoi Diagram*

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Many landscape features are represented as polygons in GIS. This paper characterizes polygon shapes with the multiplicatively weighted Voronoi (MW-Voronoi) diagram and improves its understanding. The MW-Voronoi diagram's composition is implemented with topological overlay, growth simulation, and vertex calculation methods. The decomposition is done by reversing a polygon to MW-Voronoi point pairs by segment. It is a new approach to record, characterize, and compare polygons with form and process. The implementation also serves as a geographic education and visualization tool. Applications of the methods are presented with precipitation, fire polygon, and population change data. **Key Words: Voronoi diagrams, polygon characterization, GIScience, wildfire detection.**

Introduction

Landscape characterization is to describe the character and quality of the landform in aggregate through time and space and to understand the process underlying the form, thus providing a baseline for human's interaction with the environment. Spatial models and metrics applied in GIScience and environmental planning can have different scales and abstraction levels and are common to land-use planning and landscape ecology. In land-use planning and environmental planning, landscape characterizations are applied in land evaluation (Stewart 1968), land inventory (Canada GIS), and ecological planning (McHarg 1969). During these researches, geometric and mathematic algorithms and metrics build foundations for GIS tools (Goodchild 1987; Van Kreveld et al. 1997). In geography and planning, Weber's Triangle, the Gravity Model, the Thiessen polygon, and Central Place Theory are well-known metrics used to measure and compare distance, neighborhood, connectivity, and functional range of the landscape under study (Miller, Aspinall, and Morrice 1992; N/A 1997; Plewe and Bagchi-Sen 2001). In landscape ecology, the studies of the structural pattern of a landscape are focused in three elements, the patch, corridor, and matrix (Dramstad, Olson, and Forman 1996). Many metrics in landscape ecology are designed to characterize the landscape by measuring and indexing (Farina 1998; Mandelbrot 1982; Metzger and Muller 1996; Plotnick, Gardner, and O'Neil 1993).

Many landscape features are represented as polygons in GIS. This paper focuses on the multiplicatively weighted Voronoi (MW-Voronoi) diagram, to reflect both the underlying process and the form of landscape polygons. The MW-Voronoi diagram can be considered a metric in the way that it can define and measure a generator point's region of influence based on its weight. The MW-Voronoi diagram overcomes the largest shortcoming of the planar ordinary Voronoi, where only location is considered, and considers both location and weights of the polygons under study.

Definition of the Multiplicatively Weighted Voronoi (MW-Voronoi) Diagram

Let S be a finite set of points in the Euclidean plane. Let p and q denote two points in the plane. Let the weights of the two points be $w(p)$ and $w(q)$. Let x be any point in the plane. The Euclidean distance between x and p is $d_e(x,p)$, and the weighted distance between x and p is $d_{mw}(x,p)$. The key to distinguishing between

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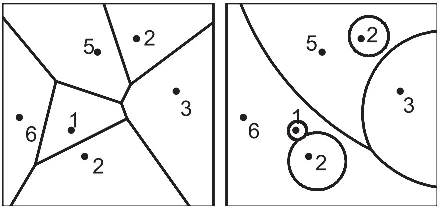

Figure 1 *Voronoi and the MW-Voronoi diagrams.*

different types of weighted Voronoi is the definition of d_{mw}. Let $region(p)$ denote the dominant region of point p; that is, p's influence region in S. In Figure 1, the weight of each point is labeled. The planar ordinary Voronoi diagram on the left of Figure 1 can be defined as

$$region(p) = \{x | d_e(x,p) \leq d_e(x,q),\ q\ in\ S\}$$

The multiplicatively weighted Voronoi diagram (MW-Voronoi) on the right can be defined as

$$region(p) = \{x | d_{mw}(x,p) \leq d_{mw}(x,q),\ q\ in\ S\},$$
$$where\ d_{mw}(x,p) = d_e(x,p)/w(p)$$

Before discussing MW-Voronoi diagrams, the Apollonius circle needs to be introduced. The Apollonius circle is a classic geometry term and is defined as the locus of a point whose distances to two fixed points follows a constant ratio. Figure 2 demonstrates that for two fixed points p and q, the locus of A keeps a ratio that meets the criteria of

$$d_e(A,p) = k\ {}^*d_e(A,q),$$
where k is a positive constant.

Therefore, the locus of A forms the Apollonius circle of point p and q.

The Apollonius circle and its geometric properties are fundamental in calculating MW-Voronoi diagrams. The locus will be the basic shape of the MW-Voronoi diagrams, which are circular arcs or lines.

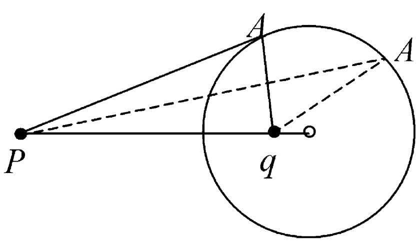

Figure 2 *The Apollonius circle.*

Applications of the Weighted Voronoi Diagrams

Voronoi has been discovered and rediscovered many times and is often referred to in different terms, including Voronoi diagram, Voronoi foam, Thiessen polygon, Dirichlet tessellation, atom domains, domains of actions, Wigner-Seitz regions, areas-of-influence polygons, areas potentially available, plant polygons, capillary domain, heavens, and so on. (Okabe, Boots, and Sugihara 1992; Okabe et al. 2000). Among these, Voronoi diagram and Thiessen polygon are the most well-known terms. Typical examples of Thiessen polygon applications are location-allocation models such as chain stores service areas, proximity analysis such as the UK's postcode boundary generation with Voronoi polygons (Boyle and Dunn 1991), and surface interpolation such as rainfall data in Kansas and Nebraska (Haining, Griffith, and Bennett 1984).

With the exploration of many applications in Thiessen polygon (Ahuja 1982; Ahuja and Tüceryan 1989; Edwards 1993; Gold 1990, 1992, 1994a, 1994b 1996; Gold, Remmele, and Roos 1995; Gold, Remmele, and Roos 1997), some concerns have also emerged. To name a few, the shape of the polygon is completely dependent on the location of the sample data points, the value of each polygon is estimated from one sample, and the Thiessen polygon method does not assume that points closer together are more similar than points far apart. Some variations in the Voronoi are the keys to solving more comprehensive problems. The planar ordinary Voronoi diagrams have very restricted factors in terms of point location, weight, rules for assignment or growth, and space (usually 2D). Relaxing any one of the factors will give us generalized Voronoi diagrams (Okabe et al. 2000). If the relaxing factor is the weight, then weighted Voronoi diagrams are generated. Depending on the variation methods, they could be multiplicatively, additively, or compoundly weighted diagrams.

Applications of the weighted Voronoi diagrams in GIScience can be divided into four periods:

1. Early prototypes (1800s to 1940s)

   Shieh (1985) tracks the earliest study of Rau (1841). According to Okabe (2000), Launhardt (1882) and Fetter (1924) and Tuominen (1949) are also forerunners. The early research starts with mathematics and geometry, and is later applied to economic markets (Fetter 1924; Launhardt 1882; Rau 1841), and engineering (Johnson and Mehl 1939).

2. Application in market and urban analysis (1950s to 1970s)

The literature on the MW-Voronoi begins to appear during this period. The MW-Voronoi was used as a geometric and mathematical modeling tool to solve problems in market and urban analysis. Table 1 accommodates cited literature from all resources.

Works in this period cover topics ranging from fundamental theoretical models to practical applications in urban systems and socioeconomic structures. Gambini designed a virtual space, an "information-seeking process," for consumers under conditions of uncertainties (Gambini, Huff, and Jenks 1967). That paper tests different scenarios of the supply points' activity. In the straight-line scenario, when two points have unequal attraction, it is actually a MW-Voronoi model, even though Gambini does not explicitly say so in his paper. Huff and his colleagues studied the MW-Voronoi diagram (Huff and Jenks 1968) and applied it to Ireland's urban system (Huff and Lutz 1979) and the U.S. national systems of planning regions (Huff 1973). Boots's 1970s work provides a foundation for his 1980s paper. His clustered work and publications from 1973 to 1975 discuss the MW-Voronoi diagram from an economic geographer's point of view, and apply the model to urban settlement and socioeconomic structure (Boots 1973, 1975a, 1975b).

3. Parallel development in computer geometry and GIS (1980s to 1990s)

Starting in the 1980s, the extensive development in computational geometry and GIS added new components to research on the MW-Voronoi. Intensive computation became less of a constraint, and, along with other computational geometry algorithms, the MW-Voronoi entered the GIS world. Four roles of the Thiessen polygons in geography are identified: models of spatial processes, nonparametric techniques in point pattern analysis, organizing structures for displaying spatial data, and information theories approaches to point patterns where they are used in calculating individual probabilities (Boots 1980). The relationship between the generalized Voronoi diagram, alpha-shape and Delaunay triangulation are shown (Edelsbrunner, Kirkpatrick, and Seidel 1983). Aurenhammer and Edelsbrun-

Table 1 *The MW-Voronoi Research, 1950s–1970s*

Author(s)	Date	Topic	Study Area
Hyson and Hyson	1950	Economic law of market areas	
Gambini et al.	1967	Market place properties	
Illeris	1967	Functional regions of urban centers	Denmark
Huff and Jenks	1968	Urban systems	
Hubbard	1970	Functional regions	Jamaica
Beckman	1971	Market potential	
Hogg	1971	Archaeology site territory define	England
Boots	1973	Subdivision of space	Great Britain
Huff	1973	Urban Spheres of Influence	
Boots	1975a	Patterns of urban settlement	
Boots	1975b	Structure of Socioeconomic Cellular Network	Great Britain
Wood	1974	Functional regions	Kenya
Cox and Agnew	1974	Theoretical counties	Ireland
Fraser	1977	Forest Sampling	
Getis and Boots	1978	Spatial Processes	
Jones	1979	Economic law of market areas	
Huff and Lutz	1979	Urban Hierarchy	Ireland

(Beckmann 1971; Boots 1973, 1975a, 1975b; Cox and Agnew 1974; Fraser 1977; Gambini , Huff, and Jenks 1967; Getis and Boots 1978; Hogg 1971; Hubbard 1970; Huff 1973; Huff and Jenks 1968; Huff and Lutz 1979; Hyson and Hyson 1950; Illeris 1967; Jones 1979; Wood 1974)

ner's (1984) work is a milestone for all of the algorithms and methods developed for the MW-Voronoi after 1984. The implementation of weighted Thiessen polygons using a GIS tool (Vincent and Daly 1990) is first found in this period. Vincent and Daly used the network analysis idea to "trick" the Arc/Info program to simulate an MW-Voronoi growth. Even though the actual implementation can no longer be found (personal communication with Vincent), their work is pioneered in bringing the MW-Voronoi into GIS. Many other discussions contribute to this field by providing alternative algorithms, improving existing algorithms, and applying it to more research fields (Radke 1999; Schaut 1991; Tanemura and Hasegawa 1980; Wang and Tsin 1990).

4. From algorithm to implementation (1990s and beyond)

   GIScience researchers want something beyond theoretical discussions and algorithm optimization. They would like to have some methods and implementation tools that allow them to actually use the model in research and teaching, the so-called off-the-shelf-tools. Gambini developed his computer program in FORTRAN IV (Gambini 1966) to calculate equilibrium lines between points. Vincent also had his Network Analysis solution (Vincent and Daly 1990). Recent software developments have made efforts to overcome this problem, including GAMBINI (Tiefelsdorf and Boots 1997) and VORONOI (Gahegan and Lee 2000).

The Composition of the MW-Voronoi Diagram and the Decomposition of a Polygon by Segment with the Reversed MW-Voronoi Process

To extend the understanding and applications of the MW-Voronoi within the field of GIScience, it is necessary to develop methods and tools because existing tools are usually designed for research purposes and have a lot of room for improvement. In addition, all the existing tools are for constructing MW-Voronoi diagrams from points; none of them does the reverse process from diagram, polygon, or polygon segment to MW-Voronoi points. Based on previous studies, this study developed three methods for composing MW-Voronoi diagrams: a topological-overlay method, a growth simulation method, and a vertex calculation method. Then a reversed method to decompose a polygon by segment to MW-Voronoi point pairs is presented. The implementation is straightforward, allowing for the transition from a model environment (virtual space) to the real world (physical space). It supports an easy exchange with common GIS software and also serves as an educational tool to visualize the abstract models. The motivation and purpose for the composition and decomposition is to characterize the landscape polygons, record the environment, and detect change over time. Polygon decomposition means "to decompose the polygon into simple component parts, solve the problem on each component using specialized algorithm, and then combine the partial solutions" (Keil and Sack 1985). The MW-Voronoi decomposition in this paper means to decompose the polygons of interest into simple segments and perform a reverse process to represent that part of the polygon with MW-Voronoi points and weights.

Three Methods to Compose MW-Voronoi Diagram

The topological overlay method is the most GIS-oriented method of the three, and it is implemented in an Arc/Info Arc Macro Language (AML). For the growth simulation and vertex calculation methods, a Visual Basic program, Composition and Decomposition of Weighted Voronoi Diagrams (CDWVD) is developed.

Method One: Topological Overlay Method

The basic idea of this concept is introduced as an observation in Aurenhammer and Edelsbrunner's 1984 paper, which states that

$$region(p) = \bigcap_{q \in S-\{p\}} dom(p,q) \qquad (1)$$

Here, S is a finite set of points, p and q are points, $dom(p,q)$ is the dominance of p over q, and $region(p)$ is the weighted Voronoi region of p. Even though the concept is easy to understand, the implementation of the topological overlay method for GIS users has not been found in the literature. This research takes the advantage of existing GIS tools and outputs the result of the MW-Voronoi diagram as an Arc/Info coverage, a common GIS format, to help GIS users understand the construction and components of MW-Voronoi diagrams.

Topological overlay method has the advantage that "combinational and topological computation is never contaminated with numerical errors" (Sugihara et al. 2000). The method presented in this paper is different than the "topology-oriented approach" discussed in Sugihara's work, which emphasizes the theoretical discussion of the algorithm. The method taken here is more focused on the implementation and the interaction between objects. This method applies the extended overlay concept broadly used in GIS, and it explains the composition of the MW-Voronoi from a different perspective, which is a topological approach that provides graph theory inputs to the problem. The conceptual model of this method is presented in equation 2. It can be seen as an alternative to equation 1, a GIS-inspired interpretation.

$$Dom_i = \bigcap_{j \in P_i} AC_{ij} - \bigcup_{k \in Q_i} Dom_k \quad \text{where } S = P \cup Q \tag{2}$$

Here, assuming S is a set of finite points, all the points in the equation are in S, and i is the point of interest, and

Dom_i = Dominant region of point i,
AC_{ij} = Apollonius circle caused by point i and j, while $W_i \leq W_j$ (W_i = weight of point i),
P_i = All the points in S with larger or equal weight than point i, and
Q_i = All the points in S with smaller weight than point i.

This means that the dominant region of a generator point can be obtained by intersecting all Apollonius circles caused by the generator point and all its larger-weight generator points, including the half planes created by any two equal weight points, and then subtracting all dominant regions from smaller-weight generator points.

Figure 3 shows that to construct the MW-Voronoi diagram, five points with their location and weights (in parenthesis) are 1, 2, 2, 3, and 5 respectively. Using the second point with weight of 2 as a demonstration, the conceptual model can be implemented as the intersection of Ac2_3, Ac2_4, and Ac2_5 and the subtraction of Ac2_1.

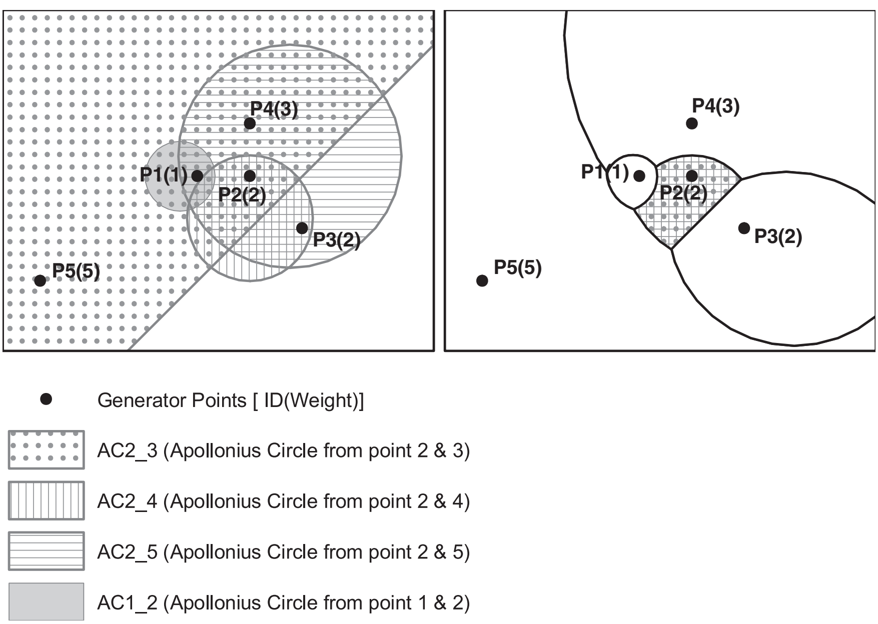

Figure 3 *Topological overlay to get MW-Voronoi dominant regions.*

Some properties of MW-Voronoi diagrams can be derived from the method. For each generator point's dominant region:

- Larger-weight neighbors contribute to it (add), smaller-weight neighbors deduct from it (subtract), and equal-weight neighbors crop it with a straight line.
- Larger-weight neighbors form convex edges, smaller-weight neighbors form concave edges, and equal-weight neighbors share straight-line boundary with it.

The method is conceptually simple. Even though its intensive computation and relatively slow processing speed do not make it a practical construction tool, the topological overlay approach behind it provides a new perspective for understanding the composition of MW-Voronoi diagrams from a polygon overlay view. The process and result also reveal the relationship between neighbors in terms of their contribution to the specified dominant region, as well as the linkage between weights and the convexity-concavity properties of the diagram.

Method Two: Growth Simulation Method

"Human intuition is often guided by visual perception. If one sees an underlying structure, the whole situation may be understood at a higher level" (Aurenhammer 1991). The growth models have been documented in literature and used in many fields (Boots 1980; Drysdale 1993; Okabe et al. 2000; Schaut 1991). However, implementations, especially those that simulate dynamically, are not easy to find. The growth simulation method developed here dynamically simulates the MW-Voronoi diagram via a simple Visual Basic program, CDWVD. It allows one to observe how points compete with one another and partition the space following the MW-Voronoi process. The implementation, even though it uses vector data, shares the same idea of Agent-based GIS analysis. Figure 4 shows that after the growth command is executed, the points expand in the space following MW-Voronoi rules.

Method Three: Vertex Calculation Method

The vertex calculation method improves the processing speed and exports the results as AML scripts to be executed to generate an Arc/Info coverage of the weighted Voronoi diagrams. The previous two methods concentrate on concept, process, and visualization, while this one focuses on practicality and efficiency. This method uses the geometric properties and relations of the Apollonius circles, their generator points' weights and locations, and their interaction and vertex points and calculates valid vertices and edges (arcs and lines). Therefore, it is named a vertex calculation method. The general steps of the implementation are summarized as: obtain all Apollonius circles (different weights) or lines (equal weights) for each pair of points; calculate intersection points of the circles; remove the nonvalid points to get valid vertex points (Figure 5); screen the arc or line connection between each pair of vertex points and only connect the valid ones; complete all the full circles and straight lines, if applicable; and finish the MW-Voronoi diagram and save the result.

Like the growth simulation method, the vertex calculation method is also implemented in the CDWVD program. The Graphic User Interface (GUI) and some of the key features related with this method are shown in Figure 6.

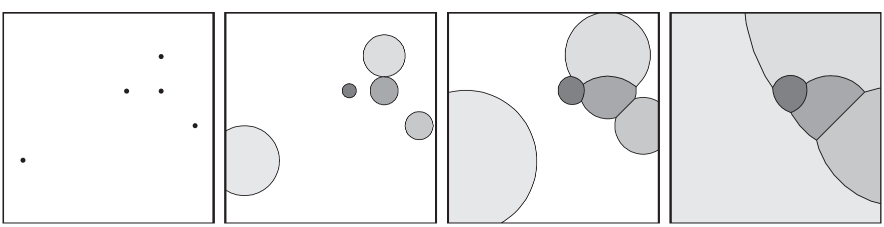

Figure 4 *The MW-Voronoi growth simulation model.*

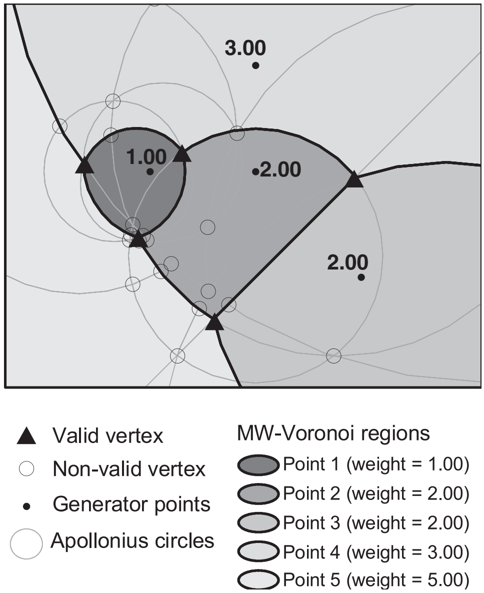

Figure 5 *Valid and non-valid vertex points with Apollonius circles.*

Method and Implementation for the Decomposition of a Polygon by Segment with the Reversed MW-Voronoi Process

Landscape is often represented by polygon features. The ability to record, characterize, and compare these polygons through space and time is critical for GIScience research. One research question is, given the boundary shape of an object (2D), can we decompose it into a set of points with weights and their generating processes? In other words, can we store polygons following the MW-Voronoi process? To some extent, the reversion, or the decomposition is more important in GIScience research.

This paper is interested in the boundary or external shape of polygons. There are two assumptions in the decomposition:

- Within a certain tolerance (fuzzy distance), any shape boundary can be approximated with circular arcs and lines (elements of MW-Voronoi diagram). This has been proved by computational geometry.
- For a known circular-arc or line segment, a pair of points with weights can be used to form it with MW-Voronoi as the underlying process.

It is known from the definition of the Apollonius circle that the second assumption has infinite solutions of many pairs of points with weights. To reduce the uncertainty, the polygon centroid is chosen as a fixed point with weight of 1 in any decomposition; thus the decomposition can be reduced to one solution. At present, the method is designed for simple and stand-alone polygons. When the centroid falls outside a polygon, it is moved inside the polygon. For contiguous polygons, the method deals with one polygon at a time.

The ratio relationships between the centroid, the approximated arc, the approximated circle center, and the other MW-Voronoi point are the keys to reversing it back from circular-arc or line segment to points and weights. The ratio relationship is depicted in Figure 7 and discussed below.

In Figure 7, q and p are two points used to generate the Apollonius circle with center C and radius r; $w(q)$ and $w(p)$ are their weights respectively, where $w(q) < w(p)$. a is the intersection point on the circle while connecting q and p. C, q, a and p are inline. From Apollonius circle, the following equations are derived:

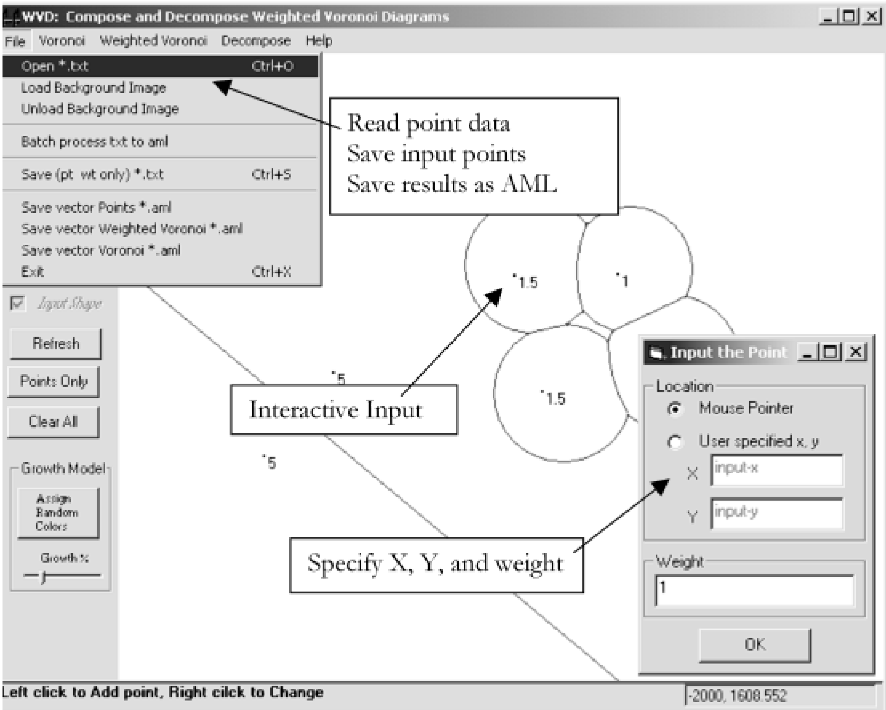

Figure 6 *The Vertex calculation method and key features of the CDWVD.*

$$|Cq| = \sqrt{\left(\frac{w^2(p)x(q) - w^2(q)x(p)}{w^2(p) - w^2(q)} - x(q)\right)^2 + \left(\frac{w^2(p)y(q) - w^2(q)y(p)}{w^2(p) - w^2(q)} - y(q)\right)^2}$$

$$= \frac{w^2(q)}{w^2(p) - w^2(q)} d_e(p,q)$$

while $d_e(p,q) = \sqrt{(x(p) - x(q))^2 + (y(p) - y(q))^2}$ and $r = \dfrac{w(p)w(q)d_e(p,q)}{w^2(p) - w^2(q)}$

therefore, $\dfrac{|Cq|}{r} = \dfrac{w(q)}{w(p)} = \dfrac{|aq|}{|ap|}$

thus $|Cq| = r\dfrac{w(q)}{w(p)}$

$$\left.\begin{aligned} q_x &= C_x + \frac{w(q)}{w(p)} r.\cos\theta \\ q_y &= C_y + \frac{w(q)}{w(p)} r.\sin\theta \end{aligned}\right]$$

On the other hand, from $\dfrac{|aq|}{|ap|} = \dfrac{|Cq|}{r}$, we have $\dfrac{r - |Cq|}{|ap|} = \dfrac{|Cq|}{r}$ and $|Cq| = \dfrac{r^2}{|ap| + r} = \dfrac{r^2}{|Cp|} = \dfrac{r}{|Cp|} r$

thus, $\dfrac{r}{|Cp|} = \dfrac{w(q)}{w(p)}$

With the above equations, the circle (center-C, radius-r) and point q, the location of p can be calculated, and vice versa with q. The circular-arc or line approximation of a polygon by

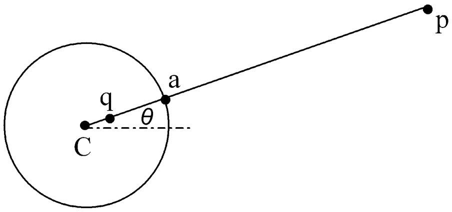

Figure 7 *The ratio between MW-Voronoi points and Apollonius circle points.*

segment is done through a heuristic process with a preset fuzzy tolerance. As shown in Figure 8, the approximation starts at point *Na*; the circle center *O* moves across the user-defined search space though loops; for each position of *O*, *O* and the radius *r*, the distance from *O* to *Na*, define the circle; vertices on the polygon next to *Na* are tested one by one, and if the distance to the circle is smaller than the fuzzy tolerance, that vertex stays, and the procedure moves to the next one . . . the approximation stops when the distance is larger than the fuzzy tolerance, which is Na + i and Na-j. For this iteration, vertices between Na + i-1 and Na-j + 1 are approximated by this circle. After O finishes sweeping the entire search space, the circle with the maximum number of vertices being included is chosen as the polygon approximation for that segment, and the responding circular-arc can be calculated and demonstrated. The process then moves on to look for the next segment's approximation. The scenario of line approximation is also considered and is performed parallel to the circular-arc approximation.

The GUI of the decomposition features and options in the CDWVD program is shown in Figure 9.

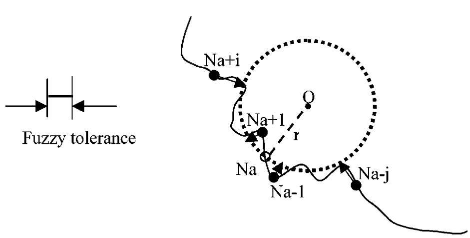

Figure 8 *The circular-arc approximation with fuzzy tolerance.*

With the above method and implementation, polygons can be decomposed by segment with circular-arcs and lines within certain fuzzy tolerances. Each approximated segment can then be reversed as a pair of MW-Voronoi points. Therefore, we can record polygons with MW-Voronoi points. By increasing the fuzzy tolerance, the simplicity of the polygon representation increases (Figure 10); the polygon can be recorded with fewer MW-Voronoi points. All of the approximation and decomposition results can be saved as a text file recording information such as segment type (0 for centroid, 1 for line, and 2 for arc), reversed MW-Voronoi point x,y and weight, and convexity (1 for convex, −1 for concave, and 0 for centroid).

Some result points may fall inside the polygon, but it does not mean that the polygon under study is formed internally. A form's underlying process in a model may not be its real-world process, but we can use the formative process to better understand and compare the forms. Since a polygon can be decomposed to MW-Voronoi points by segment, we can consider the MW-Voronoi as an underlying process to form the polygon shape, which may or may not related to the physical process of the landscape.

From Virtual Space to Physical Space—Applications

After developing methods for composing and decomposing the MW-Voronoi in virtual space, the following section will apply the algorithms and implementations to physical space. The examples will demonstrate how the MW-Voronoi methods can help to better perceive, understand, record, and compare phenomena in the environment and consider whether new perceptions can emerge from the modeling processes and results.

Application of MWVD Composition in Mean Areal Values Calculation

In geography and cartography, the mean areal value is often calculated from discrete point data. The interpolation and smoothing (IDW, isohyets, etc.) methods are often used nowadays to calculate the average areal precipitation (AAP). However, they only work well when there are enough data points. If the available

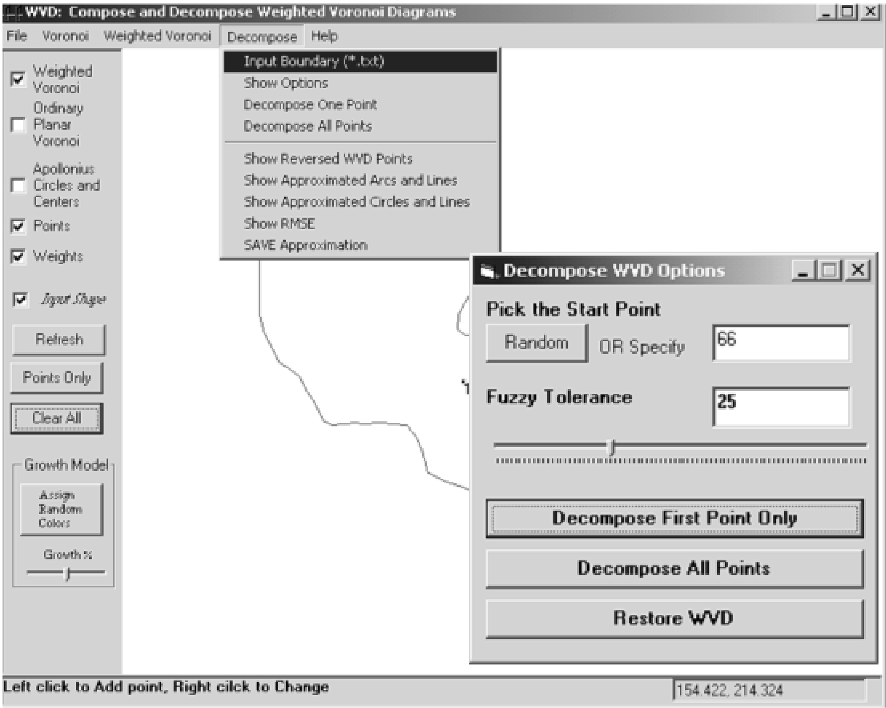

Figure 9 *GUI of the polygon segment decomposition with MW-Voronoi points.*

points are too few, many uncertainties and errors may be introduced. With few sampling sites, the challenge of estimating AAP from known points still exists. MW-Voronoi diagram is proposed here as a method for estimating AAP, compared with Thiessen polygon (the planar ordinary Voronoi) method. Daily precipitation data collected at 18 weather stations in the San Francisco Bay area on October 30, 2001, was obtained from the National Weather Service (NWS, 2001). The objectives of this case study are to estimate the AAP with the MW-Voronoi diagram, to compare the MW-Voronoi method with the planar ordinary Voronoi method in estimating point precipitation and AAP, and to evaluate the edge effect in both methods. The essential idea of the AAP estimation is

$$A\hat{A}P = \sum_{i=1}^{n} wf_i p_i$$

where wf represents the weighting factor for each input, and p represents the precipitation. For ordinary Thiessen (Voronoi) polygon method, the ratio of point i's Voronoi partitioned region's area over the total area is the wf_i. Similarly, point i's MW-Voronoi partitioned region determines the wf_i in that method. Figure 11 also shows how the different methods assign the dominant region (polygon) around each point in totally different ways.

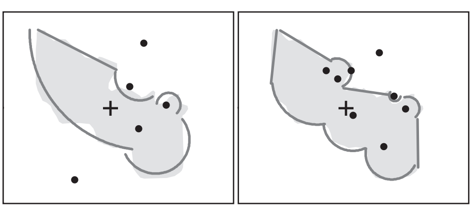

Figure 10 *The decomposition with fuzzy tolerance = 10 (Left) and 5 (Right.)*

Some tests such as point estimation, areal estimation, sensitivity, and edge effect are designed to compare the two methods in two scenarios: fixed sample size and stepwise sam-

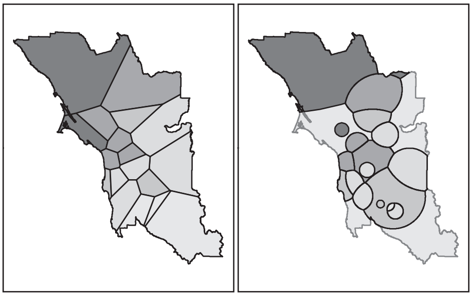

Figure 11 *The AAP estimation with Voronoi and MW-Voronoi methods.*

pling. Detailed analysis and results (Mu 2002) are omitted here since they are beyond the scope of this paper. The MW-Voronoi method always has a higher or at least equal estimation of both point precipitation and AAP than planar ordinary Voronoi method. It is observed that the MW-Voronoi method has more variability than the Voronoi method if the sample size is fixed, more sensitive to data value, less sensitive to sample size, and less edge effects. I would use the MW-Voronoi method if multiple estimations were needed and all had different sample sizes or no data were available for edge effect correction.

There is much potential for improvement in using the MW-Voronoi method to estimate AAP: spatial patterns of the sampling points, relationships with elevation, aspect, distance to ocean, wind condition, and so on. How to assign weight to each point is also a key factor. I simply use the precipitation value as the weight for that point, but it could be a combination of some or all of the factors mentioned above. Further studies related to climate and weather, physical geography, computational geometry, spatial statistics, and knowledge from other related fields are needed to better explore the phenomena.

Application of the MW-Voronoi Decomposition in Polygon Recording and Comparison

Fire studies always involve many factors including the physical landscape, weather conditions, fuel type and age, cause of fire, and season of the year. Modeling is an important approach to the study of fire. For such a complex process, it is not possible or appropriate to attempt to find a cure-all model. Different models usually focus on different aspects of the study. FARSITE™ and BEHAVE™ are two examples of popular fire simulation models. Spatial statistic models in fire study, on the other hand, focus on understanding the spatial–temporal correlations among different variables such as fuel age, time-since-fire, and meteorological data and given explanations and predictions (Flannigan and Harrington 1988; Johnson and Van Wagner 1985; Peng and Schoenberg 2001; Van Wagner 1978; Viegas and Viegas 1994). Computational geometry models help fire study by composing and decomposing the spatial forms and patterns of fire polygons and by applying the underlying process to help recognize, characterize, and compare the spatial shapes.

The landscape geometry caused by wildfire has drawn researchers' attention. Voronoi diagrams have been used to study the formation of Green Island, the term for the refuge area inside a forest fire (Roque and Choset 1998). As the authors of the Green Island study pointed out, the simple Voronoi model can actually help "simulate the landscape geometry of the burned areas and the dynamics of the growth process of forest fires."

The objective of this case study is to design a method to represent history fire polygons with MW-Voronoi decomposed points, to simplify polygon storage with a virtual process, MW-Voronoi, and thus to provide an alternative way to compare history fire polygons. The 12 largest fire history polygons (greater than 40 square kilometers or 10,000 acres) identified by California Department of Forestry and Fire Protection or U.S. Forest Service from 1950 to 1999 in the San Francisco Bay area were selected for the study, as shown in Figure 12.

Fire polygons are input into CDWVD program. After approximating fire polygons by segment with circular arcs and lines, each segment can then be decomposed as a pair of MW-Voronoi points, with the centroid as the fixed point for all of the pairs. Within a fixed fuzzy tolerance, all 12 polygons can be decomposed from three to ten pairs of weighted points and dramatically simplify the polygon representation. Using the 1954 Charles fire as an example, the decomposition results are demonstrated in Figure 13. The saved results for that polygon are shown in Table 2.

Like the Wind Rose for climate data, an MW-Voronoi vector (an MW-Voronoi rose) graph is proposed here to present the decomposition

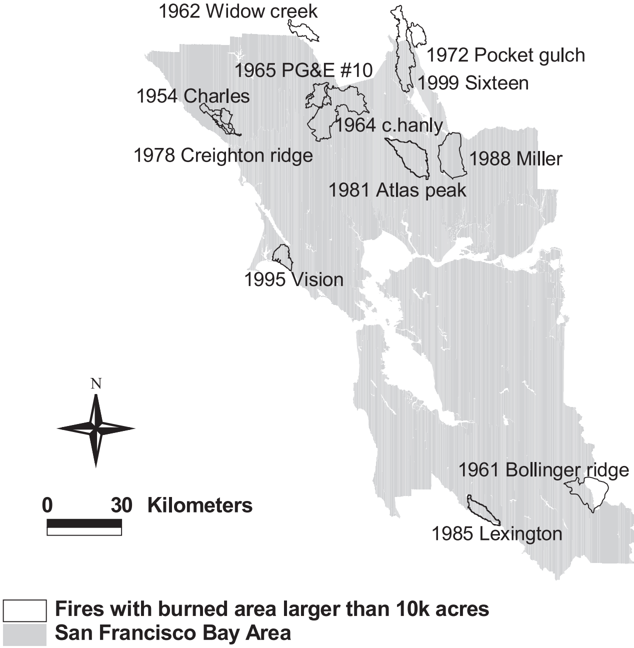

Figure 12 *Fires with burned area larger than 40 square kilometers (10,000 acres).*

results. With the MW-Voronoi decomposition information, any polygon can be graphically depicted and characterized with MW-Voronoi vectors (Figure 14) by

- Angle, the direction of the vector between the centroid and the decomposed MW-Voronoi point,
- Thickness of line, the weight of the decomposed point,
- Length, the Euclidean distance between the centroid and the point, and
- Arrow, the convexity or concavity; away from the centroid means convex segment, and toward the centroid means concave segment.

There is always a balance between the amount of information preserved and the simplicity of the results. Different fuzzy tolerances will get varied decomposition results. Figure 15 shows a decomposed polygon with fuzzy tolerance of 500 and 1000 map units respectively. With a 500-unit fuzzy tolerance, the polygon can be decomposed as six pairs of MW-Voronoi points, and four pairs with a 1000-unit fuzzy tolerance.

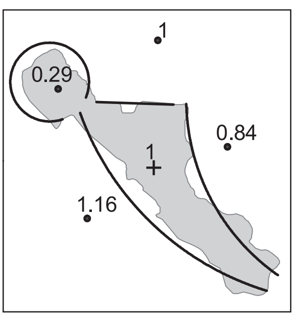

Figure 13 *The decomposition of 1954 Charles Fire polygon.*

Table 2 *Decomposed MW-Voronoi Point Representation of the 1954 Charles Fire Polygon*

Weight	X	Y	Type	Convexity
1	277326.00	62964.00	0	0
0.84	273579.00	64001.92	2	−1
1.16	280688.00	60368.93	2	1
0.29	282157.00	66920.08	2	1
1	277102.00	69383.72	1	1

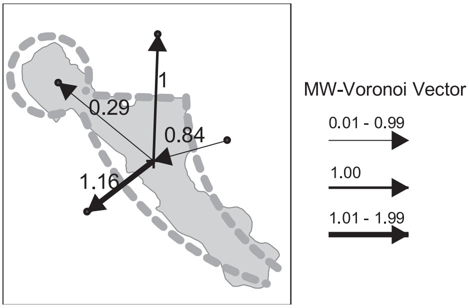

Figure 14 *The representation of the MW-Voronoi vector.*

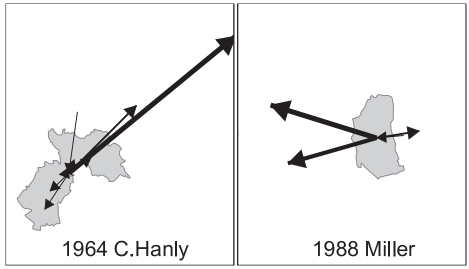

Figure 16 *The MW-Voronoi vector comparison.*

With the MW-Voronoi decomposition and polygon approximation, any fire polygon can be recorded as pairs of points with weights and be further stored and characterized as tables and MW-Voronoi vector graphs. This method reduces the storage but carries all the key information of the polygon within a certain fuzzy tolerance such as direction, convexity versus concavity, and magnitude. The vector graph also provides the visual effect of the polygon characterization. With the decomposition results, fire polygons can also be compared globally or locally. Polygon shapes can be compared via the MW-Voronoi vector graph. For instance, the 1964 fire polygon is more complex than the 1988 fire polygon because, with the same parameters, the former polygon (1964) needs ten MW-Voronoi vectors to characterize and the latter (1988) only needs four (Figure 16).

The MW-Voronoi vector graph can also be used to compare any two polygons locally by direction and magnitude. Besides the number of vectors, all the vector properties, including angle, thickness of line, length of line, and arrow direction, can be used to compare polygons in terms of orientation, magnitude, distance, and convexity of the assumed underlying MW-Voronoi process. Figure 17 maps out the 1954 and 1978 fire polygons. The comparison at the same fuzzy tolerance between these two can be made as follows: The 1978 fire is to the southeast of the 1954 fire (centroid locations); they both can be decomposed to four major influential MW-Voronoi points; their vectors' directions are very similar, except that the 1978 one is rotated clockwise a little; their convex properties are mostly similar except for the direction to the northwest, where the 1954 one is convex, and the 1978 one is concave; and their strongest forming points are both from the southwest.

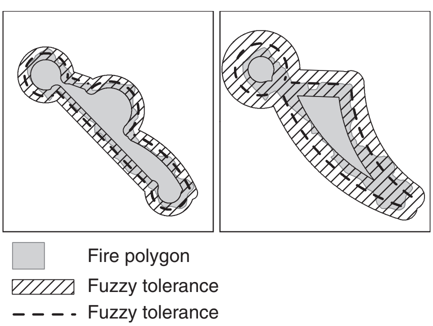

Figure 15 *The Polygon decomposition with different fuzzy tolerances.*

Figure 17 *1954 and 1978 fire polygons comparison.*

To sum up, the examples bring the discussion of MW-Voronoi diagram's composition and decomposition from virtual space to physical space. Inspired by the work of characterizing landscape change with inadequate spatial information (Radke 1998), this study provides another method. The example uses historic fire polygons identified in the San Francisco Bay area from 1950 to 1999. The flexibility of decomposing polygons with MW-Voronoi points for different fuzzy tolerances allows for the storage, characterization, and comparison of polygons even with coarse spatial accuracy.

Application of the MW-Voronoi in Education and Other Geographic Research

As a byproduct of this research, a Visual Basic program, CDWVD, is created to implement the MW-Voronoi composition and decomposition and to serve as an off-the-shelf educational and research tool (Figure 6). The main features of the CDWVD program are: MW-Voronoi composition with vertex calculation and growth methods; polygon decomposition with MW-Voronoi point pairs; point input either interactively or read from a text file; point modification such as add, move, remove, and weight change; output as MW-Voronoi, ordinary Voronoi, Apollonius circles, text file, and Arc/Info AML file.

The MW-Voronoi diagram has been used for delineating urban sphere of influence (Huff, 1973). Besides the static form at one time, the application can be extended to observing dynamic changes by constructing a series of MW-Voronoi diagrams following a time sequence. Figure 18 shows the MW-Voronoi diagram of the U.S. state population (as weight) change from 1900 to 2000. California's population increase can be visualized from its MW-Voronoi dominant region's spreading from the west to the east.

Conclusion

To characterize polygons in GIScience research, and to improve the understanding and application of the MW-Voronoi diagram, this paper develops three methods to implement the composition and one reversed method to decompose polygons. Applying the methods to examples of precipitation and fire polygon data, the paper provides another approach to

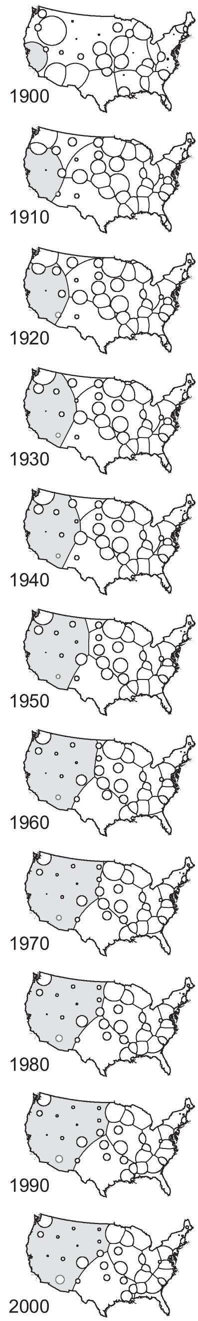

Figure 18 *MW-Voronoi diagrams for the U.S. state population 1900–2000.*

estimate weighted mean areal values from points and demonstrates how the decomposition can help record and characterize polygons with form and process and reduce data storage. The same method can also help compare polygons globally and locally, even though there are still many questions to ask. The tools developed can help the research and education in geography and cartography.

The limitations and concerns need to be addressed here. The MW-Voronoi diagram enhances the strictly location criteria of the planar ordinary Voronoi diagram by considering weights associated with points. However, there are still many other variations that need to be considered. Accommodating more factors may make the model better approximate real-world situations. In this study, polygons can only be decomposed with the reversed MW-Voronoi process segment by segment. It would be better, more consistent, and more productive if the whole polygon could be decomposed with a single process and more powerful to record contiguous polygons than the stand-alone examples demonstrated in this study. More applications are needed to test and examine the MW-Voronoi diagram to interpret the weights with appropriate context, to deepen the form–process understanding, and to bring out more perceptions. ■

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