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Characterizations of real-world Polygons

Bachelor Thesis

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Contents

Abstract

In computational geometry it is common to work on polygons. Often it is difficult to perform well on some special cases of oddly shaped polygons that are rarely or never found in real data sets.

Recent studies tried to find out which characteristics lead to this oddness - or vice versa, which make a polygon seem realistic. This work attempts to provide an overview of these characteristics.

In the last years several research groups have independently worked on and used different characterizations. Therefore the information is quite scattered. This work tackles this problem and tries to serve as a collection that summarizes this knowledge.

This thesis presents the currently existing characterizations in a structured way, discusses them in the context of realistic polygons and, if possible, relates them to each other.

1 Introduction

1.1 Realistic polygons

Realistic polygons are polygons that we would intuitively assume to exist in the real world or that simply exist in the real world. They can come from a wide range of sources, such as geography, robotics, image understanding or architecture.

They are to be contrasted against non-realistic polygons, e.g. random generated polygons, which tend to have long and skinny parts, that make them seem contorted.

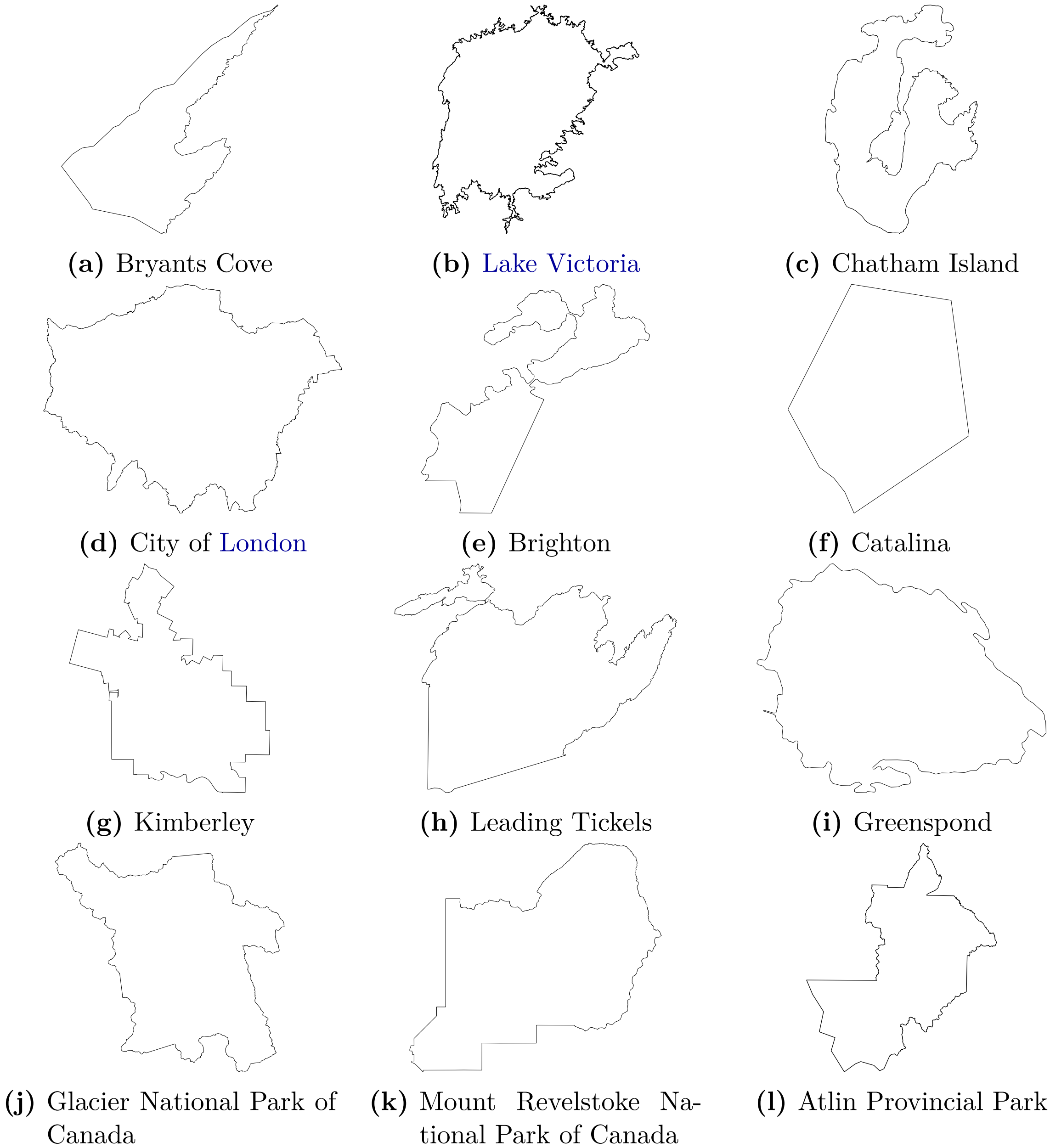
Some common application areas include:

- Geography (geographic information systems): level curves, borders (e.g. of states, cities or lakes)
- Robotics: description of robot or obstacle shapes
- Architecture: ground plans
- Image understanding: shapes of different objects, from biology (animals, leaves), calligraphy (letters) or advanced driver assistance systems (road signs), to name only a few examples.

However, this work focuses on cartographic polygons, e.g. from geography and architecture, because these are the most common areas. An assortment of geographical polygons is shown in Figure 1.1.1, as opposed to generated polygons in Figure 1.1.2.

In the following some characteristics of real world polygons are listed. The problem with attempting to define what a realistic polygon looks like, is that most likely there exists a counter example to any definition, due to the immense variety of forms in which polygons appear in the real world. Then again a very loose definition would include non-realistic polygons. So, the following list barely contains common, regularly observed and vague features, which by far do not hold for every real world polygon:

- Main region(s): mostly, realistic polygons consist of one or only a few prominent main regions.
- Offshoots and spikes: realistic polygons, especially from geographical data, tend to have uneven borders. This unevenness often comes in form of small scraps. Offshoots and spikes, roughly described, are part of the border of a polygon that start at a reflex vertex (as defined in Section 1.4), followed by a chain of short edges and end at a reflex vertex, that is near the start vertex.
- Long edges: relatively long edges, that are often coupled with right or blunt angles, are found in ground plans and human-made borders (e.g in Canada and the USA).



(a) Bryants Cove
(b) Lake Victoria
(c) Chatham Island
(d) City of London
(e) Brighton
(f) Catalina
(g) Kimberley
(h) Leading Tickels
(i) Greenspond
(j) Glacier National Park of Canada
(k) Mount Revelstoke National Park of Canada
(l) Atlin Provincial Park

Figure 1.1.1: Examples of real world polygons

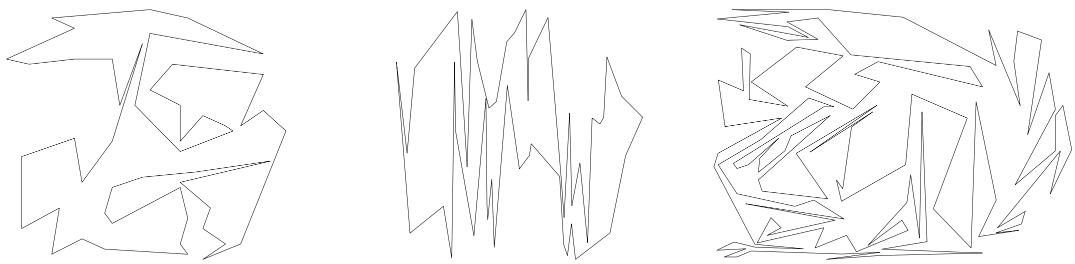


Figure 1.1.2: Examples of nonrealistic polygons

1.2 Realistic Input Models

The first characterizations that are reviewed here evolved naturally from researching polygons. Some concepts reappeared and those findings proved their usefulness in mathematics and algorithmics (such as convexity Section 2.1). They were used as tools to find solutions to problems in those fields.

Then, over the last years the attempt to characterize polygons has become more conscious. Alt et al. were among the first to consider fat objects [AFK+92] (in their case in the context of motion planning for rectangular shapes, such as robots) and shortly after the term 'fatness' came up [Ove92]. As a generalization the term realistic input model became wide spread, around 1997 [DBKVdSV97], to describe properties of real world data. This field of research is by far not restricted to general polygons. Among others are characteristics of other geometric shapes [PT02, ERS93, AEKS06] or the distributions of shapes [DBKVdSV97].

However, the core motivation stays the same: "by making assumptions about the input, certain hypothetical worst case scenarios can no longer occur, and a more efficient solution to a problem can be shown for all inputs that satisfy the assumptions. Either an existing algorithm can be shown to be more efficient for realistic inputs, or a special algorithm is designed that assumes that it will run on realistic inputs only." [MVKvdS06]

This work focuses on general polygons and has collected the characterizations that were developed over the years by several research groups. The following chapters offer the reader a structured overview on the range of existing characterizations.

1.3 Acquisition of Polygonal Data

Polygonal data can come from a variety of sources. The obvious ones are real world data sets. Someone measured something in the real world, saved it and made it available.

However in practice these data sets are often too small (e.g. for testing an algorithm) or simply unavailable. In such a case more data can be created by one or both of the following variants: hybrid or generated data.

Hybrid data takes existing data and modifies it. This increases the sheer amount of data samples, but the variety within that set depends on the original data set and therefore it is limited to a certain degree.

Generated data on the other hand creates completely new samples. This can create a huge amount of samples with a wide spectrum of features, but the downside is that those samples are far more likely to be unrealistic, for they are not based on real data.

The examples used for this thesis come from following sources: The used real world polygons were mainly extracted from OpenStreetMap [Saa15] plus some that were available on the Internet[1]. For the unrealistic polygons a random polygon generator[2] [DWI] was used

[1] links are provided correspondingly

[2] http://srufaculty.sru.edu/david.dailey/svg/polygons7.svg (Status as of 30.05.2016)

to provide templates for the polygons which then were manually created. Beyond that polygons that were available online [Hel] were recreated.

Several Heuristics for the generation of random polygons have been studied for general polygons [DW08] as well as for sub-classes of polygons [AH96, ZSSM96] that will be viewed in this work, too.

1.4 Basic Terms

In this section, some knowledge is provided, that is essential or useful for the remainder of this thesis.

Simple Polygon A polygon is simple, if its edges intersect only at their end points.

Throughout this thesis the polygons are assumed to be simple.

Visibility Given a polygon P and two points p, q inside P. p is visible to q, if the line section $\overline{pq}$ is completely inside P. It is common to say p sees q.

The concept of visibility is fundamental for several of the characterizations and explicitly or implicitly used in their definitions (Sections 2.1, 2.2, 2.3, 2.5, 2.6, 2.11, 2.13, 2.14, 2.15).

Convex and Reflex Vertices Given a vertex v of a polygon. v is convex, if its internal angle is less than π, otherwise v is reflex.

Reflex vertices are also called concave vertices.

Convex Chains A path that consists of points $p_1, ..., p_n$ is called a convex chain, if for every edge $\overline{p_i, p_{i+1}}$ (with $1 < i < n-1$) the adjacent edges $\overline{p_{i-1}, p_i}$ and $\overline{p_{i+1}, p_{i+2}}$ lie on the same side of the supporting line of $\overline{p_i, p_{i+1}}$.

The more intuitive term left-turning path (right-turning path) can be used, if all adjacent edges lie on the left (right) side.

Diameter The diameter of a set of points S is defined as the maximum distance between two points of S. It is denoted as $diameter(S)$.

Since 2D polygons are compact sets of points in the plane, this definition also holds for polygons.

2 Characterizations

In this chapter the characterizations are presented, whereby every section is structured like this: first the definition is given, then relations to other characterizations are listed and finally the characterization is discussed. The discussion will serve to (1) give an intuition for the characterization and (2) to evaluate the characterization in the context of realistic polygons.

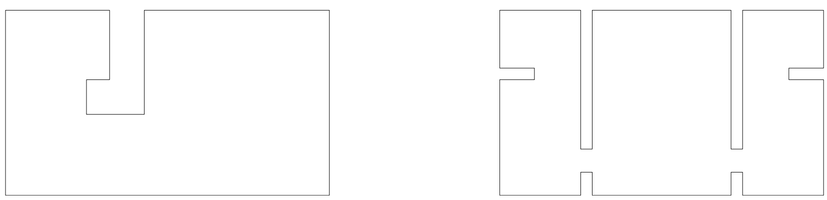
In this work the characterizations are looked at separately. Although it would be desirable to have a rating in the end that lists the characterizations according to their usefulness to measure how realistic a polygon is, in general they are not comparable. And even for single characterizations, it is often possible to find both realistic and non-realistic examples, which shows that even within a single characterization it is difficult to determine whether it is useful as a measurement for how realistic a given polygon is. So an isolated view on the characterizations needs to suffice.

Comparability of characterizations

Unfortunately, when handling characterizations of polygons it becomes apparent, that they are difficult to compare to each other. There is no universal way to decide if one is strictly better than another.

The problem is, that for two given characterizations $char1$, $char2$ it is usually possible to find both a real world example, that fulfills $char1$ but not $char2$ and another that fulfills $char2$ but not $char1$. One example for this is given in Figure 2.0.1. The given polygons can easily be imagined to be ground plans of buildings, and are therefore realistic. The polygon in Figure 2.0.1a is palm-shaped (Section 2.7) but not externally visible (Section 2.8) and vice versa for the polygon in Figure 2.0.1b.

Nevertheless relations can be found between some of the characterizations. For example some non-parametrized characterizations are sub-classes of others or special cases of parametrized characterizations, for a certain fixed parameter. A summary of that can be seen in the conclusion (Chapter 3).



(a) A palm-shaped polygon **(b)** An externally visible polygon

Figure 2.0.1: Non comparable polygons in terms of Palm-Shape and External Visibility

2.1 Convexity

Definition Given a Polygon P and any two points p, q inside P. P is convex, if the intersection of P with the line, that connects p and q, is one single connected segment.

Relations to other Characterizations

- Convexity is related to many of the following characterizations. Therefore the specifics are addressed in the respective section.

The property of convexity is by far the most basic and most extensively studied of all the characterizations covered in this thesis (example in Figure 2.1.1). Furthermore many of the following characterizations can be traced back to convexity.

Nevertheless convexity is not a good measurement for realness of polygons, since it is a too narrow characterization. Many real world polygons tend to be the union of multiple bulky polygons, while others (especially those with high resolution) have little offshoots on their border.

However, it is very difficult to come up with an unrealistic convex polygon. Since these shapes are so basic, they can usually be imagined to exist in the real world. An arbitrarily long and skinny polygon would be unusual, but generally speaking convexity is a good indicator for a polygon to also be realistic.

Related to convexity the concept of the convex hull of a polygon (Figure 2.1.2) should be mentioned here:

Definition Given a Polygon P. The convex hull of P $ch(P)$ is the smallest convex polygon that contains P.

Based on this definition, simple characterizations could consider the ratios between area or perimeter of P and $ch(P)$. However, as far as I am aware, this has not yet been studied in the context of realistic input models.

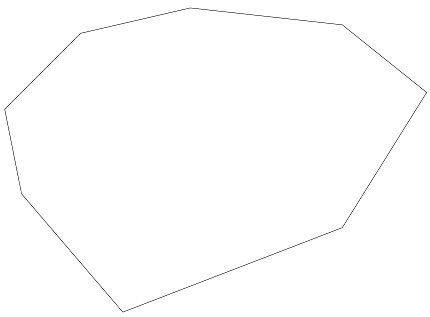


Figure 2.1.1: A convex polygon

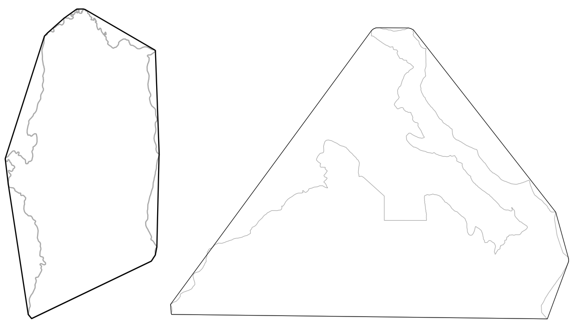


Figure 2.1.2: Convex hulls

2.2 Directional Convexity

Definition Given a Polygon P and a (possibly infinite) set of vectors D. P is directionally convex (also called D-convex) with respect to a set of directions D, if the intersection of P with any line, parallel to a vector $d \in D$, is one single connected segment.

Relations to other Characterizations

- Convexity (Section 2.1), ortho-convexity (Section 2.3) and monotony (Section 2.4) are special cases of directional convexity.

This characterization is mentioned here mainly for completeness and because it nicely covers the middle ground between convex and monotone polygons.

For directional convexity there are several problems as a measurement to decide if a polygon is realistic or not. The choice of a fixed set D would generally result in rotation-sensitivity and even then, the most general case, namely monotone polygons, is still very restricting.

2.3 Ortho-Convexity

Definition Given a Polygon P and two orthogonal vectors (e.g. the coordinate axes). P is ortho-convex with respect to the vectors, if the intersection of P with any line parallel to one of the orthogonal vectors is one single connected segment.

Relations to other Characterizations

- Ortho-convexity is a relaxation of convexity (Section 2.1).
- Monotony (Section 2.4), external visibility (Section 2.8) and crab-shape (Section 2.9) are relaxations of ortho-convexity.
- Ortho-convexity is a special case of directional convexity (Section 2.2).

Like directional convexity, ortho-convexity is mentioned here mainly for completeness.

Intuitively, the shapes of ortho-convex polygons are either stair-shaped, cross-shaped or something in between(see Figure 2.3.1).

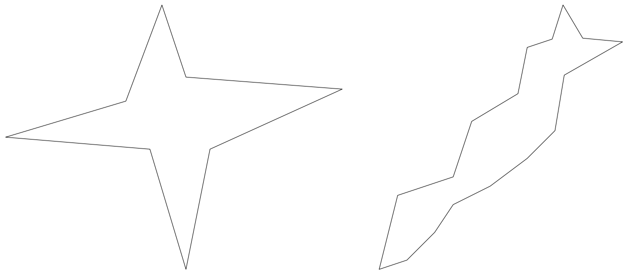


Figure 2.3.1: Ortho-convex polygons

2.4 Monotone Polygons

Definition Given a polygon P. P is monotone, if there exist two extreme vertices in a preferred direction d (such as p_{ymin}, p_{ymax} if the y direction is preferred) such that they are connected by two polygonal chains monotonic in this direction.

Relations to other Characterizations

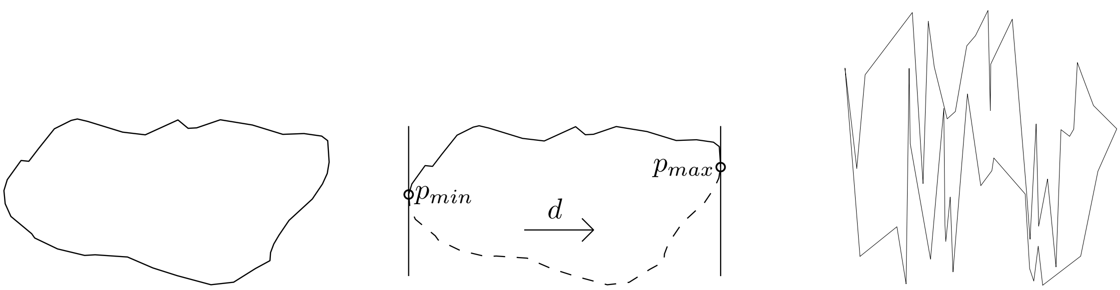
- Monotony is a relaxation of ortho-convexity (Section 2.3) and therefore also of convexity (Section 2.1).
- External visibility (Section 2.8) and crab-shape (Section 2.9) are relaxations of monotony.
- Monotony is a special case of directional convexity (Section 2.2).

The monotone polygons are the same as the directionally convex polygons, if the vector, perpendicular to the preferred direction, is the only vector, in the set of directions D of the directionally convex polygons.

It is interesting to mention, that the characterizations so far - convexity, directional convexity and ortho-convexity - were restrictive to a point that real world examples seemed rather generic. For instance the borders of the states Colorado, Wyoming or North Dakota which very similar to rectangles.

Monotony is the first characterization I found a more complex real-world example for (see Figure 2.4.1a). On the other hand monotony already allows enough freedom that unrealistic examples can be found (see Figure 2.4.1c), which was difficult for the other characterizations.

Also, the subclass of monotone polygons, called monotone-separable polygons, should be mentioned here, because it connects to another characterization: If P completely contains the line segment $\overline{p_{min}p_{max}}$ (as in Figure 2.4.1b), P is called monotone-separable and $\overline{p_{min}p_{max}}$ separates P into two edge-visible polygons (Section 2.5)[Tou84].



(a) A monotone real world polygon **(b)** Monotone chains from p_{min} to p_{max} **(c)** A monotone, non-realistic polygon

Figure 2.4.1: Monotone polygons

2.5 Edge-Visibility

Definition Given a Polygon P. P is said to be weakly-visible from an edge $\overline{pq}$, if any point in P sees some point on $\overline{pq}$. P is edge-visible, if such an edge exists on the boundary of P.

Relations to other Characterizations

- Edge-Visibility is a relaxation of Convexity (Section 2.1).
- Palm-shape (Section 2.7) and crab-shape (Section 2.9) are relaxations of edge-visibility.

This is one of the characterizations that is a tool for the fields of mathematics and algorithmics, instead of being a realistic input model. Therefore edge-visibility also is listed here mainly for completeness. When using this characterization in the context of general polygons they are usually split into edge-visible sub-polygons (as mentioned in Section 2.4) [TA82] so it is to assume that for whole polygons it is not a useful characteristic.

Polygons that are convex except for a small dent, are nicely covered by this characterization (Figure 2.5.1), though by far not every edge-visible polygon is realistic, as shown by Figure 2.5.2.

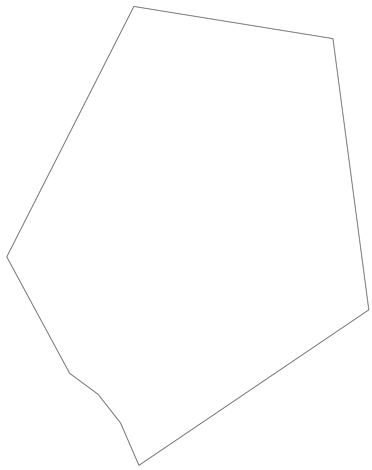


Figure 2.5.1: A real world polygon that is not convex, but edge-visible

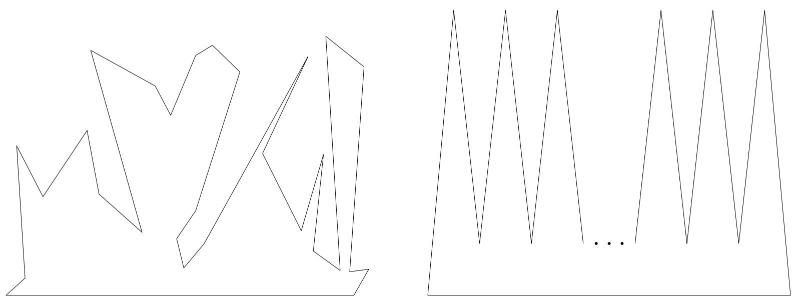


Figure 2.5.2: Examples of edge-visible polygons

2.6 Star-Shaped Polygons

Definition Given a Polygon P. P is star-shaped, if it contains a set of points - the (convex) kernel - each of which can see the entirety of P.

Relations to other Characterizations

- Star-shape is a relaxation of convexity (Section 2.1).
- Palm-shape (Section 2.7), external visibility (Section 2.8) and crab-shape (Section 2.9) are relaxations of star-shape.
- The star-shaped sets are precisely the 1-guardable sets (Section 2.13).

The notion of a star-shape is not useful as a characterization of real world polygons, but it is listed here for completeness and because it is widely known and used throughout many papers.

For realistic polygons uneven borders are quite common, like offshoots or small right angled dents (e.g. from geography or architecture). Figure 2.6.1 shows that even on rather simple polygons these characteristics may prevent them from being star-shaped.

Furthermore star-shaped polygons already have enough degrees of freedom to offer unrealistic examples with long, skinny and narrow parts (rightmost example in Figure 2.6.2).

However, the ratio of the size of the biggest star-shaped sub-polygon of P and P might be a measurement, that could be taken into account. The sub-polygon that can be seen from a point p within P is called visibility polygon with respect to p. ε-good polygons have a similar approach and are presented in Section 2.15.

Finally, it is interesting to note that every line through the kernel divides P into exactly two edge-visible polygons (Section 2.5).

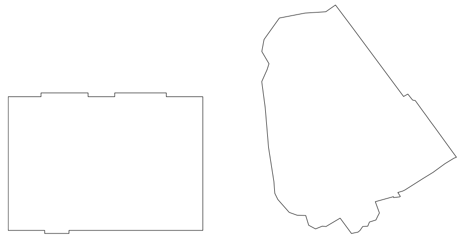


Figure 2.6.1: Realistic, but not star-shaped polygons

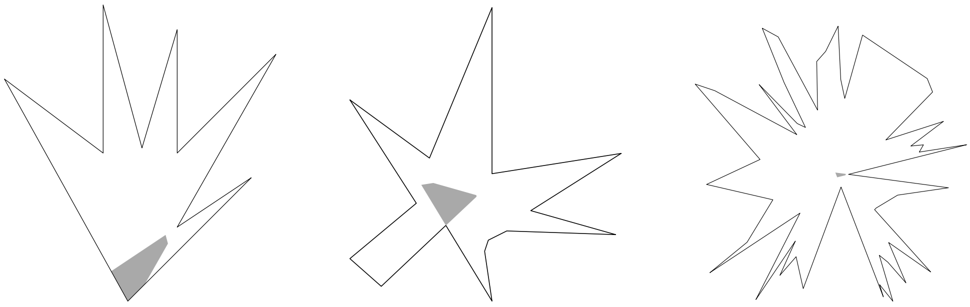


Figure 2.6.2: Star-shaped polygons with their kernel (gray)

2.7 Palm-Shaped Polygons

Definition Given a Polygon P. P is palm-shaped, if it contains a point p, such that the shortest interior path from p to any other point in P is a convex chain (as defined in Section 1.4).

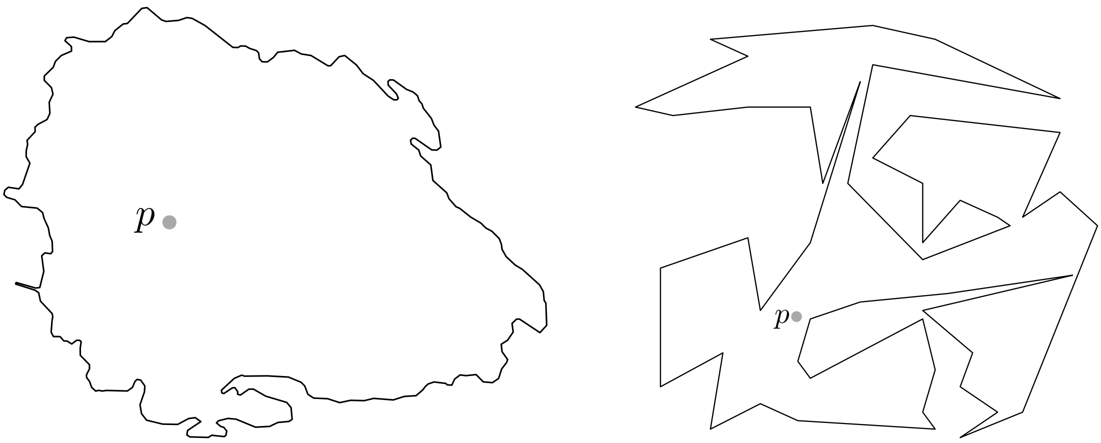
Relations to other Characterizations

- Palm-shape is a relaxation of star-shape (Section 2.6) and of edge-visibility (Section 2.5) and therefore also of convexity (Section 2.1).
- Crab-shape (Section 2.9) is a relaxation of palm-shape.

Palm-shapes were introduced by ElGindy and Toussaint [ET89] and, in contrast to star-shapes, they include cases with uneven borders. For example the polygons in Figure 2.6.1 are palm-shaped, just like even more complex real world polygons with offshoots (Figure 2.7.1a). In fact, if an offshoot by itself has no narrow "zig zag"-shapes only one point of the offshoot needs to be seen by p to qualify as palm-shaped.

However, non-realistic palm-shaped polygons can easily be found, too. Figure 2.7.1b shows an example with one possible p and illustrates that long and skinny parts are not excluded.

Moreover it is worth mentioning, that in the original work of ElGindy and Toussaint [ET89] a sub-class of palm-shaped polygons was presented. Palm-shaped polygons that are restricted to left-turning (right-turning) paths are called left (rigth) palm polygons.



(a) A real world polygon **(b)** A non-realistic polygon

Figure 2.7.1: Palm-shaped polygons with a possible p

2.8 External Visiblity

Definition Given a Polygon P. P is externally visible, if for every point x on the boundary on P there exists a Ray starting in x, that intersects P only in x.

Relations to other Characterizations

- External visibility is a relaxation of both star-shape (Section 2.6) and monotony (Section 2.4), and therefore also of convexity (Section 2.1).
- Crab-Shape (Section 2.9 is a relaxation external visibility.

When looking at realistic polygons, features like coves tend to contradict external visibility, particularly if they have offshoots within them. This means there are many real-world counterexamples (Figure 2.8.1). So obviously not every realistic polygon is externally visible, but it stands to reason that external visibility is an indicator for a polygon to be realistic. Again there exist non realistic, externally visible polygons, but randomly generated polygons frequently create inclusion-like pockets that in turn are not externally visible (see Figure 2.8.2a). That is because of the long skinny edges and especially holds true for a bigger count of vertices, despite the possibility of constructing externally visible polygons with arbitrarily many vertices (Figure 2.8.2b).

A further thought would be a parameterized variant of external visibility, say only a certain amount of the border needs to be externally visible. No such characterization was proposed earlier to my knowledge, so there are no results yet to be presented here.

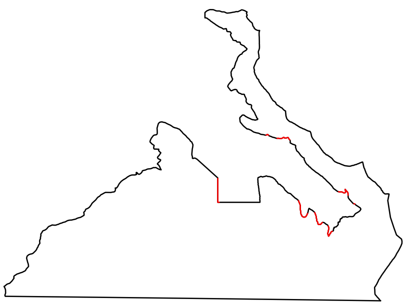
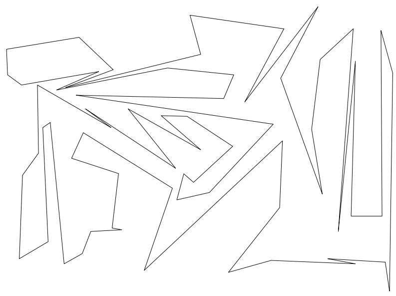
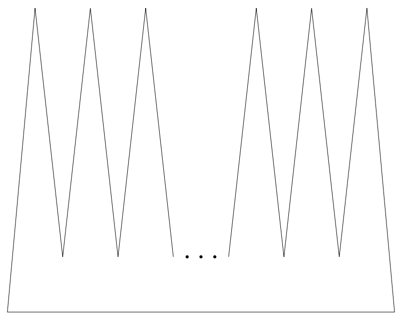


Figure 2.8.1: Non externally visible parts in a cove (marked red)



(a) Long, narrow parts can cause inclusion-like pockets



(b) An externally visible Polygon with arbitrarily many vertices

Figure 2.8.2: Unrealistic polygons with long and narrow features

2.9 Crab-Shaped Polygons

Definition Given a Polygon P. P is crab-shaped, if there exists a point p in the plane such that the shortest path between p and any point on the boundary of P, not properly intersecting the boundary of P, is a convex chain (as defined in Section 1.4).

Relations to other Characterizations

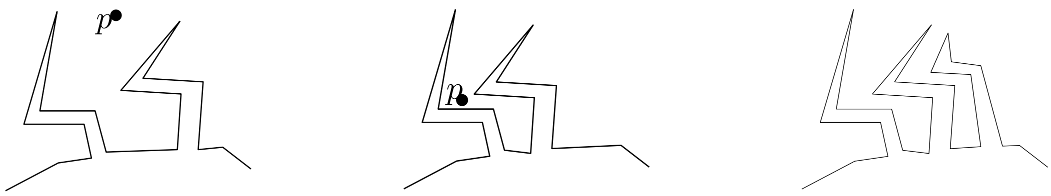
- Crab-shape is a relaxation of external visibility (Section 2.8) and palm-shape (Section 2.7) and therefore also of star-shape (Section 2.6), edge-visibility (Section 2.5), monotony (Section 2.4), ortho-convexity (Section 2.3) and convexity (Section 2.1).

Crab-shapes were introduced by ElGindy et al. [ET89] and shown to be the generalization of many known classes of polygons. Figure 2.9.1 illustrates that crab-shapes are not the union, but a generalization of palm-shaped and externally visible polygons.

Therefore crab-shaped polygons inherit the property of palm-shaped polygons to include uneven borders. Additionally crab-shaped polygons allow features that were excluded by palm-shapes, like narrow, 'zig zag'-shaped offshoots. Figure 2.9.2 shows that several of these offshoots are allowed by crab-shapes, as long as only one such structure occurs outside of P. In terms of realistic polygons these structures can be compared to curvy coves. Thus crab-shaped polygons may have several curvy offshoots but only one such cove or vice versa several curvy coves but only one such offshoot.

Unrealistic polygons (like in Figure 1.1.2) are likely to contain 'zig zag'-structures, due to their long, narrow features (especially those with many vertices). To show that these structures also appear outside of unrealistic polygons imagine the following: take a random point inside the polygons convex hull and try to tell if it is on the polygons inside or outside. It is often difficult to determine by merely looking. Consequently outside and inside of a random polygon seem to have similar features and that in turn shows that unrealistic polygons are similarly likely to have 'zig zag'-structures at the outside which eventually indicates that crab-shapes tend to reject unrealistic polygons.

Figure 2.9.1: Crab-shaped polygon that are neither palm-shaped, nor externally visible



(a) crab-shaped **(b)** crab-shaped **(c)** non crab-shaped

Figure 2.9.2: 'Zig zag'-shaped structures inside and outside P.
(a) 2 inside, 0 outside, (b) 2 inside, 1 outside, (c) 3 inside, 2 outside

2.10 k-Convexity

Definition Given a Polygon P and any two points p, q inside P. P is k-convex, if the intersection of P with the line through p and q results in k connected segments at most.

Relations to other Characterizations

- The 1-convex sets are precisely the convex sets (Section 2.1).
- Every k-point convex (Section 2.11) polygon is $(k-1)$-convex, as shown in [AAHD+09].

Aichholzer et al. ([AAHD+09, AAD+12]) introduced and discussed k-Convexity as an intuitive characteristic and a generalization of convexity. They further researched the special case of 2-convexity thoroughly, for which two examples are shown in Figure 2.10.1.

Even though k-Convexity restricts the complexity of a polygon quite nicely, it achieves this in a way that does not necessarily favor realistic polygons. Small bumps can rapidly increase the minimum k for a polygon. An example is given by the rather simple polygon in Figure 2.10.2a, which is already a 5-convex polygon, due to some unevenness on its bottom side. The comparison in Figure 2.10.2b illustrates even more, that this characterization favors straight edges over uneven ones. For both polygons a traversal line is shown, that decomposes into eight segments. In fact, there exist other traversal lines for the realistic (left) polygon that lead to more than eight segments, but those are located very closely along the edges and would be difficult to visualize.

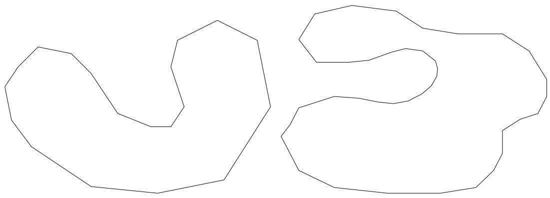
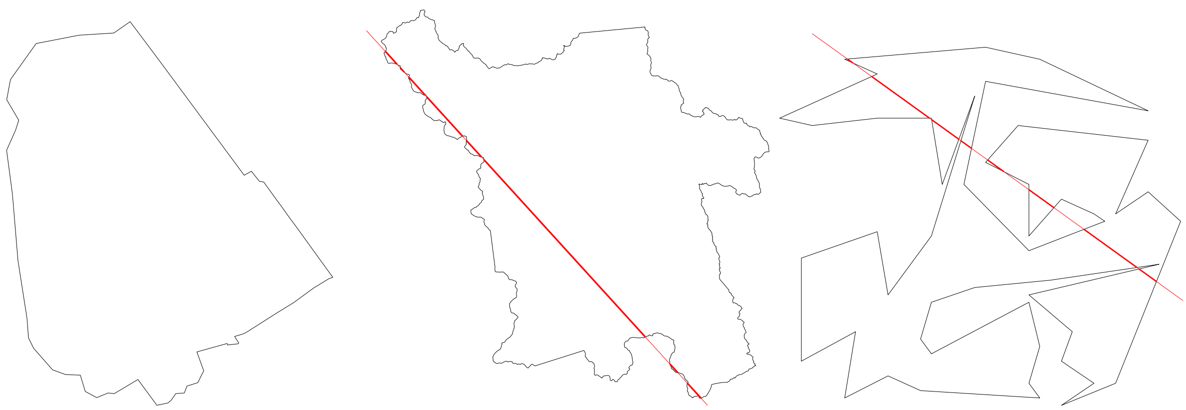


Figure 2.10.1: 2-convex polygons



(a) A 5-convex polygon **(b)** k-convex polygons, with $k \geq 8$

Figure 2.10.2: k-convex polygons, with according k

2.11 k-Point Convexity

Definition Given a Polygon P. P is k-point convex, if for any k points in P, at least one of the line segments they span is contained in P.

Corollary - More intuitively, in a k-point convex Polygon, it is impossible to find k points, such that none of those sees at least one other point.

Relations to other Characterizations

- The 2-point convex sets are precisely the convex sets (Section 2.1).
- Every k-point convex polygon is $(k-1)$-convex (Section 2.10), as shown in [AAHD+09].

There are two types of characteristics in the border of polygons that can quickly increase the necessary k. One are offshoots and the other are chains of concave vertices. The former may function as hiding places, which is shown in Figure 2.11.1 and quite intuitive. Each offshoot can be occupied by one point, with being visible from only a few places in the remaining polygon. Whereas for the latter on every segment on the chain one point can be placed, without seeing each other (see Figure 2.11.2). However in this case the remainder of P could be mostly covered. Either way this shows that the k might be big for real world polygons.

Another thought on k-point convexity is that the union of $k-1$ convex polygons is obviously k-point convex. So maybe k gives a clear insight in the complexity of P? Unfortunately the converse is false and only under certain circumstances are k-point convex polygons decomposable into $k-1$ polygons [BK76].

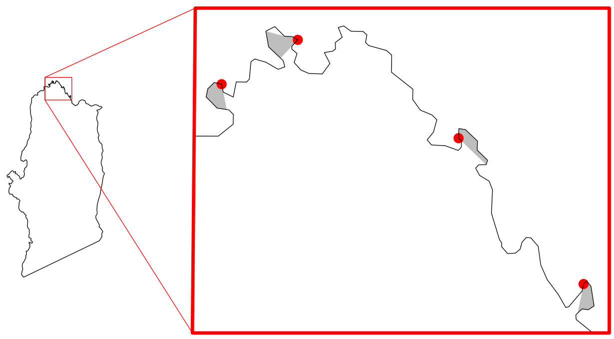
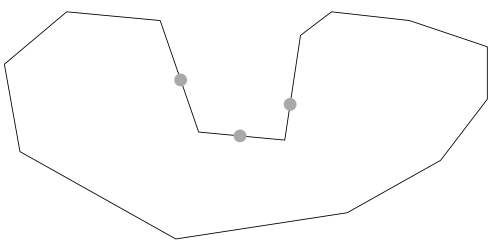
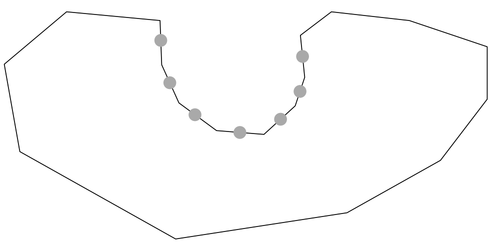


Figure 2.11.1: Points in offshoots cover only small areas



(a) A 4-point convex polygon



(b) An 8-point convex polygon

Figure 2.11.2: Points placed along concave chains on the border of P

2.12 k-Link Convexity

Definition Given a Polygon P and any two points p, q inside P. P is k-link convex, if the geodesic path (shortest polygonal path, that is contained in P) connecting them inside P consists of at most k edges.

Relations to other Characterizations

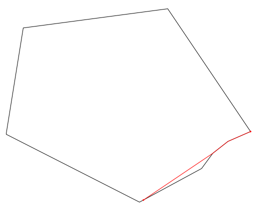
- The 1-link convex sets are precisely the convex sets (Section 2.1).

The intention behind this clearly is to exclude polygons that have either lots of branches or narrow parts with a "zig zag"-shape.

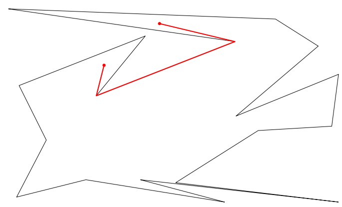
However, since only the number of edges is considered and not their length, k-link convexity does not prevent a polygon from having long and skinny parts (see Figures 2.12.1b and 2.12.2b).

Furthermore, very small alterations of P can cause a change of k. By adding a vertex at a position, where the geodesic intersects the border of P, k would increase by one (see Figure 2.12.1a). So, this characteristic could indicate the resolution of a polygon, since it can scale with the number of vertices (see Figure 2.12.2a).

In terms of a measurement for realistic polygons it could be worth to try exchanging the geodesic path by the edge-minimizing path to favor free areas.

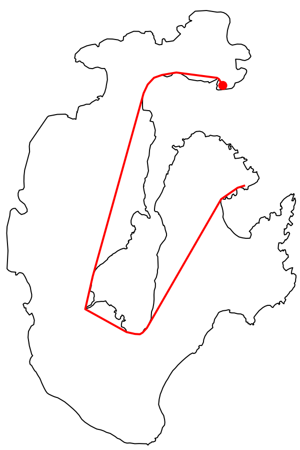


(a) A real world polygon

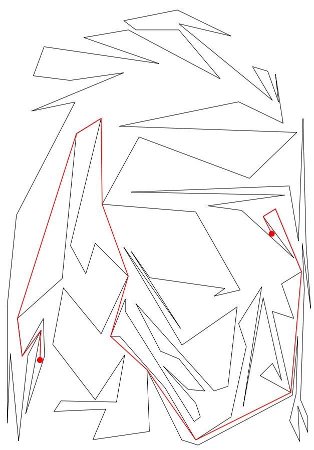


(b) A non-realistic polygon

Figure 2.12.1: 3-link convex polygons with geodesic example path



(a) A (at least) 17-link convex real world polygon



(b) A (at least) 15-link convex non-realistic polygon

Figure 2.12.2: k-link convex polygons for bigger k with geodesic example path

2.13 k-Guardability

Definition Given a Polygon P. P is k-guardable, if there exists a set of k points - guards - in P, such that every point of the boundary of P can be seen by at least one of the guards.

Relations to other Characterizations

- The 1-guardable sets are precisely the star-shaped sets (Section 2.6).
- ε-good polygons (Section 2.15) and (α, β)-covered polygons (Section 2.18) are $O(1)$-guardable for constant parameters (ε resp. α, β), as shown by [Val98, ABD+08]. A polygon is $O(1)$-guardable, if the minimum k does not depend on its number of vertices.

Aloupis et al. [ABD+08] proposed k-guardability in terms of realistic input models. Some examples are illustrated in Figure 2.13.1.

On one hand, non-realistic polygons with long and skinny parts can be assumed to need one guard for each of those parts. On the other hand, that statement also holds for offshoots of realistic polygons (Figure 2.13.2).

Closely related to this characteristic is the extensively studied art gallery problem [U+00], which is the problem of determining how many guards are needed to see the whole interior of P. Interestingly for k-guardable polygons $O(k)$ guards suffice [ABAST08].

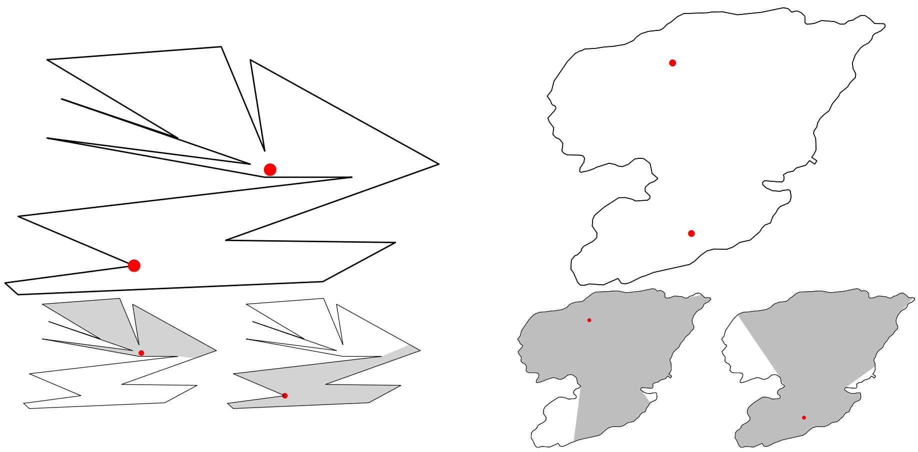


Figure 2.13.1: 2-guardable polygons, with guards and their visibility polygons (gray)

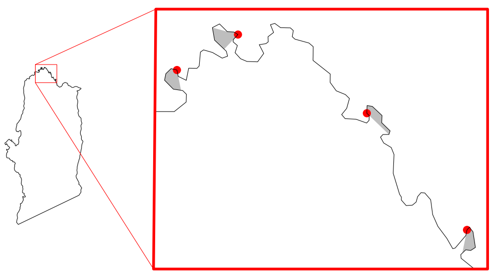


Figure 2.13.2: Guarding the red points, requires at least one guard per gray area

2.14 k-Border-Guardability

Definition Given a Polygon P. P is k-border-guardable, if it is possible to find a set of points Q consisting of interior points of edges of P such that the following two conditions are fulfilled:

1. every point of P is visible from at least k elements in Q
2. no edge of P has more than one element in Q

Relations to other Characterizations

- The n-border-guardable sets are precisely the convex sets (Section 2.1), for polygons with n vertices.

This characterization was introduced as 'k-Guardability'. To avoid confusion with the characterization of Section 2.13 the term k-border-guardability is used here.

There are two ways to approach this characterization: (1) determine the maximum k for P to be k-border-guardable, (2) find the smallest Q for a fixed k.

For the first approach, on the one hand a big k could indicate few narrow spikes, because such a spike would require a guard on a narrow section across from it. That in turn would limit the placement of the guards and probably cause an overall smaller k. Furthermore if P is compounded of several main regions, it demands them to have a likewise roundish shape. On the other hand, small features of a polygon could determine a small k for a realistic polygon. For example a structure like in Figure 2.14.1 would restrict a polygon to be 3-guardable.

The second approach could be problematic, since not every polygon is k-border-guardable for $k \geq 3$ (like the comb polygon in Figure 2.14.2). However any polygon with n vertices can be 2-border-guarded by $n-1$ guards and 1-border-guarded by $\lfloor \frac{n}{2} \rfloor$ guards [BBC+94].

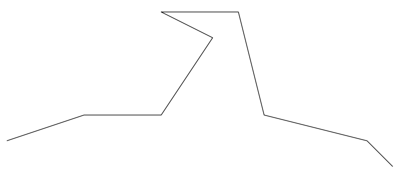


Figure 2.14.1: This offshoot restricts a polygon to be 3-border-guardable

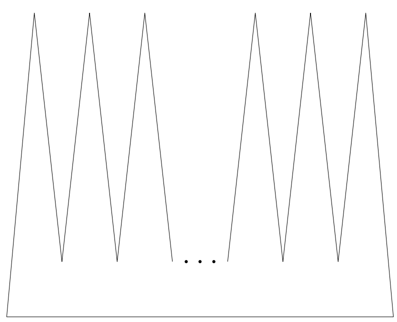


Figure 2.14.2: A polygon which is not 3-border-guardable

Interlude: Fatness

Occasionally polygons are grouped into a class of characterizations, that are referred to as 'fatness'.

There are many definitions of 'fatness' and the following four characterizations present those which apply to 2D polygons. Namely they characterize polygons as ε-good (Section 2.15), α-fat (Section 2.16), β-fat (Section 2.17) and (α, β)-covered (Section 2.18).

Other characteristics exist, but apply only to other geometric forms. For instance triangles can be δ-fat [PT02]. This means each of its inner angles needs to be bigger than δ. Respectively in 3D, wedges are δ-fat, if their opening angle is at least δ [ERS93].

Aronov et al. [AEKS06] discussed κ-round objects. For every point p on their boundary those objects have to fit a ball with certain properties inside them. The balls need to (1) have a radius of at least κ times the diameter of the object and (2) touch the boundary at p. The second condition forbids convex vertices, so κ-Roundness applies to objects with curved borders only.

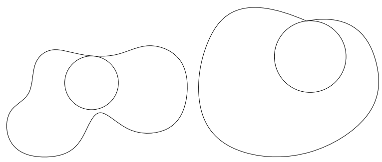


Figure 2.14.3: κ-round objects

2.15 ε-Good Polygons

Definition Given a Polygon P. P is ε-good, if each point in P sees at least a certain fraction ε of P.

Relations to other Characterizations

- The 1-good sets are precisely the convex sets (Section 2.1).
- ε-good polygons are $O(1)$-guardable (Section 2.13) for constant ε, as shown by [Val98]. A polygon is $O(1)$-guardable, if the minimum k does not depend on its number of vertices.

ε-good polygons can not have narrow corridors, because points in these parts would have only a restricted view into the polygon (That is unless the polygon is long and narrow as a whole). This results in an overall good connectivity within the polygon.

It is worth mentioning that typical features of real world polygons (like offshoots) can not be ε-good, but that should be less a concern when talking about 'fat' polygons.

Lastly, it should be mentioned, that Kavraki et al. [LKL+95] introduced this characterization in the context of motion planning for robots and that Kirkpatrick used a similar notion, that considers only points on the boundary of P, which need to see at least a fraction ε of the boundary of P [Kir00].

2.16 α-Fatness

Definition Given a Polygon P. P is α-fat, if P is contained in some axis-parallel square S^+ and P contains another axis-parallel square S^-, so that the ratio between the edge length of S^+ and S^- is at most α.

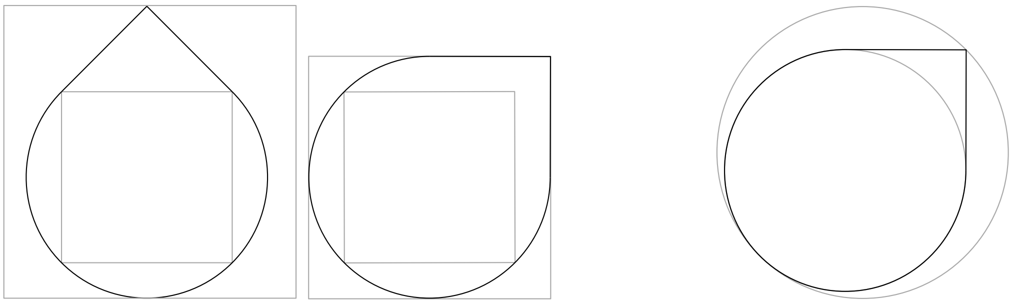
Relations to other Characterizations

- α-fatness is not related to any other characterization within this work.

Agarwal et al. [AKS95] originally used this notion for 3D objects whose xy-Projection had to suffice the definition.

The first thing to note is that α-Fatness is rotation-sensitive (Figure 2.16.1a), which is an unfavorable property in the context of measuring how realistic a polygon is. This can easily be avoided by using circles instead of axis-aligned squares (Figure 2.16.1b).

Furthermore this characterization ignores details on the polygons borders. The only relevant property is whether there exists a big enough free area inside P with respect to the size of P. Therefore it might be a good measure of the overall shape of P.



(a) α depends on the rotation of shape **(b)** Using circles avoids rotation-sensitivity

Figure 2.16.1: An object with S^+ and S^-

2.17 β-Fatness

Definition Given a Polygon P. P is β-fat, if for every circle b whose center is inside P and which does not contain P completely, the volume($b \cap P$) is at least $\beta \cdot$ volume(b).

Relations to other Characterizations

- β-fatness is not related to any other characterization within this work.

This characterization was introduced by Overmars [Ove92] for d-dimensional spaces. The original definition also used a parameter k for which the intersection needs to be as least $\frac{1}{k} \cdot volume(b)$.

The definition of β-fatness tries to exclude polygons with long and skinny parts and "if P is convex, then β-fatness captures the intuition of 'fat' polygons nicely. However, non-convex 'comb' polygons with very thin spikes that are very close together also fall into the class of β-fat polygons." [ABD+08] A similar example with that effect is shown in Figure 2.17.1.

Therefore it is worth mentioning that a simplified definition exists for convex polygons: "an object is fat if its volume is at least β times the volume of its minimal enclosing d-dimensional sphere for some constant β" [dB95]. The polygons covered by this simplified definition however are not exactly the same as the convex, β-fat polygons. Imagine a polygon with many vertices, nearly a circle. It has a minimum β of close to 1 according to the simplified definition, but a β less than 0.5 according to β-fatness.

Despite the seemingly big difference to the original definition, the simplification makes sense in the context in which β-Fatness is usually used. This definition is often utilized when looking at sets of polygons; and for non-intersecting β-fat polygons some favorable properties can be shown for the entirety of the set [DBKVdSV97].

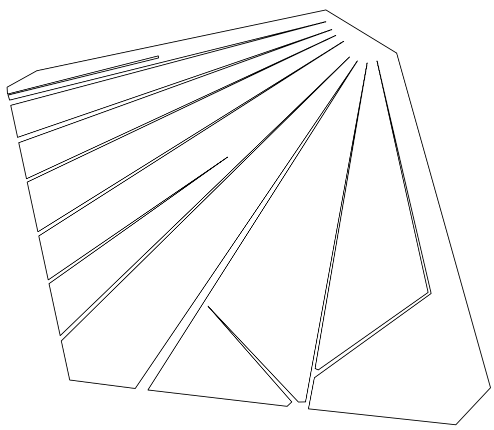


Figure 2.17.1: A β-fat polygon with thin parts

2.18 (α, β)-Covered Polygons

Definition Given a polygon P. P is (α, β)-covered, if for each point p on the boundary of P there exists a triangle $T(p)$, called a good triangle of p, such that:

1. p is a vertex of $T(p)$,
2. $T(p)$ is completely contained in P,
3. each angle of $T(p)$ is at least α, and
4. the length of each edge of $T(p)$ is at least $\beta \cdot diameter(P)$.

Relations to other Characterizations

- (α, β)-covered polygons are $O(1)$-guardable (Section 2.13) for constant α and β, as shown by [ABD+08]. A polygon is $O(1)$-guardable, if the minimum k does not depend on its number of vertices.

The inner angles of the vertices of P are obviously restricted to at least α, too. Furthermore P can have no thin corridors, because a good triangle would not fit into those.

The notion of (α, β)-covered polygons combines the ideas from δ-fat triangles and κ-round objects (mentioned in the overview on Fatness) and fits them into the context of polygons (Figure 2.18.1). As consequence thereof "(α, β)-covered polygons model the intuitive notion of fatness for non-convex input" [ABD+08].

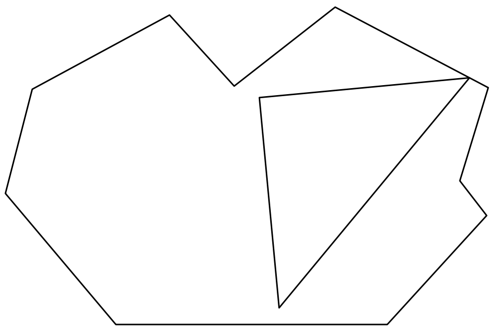


Figure 2.18.1: An (α, β)-covered polygon

3 Conclusion

3.1 Summary

This thesis collected and presented the existing characterizations related to realistic polygons. It was demonstrated that they are generally not comparable in terms of being measurements of how realistic a given polygons is. Nevertheless some can be shown to be generalizations or special cases of others.

The non-parametrized characterizations and their relations to each other are illustrated in the Venn-Diagram in Figure 3.1.1.

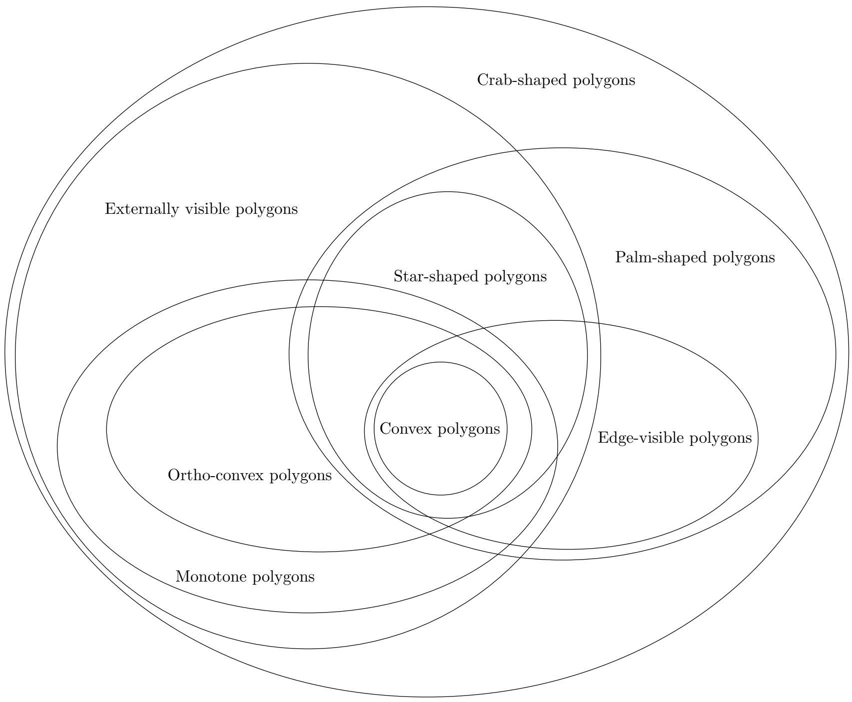


Figure 3.1.1: Relations of non-parametrized Characteristics

The parametrized characterizations can often be traced back to non parametrized characterizations for specific parameters. It can be shown that the following sets of polygons with n vertices are precisely the same:

Convex polygons (Section 2.1)
= Directionally convex polygons, with D containing all directions (Section 2.2)
= 1-convex polygons (Section 2.10)
= 2-point convex polygons (Section 2.11)
= 1-link convex polygons (Section 2.12)
= n-border-guardable polygons (Section 2.14)
= 1-good polygons (Section 2.15)

Star-shaped polygons (Section 2.6)
= 1-guardable polygons (Section 2.13)

Ortho-convex polygons (Section 2.3)
= Directionally convex polygons, with D containing only two orthogonal directions

Monotone polygons (Section 2.4)
= Directionally convex polygons, with D containing only one direction

Individually none of the examined characterizations serves as classifier that can decide whether a polygon is realistic or not. However some of them can serve as indicators for certain features of polygons. This could prove useful to evaluate polygonal data of specific application fields.

3.2 Evaluation

This work not only collected but evaluated realistic input models for 2D polygonal data.

Normally when new realistic input models are proposed it is briefly mentioned that many realistic data sets share the characteristics covered by the proposed model. Often some application areas are listed and afterwards the focus quickly shifts to their algorithmic aspects. The models are rarely disputed, for they are usually intuitively agreeable. This circumstance makes them difficult to compare without applying large test sets on them.

This work tried to reevaluate known characterizations with respect to concrete characteristics of cartographic real world polygons (like main regions and offshoots). For the individual characterizations reasons have been pointed out that make them favor certain types of characteristics and neglect others. Therefore many were found to be good indicators for certain types of polygons.

Furthermore it was discussed whether the characterizations cover unrealistic polygons. Again the focus was on specific characteristics (like long and narrow parts) and eventually several characterizations tend to favor realistic over unrealistic polygons.

3.3 Future Work

It is uncertain if a combination of several characterizations exists that would vastly improve the measurement of how realistic a polygon is compared to the single characterizations.

Another possibility to look at this problem is to focus on sub-sets of realistic polygons and figure out which characterization works best to describe their features.

Of course these two approaches can be coupled to come up with new measurements for application-specific polygons, that is the measurement of realness could strongly differ for e.g. ground plans and city borders.

Furthermore some ideas for new characterizations were proposed in this work that can be studied from a perspective of realistic input models.

One newly proposed characterization is the ratio between the area (or perimeter) of a polygon and its convex hull. 'Fat' polygons would by tendency have a bigger ratio compared with polygons that have coves or thin and branching parts.

Another proposal is a parametrized version of external visibility that requires only a certain amount of the border to be visible. This characterization assumes that realistic features are generally less covered than unrealistic ones. The examples given in Section 2.8 were coves as realistic features and inclusion-like pockets as unrealistic features.

In Section 2.12 a new measurement was suggested that is similar to k-link convexity, but considers the edge-minimizing path instead of the geodesic path. This could create challenges to the field of computational geometry, since not only the vertices of the polygon need to be considered. However the effort could result in a viable characterization, since the number of path-edges can usually be decreased for realistic polygons with wide areas but not for unrealistic polygons with their long and narrow parts.

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