# Analyzing and Modeling non-equilibrium Brain Dynamics in Schizophrenia

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# MOU process

- **Multivariate Ornstein-Uhlenbeck Process:**

$$\frac{d\mathbf{x}(t)}{dt} = -\mathbf{B}\mathbf{x}(t) + \boldsymbol{\eta}(t) \quad \textit{with} \quad \langle \boldsymbol{\eta}(t)\boldsymbol{\eta}^T(t')\rangle = 2\mathbf{D}\delta(t-t'). \tag{1}$$

where $\mathbf{B}$ is the friction matrix and $\mathbf{D}$ is the covariance matrix

- **Entropy production rate for an non-equilibrium stationary state of MOU process:**

$$\Phi = \mathrm{tr}(\mathbf{B}^{\mathrm{T}}\mathbf{D}^{-1}\mathbf{Q}) = -\mathrm{tr}(\mathbf{D}^{-1}\mathbf{B}\mathbf{Q}) \tag{2}$$

where $\mathbf{Q}$ is defined through the stationary matrix $\mathbf{S}$:

$$\mathbf{BS} = \mathbf{D} + \mathbf{Q}, \quad \mathbf{SB}^{\mathrm{T}} = \mathbf{D} - \mathbf{Q} \tag{3}$$

if we are in equilibrium $\mathbf{Q} = 0$

# Fit MOU process to fMRI data

- **Empirical covariance matrices:**

$$\hat{S}_{ij}(0) = \frac{1}{T-2} \sum_{1 \leq t \leq T-1} (x_i(t) - \bar{x}_i)(x_j(t) - \bar{x}_j) \tag{4}$$

$$\hat{S}_{ij}(1) = \frac{1}{T-2} \sum_{1 \leq t \leq T-1} (x_i(t) - \bar{x}_i)(x_j(t+1) - \bar{x}_j) \tag{5}$$

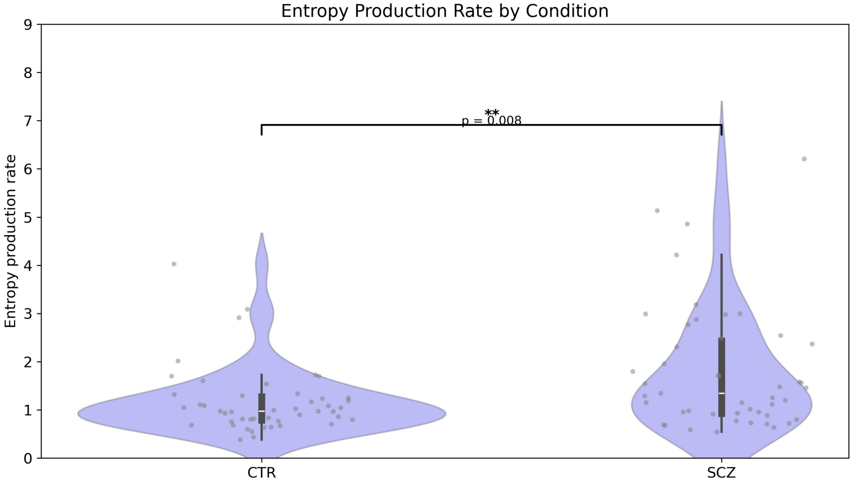
- **Fitting the model through gradient descent:**

$$\Delta \mathbf{B} = \epsilon_B \, \mathbf{S}(0)^{-1} \left[ \Delta \mathbf{S}(0) - \Delta \mathbf{S}(1) e^{\mathbf{B}^T} \right] \tag{6}$$

$$\Delta \mathbf{D} = \epsilon_D \left( \mathbf{B} \Delta \mathbf{S}(0) + \Delta \mathbf{S}(0) \mathbf{B}^T \right) \tag{7}$$

# Entropy Production Rate per condition

Since the p-value is lower than 0.05,
the EPR for the two conditions shows a statistically significant difference:



# Violation of Fluctuation Dissipation Theorem

- For stochastic dynamical systems out of equilibrium we have the violation of fluctuation dissipation equality:

$$\mathbf{R}(\tau)\mathbf{D} = -\frac{d\mathbf{C}(\tau)}{d\tau}\mathbf{X} \tag{8}$$

  where **C** is the correlation matrix, **R** is the response matrix and **X** is the fluctuation-dissipation ratio. If we are in equilibrium $\mathbf{X} = \mathbf{1}$.

- It can be shown that there is a connection between FDT-violations and EPR:

$$\mathbf{X}^{-1} = \mathbf{D}^{-1}\mathbf{B}\mathbf{S} \tag{9}$$

- **AIM:** Try to localize which brain region or which connections between brain regions contributes the most to the difference of EPR by studying rispectively matrix **X** and vector **V** computed by summing the rows of **X**