Stellar Physics

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2024-01-01

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Preface

This is a set of Stellar Physics notes for the Astronomy 1 course at the University of Glasgow (2023/2024).

Recommended textbook for this course: - “An introduction to Modern Astrophysics”, Carroll & Ostlie, 2nd edition

The slides are for guidance and are not sufficient for studying. You are supposed to read the notes, complementing them with your own notes taken during the lectures and the book.

Introduction

Astronomy is a *observational* science. We cannot bring a star down to Earth to study it in the laboratory. Unlike other branches of Physics, we cannot experiment directly on stars and what happens in the universe is beyond our control, but we can use our knowledge of Physics to make predictions and we can use observations to validate our theories. This can allow us to answer many questions about the stars. Some of these questions date back to the most ancient times, and through history have inspired philosophers and scientists to develop our understanding of the cosmos. This interplay between Astronomy and Physics is today called *Astrophysics*, which has grown to encompass phenomena from planets to galaxies to the evolution of the universe.

Astrophysics is very multidisciplinary

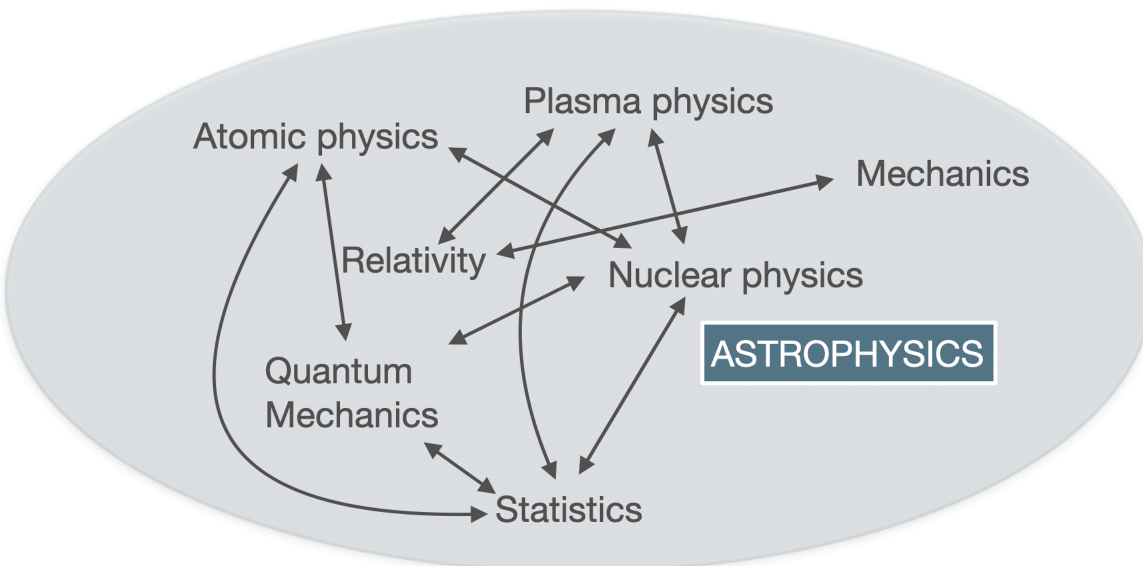

Figure 1: Multidisciplinarity in astrophysics

Astrophysics is very multidisciplinary because of 'several effects' that come into play in the physics of stars, galaxies, the universe… it is an 'arena' of different fields of physics (but also mathematics and chemistry), spanning across classical mechanics, relativity, nuclear and particle physics, statistics, atomic and molecular physics, plasma physics, quantum mechanics and more (the list above and in the picture is not exhaustive)! In this course you will see how some of these fields play a role in the physics of stars.

1 Lecture 1 - Stellar properties

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2 Look at the night sky, what do we see?

Let's start with some simple observations of the night sky… "Look at the night sky, what do we see?" Here in Scotland, the answer to this questions is likely 'clouds', but if you are curious to know what is beyond those clouds, you can check out Stellarium (you can download it or you can use the web version: https://stellarium-web.org/).

All that glitters is not (only) stars!

Figure 2.1: An example from Stellarium

Looking at the night sky we may be able to see planets, meteors, galaxies, satellites, comets, star clusters and many stars!

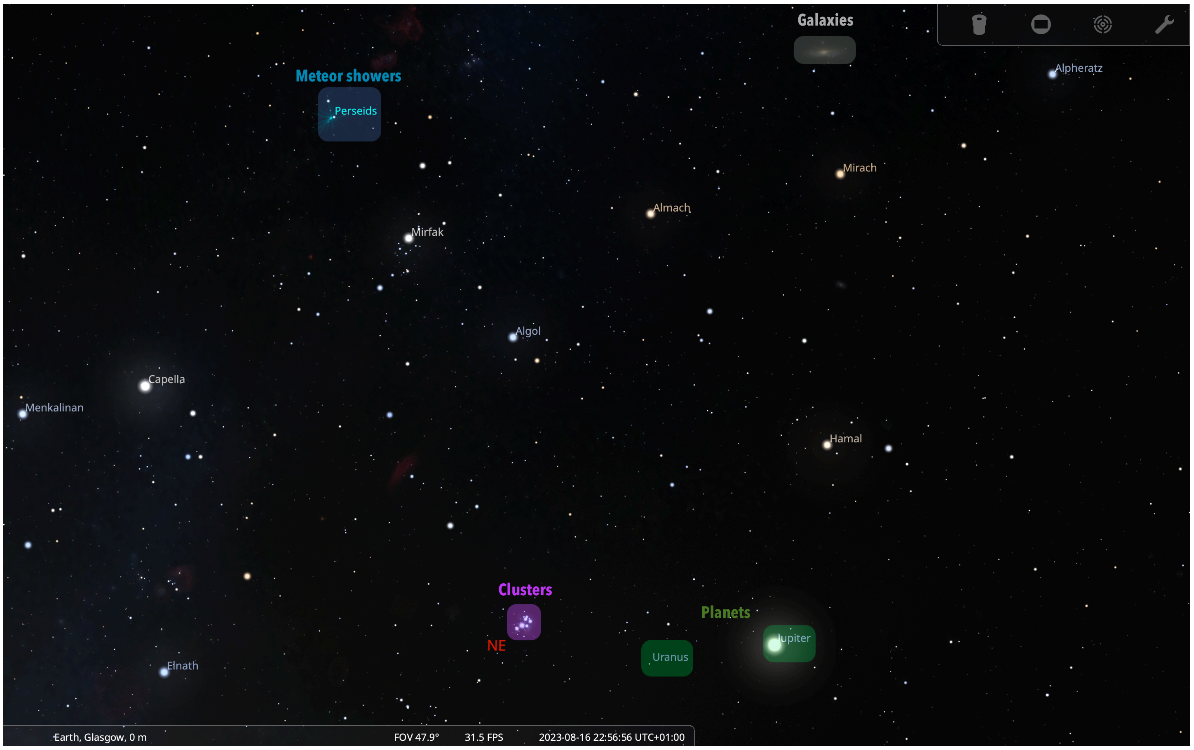

Figure 2.2: An example from Stellarium (with marked typical objects)

3 What is a star?

A star is an astronomical spheroid body: - held together by *self-gravity* - *radiating* energy from an internal source (tipycally from nuclear fusion reactions, and occasionally from the release of gravitational potential energy during contraction or collapse)

Having this definition you should be able to answer this question: "is the Death Star a (fictional) star"?

The answer is "no", because it does not radiate energy! The Death Star is actually a (fictional) space station orbiting the (fictional) planet moon of Endor. Enough with the fictional stuff now.

If you look at the sky you may see that planets and comets are also shining in the sky, so why can't they be considered stars, based on the definition given above? Do they radiate energy? Are they self-gravitating?

Planets actually do not *radiate*, they mostly reflect the light coming from the Sun, and the same holds for comets. In addition to that, comets do not have enough mass to be considered self-gravitating objects.

3.1 Life and death of a star

A star is not a static object. During its life it will evolve, changing its internal structure and chemical composition as it 'burns its *fuel*' and changing its mass and size, eventually resulting in its death when one of the conditions that define a star is no longer valid, i.e. when the star is no longer self-gravitating or when it stops radiating having exhausted its nuclear fuel. We will talk more about the evolution of stars in the following lectures. Now let's go back to what we can observe having a look at the night sky and let's see what we can tell from observations of stars.

3.2 Do all stars look the same?

- There are a lot of stars, not uniformly distributed
- Some look brighter than others
- Some are bigger than others

- They have different colours
- Sometimes there are coloured or dark areas around stars
- Some stars have fluctuating brightness: repeating and irregular

3.3 A comparison between two stars on Stellarium

Let's check the properties of two stars on Stellarium. What can you notice after a first look at these stars? You may notice that they have different colours (one looks white, the other one looks white-blue). The boxes contain some information about these stars, and you may recognise many of these fields in the blue boxes from positional astronomy. We will find out what the properties in the red boxes mean during this course.

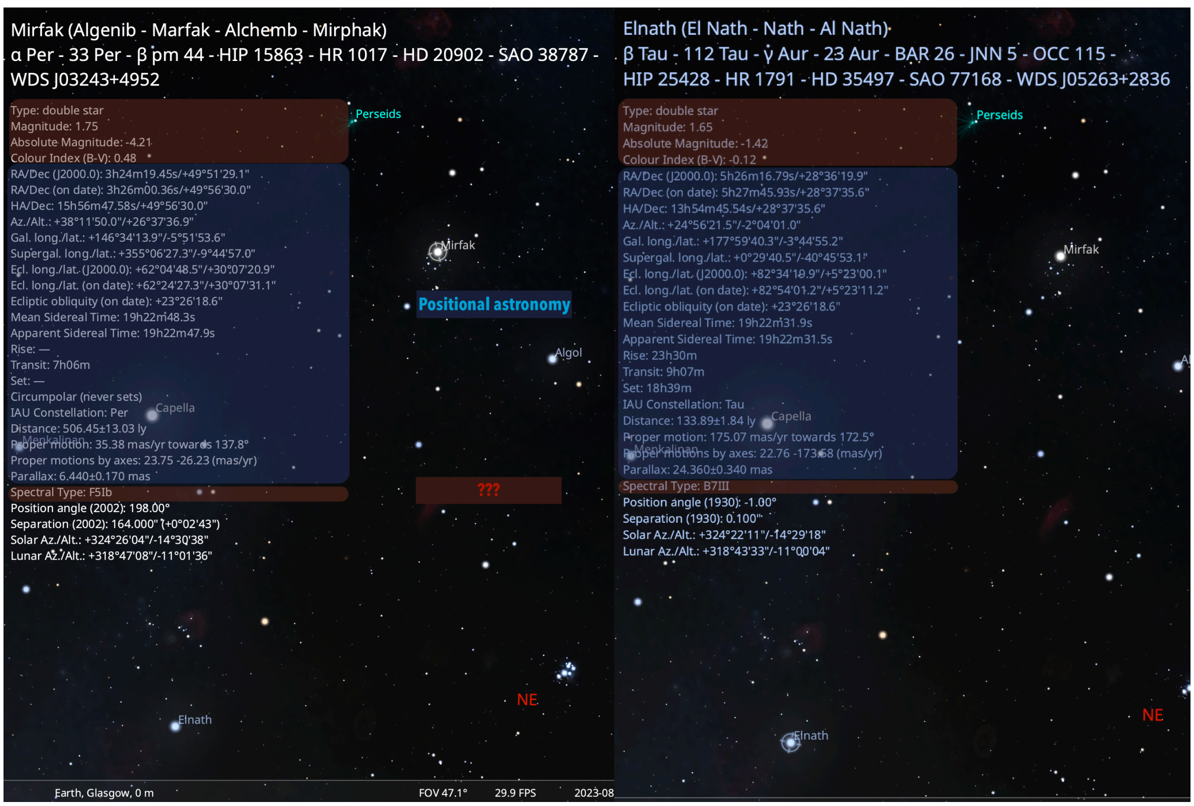

Figure 3.1: Comparison between two stars on Stellarium

3.4 What can we tell from observations?

- How distant are the stars?

- How big are they?
- How bright?
- How, when and where do they form?
- What are their most important characteristics?
- What are they made of?
- What is their energy source?
- What happens when it runs out?

4 How distant are the stars?

4.1 An easy case: the distance to the Sun

How do we find the distance to the Sun?

Easy! - Just define The Astronomical Unit (a.u.) as the average distance between the Earth and the Sun!

4.2 The astronomical unit

Ok, but how big is an au? In 1976 the International Astronomical Union (IAU) adopted a standard definition whose value has been updated with increasing precision during the years. At the current date, the astronomical unit is: - 1 a.u. = 149,597,870,700 m

4.2.1 Question

Can you now work out how long it takes for the sunlight to reach Earth?

```
import warnings
warnings.filterwarnings('ignore')
import math
AU = 149597870 # km
c = 299792.458 # km/s
ts = AU/c # s
tm = ts/60 # min
print(f"Time taken by light to reach Earth from the Sun = {ts:1.4} s = {tm:1.4} min = 8' 1
```

```
Time taken by light to reach Earth from the Sun = 499.0 s = 8.317 min = 8' 19''
```

4.2.2 A remark on the Astronomical Unit

Note: 1 a.u. is an *average* distance, because the distance between the Earth and the Sun varies during the year.

```
import warnings
warnings.filterwarnings('ignore')
import math
AU = 149597870 # km
aphD = 152100000 # km
perD = 147100000 # km
AU_avg = (aphD+perD)/(2*AU) # Average distance, gives AU, check: print(f"{AU_avg}")
print(f"Distance Earth-Sun at aphelion (most distant) = {aphD:1.3e} km = {aphD/AU:1.4} au"
print(f"Distance Earth-Sun at perihelion (closest) = {perD:1.3e} km = {perD/AU:1.3} au")
print(f"Average distance Earth-Sun = {AU:1.3e} km = 1 au")
```

```
Distance Earth-Sun at aphelion (most distant) = 1.521e+08 km = 1.017 au
Distance Earth-Sun at perihelion (closest) = 1.471e+08 km = 0.983 au
Average distance Earth-Sun = 1.496e+08 km = 1 au
```

```
from IPython.display import display, HTML
display(HTML("<style>.container { width:100% !important; }</style>"))
```

```
<IPython.core.display.HTML object>
```

4.3 Size of our solar system

Planet	Distance from Sun (a.u.)
Mercury	0.39
Venus	0.72
Earth	1
Mars	1.52
Jupiter	5.20
Saturn	9.54
Uranus	19.2
Neptune	30.06

4.4 How do we measure the distance to the stars?

4.4.1 The parallax method

We started with the easy example of answering how distant is the Sun, but what about other stars? One way to determine the distance to nearest stars is to use the *parallax method*.

If you look at the position of a closer object against a further background, you will notice that the image of the near object will shift in the background as you move. This effect is called parallax and you can experience it even when you close one eye at the time looking at an object against a background.

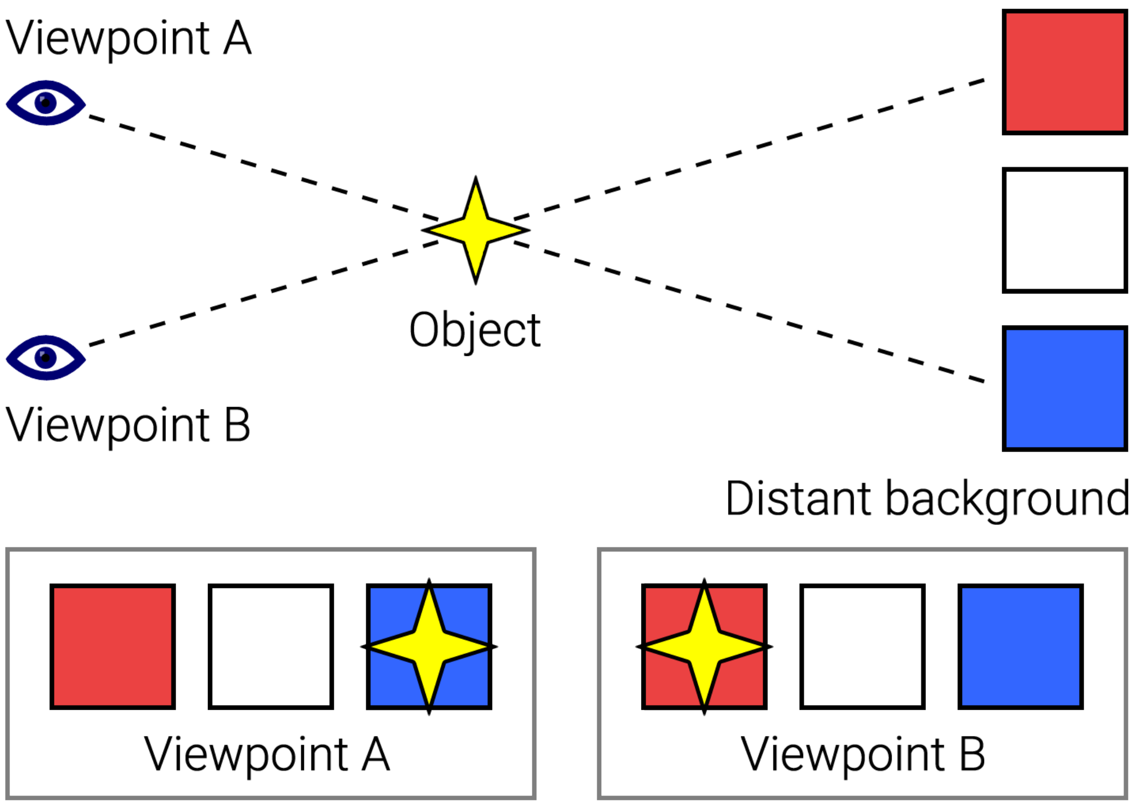

Figure 4.1: Parallax1

Analogously, in a background of distant stars, a closer star will appear to move against them as the Earth orbits the Sun. So how do we use this to determine the distance to a start? We use trigonometry.

The angle formed between the lines of sight of a star from two positions of the observer is twice the parallax angle. In the example below, the parallax is the angle between the lines of sight of the observer on Earth and from the Sun (the middle point between the position of Earth six months apart).

The number of stars that can be observed from Earth is limited and nowadays *direct* observations of the distance using the parallax method are done by satellites orbiting the Sun in sync with Earth. Hipparcos gathered data on more than one million stars, and its successor Gaia has already observed 2 billion stars!

- The parallax is generally measured in arcseconds (as). 1 arcsecond $= \dfrac{1}{3600}\,\mathrm{deg} = \dfrac{1}{206,265}$ radians. You can show these conversions by recalling that

$$1\ \mathrm{rad} = 57.3\,\mathrm{deg} = (57.3 \times 60 \times 60)\ \mathrm{as} = 206,265\ \mathrm{as}$$

  .

- The parsec (**par**allax-**sec**ond, pc) is defined as:

$$1\ \mathrm{parsec} = \frac{1 au}{\text{arcsecond in radians}} = 206,265\ au = \frac{648000}{\pi}\mathrm{au} \approx 3.086 \times 10^{16}\mathrm{m}$$

  .

- A parallax of 1 arcsecond corresponds to a distance to the star of 1 parsec (3.26 light-years).

- Using trigonometry, the distance d to the star is given by:

$$d = \frac{1\mathrm{au}}{\tan p} \simeq \frac{1}{p}\mathrm{au}$$

where p is the parallax, and for this approximation we used p measured in radians. Converting the parallax p to p'', from radians to arcseconds, we obtain the distance in parsec:

$$d \simeq \frac{206,265\mathrm{au}}{p''} = \frac{1}{p''}\mathrm{pc}$$

```
import warnings
warnings.filterwarnings('ignore')
import numpy as np
import math
AU = 149597870 # km
pc = 648000/np.pi*AU # km
print(f"1parsec = {pc:1.3e} km")
```

```
1parsec = 3.086e+13 km
```

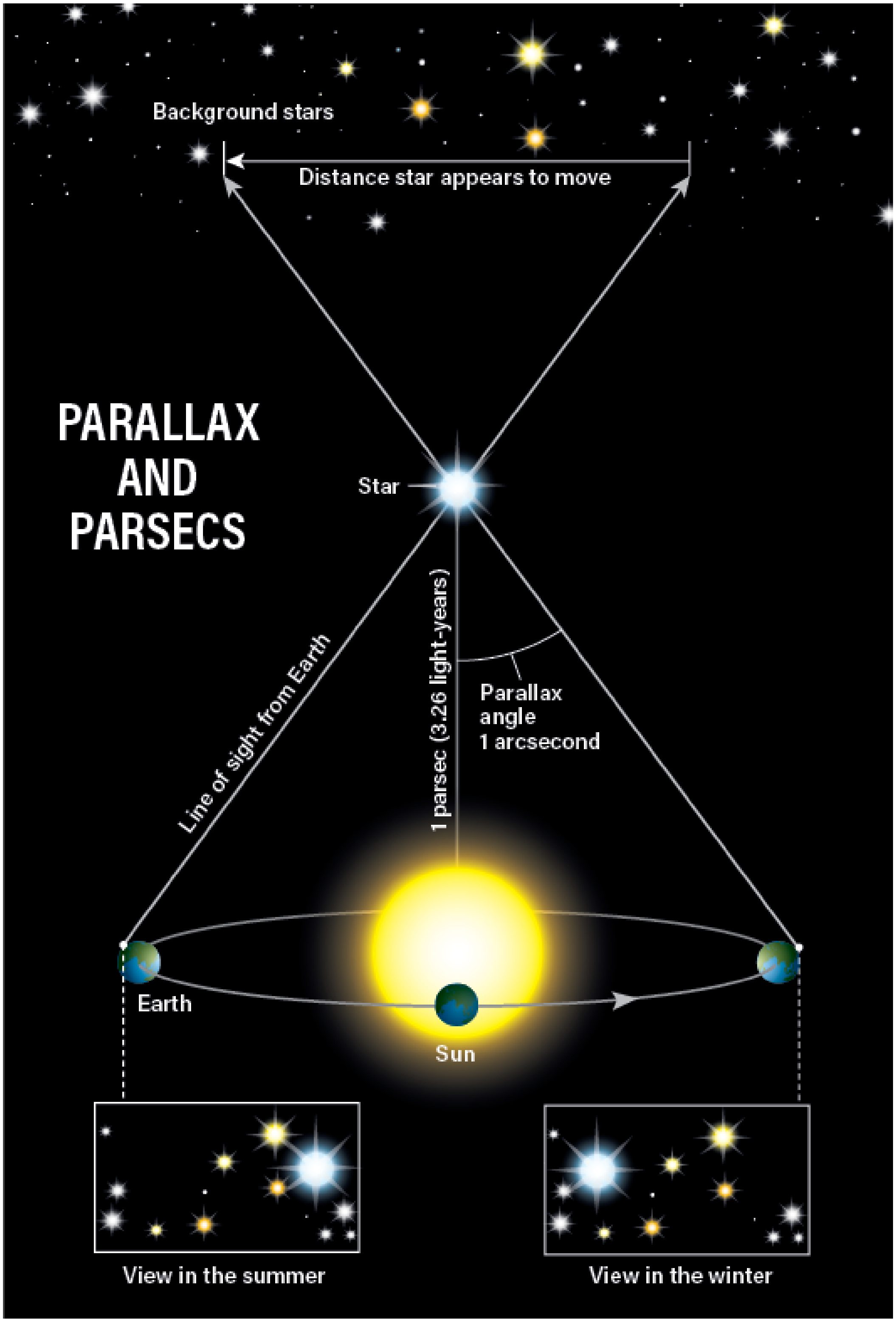

Figure 4.2: Parallax and parsecs representation. Image readapted from: https://www.astronomy.com/wp-content/uploads/sites/2/2023/02/parsec.png

4.4.2 The nearest star: Proxima Centauri

The further a star is, the smaller the parallax angle.

The nearest star (Proxima Centauri) has a parallax of 0.7681 arcseconds. This is *very* tiny, but still, we are able to measure it! To give you an idea of how small this is: - It is approximately the angle subtended by an object having a diameter of 2 centimeters, positioned at a distance of 5.3km! - The shift produced by the parallax against the background would be $1/2000^{th}$ the width of the Full Moon!

The parallax method is covered in the 'Observational Methods' course, so this is just a taster, but now knowing the parallax of Proxima Centauri (or another star), we can find the distance to that.

```
# The correct version
import warnings
warnings.filterwarnings('ignore')
import math
import numpy as np
def arcsec_to_rad(p):
    rad = p/206265
    return rad

p = 0.7681 # as
d_pc = 1/p # pc

p_rad = arcsec_to_rad(p)
d_au = 1/p_rad # au
d_km = d_au*AU # km
print(f"p = {p} as")
print(f"d = 1/p pc = 1/{p} pc = {d_pc:1.1e} pc")
print(f"d = 1/p au = 20625/p au = 20625/{p} au = {d_au:1.1e} au")
print(f"d = {d_au:1.1e} au * 149597870 km = {d_km:1.1e} km")
print(f"The distance from Proxima Centauri is = {d_pc:1.1e} pc = {d_au:1.1e} au = {d_km:1.
```

```
p = 0.7681 as
d = 1/p pc = 1/0.7681 pc = 1.3e+00 pc
d = 1/p au = 20625/p au = 20625/0.7681 au = 2.7e+05 au
d = 2.7e+05 au * 149597870 km = 4.0e+13 km
The distance from Proxima Centauri is = 1.3e+00 pc = 2.7e+05 au = 4.0e+13 km
```

4.4.3 'Spot the mistake' question:

An unknown star has a parallax of 379.21 mas. Your messy lecturer tried to calculate the distance to the star but she must have made some mistakes because something looks wrong. Can you tell what?

Hints

Hint 1

Check the orders of magnitude, do they seem reasonable?

Hint 2

Compare these orders of magnitude with the ones found for Proxima Centauri, any red flags?

Hint 3

The units are important!

```python
# This is the version with mistakes
import math

p = 379.21/10000 # as
d_pc = 206265/p # pc

d_au = 1/p # au

AU = 149597870 # km
d_km = d_au*AU # km
print(f"The distance to the star is = {d_pc:1.1e} pc = {d_au:1.1e} au = {d_km:1.1e} km? So
```

The distance to the star is = 5.4e+06 pc = 2.6e+01 au = 3.9e+09 km? Something does not look right… find the mistakes!

Solution

Below is the solution to the question. Something was clearly wrong with the orders of magnitudes of the values obtained in the wrong solution: - the distance in pc can not be larger than the distance in au! Those orders of magnitude are clearly off! - if you compare this with the values obtained for Proxima Centauri, you should expect approximately the same orders of magnitude - 10^6 pc would be way too far to be able to use the parallax method to measure the distance: the most precise and reliable measurements of the parallax (with the Hubble Space Telescope and Gaia space mission), can measure distances of the order of 10^3pc.

```
# The correct version
import math

p = 379.21/1000 # as <- the units conversion was wrong
d_pc = 1/p # pc <- This formula for the distance in pc was swapped with the one below for

d_au = 206265/p # au

AU = 149597870 # km
d_km = d_au*AU # km
print(f"The distance to the star is = {d_pc:1.1e} pc = {d_au:1.1e} au = {d_km:1.1e} km")
```

```
The distance to the star is = 2.6e+00 pc = 5.4e+05 au = 8.1e+13 km
```

4.5 Transit method

The parallax was also originally proposed as a way to determine the distance to the Sun by the Scottish astronomer James Gregory, in 1663, using the transit of Venus. Gregory is seldom given any credits for this idea and usually all the credits goes to Edmund Halley for some reason (can anyone find out why?).

In 1677, at the age of 20, Halley travelled to St Helena to map the Southern Skies and observed a transit of Mercury on November 7th. Like Gregory, he suggested that the transit of a planet like Venus could be used as a method to measure the distance to the Sun, using parallax.

Here is the translation of Halley's publication about this idea: https://eclipse.gsfc.nasa.gov/transit/HalleyParalla

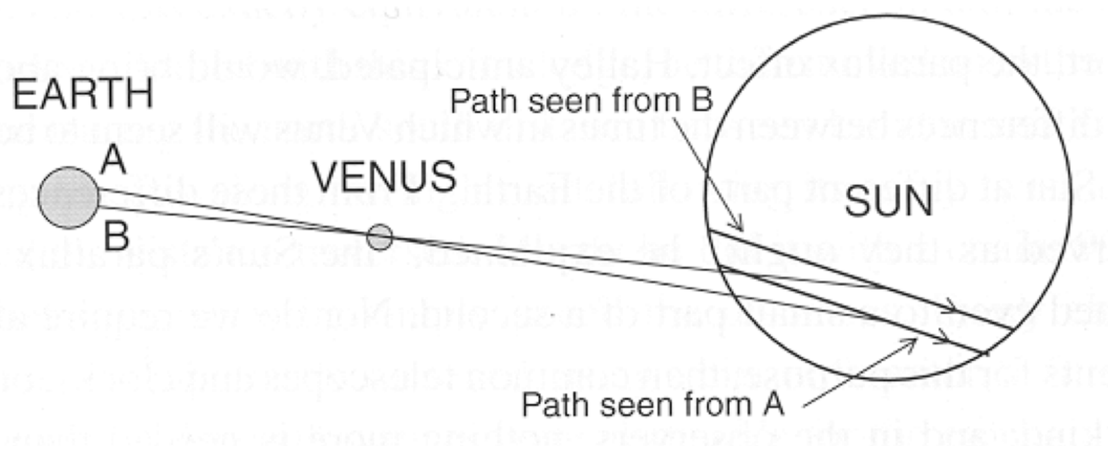

Figure 4.3: Transit Method

According to ‘Halley’s method’, two observers (A and B in the figure above) have to be positioned at a great distance from each other, but both located on the side of Earth facing the Sun at the time of the transit. Each observer would precisely measure the times that Venus appear to be in different positions (contact points) relative to the Sun. The observers would see different chords of the planet’s trajectory across the disk of the sun due to parallax. Venus forms two similar triangles with the Earth and the Sun at their opposite edges. Since the triangles are similar, the projected distance on the Sun can be found by comparing it to a known baseline on Earth. Once the angle shift is measured, the distances to Venus and then to the Sun can be determined using trigonometry.

Modern techniques use radar echo ranging to measure distances in the Solar System very precisely.

5 How big are stars?

5.1 How big is the Sun?

We can deduce the width of the sun if we know its angular diameter α and the distance D.

For small angles, $\alpha \approx \frac{2R_\odot}{D}$.

WARNING: This formula uses *radians* for α

Since the Sun's angular diameter is $\alpha \approx 0.5^\circ \approx 32$ arcmin $\approx 9.3 \times 10^{-3}$rad, and we know its distance (from Earth) $D = 1\mathrm{au} = 149,597,870$ km, we can now calculate the size of the Sun.

```
import warnings
warnings.filterwarnings('ignore')
import math
AU = 149597870 # km
ang_diameter_arcm = 32 # angular diameter of Sun is about 32 minutes of arc
ang_diameter = ang_diameter_arcm/60 * math.pi/180
# ang_diameter = ang_diameter_rad
diameter = AU * ang_diameter
print(f"1 au = {AU:1.3e} km")
print(f"angular diameter of the sun =  {ang_diameter:.3e} radians")
print(f"Diameter of Sun = {diameter:.0f} km")
print(f"Radius of Sun = {diameter/2:.0f} km")
```

```
1 au = 1.496e+08 km
angular diameter of the sun =  9.308e-03 radians
Diameter of Sun = 1392520 km
Radius of Sun = 696260 km
```

The accepted radius is 6.957×10^5 km.

5.2 What about other stars?

Here is a video showing a comparison between star sizes: https://www.youtube.com/watch?v=HEheh1BH34Q

5.2.1 A comparison of some star sizes

Here is a list of some selected examples of star radii, compared to the Sun. Some stars, such as Sirius B, are much smaller than the Sun. (The picture below shows an artistic impression of Sirius A and Sirius B.)

The typical range of star radii is roughly $10^{-3}R_\odot < R < 10^3 R_\odot$.

Name	Radius
Sirius B	0.008 $R_\odot$
Sirius A	1.71 $R_\odot$
Arcturus	12 $R_\odot$
Aldebaran	22 $R_\odot$
Rigel	78.9 $R_\odot$
Mira	210 $R_\odot$
Betelgeuse	320 $R_\odot$

Image credit: NASA, ESA and G. Bacon (STScI)

- Sirius B is a very small but very hot white dwarf. It is even smaller than Earth, and it cannot be observed by the naked eye. Its companion star Sirius A, in contrast, is the brightest star in the sky
- Betelgeuse – at a distance of 650 l.y. is a red supergiant and rather short-lived star, the second brightest in Orion and the ninth brightest in the sky
- It's close to the end of its life and en-route to a rather spectacular death within a million years or so
- For Betelgeuse, the angular diameter α is about 0.000014 degrees (1.4×10^{-5} degrees)
- Most stars have a much smaller angular diameter and cannot be directly imaged
- We have to deduce their size indirectly! (more on this later)

Image credit: ALMA (ESO/NAOJ/NRAO)/E. O'Gorman/P. Kervella

5.3 A handy guide to estimating angular diameters

You can estimate angular diameters using your hand! Extend your arm, place the hand at a right angle and follow the intstructions of the following figure to estimate different angles. This may not be useful to determine the size of objects that are very far and appear too small, such as stars, but can be useful to measure the angular distance between two stars.

Figure 5.1: Artistic impression of Sirius A (large blue-white star on the left) and Sirius B (small blue white-dwarf star on the right)

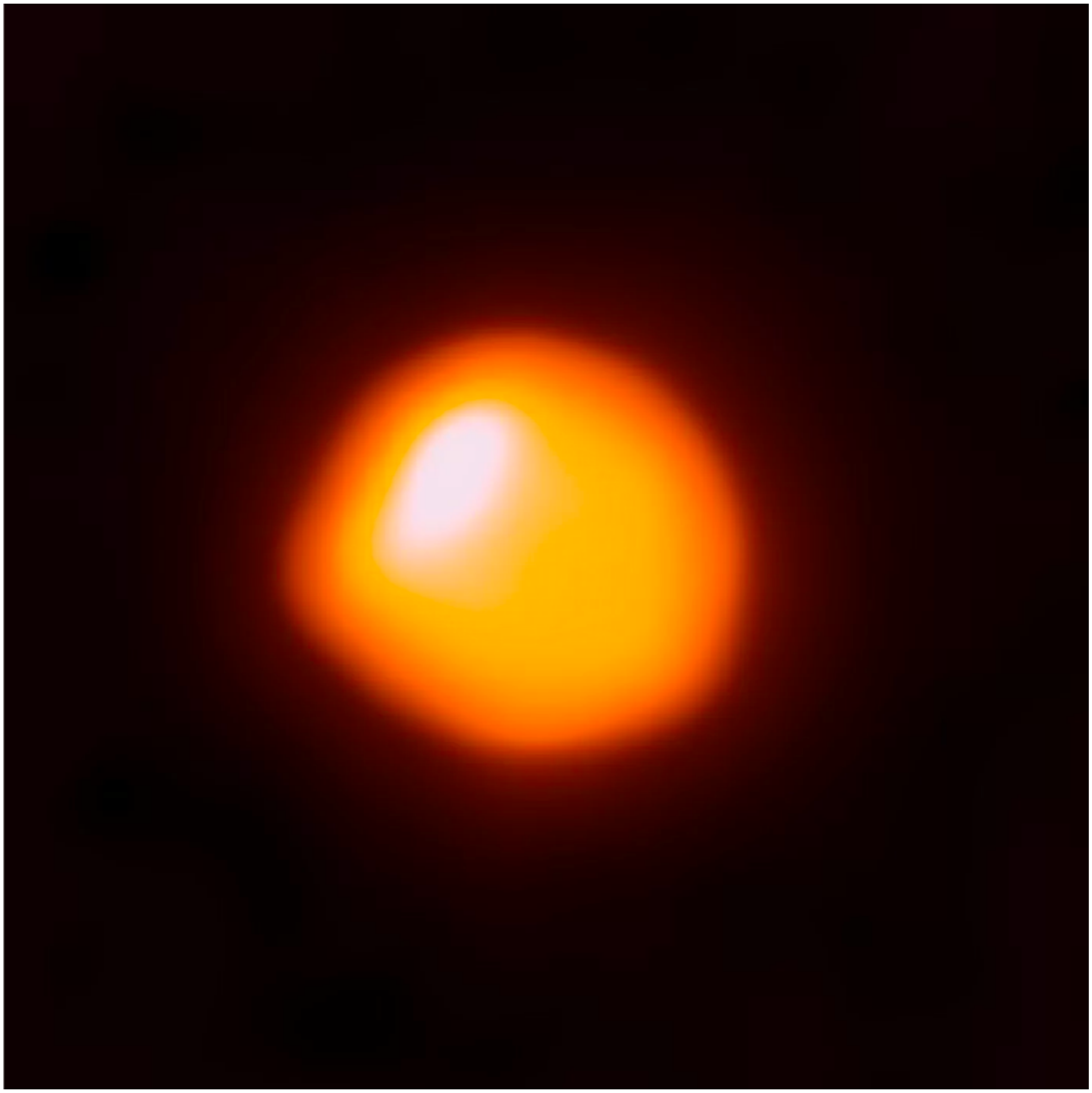

Figure 5.2: Betelgeuse

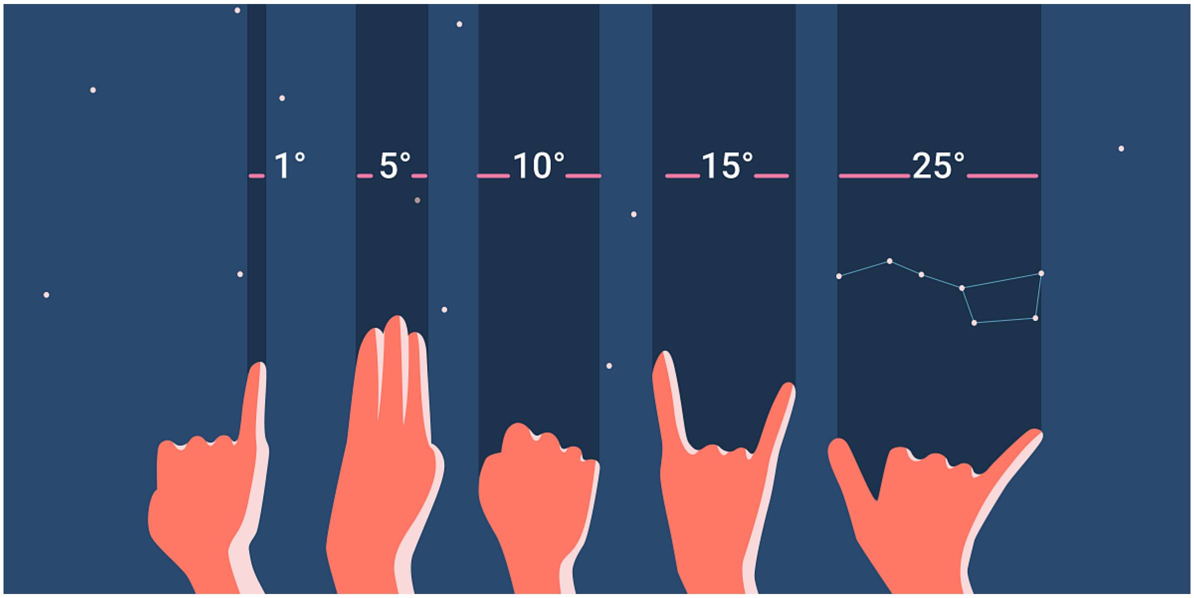

Figure 5.3: How to use hands to measure angular distances in the sky

5.4 How far apart are the stars?

Let's have a look at some orders of magnitude (this is a good resource if you want to visualise them: https://scaleofuniverse.com/en-gb).

- The nearest star (Proxima Centauri) is about 10^{16} m away.
- Diameter of the Sun ~ 10^{9} m.
- The difference in scale is 10^{7}m in distance, or 10^{21}m^{3} in volume.
- Relative to their size, stars are the most isolated structures in the universe.
- If stars were grains of sand (~1 mm), they would be ~10 km apart!
- However, we find them in clusters and binaries - what explains this? (more on this later)

6 How bright are stars?

If we try to answer this question based on our observations, we have to keep in mind that the apparent brightness of a light source will vary depending on its distance from the observer. The brightenss we observe is therefore not an *intrinsic* property of the stars, as it depends on the distance between the observer and the stars.

This is the reason why we need to differentiate between the *apparent brightness* of a star and its *luminosity*.

6.1 Apparent brightness and luminosity

The **apparent brightness**, F (units [W/m^2]), also referred to as the *radiant flux* or *power flux*, is the total amount of energy emitted by a star, per unit time, crossing an unit area. It is *not* an intrinsic property of a star, because it depends on the distance from the star.

The following figure can give you a better understanding of that. Each square represents an unit area. The lightbulb, like a star, radiates equally in every direction (i.e. isotropically). The 'rays' coming out of the light source represent the flux. The farther the position of the observer is from the source, the less flux the observer, in one unit area, will receive. The relationship between the power flux, F, and the distance to the source, r, is given by the inverse square law:

$$F \propto \frac{1}{r^2}.$$

Since the light source radiates isotropically (even though the figure below shows only a portion of the rays), the power flux F is equal across all parts of the sphere.

The **Luminosity** of a star, L, measured in Watts (W) or equivalently Joules per second (J/s), is its energy output per unit time. The luminosity is the *total* energy radiated in all the directions in an unit time and it does not depend on the distance from the observer: it is an intrinsic property of the star.

How are these two quantities related?

Imagine a sphere of radius r centered on a star, which is isotropically radiating. Since the power flux F is equal across all parts of the sphere and is the energy output, per unit time,

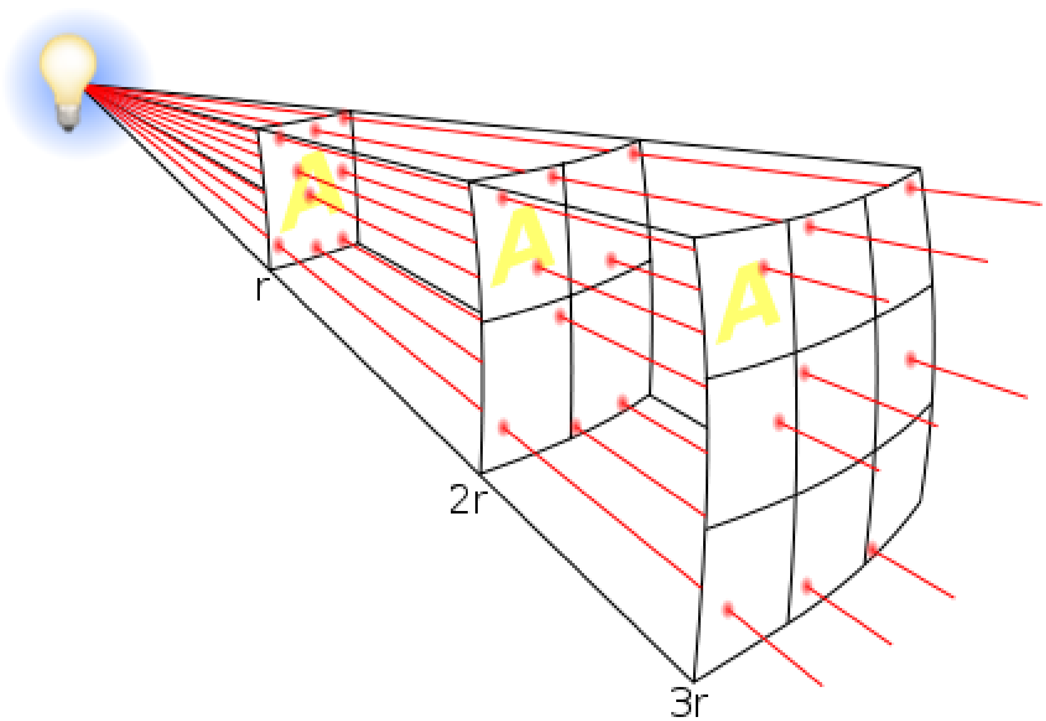

Figure 6.1: Inverse square law of the radiant flux of a light source

crossing an unit area, we need to divide the luminosity equally over the area of the sphere, $4\pi r^2$. Therefore

$$F = \frac{L}{4\pi r^2} \tag{6.1}$$

The apparent brightness depends on the luminosity and on the distance of the observer from the star. From the formula above, you can see how two stars of equal luminosity, located at different distances from Earth, would have different apparent brightness.

Photometry is the measurement of the quantity of light from a star, i.e. the flux of energy per unit time, per unit area, that we receive at Earth.

For a particular star, if we know the luminosity (L) of the star and its distance from Earth (r) we can calculate the flux using the equation above.

6.1.1 The Sun's luminosity

For our Sun, we have:

$F_\odot = 1361\,\mathrm{W/m^2}$ (The "Solar Constant", or "Solar irradiance", sometimes indicated with the symbol $S_\odot$ in textbooks)

$r = 1\,\mathrm{a.u.} \approx 1.5 \times 10^{11}\,\mathrm{m}$

so

$L_\odot = 4\pi r^2 F = 3.8275 \times 10^{26}\,\mathrm{W}$

```python
import scipy
from scipy.constants import pi, au, parsec

F = 1361.0 # W/m/m
r = au # m
L = F*4*pi*r**2

print(f'Power flux: F = {F} W/m/m')
print(f'r = 1au = {r:1.4e} m')
print(f'Luminosity: L =  F*4*pi*r**2 = {L:1.4e} W')
```

```
Power flux: F = 1361.0 W/m/m
r = 1au = 1.4960e+11 m
Luminosity: L =  F*4*pi*r**2 = 3.8275e+26 W
```

For other stars there is a large range of luminosities:

$10^{-4}\, L_{\odot} < L < 10^{6}\, L_{\odot}$

If we think of the least luminous star as a 100W incandescent lightbulb, this huge range tells us that the most luminous star would be like the entire world's power output

Future question: **Why is there this very large range?**

6.1.2 'Spot the mistake' question

What would the apparent brightness be for a star like the Sun at a distance of 10 pc? Something does not seem to make sense in the answer below, why?

```
# This is the version containing a mistake
L = 3.8275E26 # W
r = 10 #pc
F = L/(4*pi*r**2)
print(f'Solar luminosity = {L} W')
print(f'Distance r = {r} pc')
print(f'Apparent brightness of the Sun at 10 parsec: F = {F:1.4e} W/m/m')
```

```
Solar luminosity = 3.8275e+26 W
Distance r = 10 pc
Apparent brightness of the Sun at 10 parsec: F = 3.0458e+23 W/m/m
```

Does this make sense?

Hints

Hint 1: compare this apparent brightness obtained below with the value of the power flux used (the Solar constant) in the previous example. What is wrong?

Hint 2: Do a 'sanity check' of the units in the quantities used above. Does it seem correct?

If we compare this value obtained for the apparent brightness, $F = 3.046 \times 10^{23}\mathrm{W/m}^2$, with the Solar constant used in the previous example, $F_{\odot} = 1361\mathrm{W/m}^2$, we can immediately notice that something is wrong. If we are observing the Sun from a further position of 10pc instead of 1au, we cannot expect it to appear brighter!

Let's do a dimensional analysis 'sanity check' by writing down the units of the quantities given above, and by replacing them in the formula to determine the flux from the luminosity:

$$F = \frac{L}{4\pi r^2} \implies [W/m^2] = \frac{[W]}{[pc]^2}$$