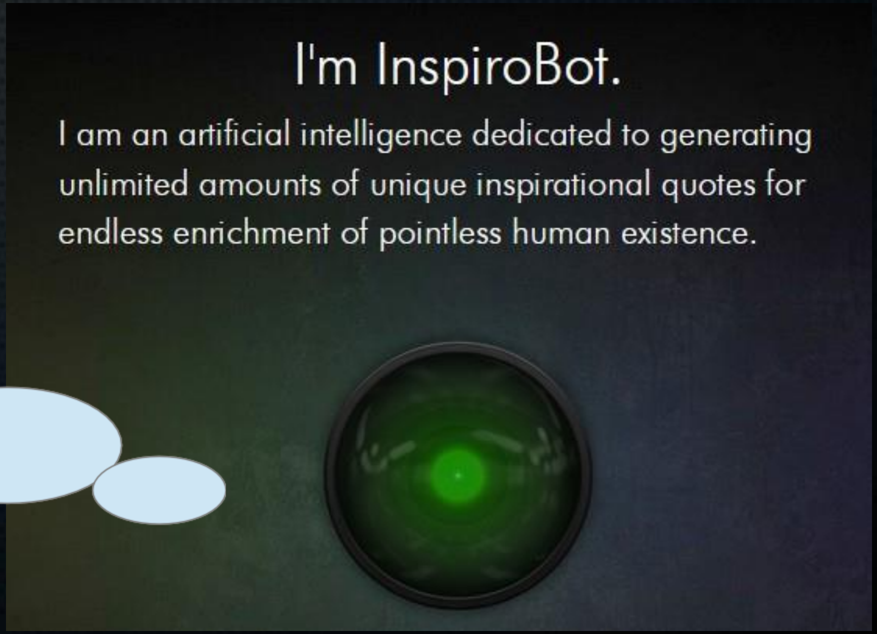
Daily Inspiration



Today

- Recap yesterday
- Logistic regression: using regression tools for classification
- Neural network basics

Yesterday

- Cost function: (differentiable) function that shows how wrong an estimate is for given parameters.
- Gradient descent: one common way to minimise the cost function automatically, i.e. to get optimal parameters
- Linear regression: very simple model that assumes that value to predict is linear combination of input features.
- Overfitting and underfitting, bias and variance: want our model to work wel for unseen data. Need just enough model freedom given the complexity of our problem. How:
  - Cross-validation to measure ability to generalise + get best hyperparameters
  - Use learning curves to diagnose bias vs. variance

Logistic regression

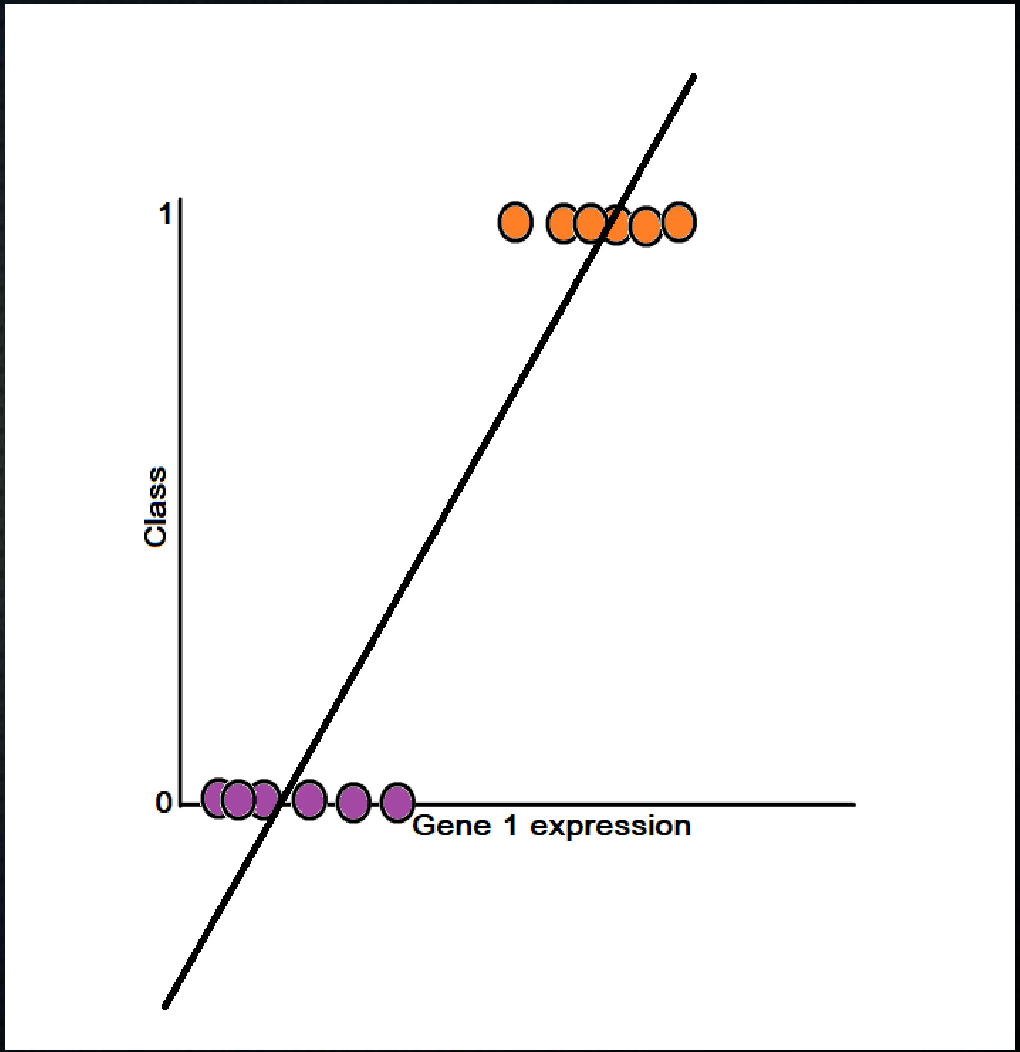
- You tell me: what is logistic regression?

Logistic regression

- Use regression-like framework for classification

Logistic regression

- Naïve idea:
  Train a linear regression. If Class >= 0.5, predict class 1. Otherwise, class 0.



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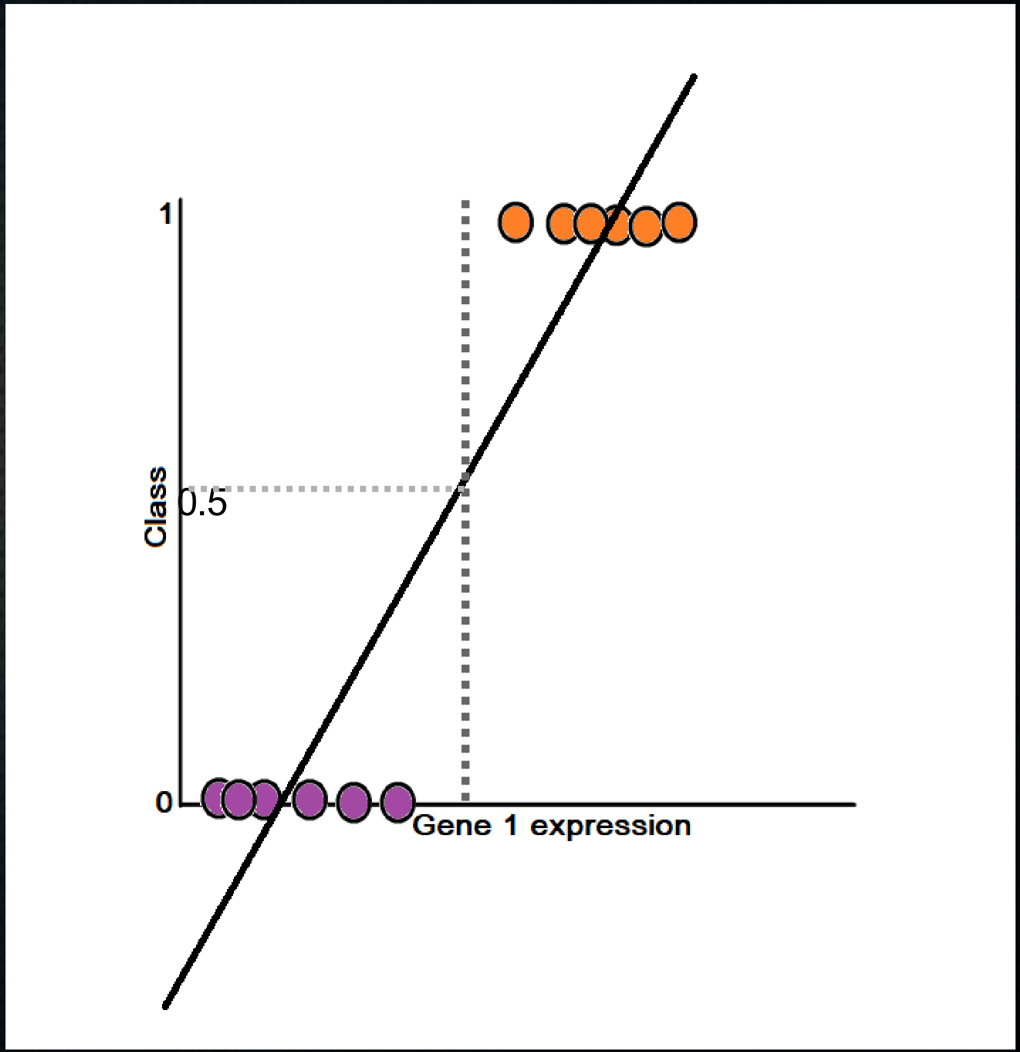

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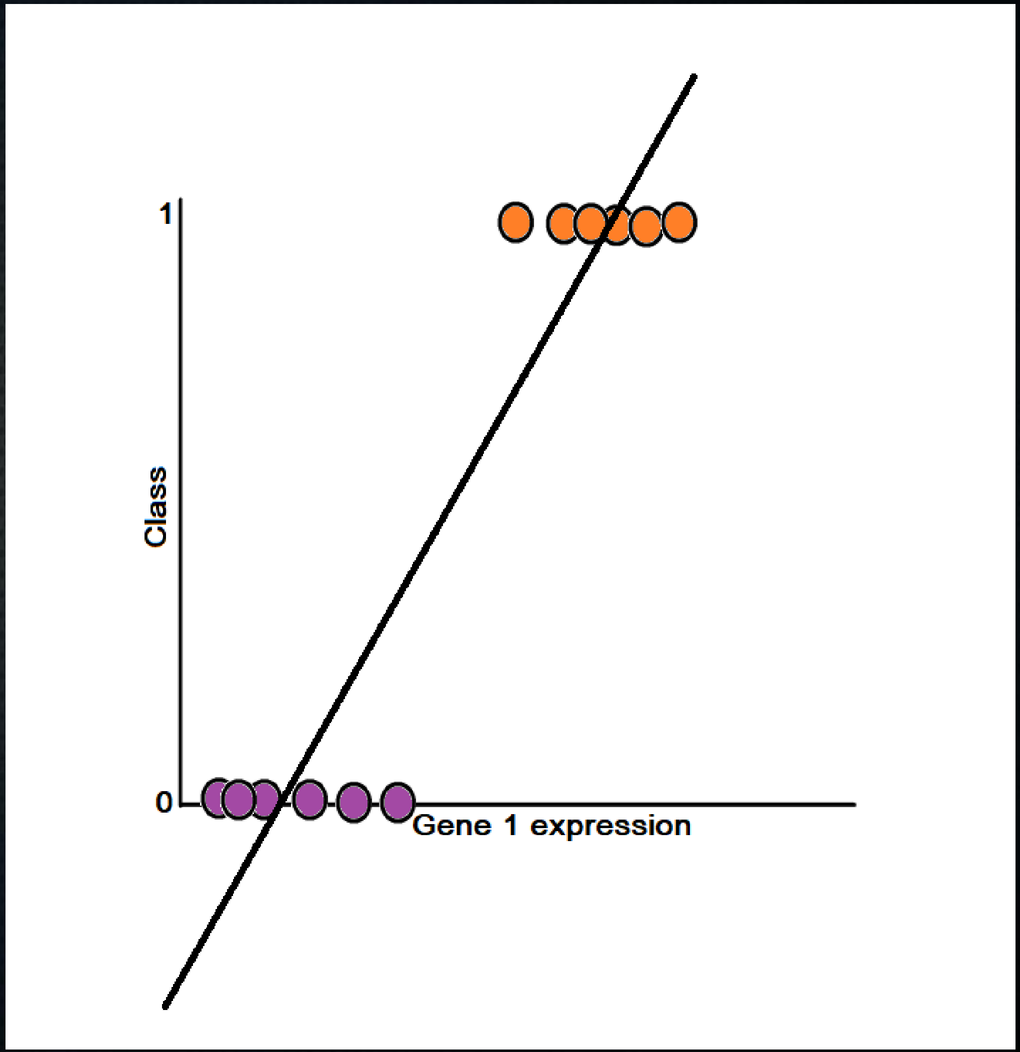

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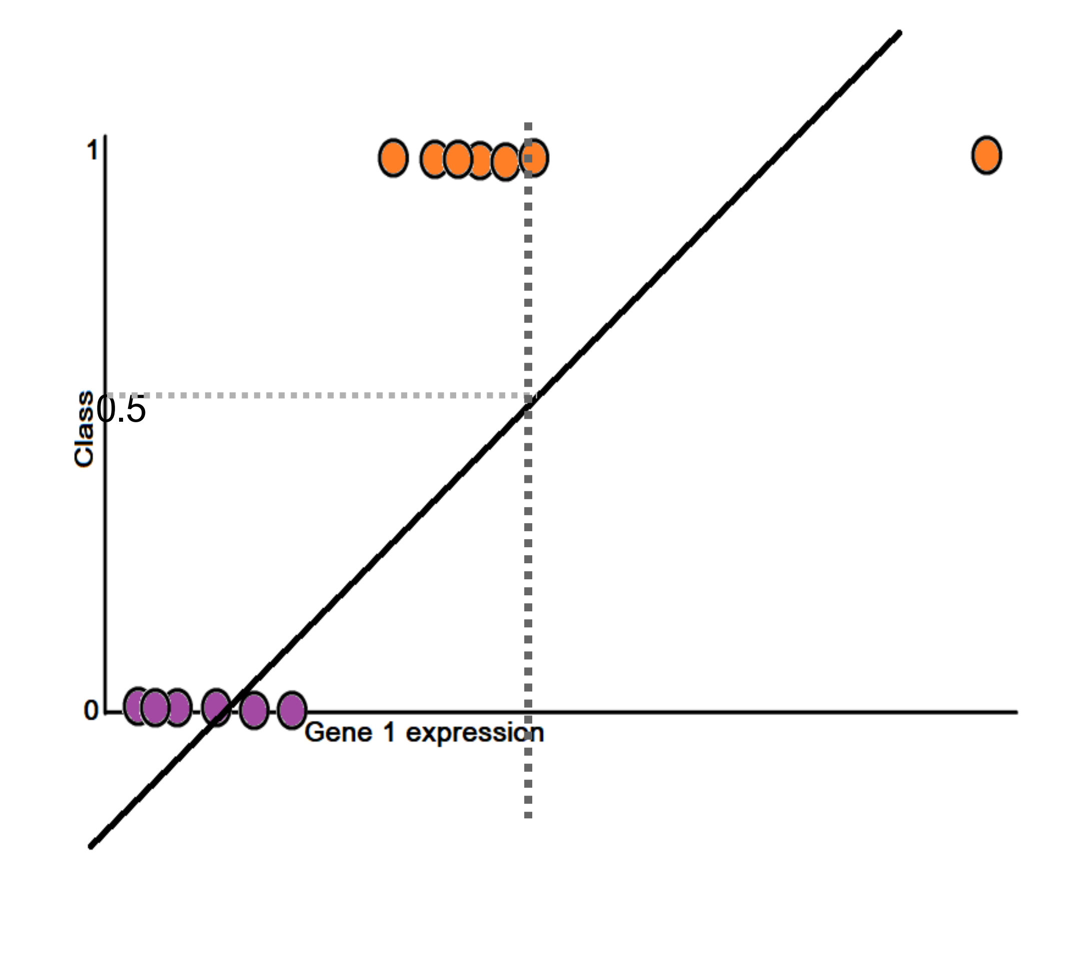

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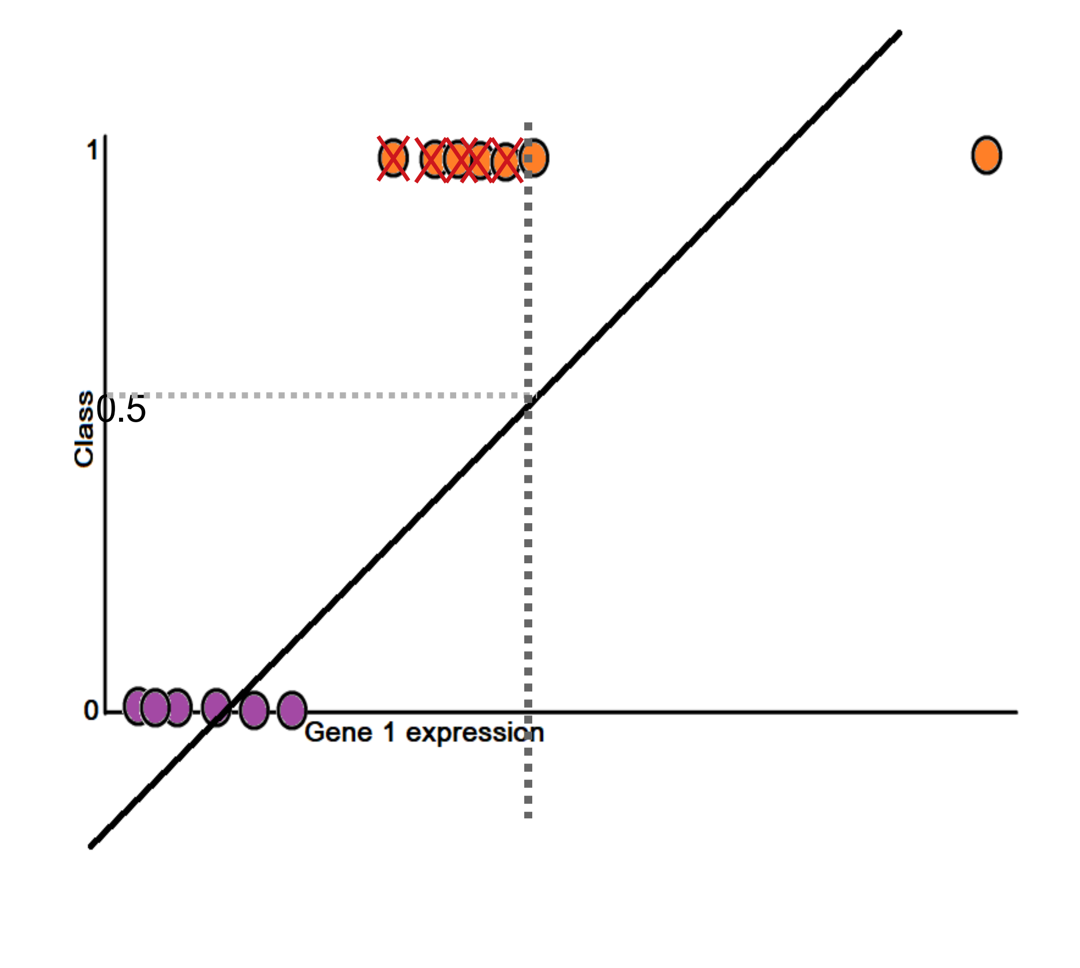
  -This example seemed to work, but quickly breaks down →
  get what is basically confirmation of hypothesis, but perform worse!



Logistic regression

- What we want:
  - Use the information that we only have two classes, 0 or 1.
  - Hypothesis function should output only numbers between 0 or 1.

Sigmoid or logistic function

- Before, our hypothesis function was of the form:

$$h_\theta(x) = \theta^T \cdot x$$

Sigmoid or logistic function

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$$h_\theta(x) = \theta^T \cdot x \longrightarrow [0.5 \;\; 3 \;\; -1.5] \cdot \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}$$

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Features for one sample (x0 = 1, intercept term, 2 data-derived features x1 and x2)

Sigmoid or logistic function

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Sigmoid or logistic function

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- Change that to the following:

$$h_\theta(x) = g(\theta^T \cdot x)$$

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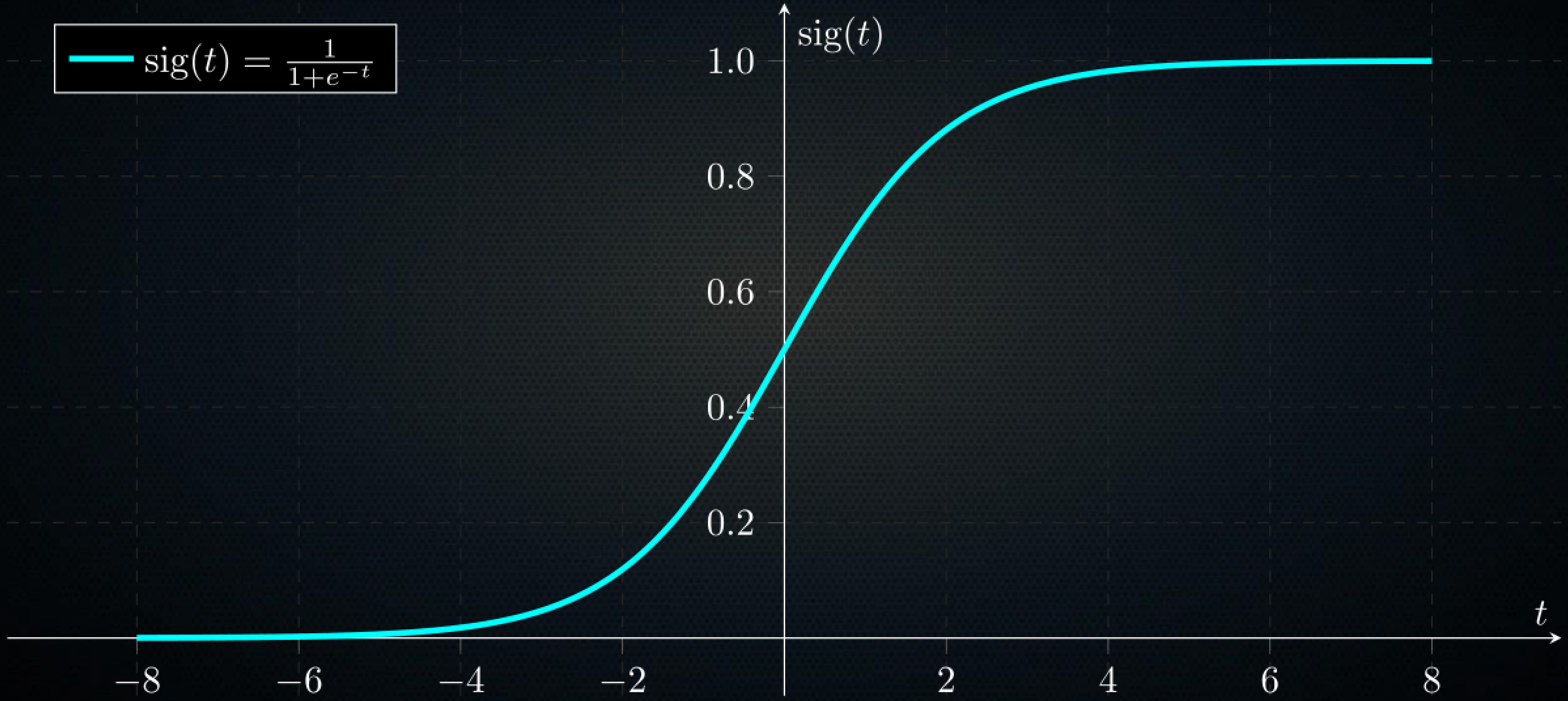
- What does that look like?

$$z \to \infty, e^{-z} \to 0$$

$$z \to -\infty, e^{-z} \to \infty$$

What does the sigmoid function look like?



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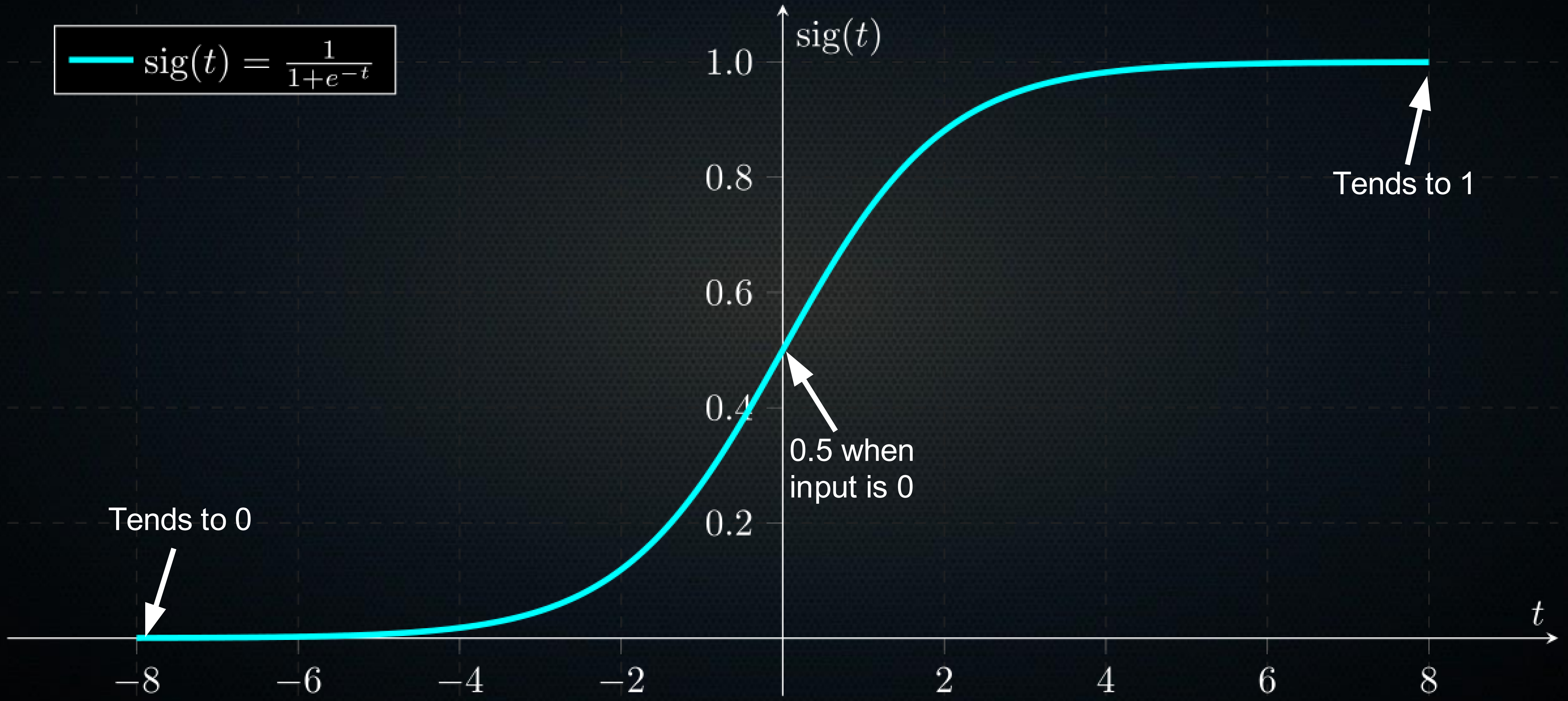

Sigmoid or logistic function

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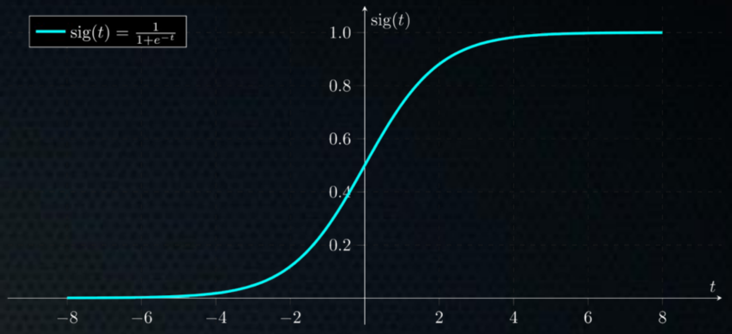

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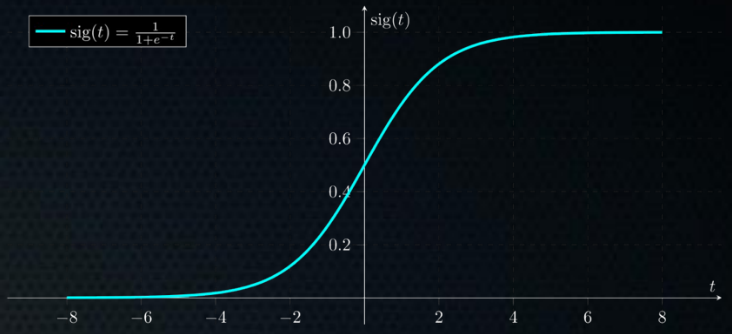

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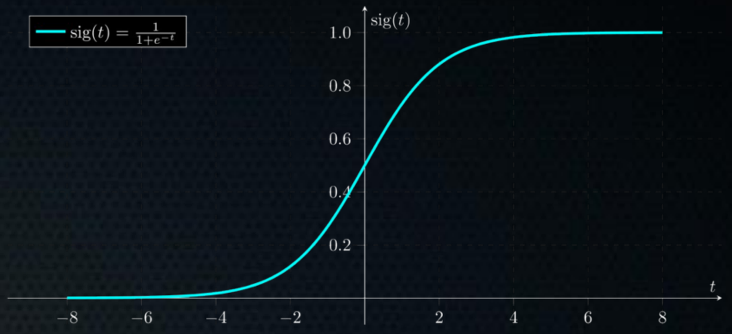

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$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{Tumor size} \\ \text{Neovascularisation level} \end{bmatrix}$$

$h_\theta(x) = 0.8$ → 80% chance of tumor being malignant

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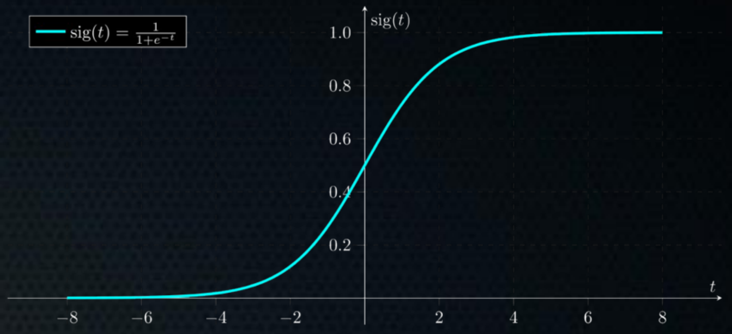

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$h_\theta(x) = 0.8 \longrightarrow$ 80% chance of tumor being malignant (class 1)

100% - 80% → 20 % chance of being benign (class 0)

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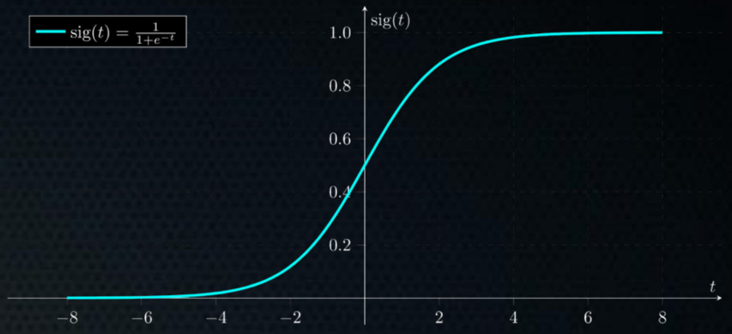

  - Interpret outcome of $h_\theta(x)$ as probability that class = 1 given the features.
  - Formally:

$$h_\theta(x) = \frac{1}{1+e^{-(\theta^T \cdot x)}} = p(y=1|x;\theta)$$

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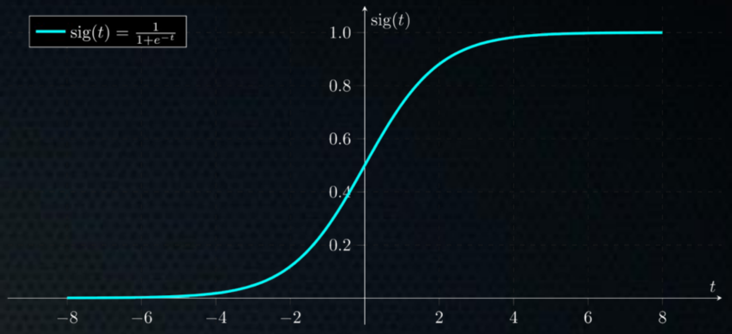

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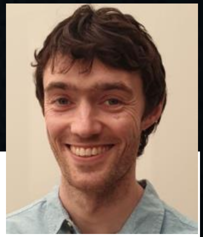
$$\frac{p(y=1)}{1-p(y=1)}$$

Log odds



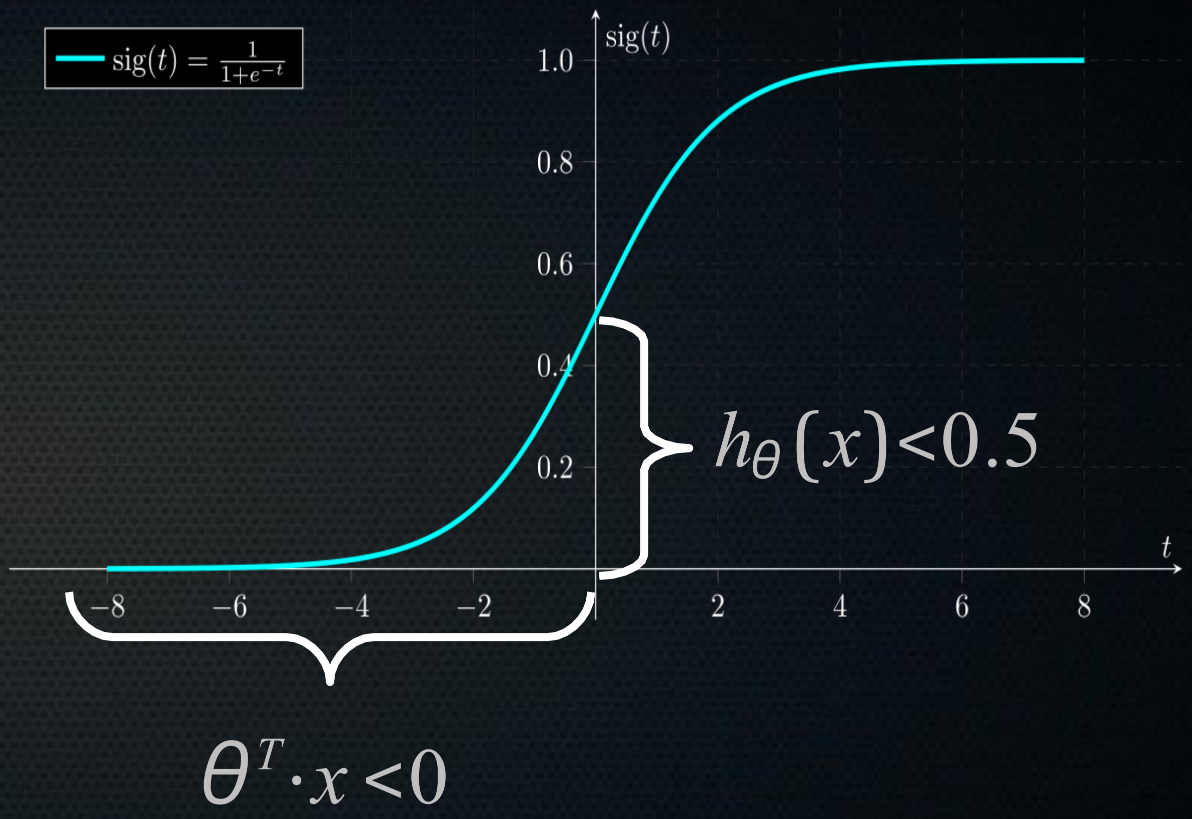
Coefficients:

	Estimate	Std. Error	z value	Pr(>|z|)	
(Intercept)	-8.922213	0.879485	-10.145	< 2e-16	***
pregnant	0.106275	0.039885	2.665	0.00771	**
glucose	0.036555	0.004546	8.041	8.88e-16	***
pressure	-0.008019	0.006230	-1.287	0.19801	
triceps	0.001909	0.008258	0.231	0.81721	
insulin	-0.001394	0.001098	-1.269	0.20445	
mass	0.082665	0.017927	4.611	4.00e-06	***
pedigree	0.901691	0.374350	2.409	0.01601	*
age	0.020417	0.010854	1.881	0.05995	.

logOdds(diabetes) = -8.9 + (0.106*pregnant) + (0.037*glucose)…. + (0.02*age)

Decision boundary

- Threshold:



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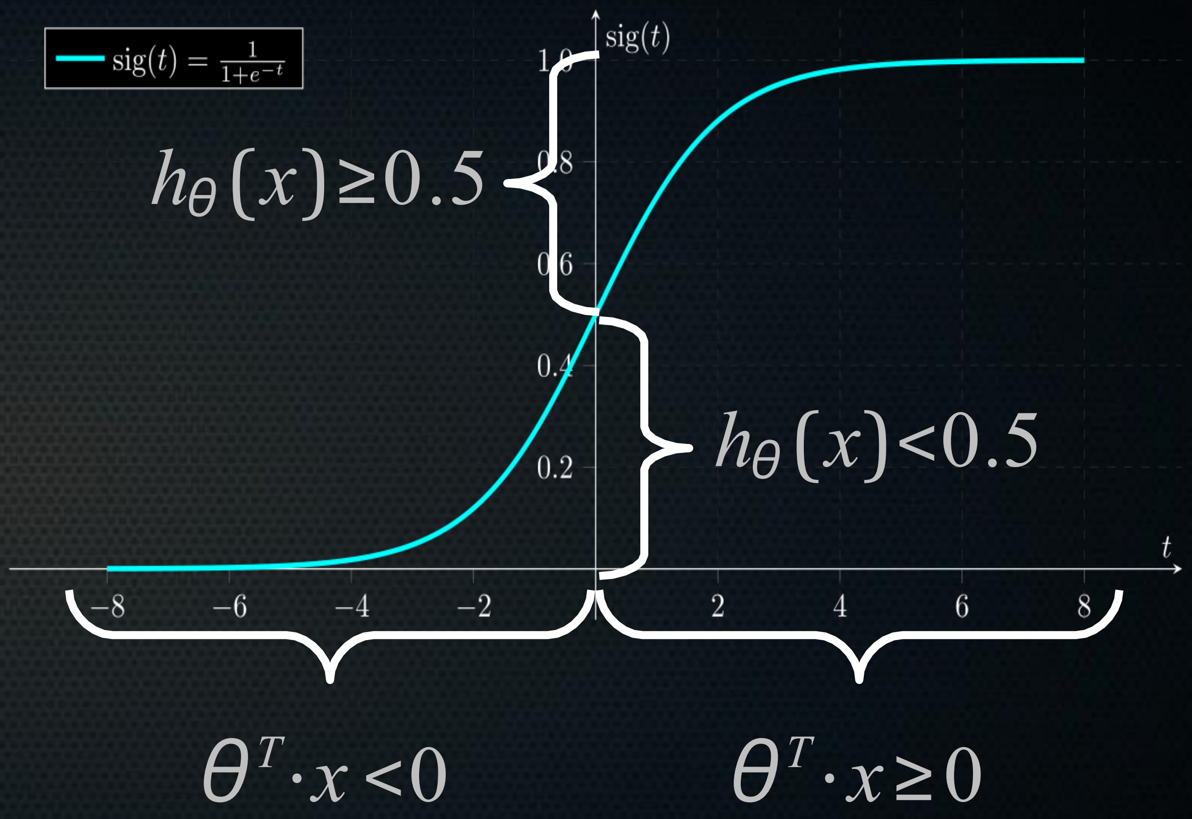

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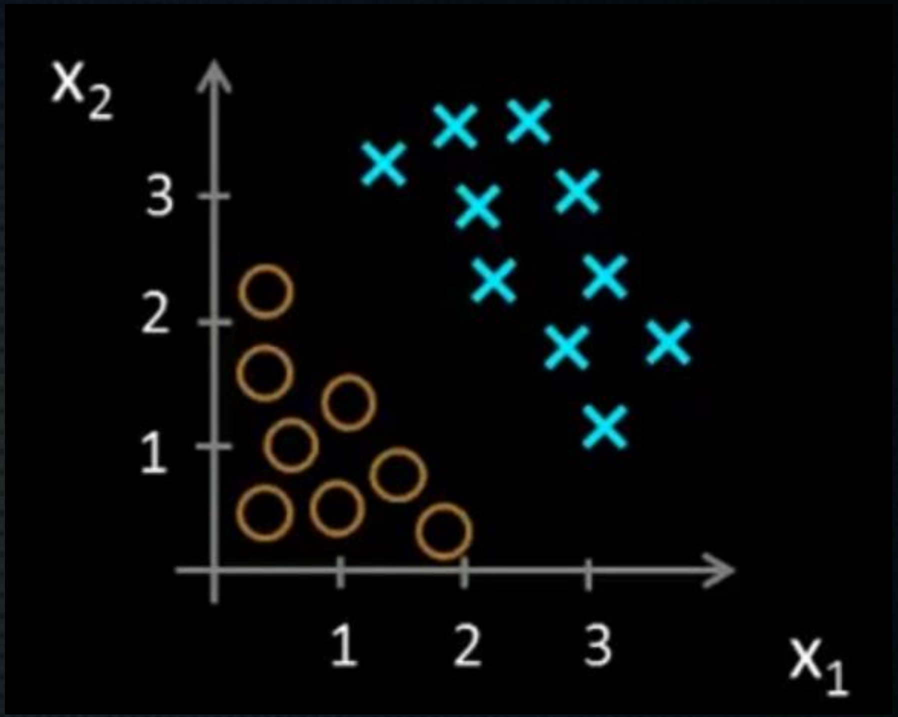
- How does it look? $g(z) = \frac{1}{1+e^{-z}}$

$$h_\theta(x) = g(\theta_0 \cdot x_0, \theta_1 \cdot x_1, \theta_2 \cdot x_2)$$



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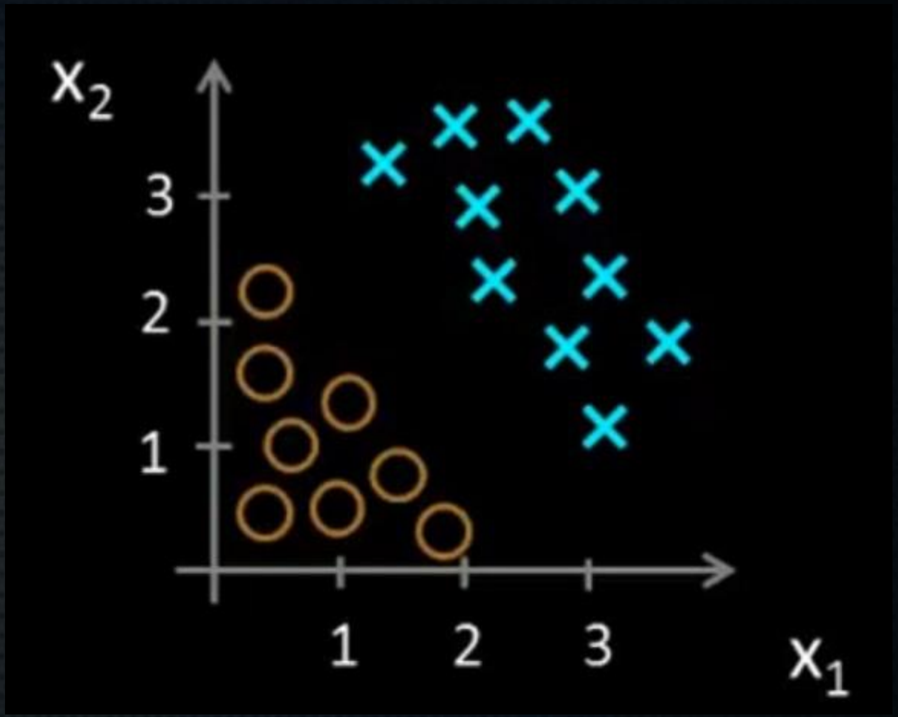

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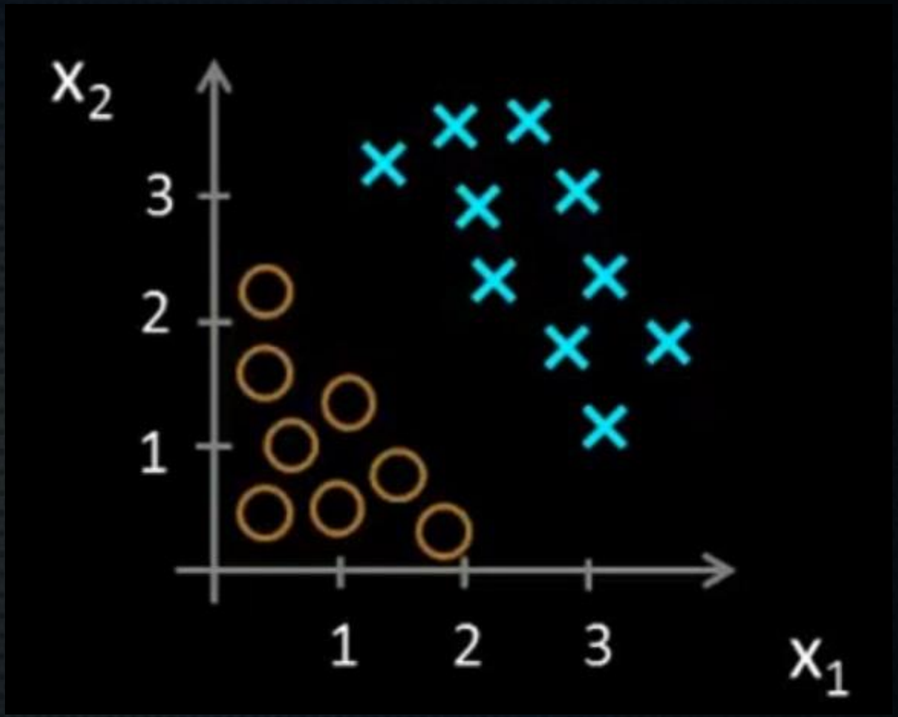

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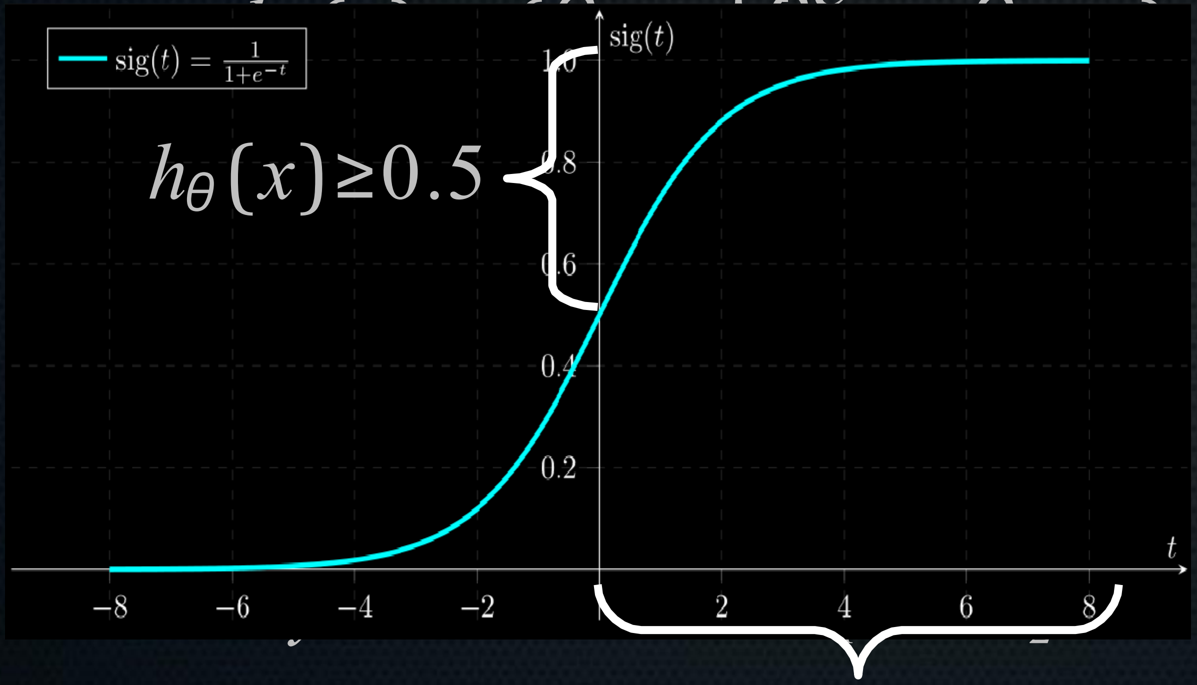
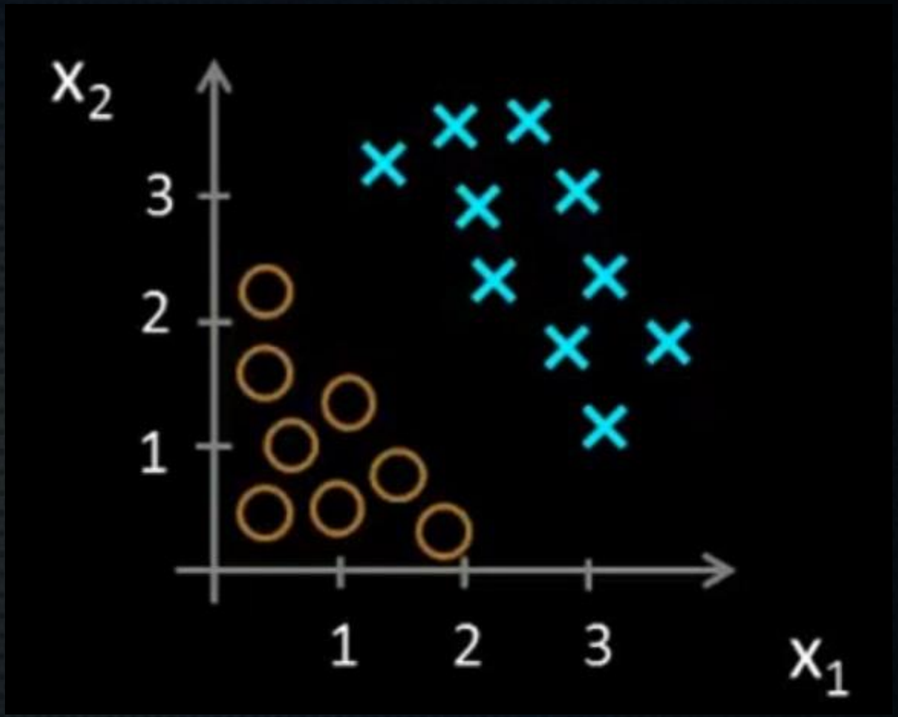


$h_\theta(x) \geq 0.5$

$\theta^T \cdot x \geq 0$



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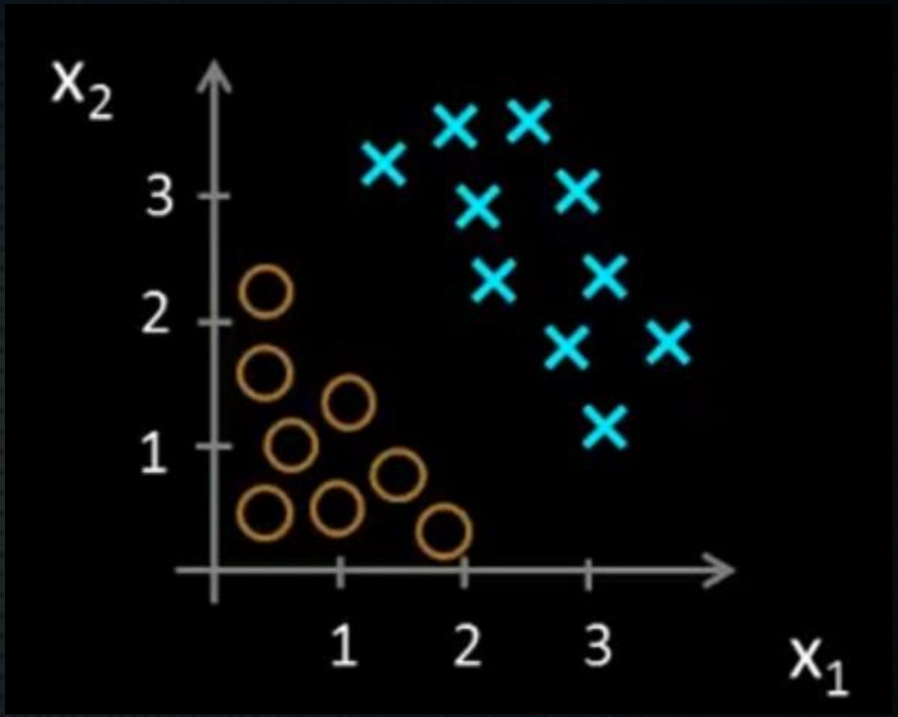
$$x_1+x_2=3$$



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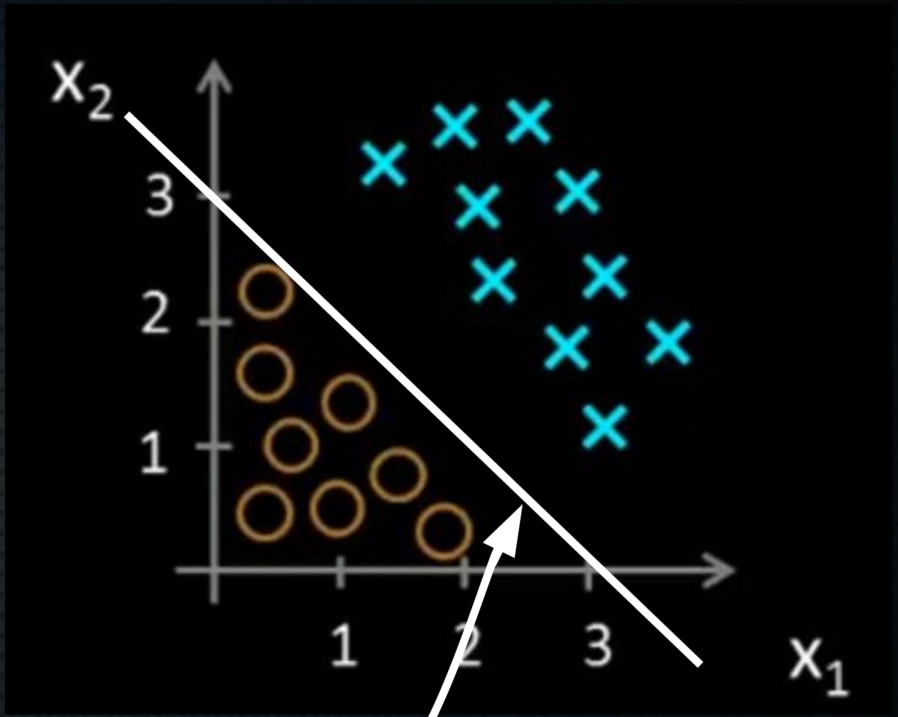

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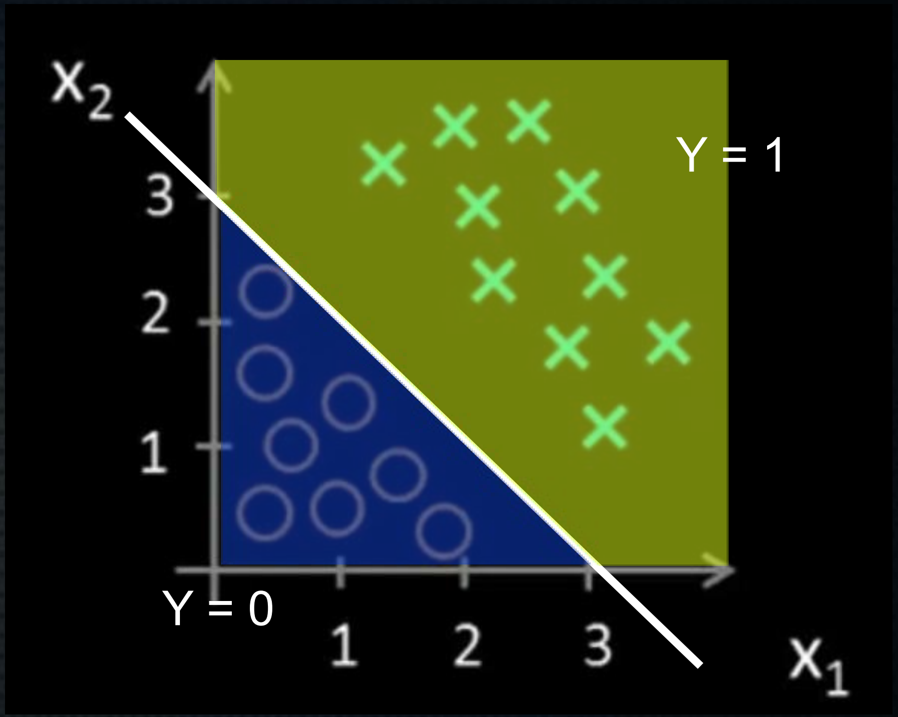

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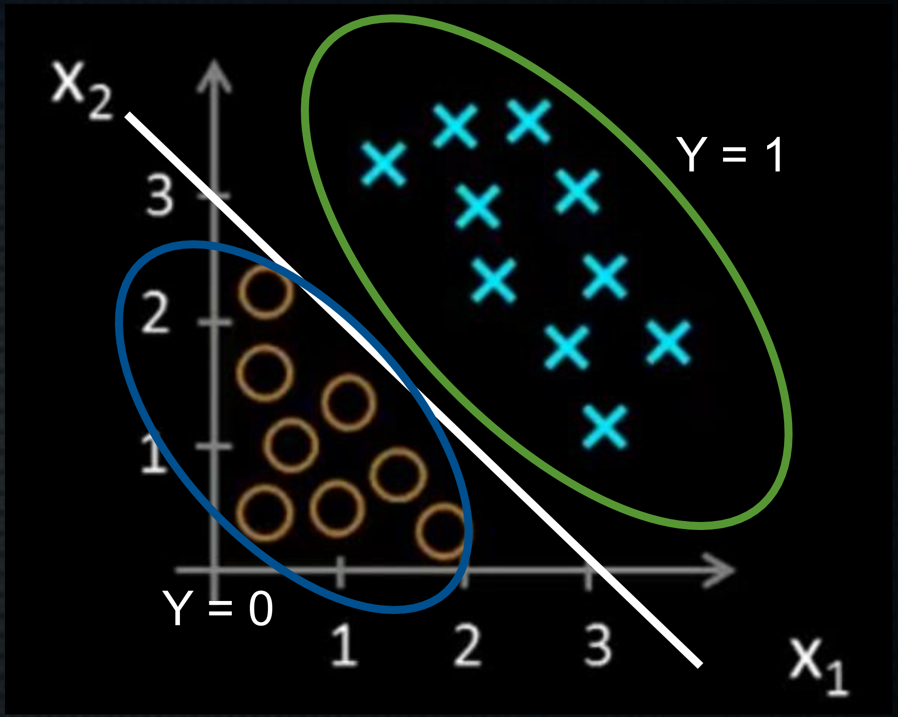

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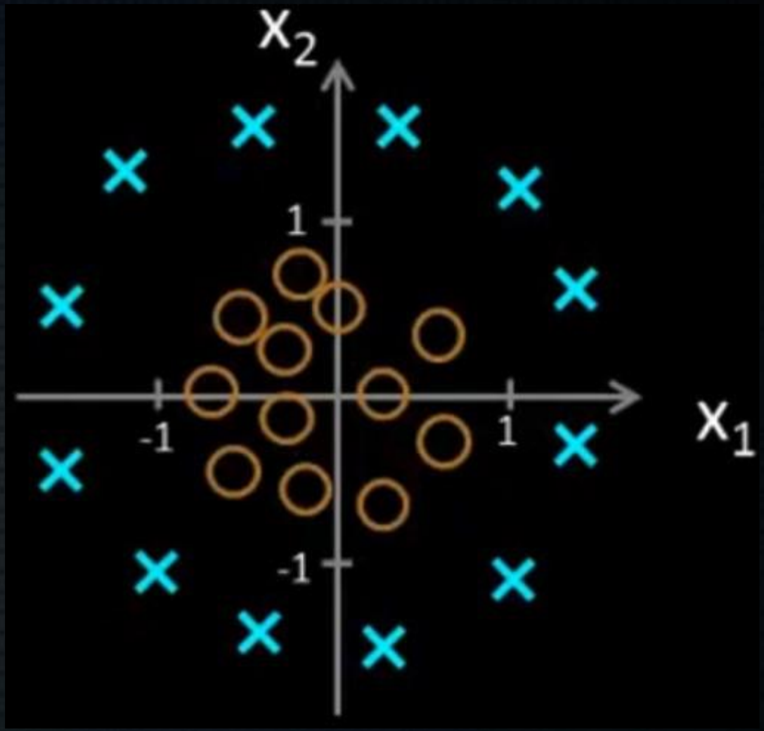

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Non-linear decision boundary

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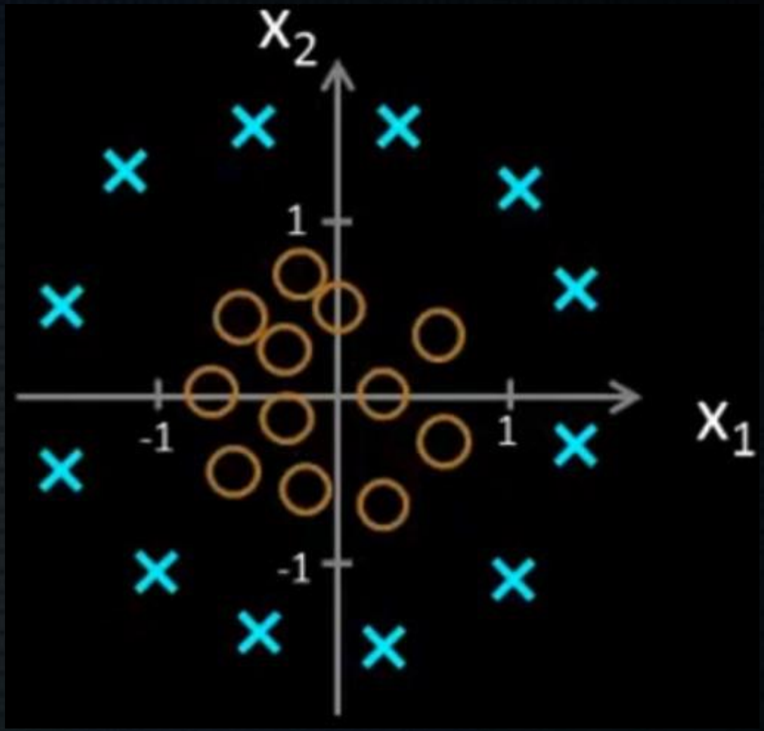
$$h_\theta(x)=g(\theta_0 x_0,\theta_1 x_1,\theta_2 x_2,\boxed{\theta_3 x_1^2,\theta_4 x_2^2})$$

- Add two polynomial features



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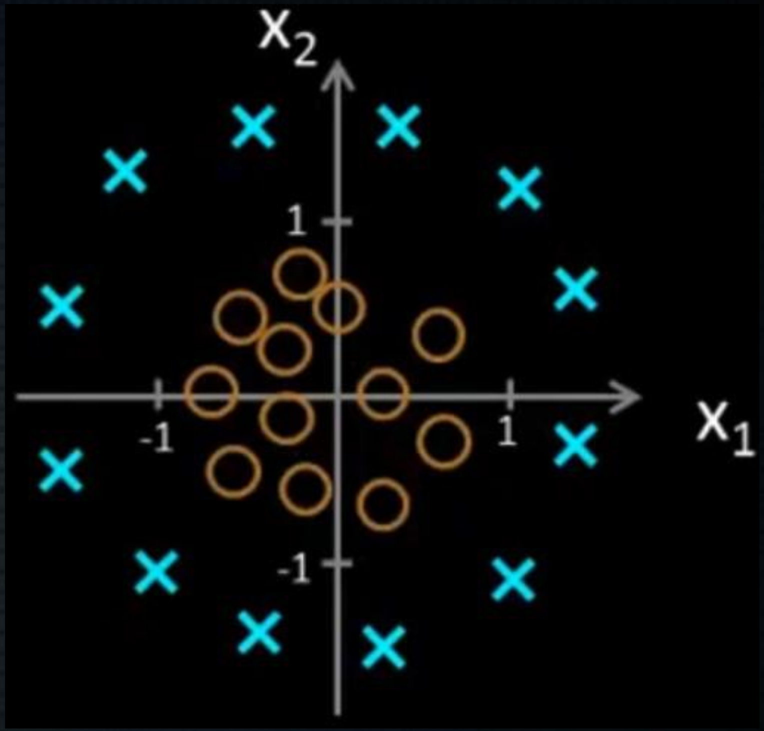

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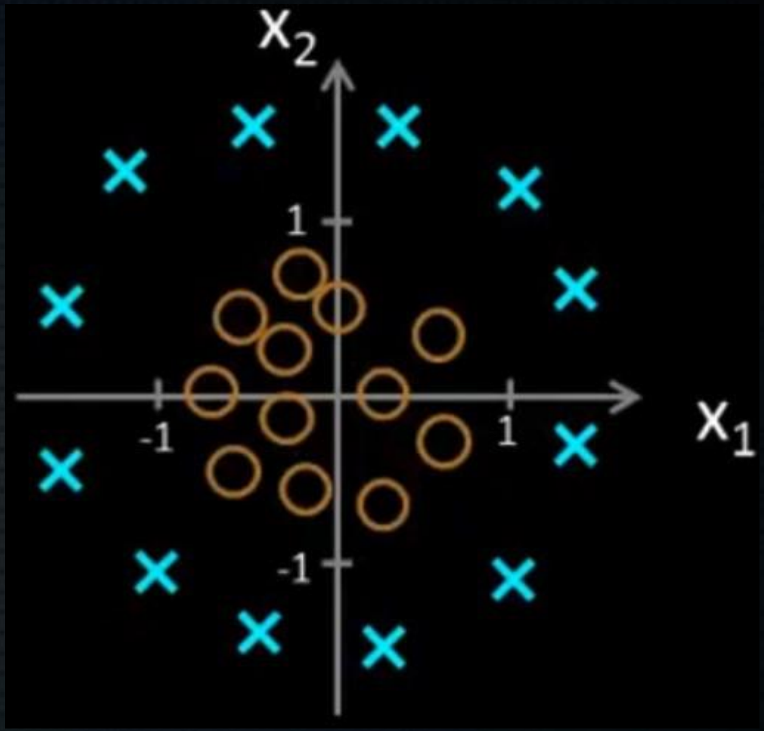
Can you work out what the decision boundary will be?



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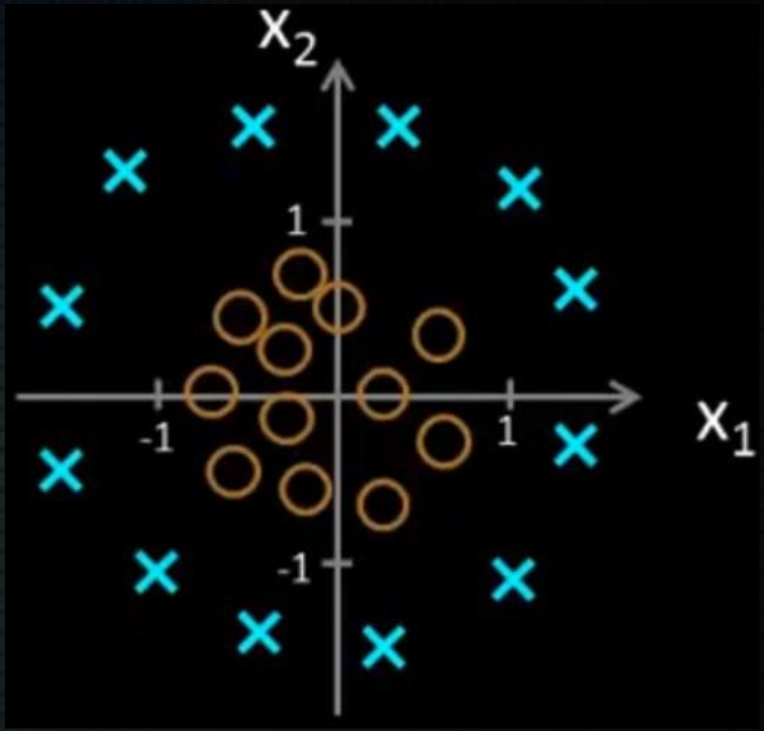

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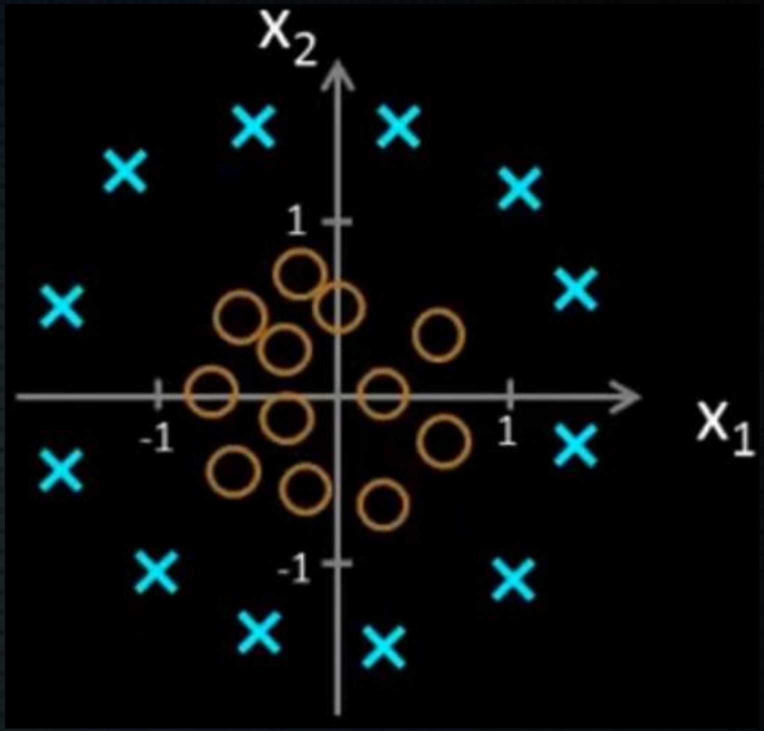

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$$x_1^2 + x_2^2 \geq 1$$

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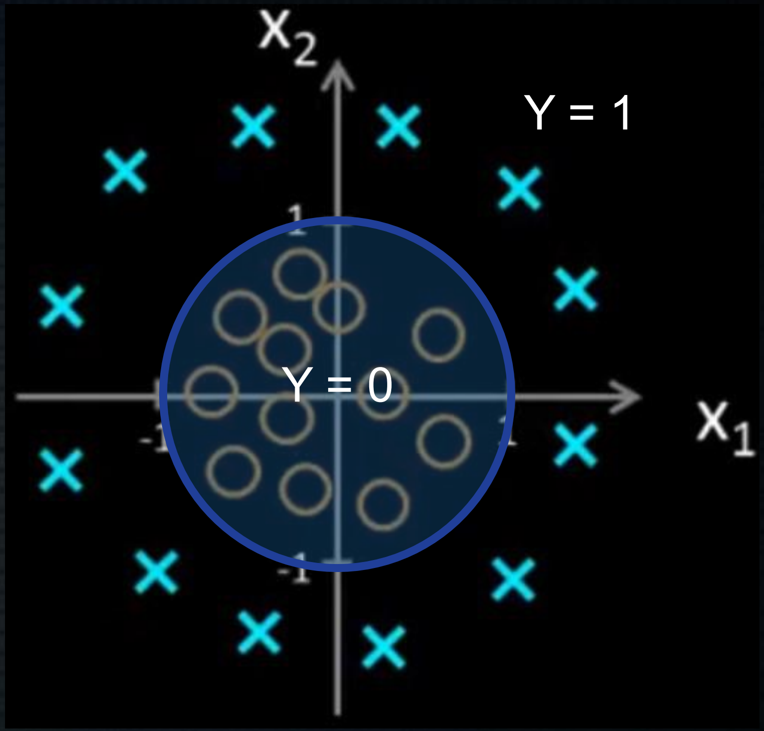
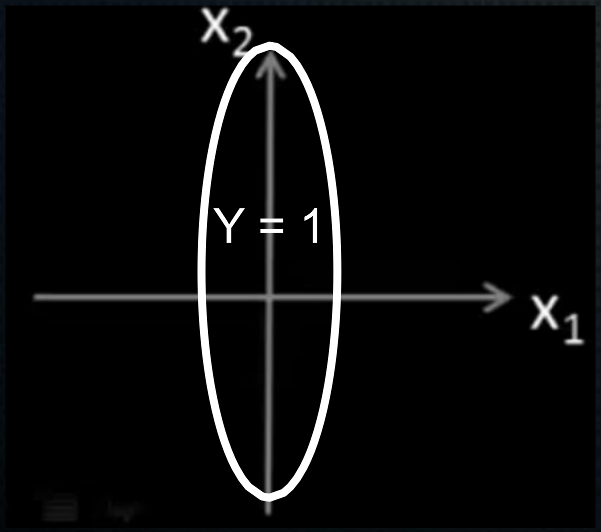
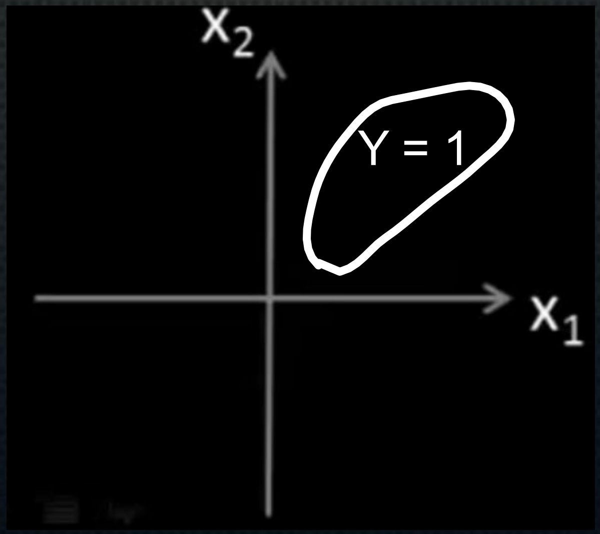
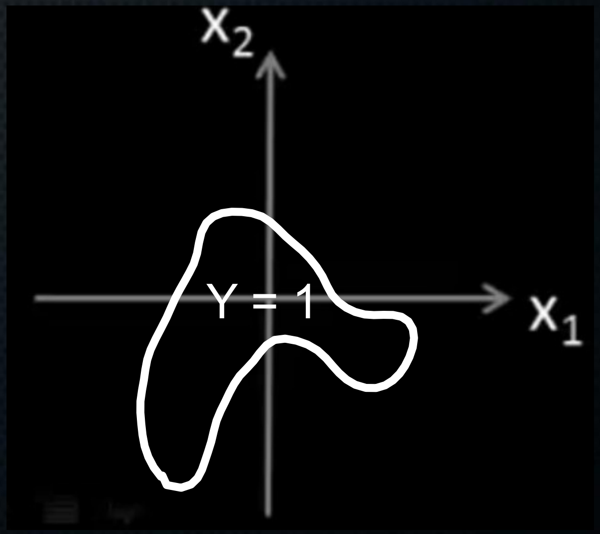


Non-linear decision boundary

- How does it look?
- If you add more and higher-order polynomial features, you can get complex boundaries:







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$$J(\theta)=\frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}(h_\theta(x^{(i)})-y^{(i)})^2$$

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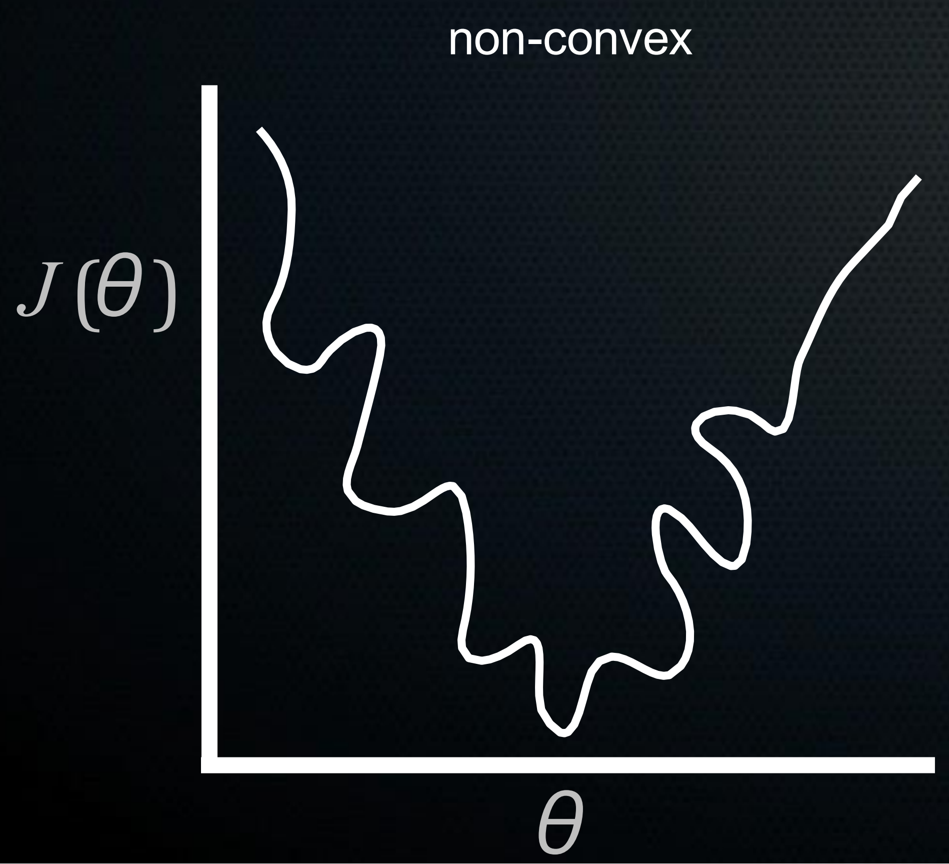

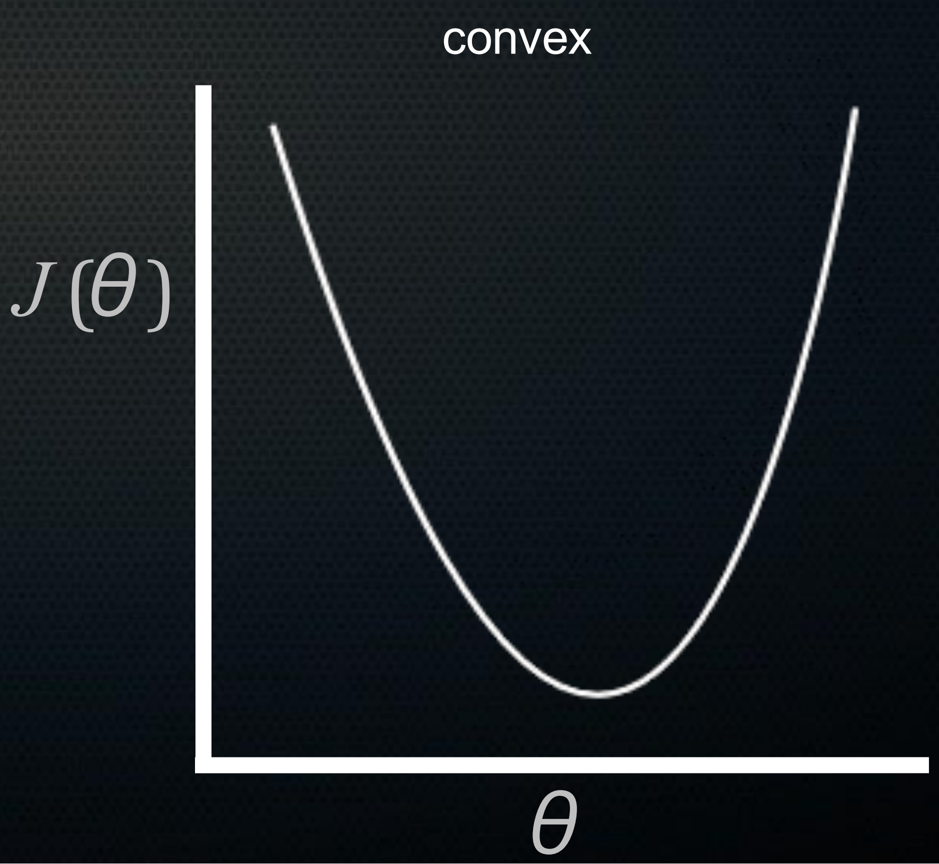

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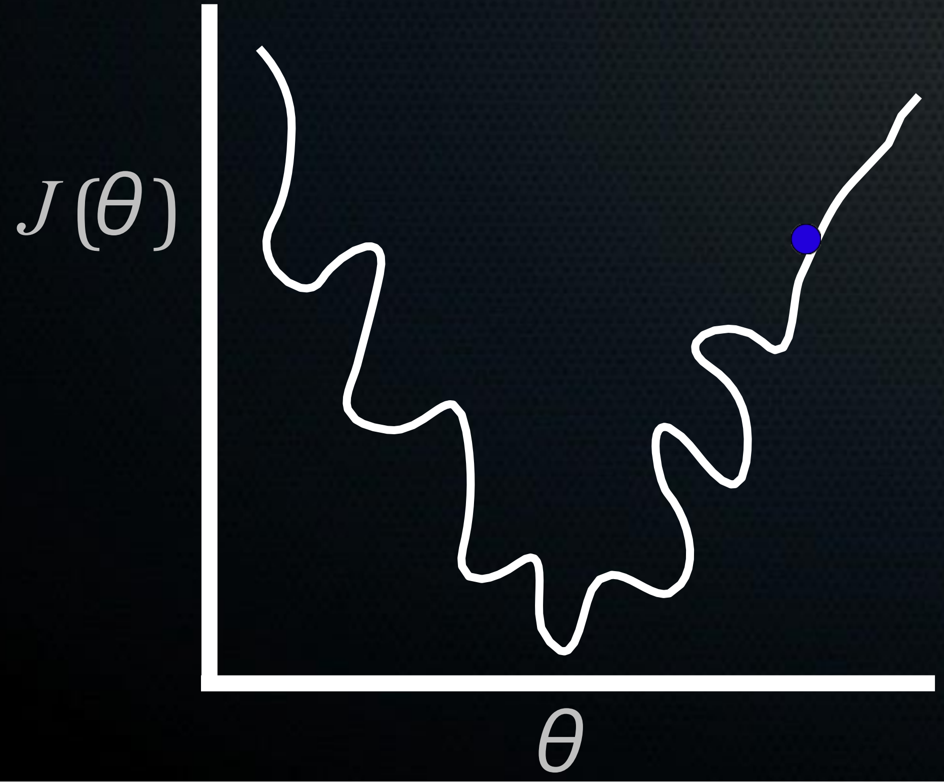
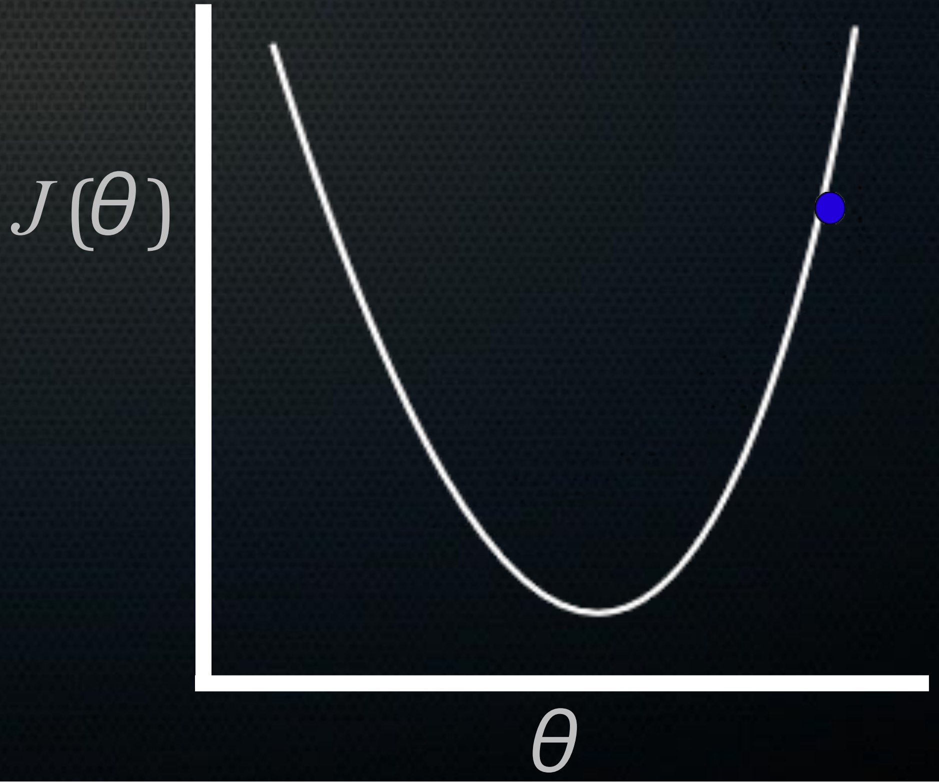


convex



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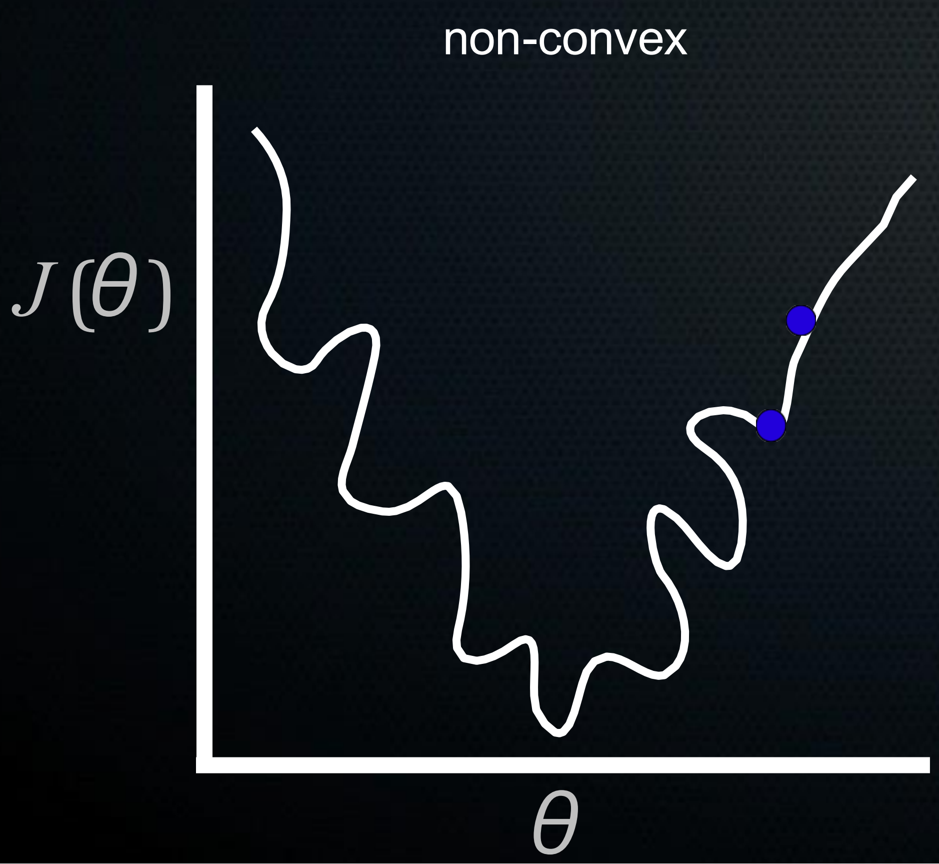

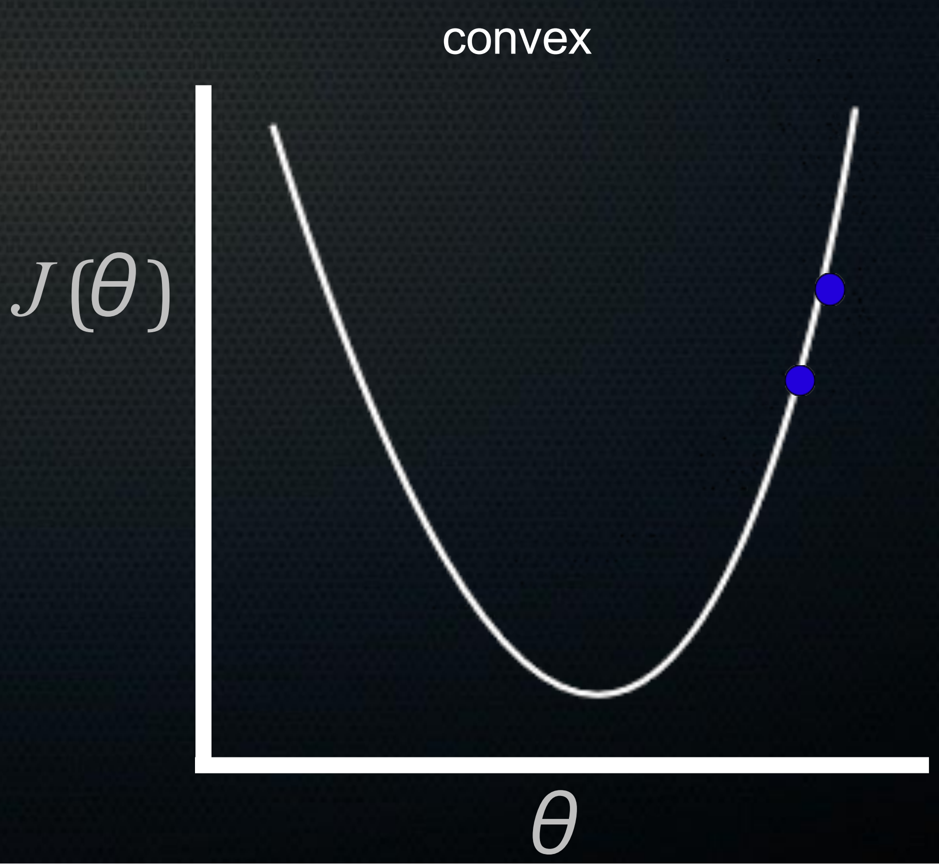

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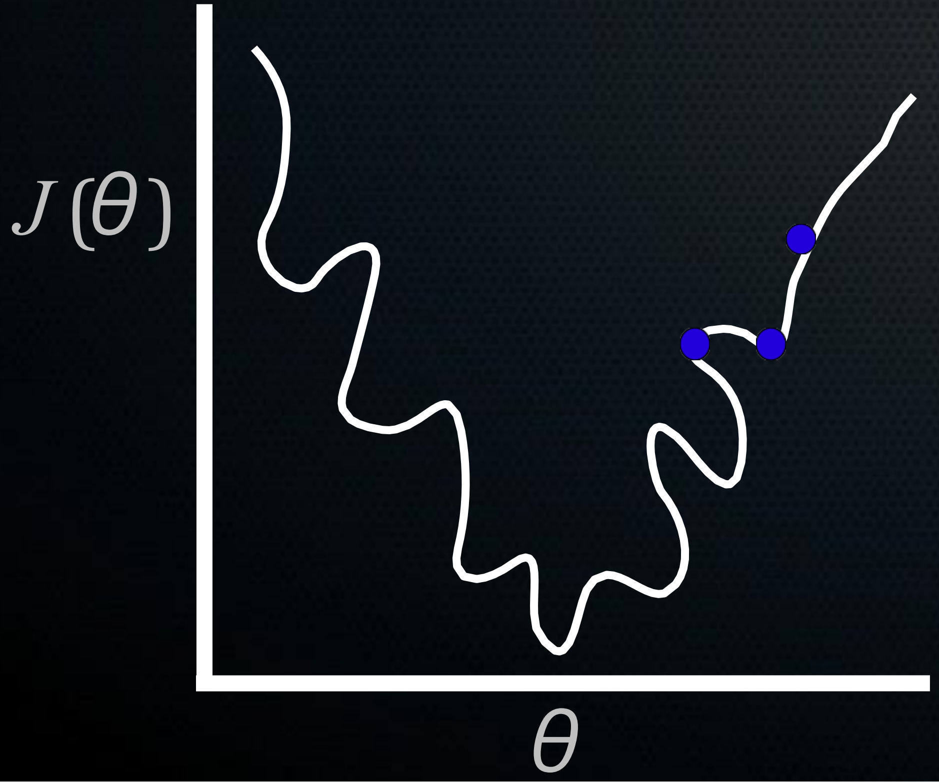

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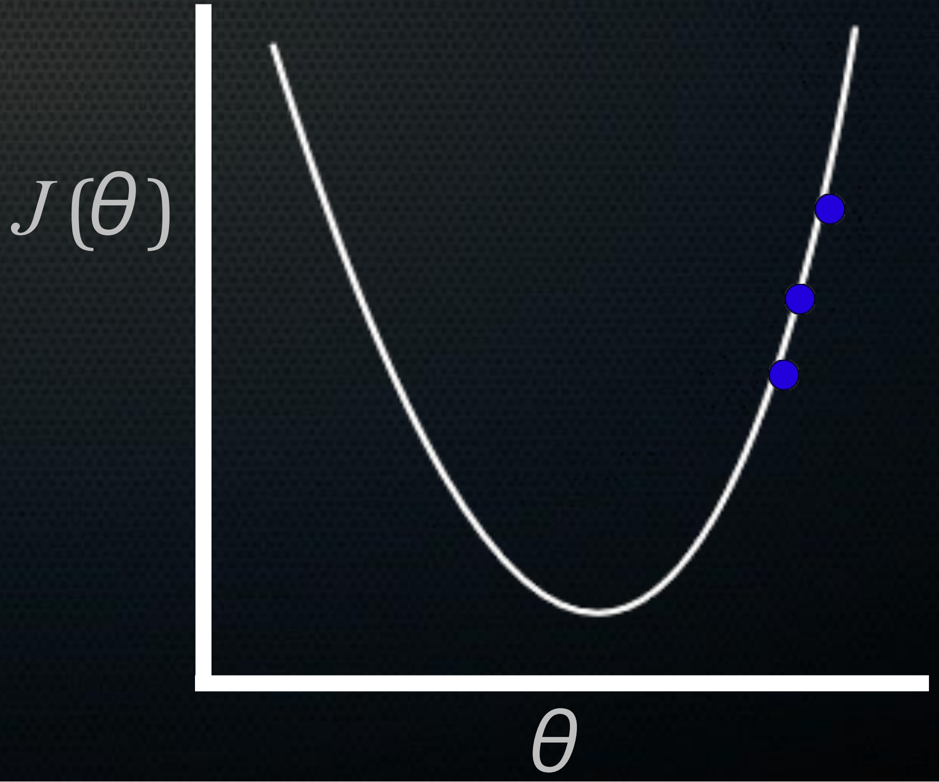

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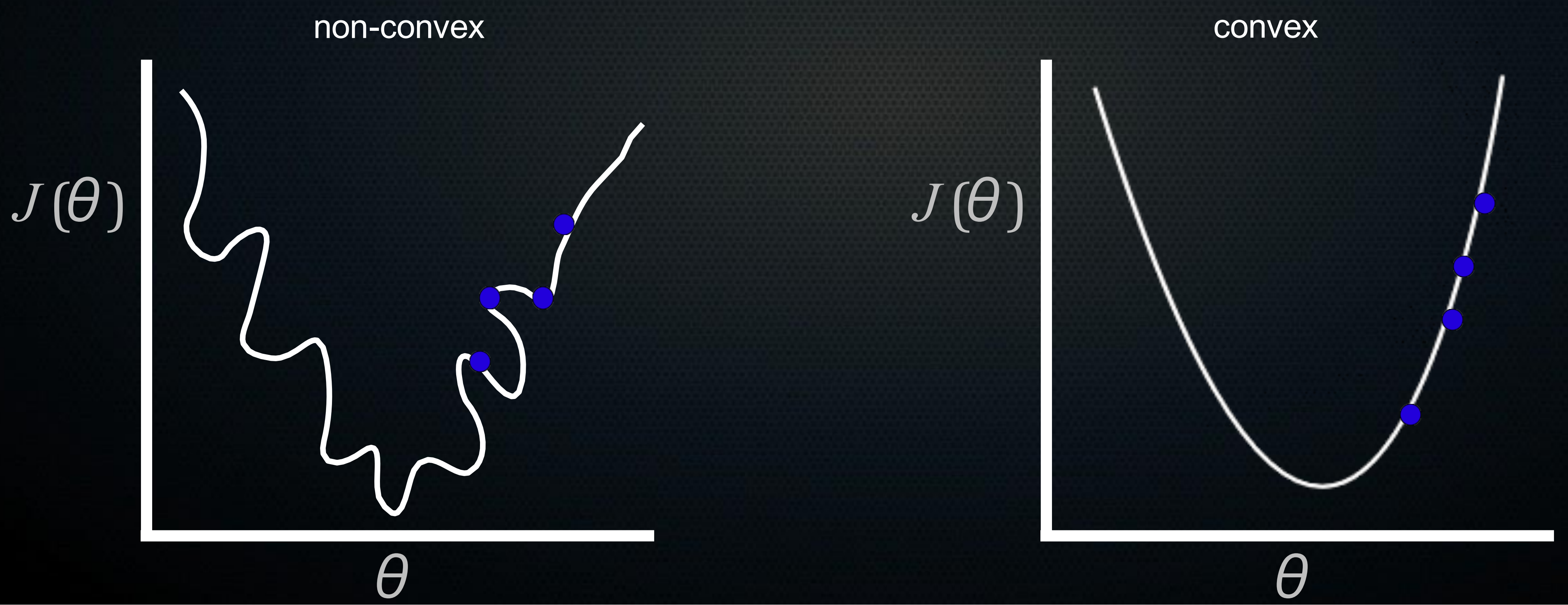

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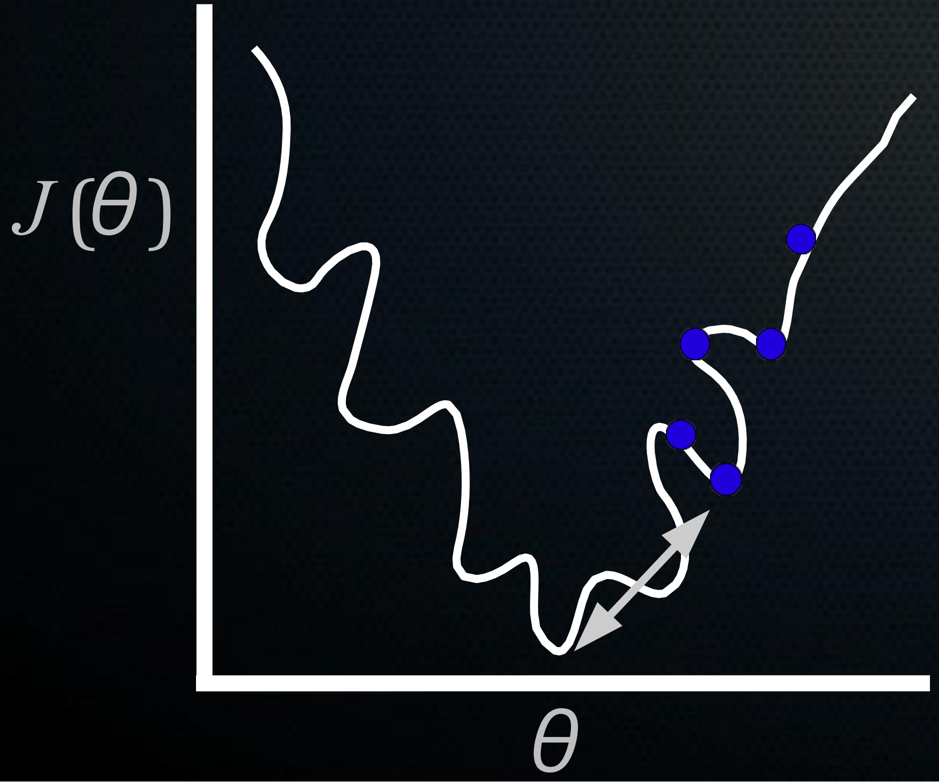

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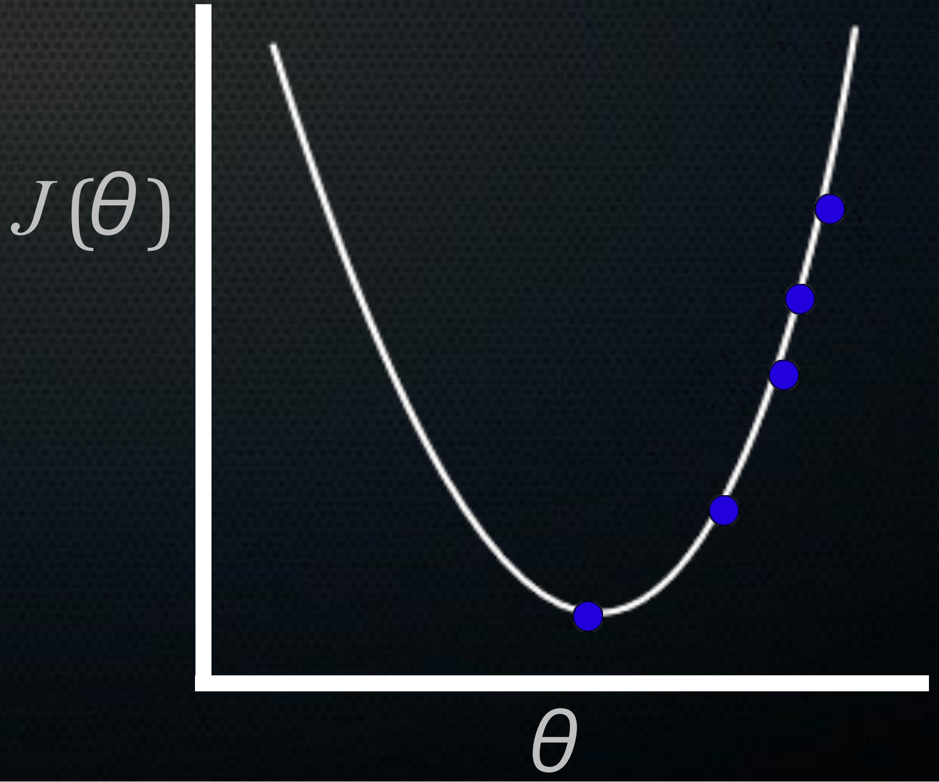

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$$\mathrm{Cost}(x)=\begin{cases}-\log(h_\theta(x))\ \text{if } y=1\\ -\log(1-h_\theta(x))\ \text{if } y=0\end{cases}$$

What does this function look like?

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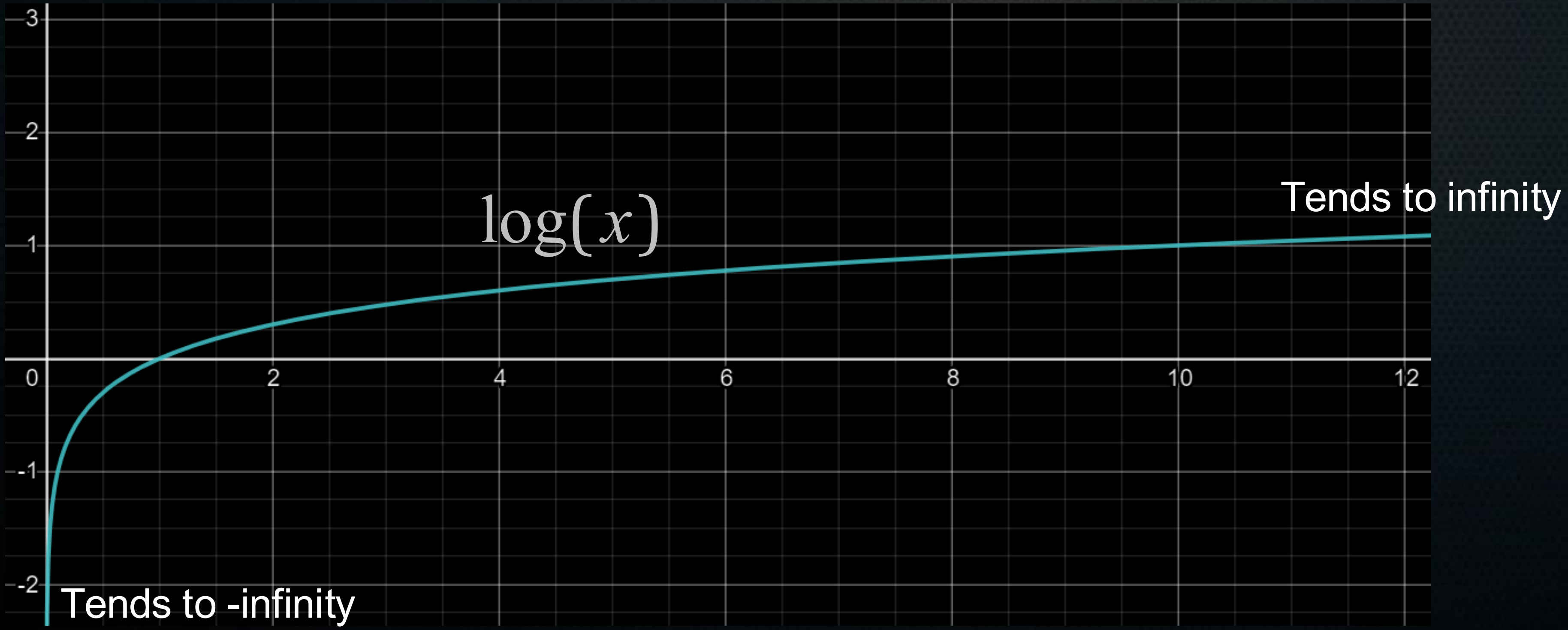

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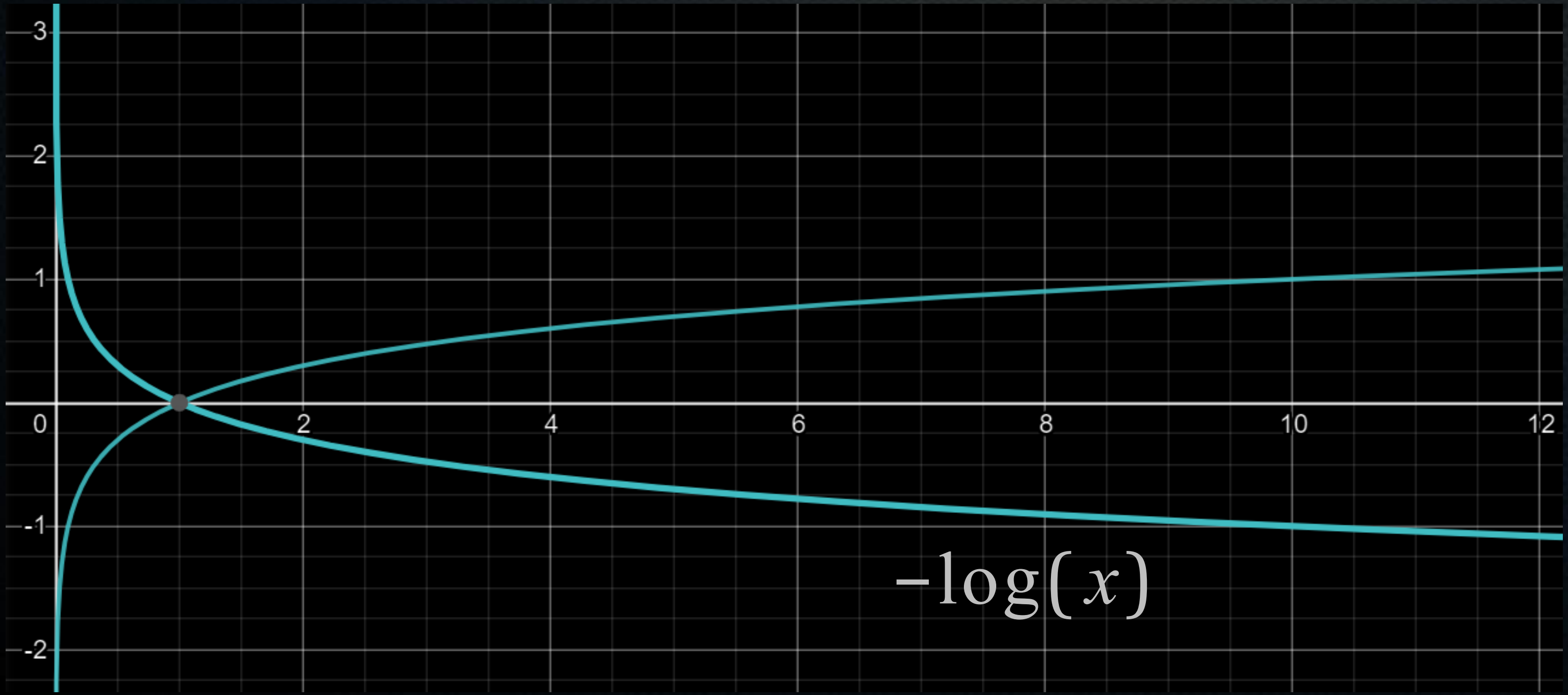

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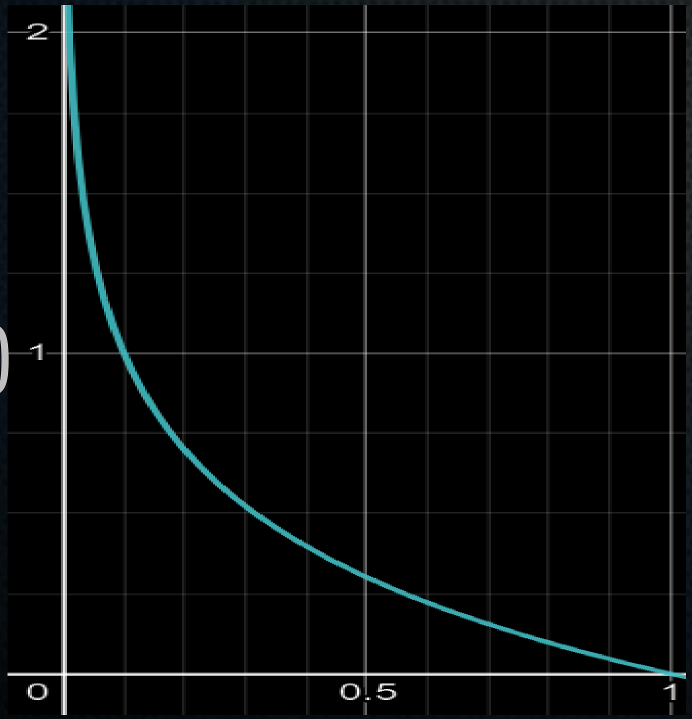

$h_\theta(x)$

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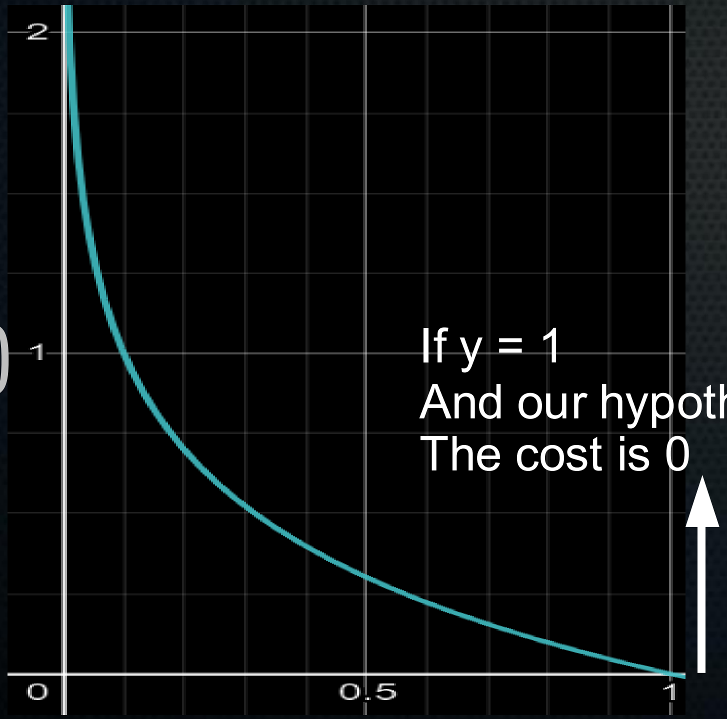

$h_\theta(x)$

If y = 1
And our hypothesis predicts 1
The cost is 0

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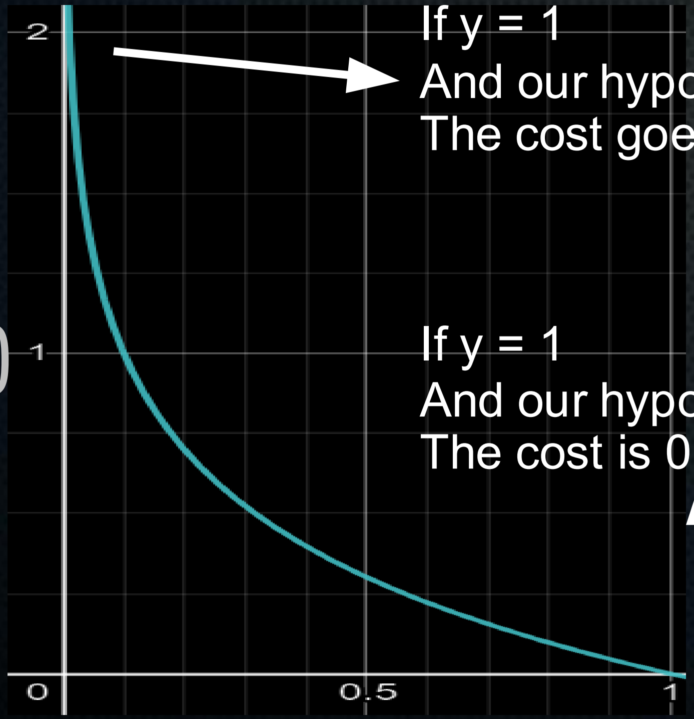

$h_\theta(x)$

If y = 1
And our hypothesis predicts 0
The cost goes to ∞

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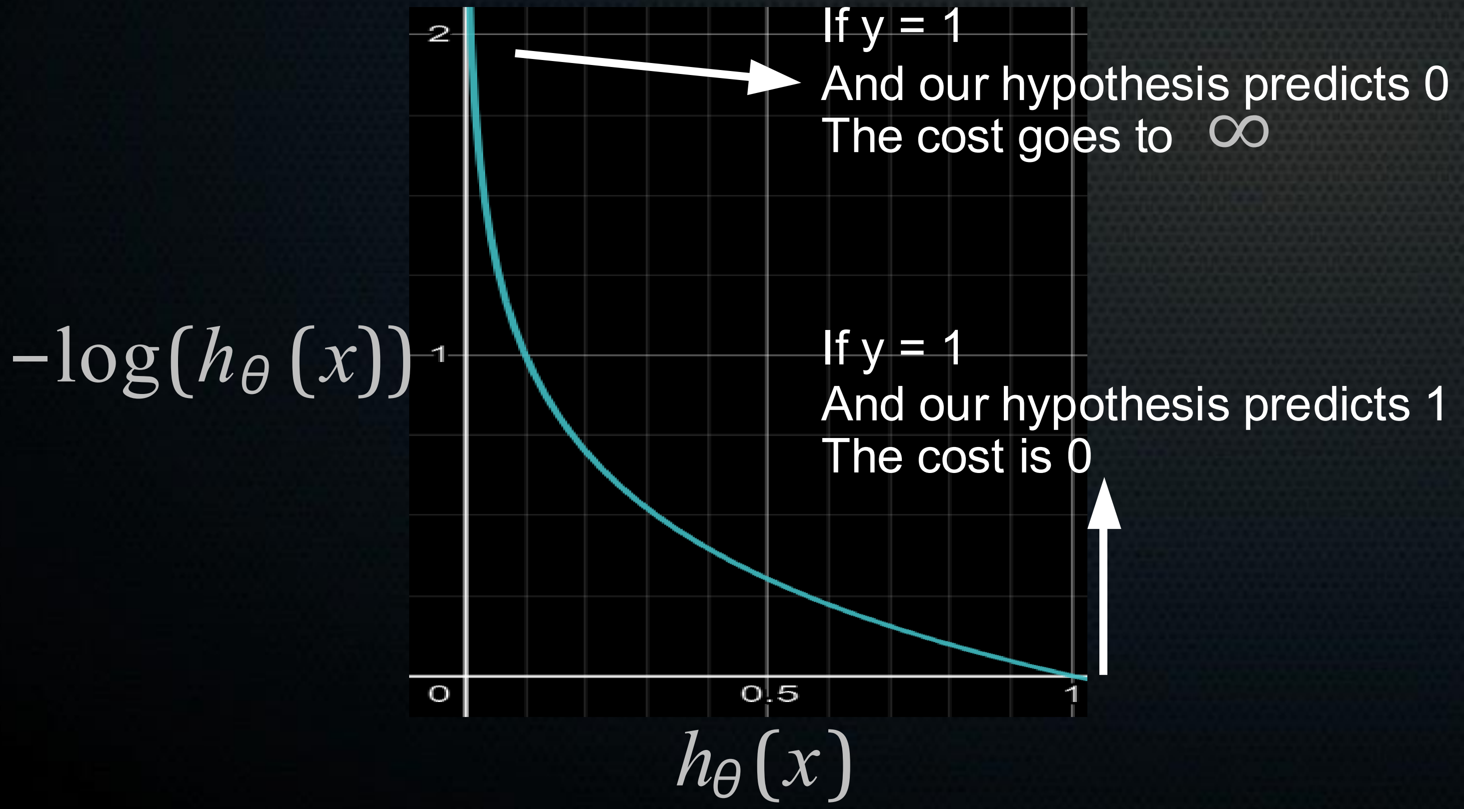

If we had used MSE

$$(0-1)^2 = 1$$

Penalised far less!

What does this function look like?

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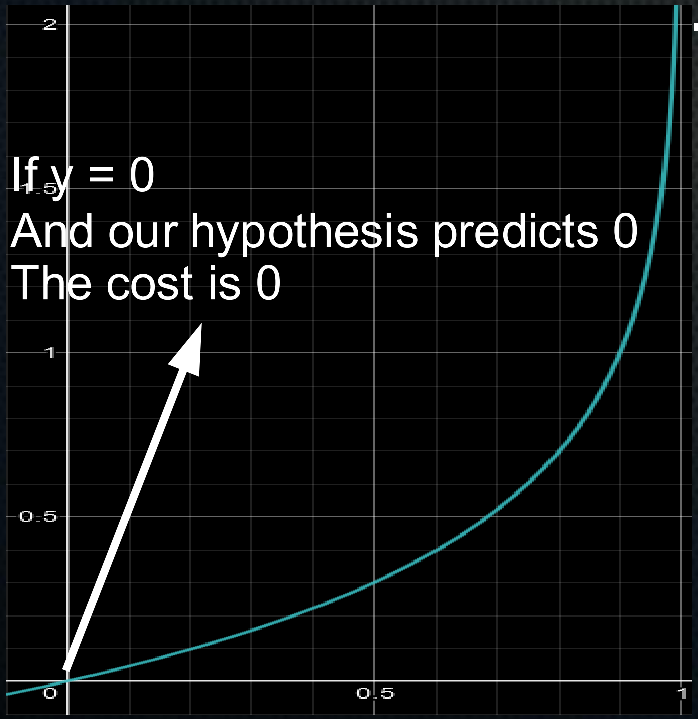

$h_\theta(x)$

If y = 0
And our hypothesis predicts 1
The cost goes to ∞

Simplified notation

$$\text{Cost}(x) = \begin{cases} -\log(h_\theta(x)) \text{ if } y=1 \\ -\log(1-h_\theta(x)) \text{ if } y=0 \end{cases}$$

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$y = 1$

$$-1 \cdot \log(h_\theta(x)) - (1-1) \cdot \log(1-h_\theta(x))$$

$$\boxed{-\log(h_\theta(x))}$$

Putting it all together

$$\mathrm{Cost}(x) = -\,y \cdot \log(h_\theta(x)) - (1-y) \cdot \log(1-h_\theta(x)) \qquad J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \mathrm{Cost}(x^{i})$$

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$$J(\theta) = \boxed{-}\,\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \cdot \log(h_\theta(x^{(i)})) + (1-y^{(i)}) \cdot \log(1-h_\theta(x^{(i)}))$$

Optimising the cost function

- Same form as for linear regression (only hypothesis function differs!)

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left(\left(\boxed{h_\theta(x^{(i)})} - y^{(i)} \right) \cdot x_j^{(i)} \right)$$

$$\theta_j := \theta_j - \frac{a}{m} \sum_{i=1}^{m} \left(\left(\boxed{h_\theta(x^{(i)})} - y^{(i)} \right) \cdot x_j^{(i)} \right)$$

$$\boxed{h_\theta(x^{(i)}) = \frac{1}{1 + e^{-\theta^T * x}}}$$

[Derivation link]

Summary

- By using the sigmoid function as a transformation of normal regression and interpreting the output as a chance of being 0 or 1 we can do classification.
- Only the form of our hypothesis function is different
- Need a different cost function: should be smooth, and give logical values for large errors.

Break for practical