Daily Inspiration

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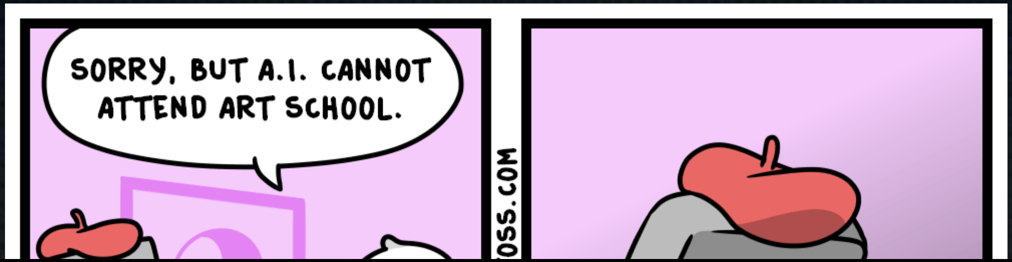


Today

- Recap yesterday
- Logistic regression: using regression tools for classification
- Neural network basics

Yesterday

- Cost function: (differentiable) function that shows how wrong an estimate is for given parameters.
- Gradient descent: one common way to minimise the cost function automatically, i.e. to get optimal parameters
- Linear regression: very simple model that assumes that value to predict is linear combination of input features.
- Overfitting and underfitting, bias and variance: want our model to work wel for unseen data. Need just enough model freedom given the complexity of our problem. How:
  - Cross-validation to measure ability to generalise + get best hyperparameters
  - Use learning curves to diagnose bias vs. variance

Logistic regression

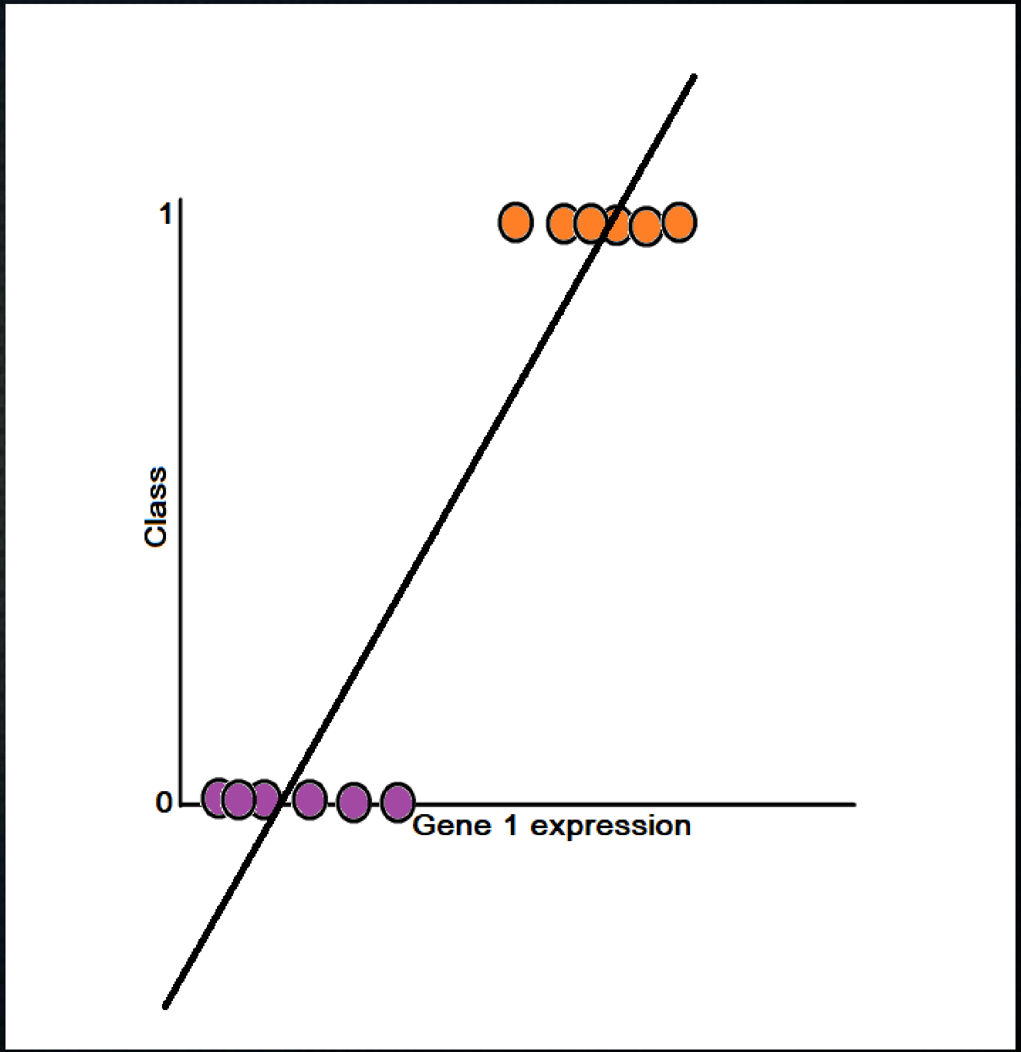
- You tell me: what is logistic regression?

Logistic regression

- Use regression-like framework for classification

Logistic regression

- Naïve idea:
  Train a linear regression. If Class >= 0.5, predict class 1. Otherwise, class 0.



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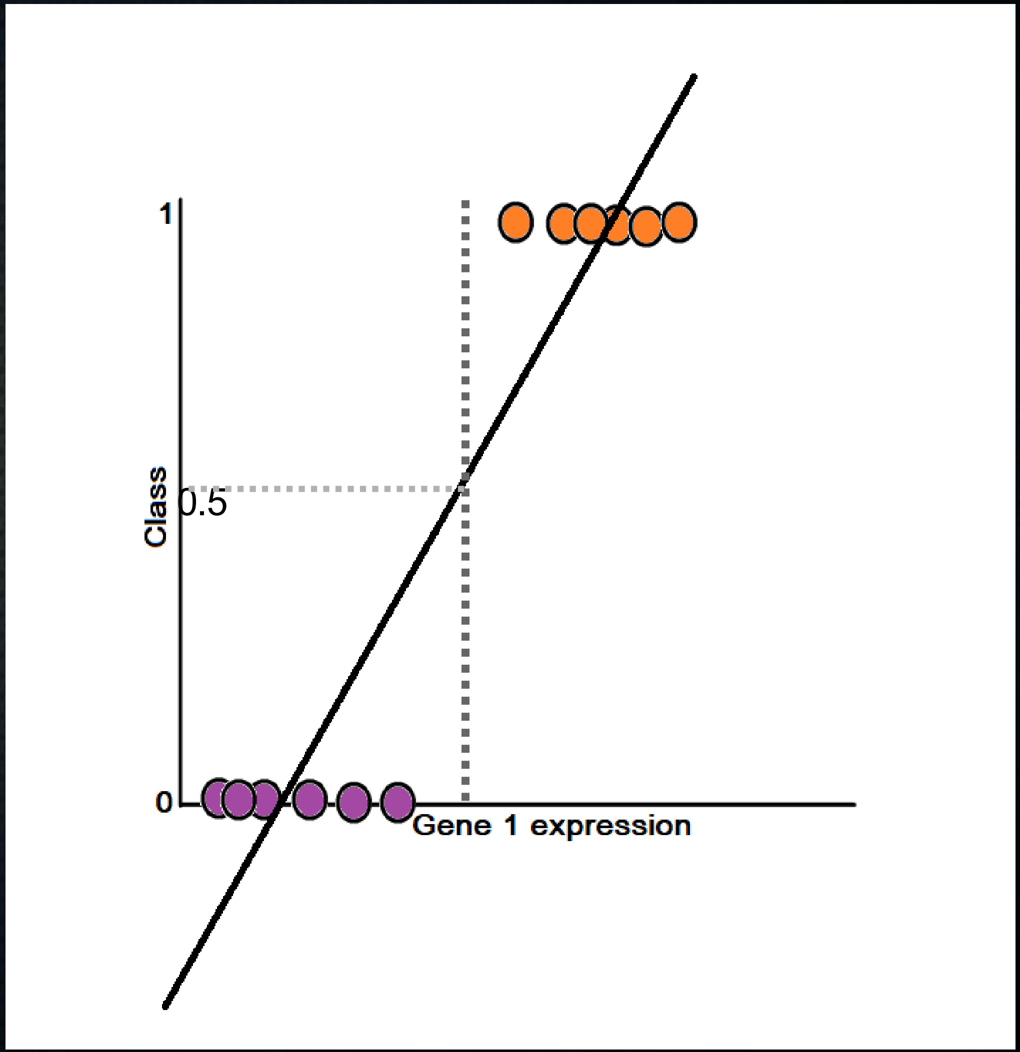

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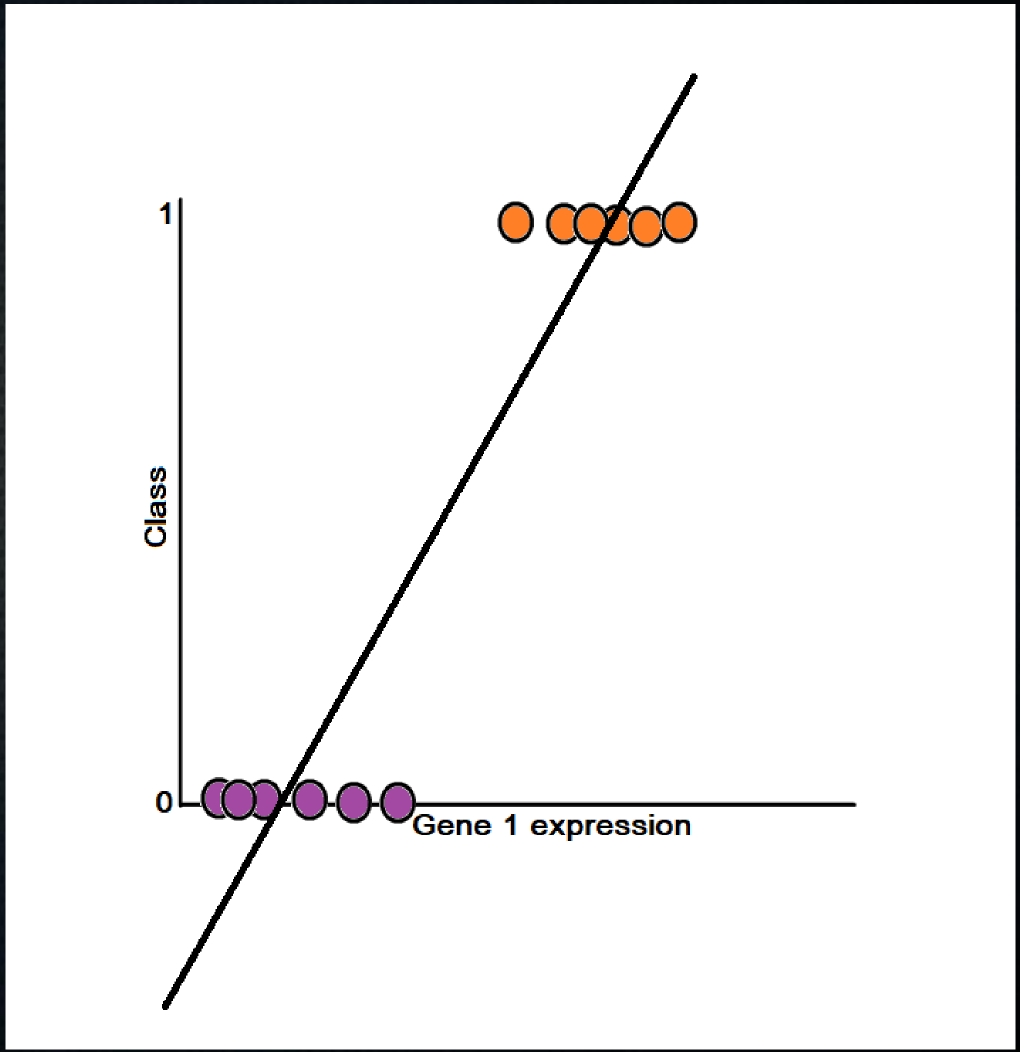
  -You can predict class > 1 and < 0, while that is not possible in reality.



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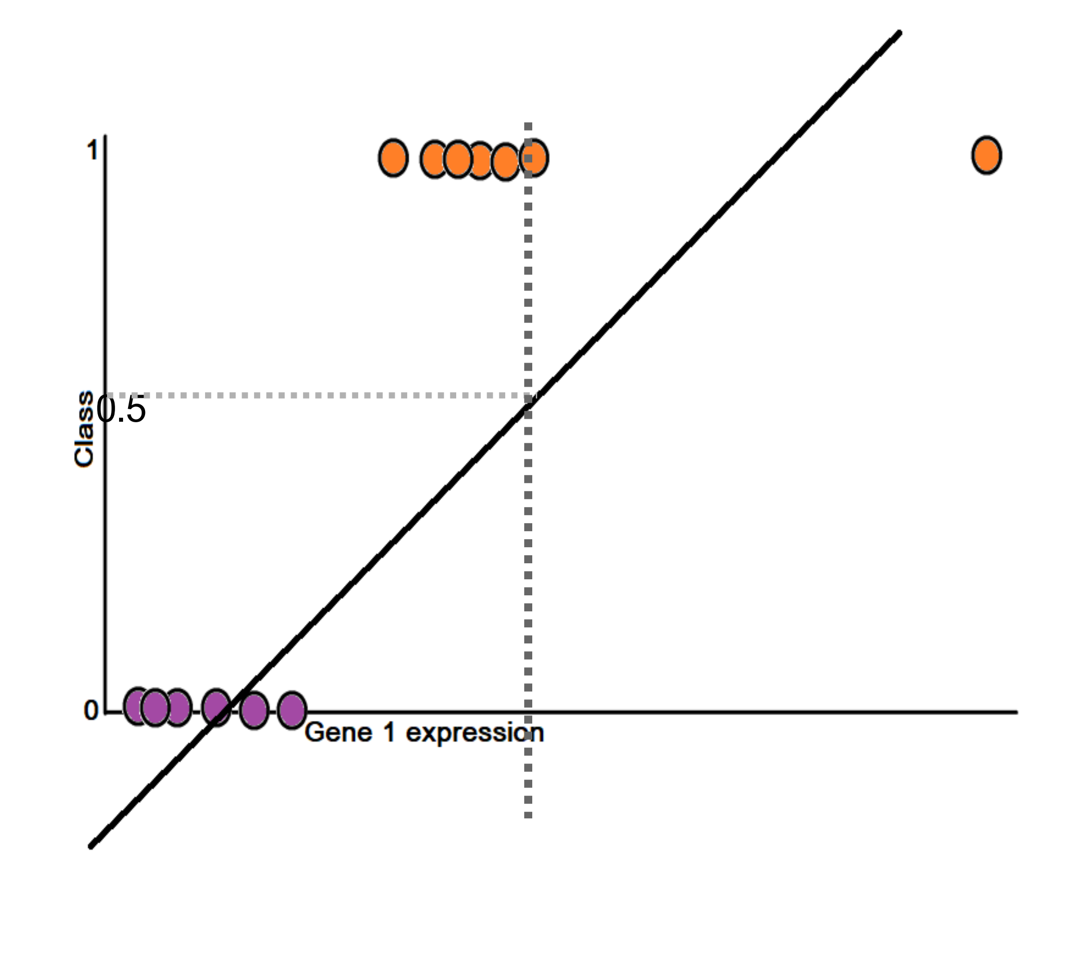

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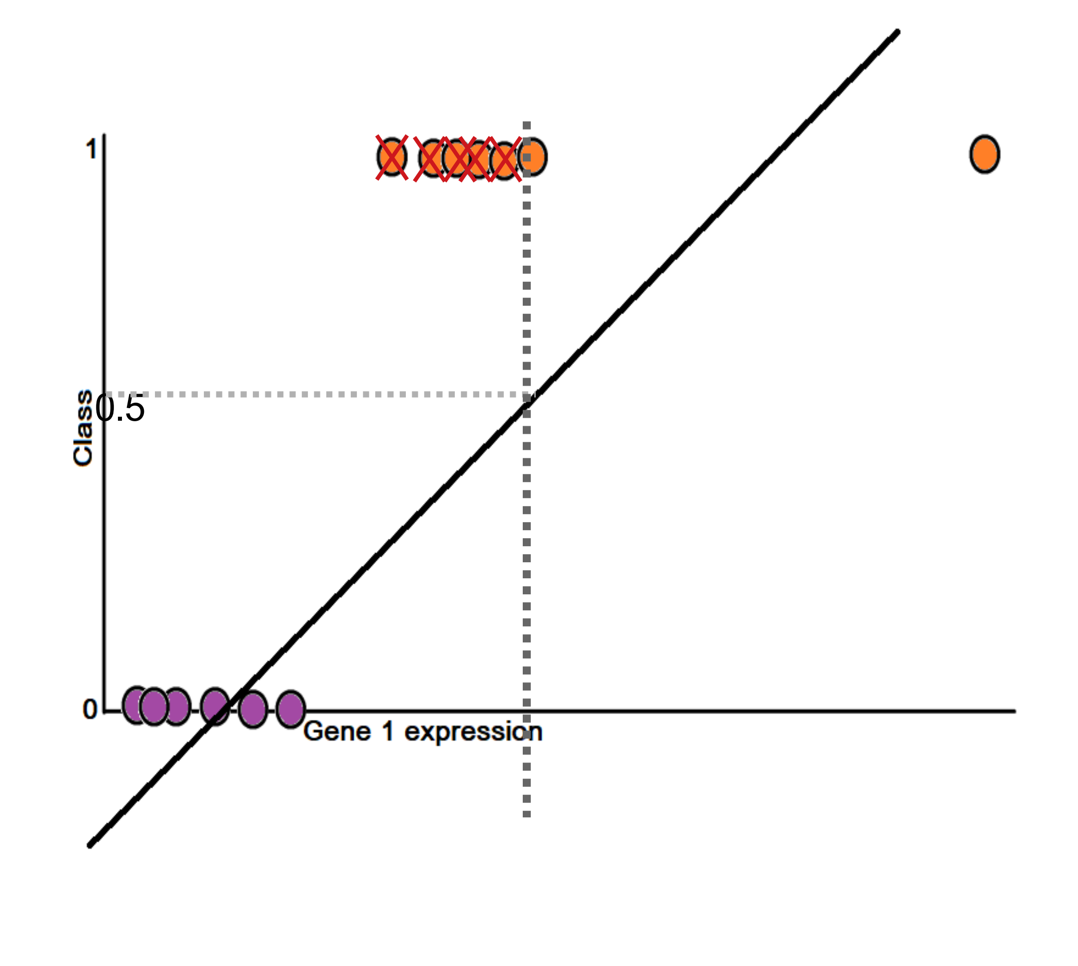
  -This example seemed to work, but quickly breaks down →
  get what is basically confirmation of hypothesis, but perform worse!



Logistic regression

- What we want:
  - Use the information that we only have two classes, 0 or 1.
  - Hypothesis function should output only numbers between 0 or 1.

Sigmoid or logistic function

- Before, our hypothesis function was of the form:

$$h_\theta(x) = \theta^T \cdot x$$

Sigmoid or logistic function

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$$h_\theta(x) = \theta^T \cdot x \longrightarrow [0.5 \;\; 3 \;\; -1.5] \cdot \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}$$

Sigmoid or logistic function

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$$h_\theta(x)=\theta^T \cdot x \longrightarrow \underbrace{[0.5 \;\; 3 \;\; -1.5]}_{\text{Learned parameters (theta 0 – theta 2)}} \cdot \left.\begin{bmatrix}1\\3\\8\end{bmatrix}\right\} \text{Features for one sample (x0 = 1, intercept term, 2 data-derived features x1 and x2)}$$

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$$h_\theta(x)=\theta^T \cdot x \longrightarrow [0.5 \;\; 3 \;\; -1.5] \cdot \begin{bmatrix}1\\3\\8\end{bmatrix} = 0.5 \cdot 1 + 3 \cdot 3 - 1.5 \cdot 8 = -2.5$$

Sigmoid or logistic function

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- Change that to the following:

$$h_\theta(x) = g(\theta^T \cdot x)$$

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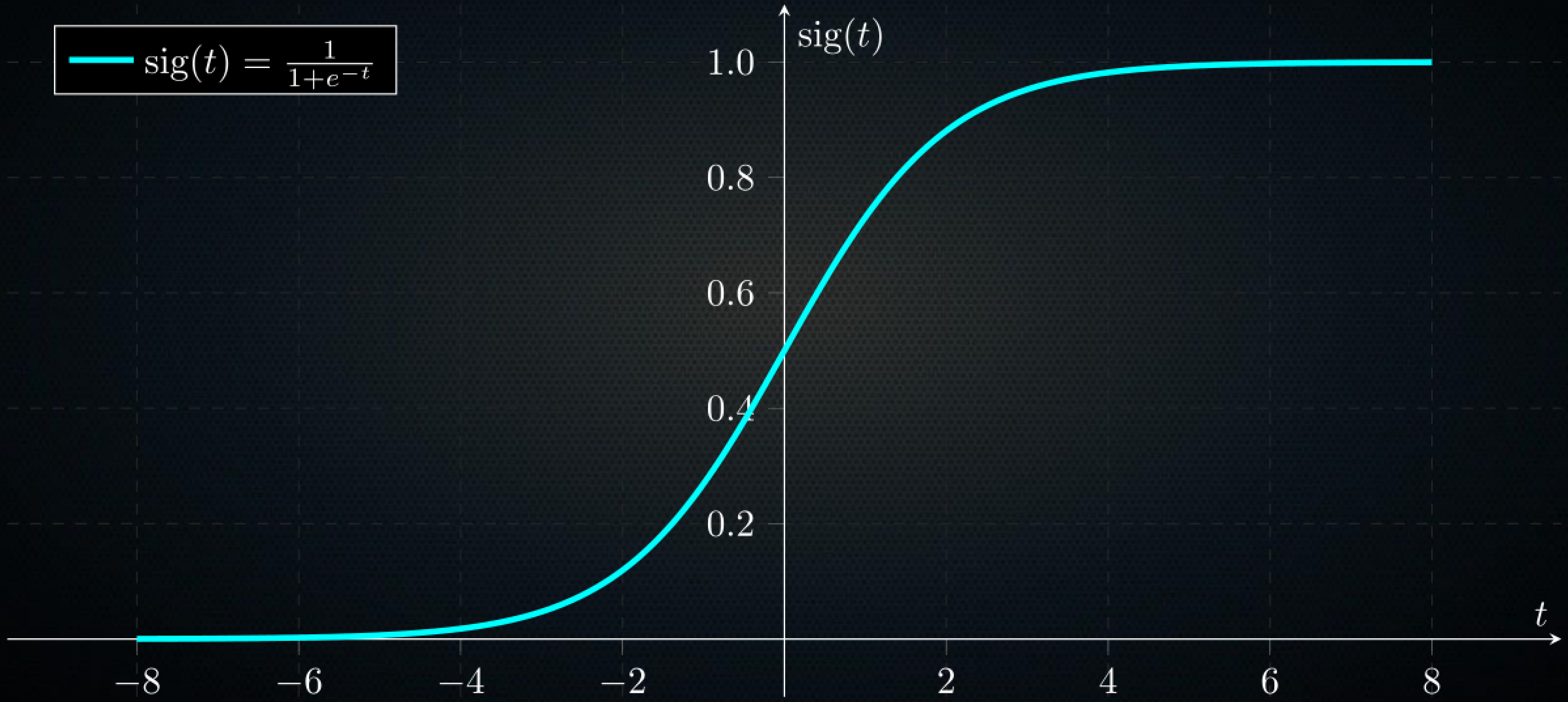
- What does that look like?

$$z \rightarrow \infty, e^{-z} \rightarrow 0$$

$$z \rightarrow -\infty, e^{-z} \rightarrow \infty$$

What does the sigmoid function look like?



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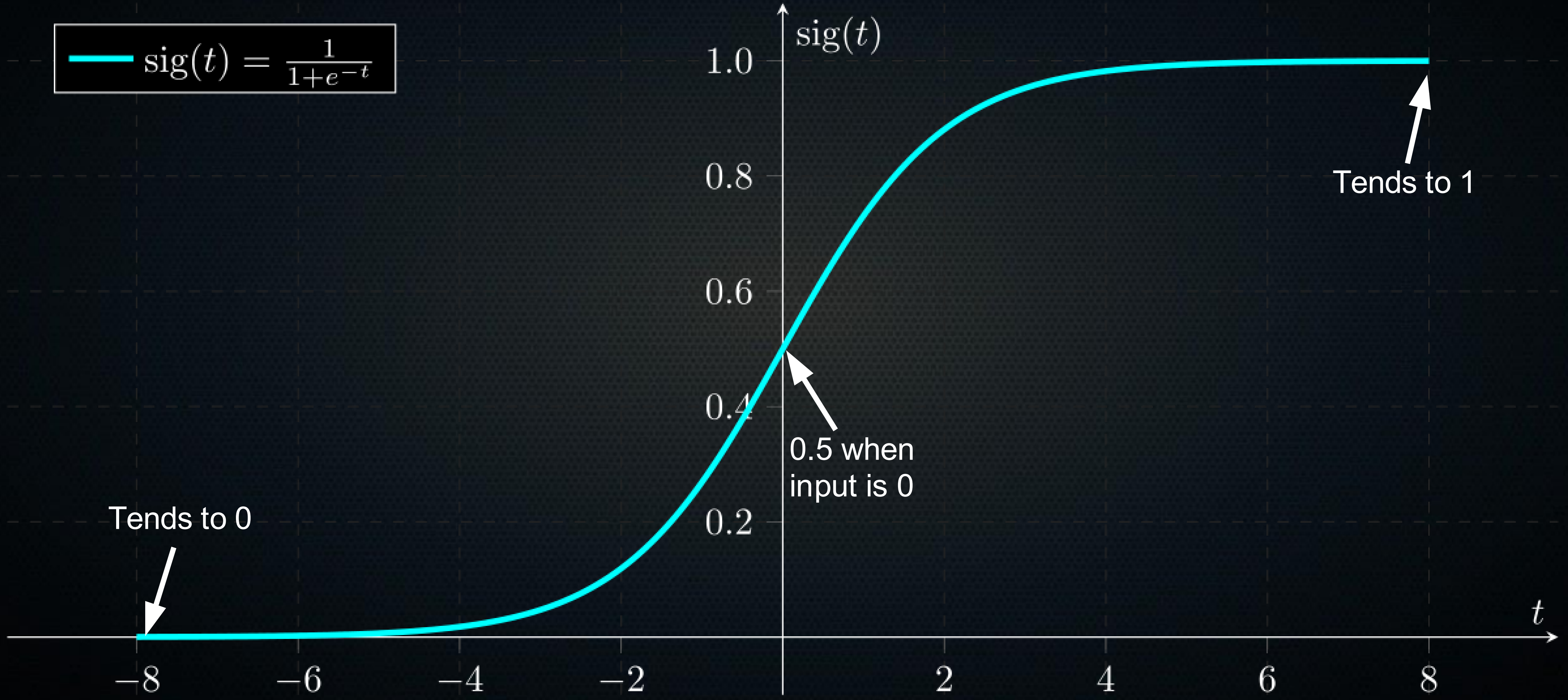
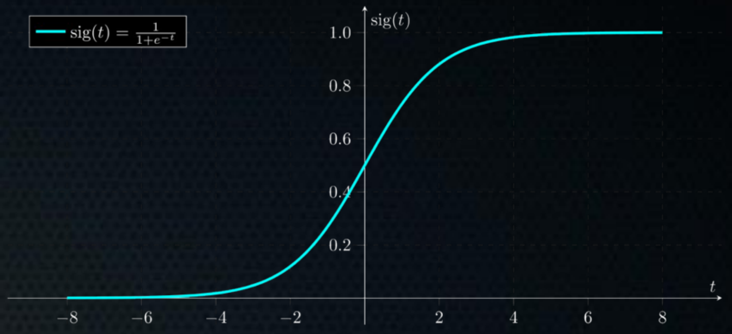


Sigmoid or logistic function

- How do we work with this? $h_\theta(x) = \dfrac{1}{1+e^{-(\theta^T \cdot x)}}$



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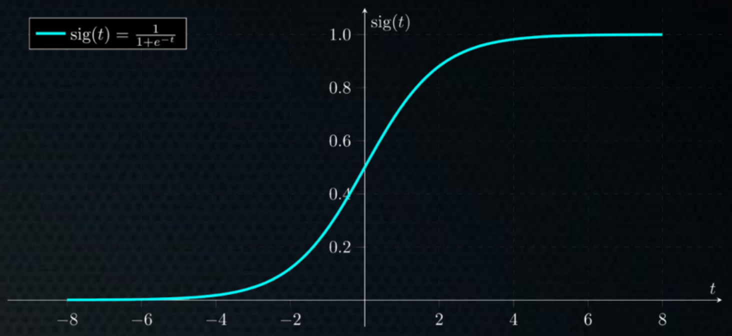

  - Interpret outcome of $h_\theta(x)$ as probability that class = 1 given the features.

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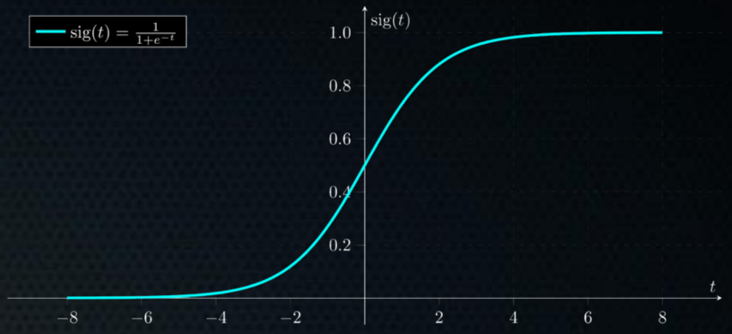

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$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{Tumor size} \\ \text{Neovascularisation level} \end{bmatrix}$$

$h_\theta(x) = 0.8$ ⟶ 80% chance of tumor being malignant

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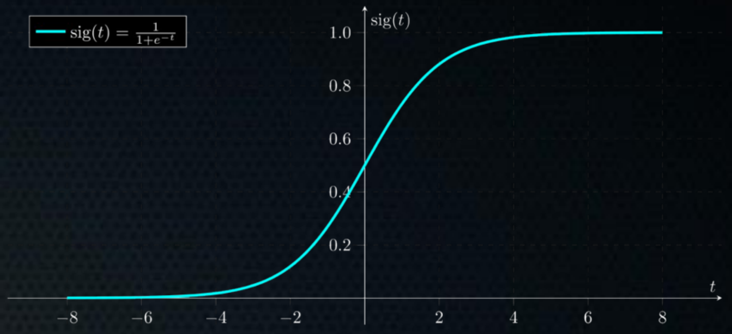

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$h_\theta(x) = 0.8 \longrightarrow$ 80% chance of tumor being malignant (class 1)

100% - 80% → 20 % chance of being benign (class 0)

Sigmoid or logistic function

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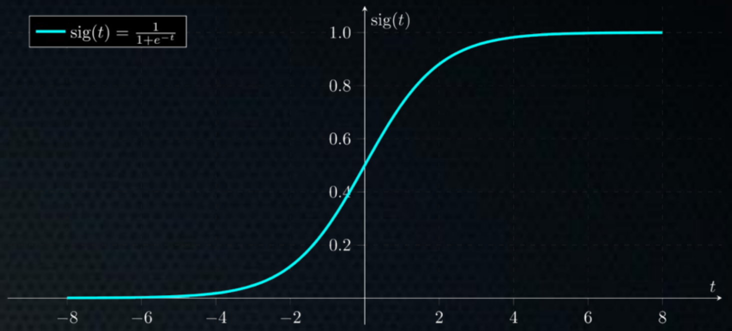

  - Interpret outcome of $h_\theta(x)$ as probability that class = 1 given the features.
  - Formally:

$$h_\theta(x) = \frac{1}{1+e^{-(\theta^T \cdot x)}} = p(y=1 | x;\theta)$$

$$p(y=0 | x;\theta) = 1 - h_\theta(x)$$

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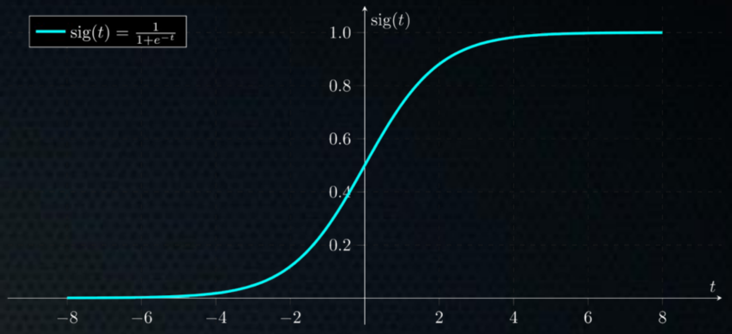

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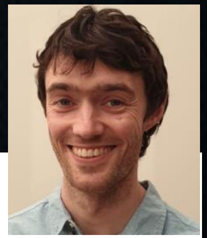
$$\frac{p(y=1)}{1-p(y=1)}$$

Log odds

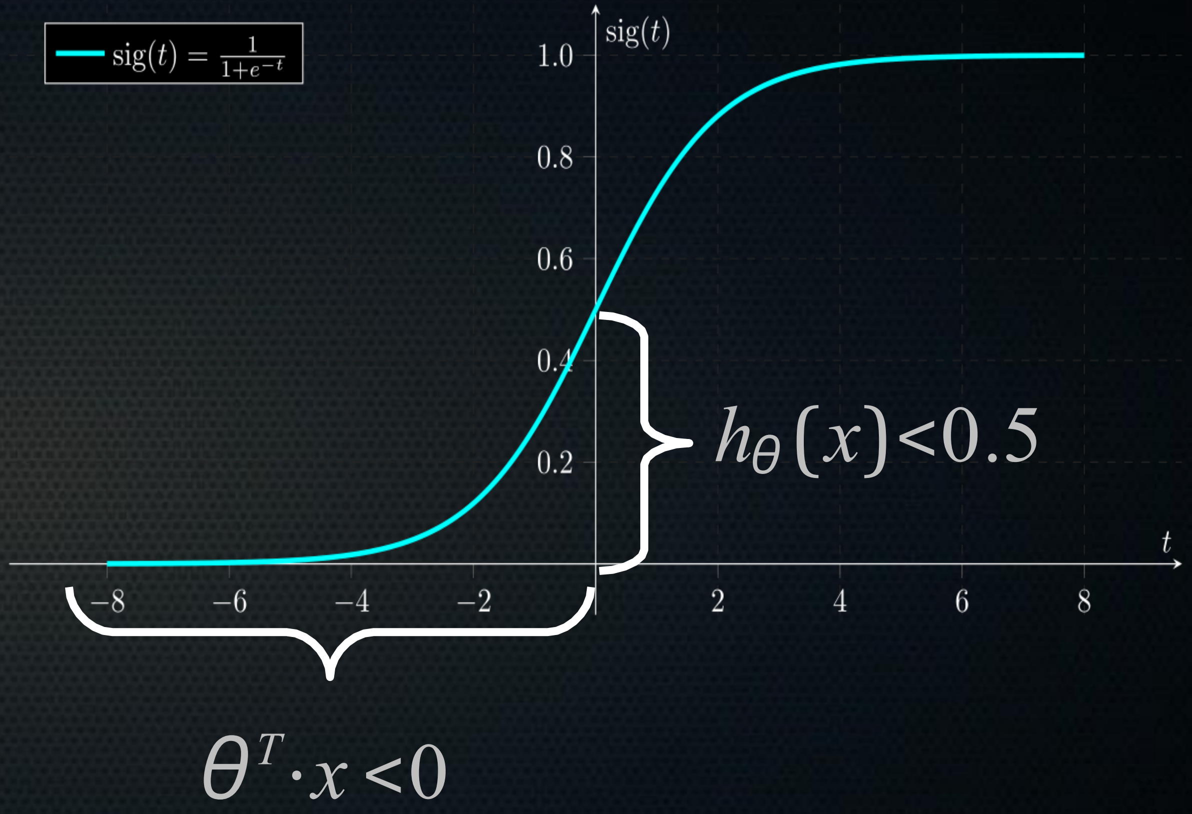


```
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -8.922213   0.879485 -10.145  < 2e-16 ***
pregnant     0.106275   0.039885   2.665  0.00771 **
glucose      0.036555   0.004546   8.041 8.88e-16 ***
pressure    -0.008019   0.006230  -1.287  0.19801
triceps      0.001909   0.008258   0.231  0.81721
insulin     -0.001394   0.001098  -1.269  0.20445
mass         0.082665   0.017927   4.611 4.00e-06 ***
pedigree     0.901691   0.374350   2.409  0.01601 *
age          0.020417   0.010854   1.881  0.05995 .
```

logOdds(diabetes) = -8.9 + (0.106*pregnant) + (0.037*glucose)…. + (0.02*age)

Decision boundary

- Threshold:



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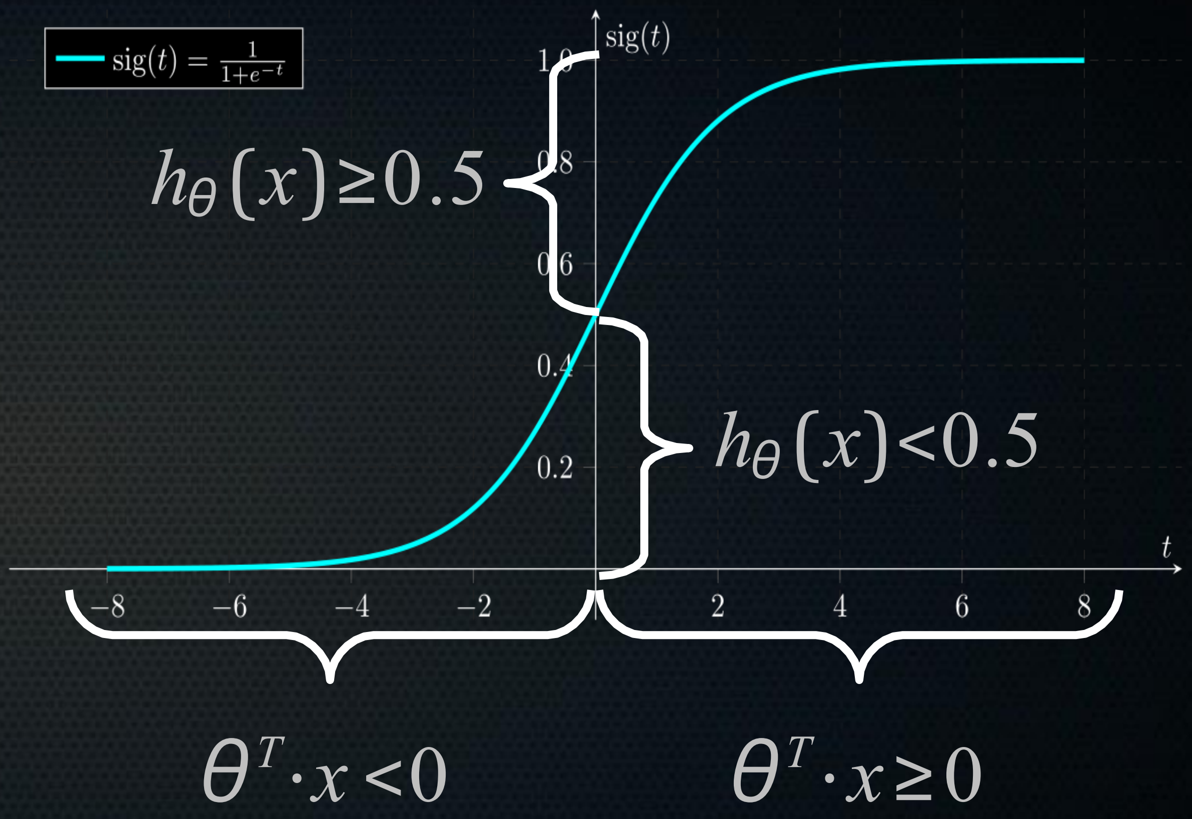
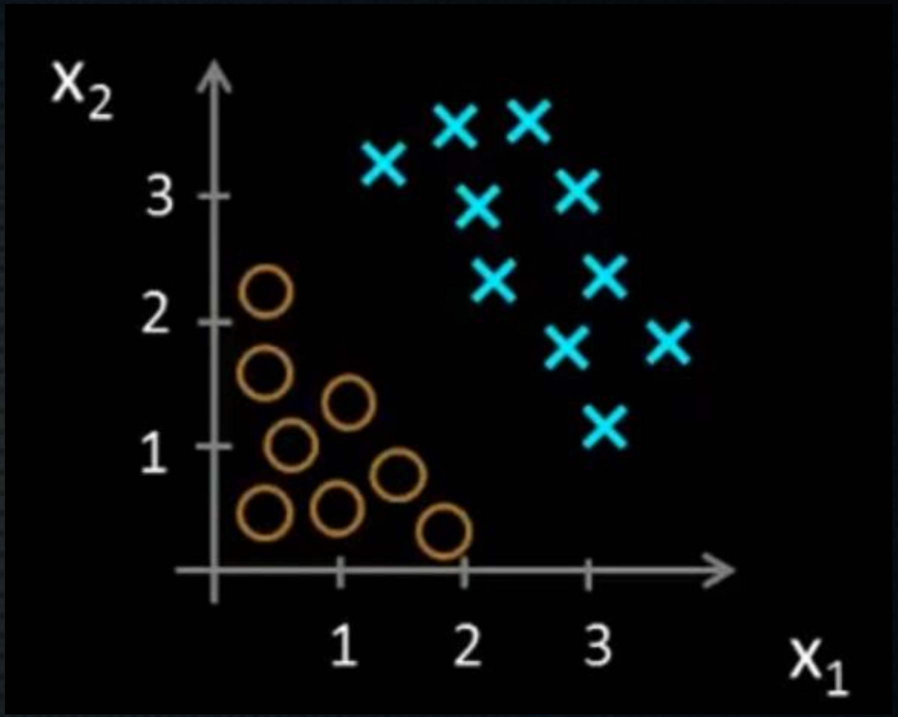


Decision boundary

- How does it look?

$$g(z)=\frac{1}{1+e^{-z}}$$

$$h_\theta(x)=g(\theta_0 \cdot x_0, \theta_1 \cdot x_1, \theta_2 \cdot x_2)$$



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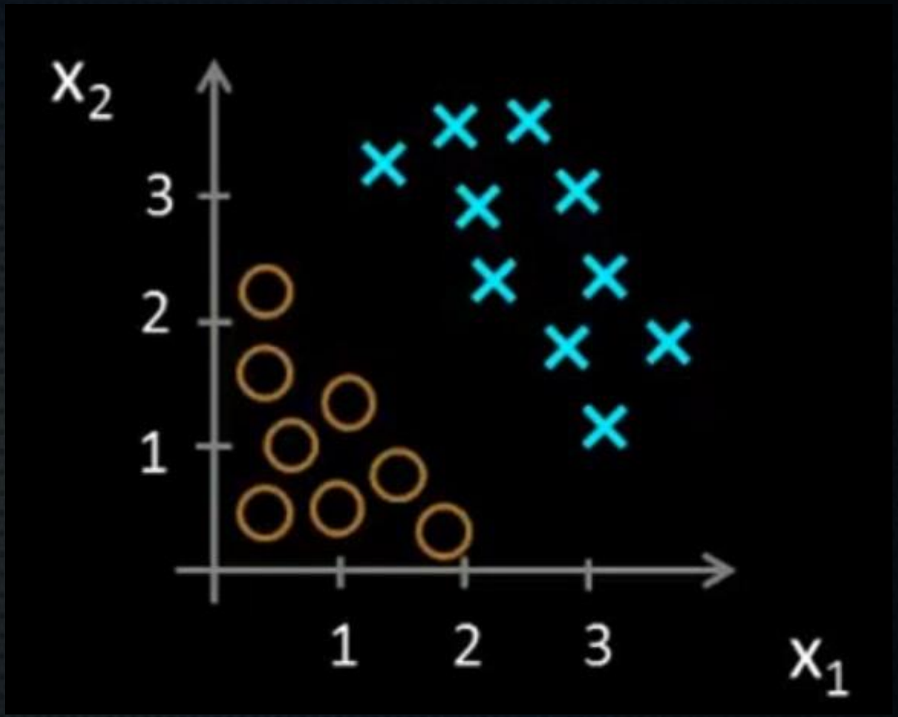

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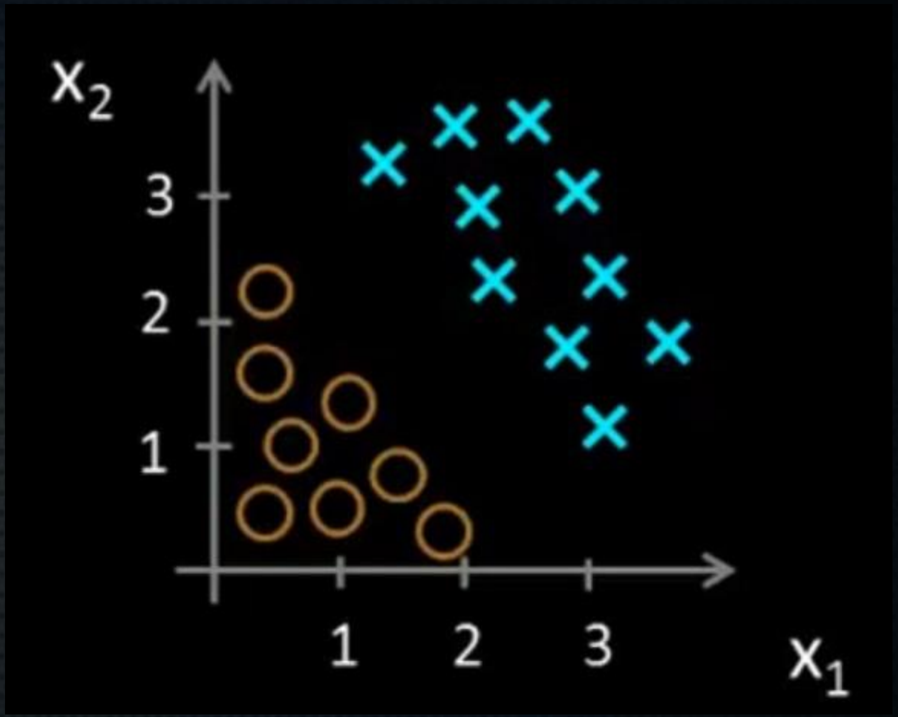

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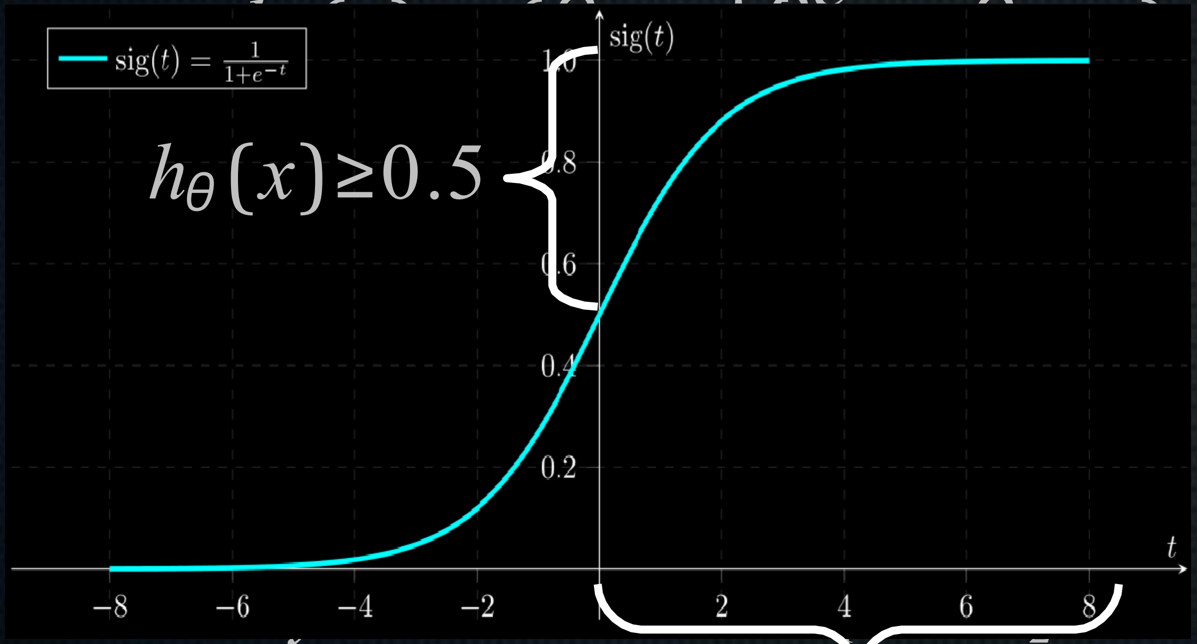

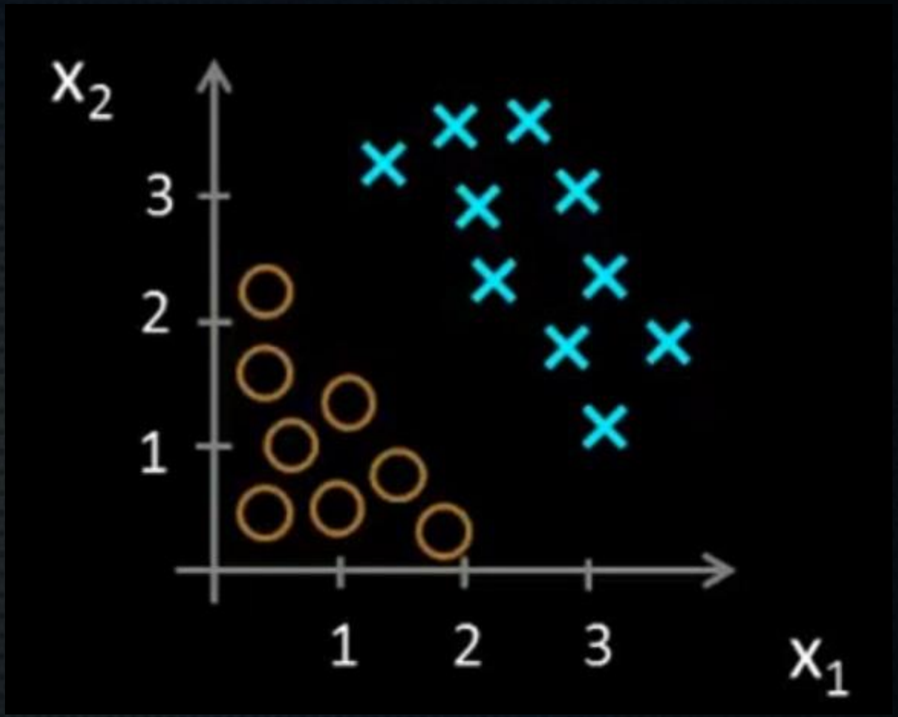

$$\theta^T \cdot x \geq 0$$

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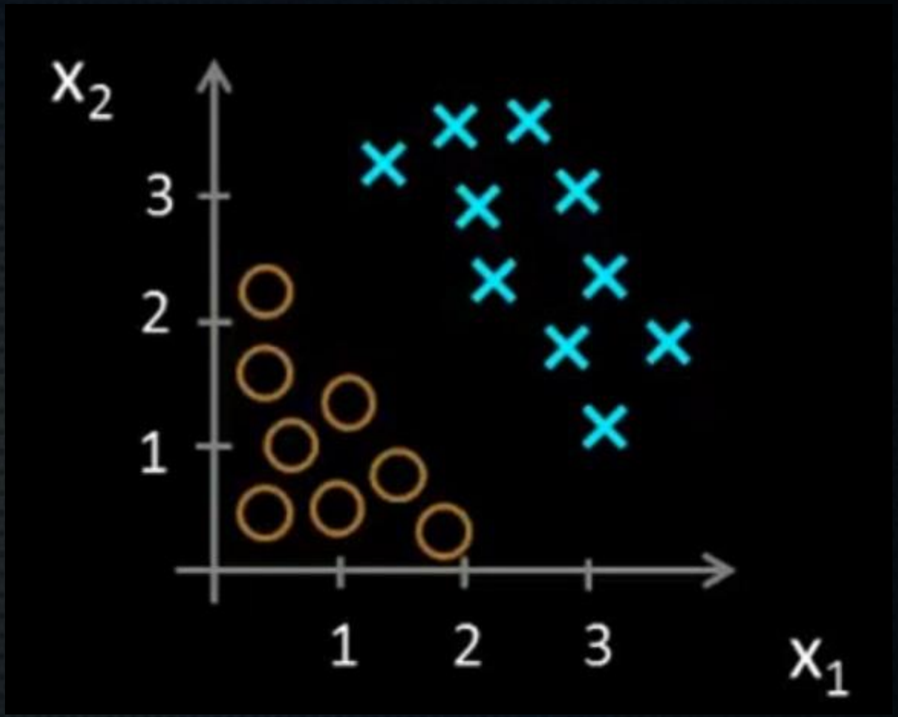
$$x_1+x_2=3$$



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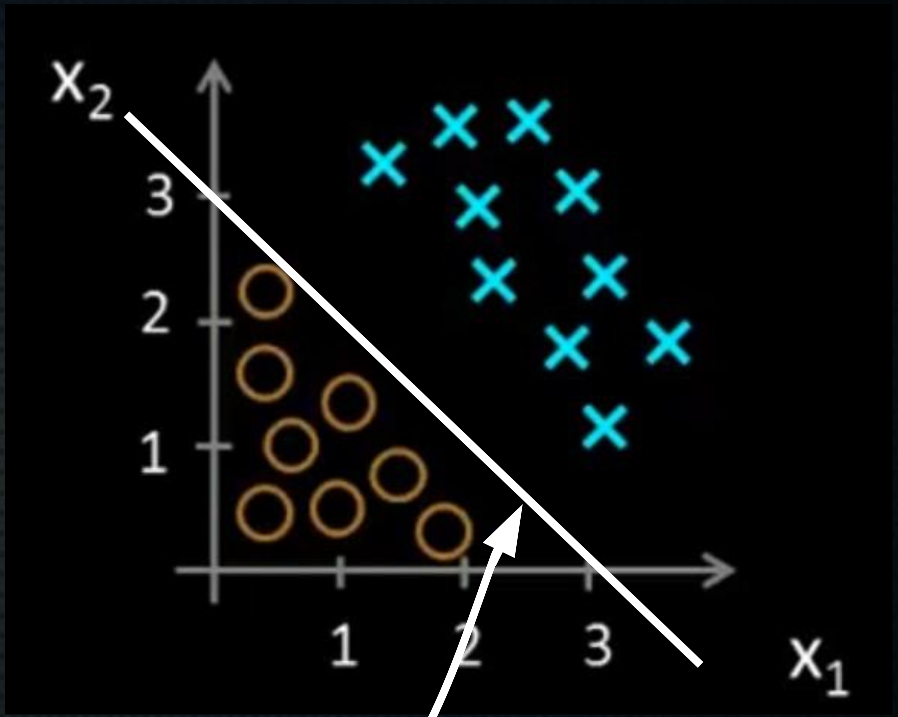

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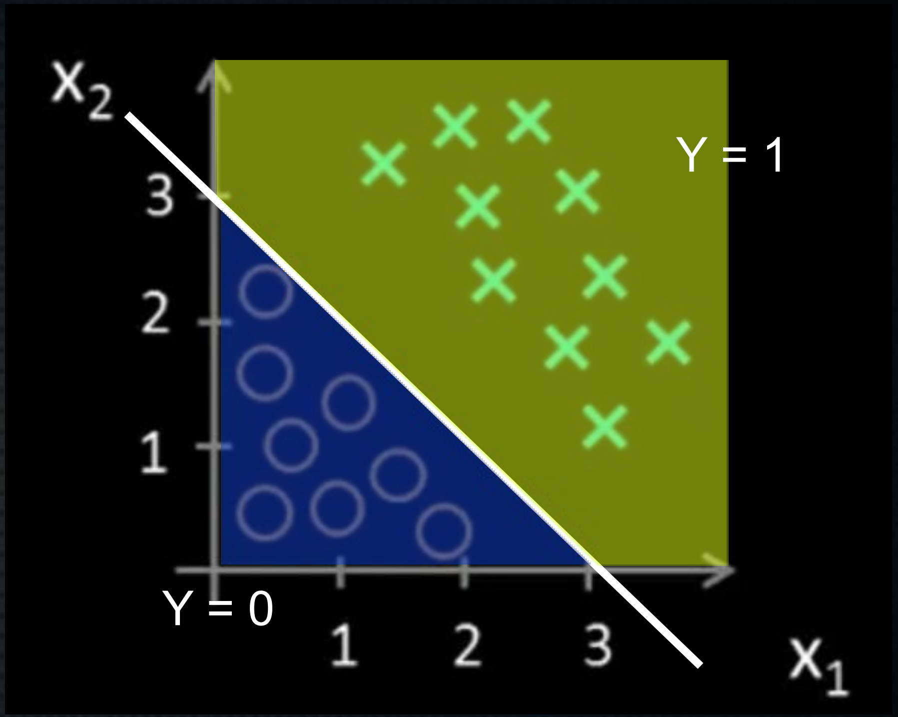

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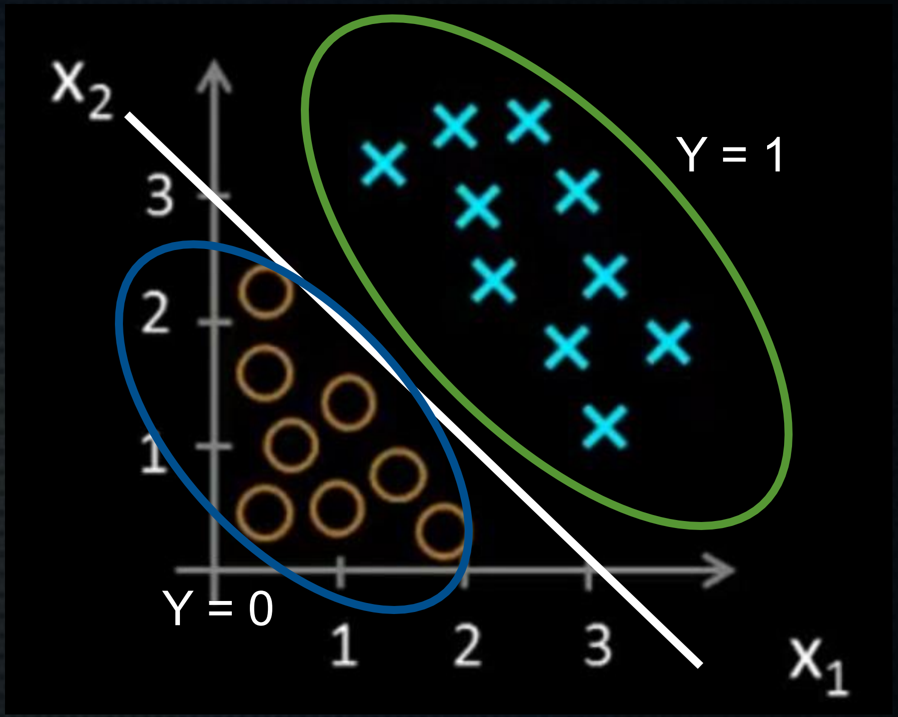

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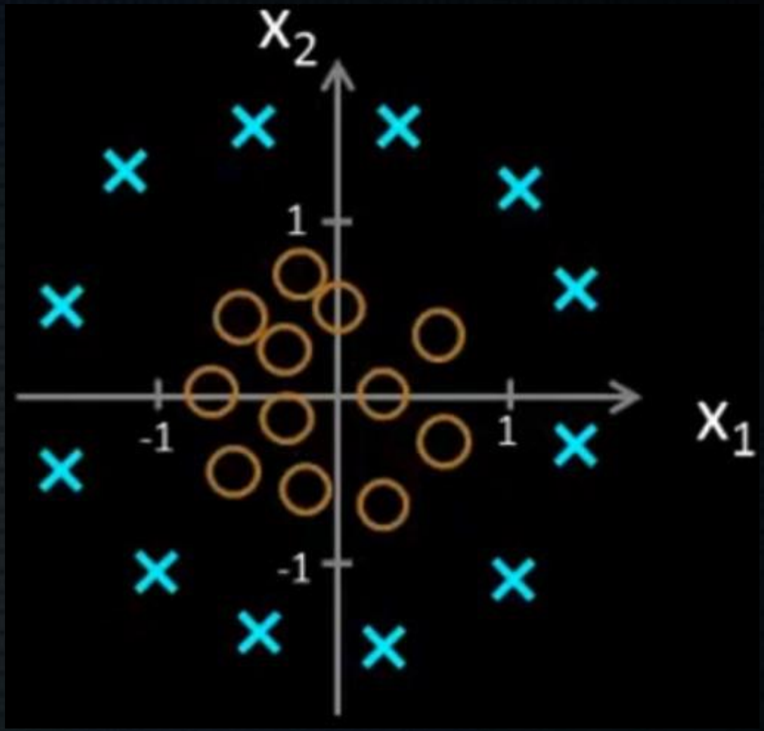

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Non-linear decision boundary

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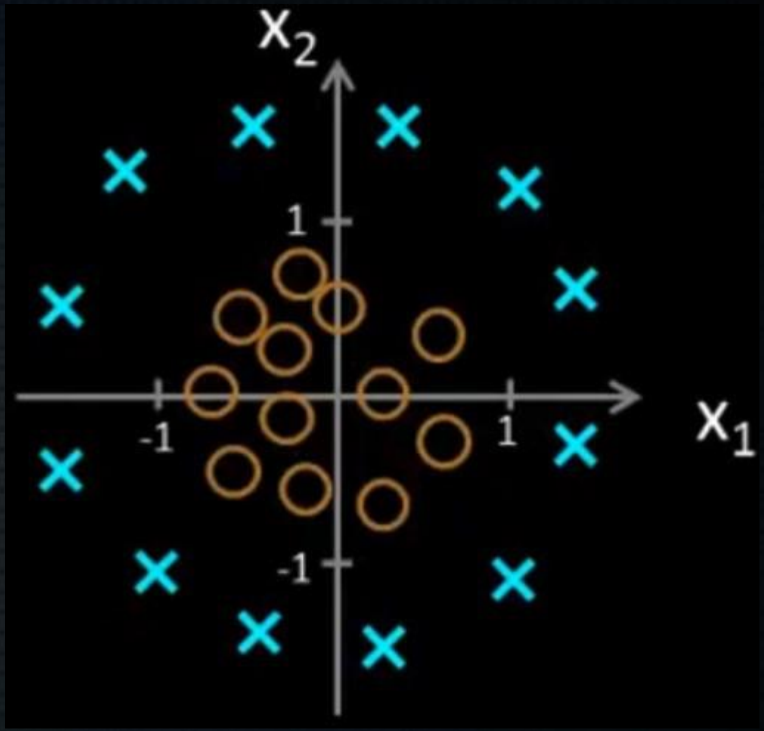
$$h_\theta(x)=g(\theta_0 x_0,\theta_1 x_1,\theta_2 x_2,\boxed{\theta_3 x_1^2,\theta_4 x_2^2})$$

- Add two polynomial features



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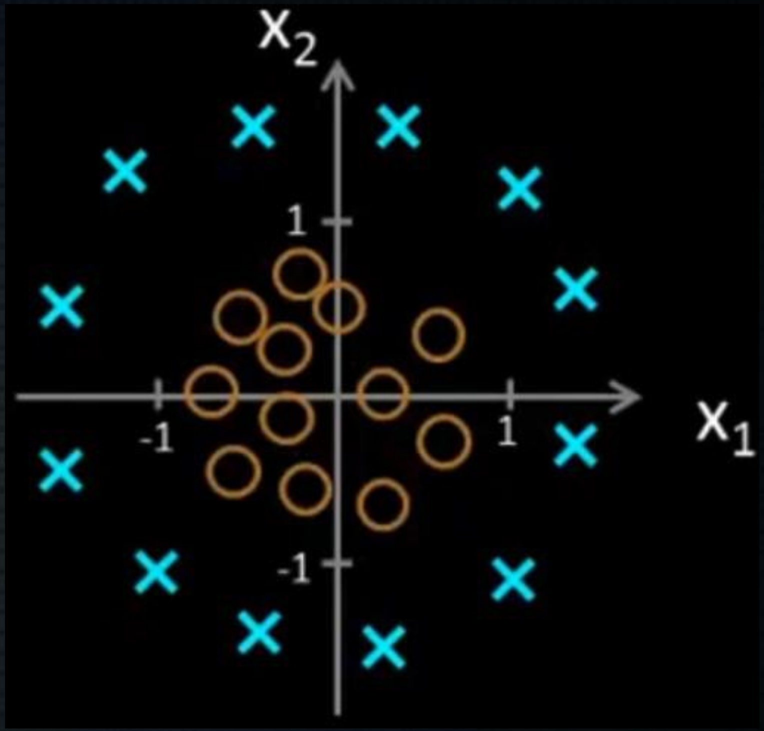
$$\theta = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$



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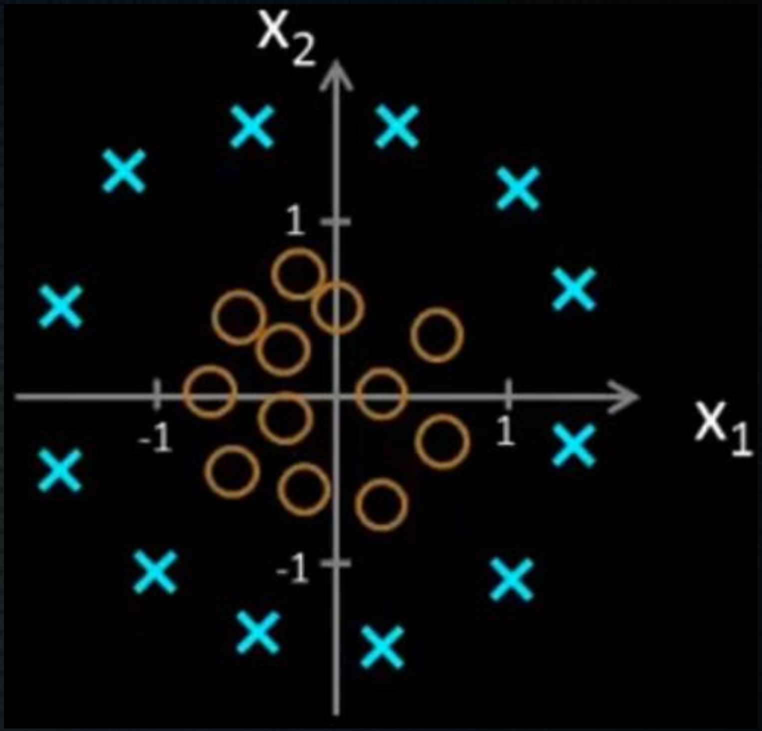
Can you work out what the decision boundary will be?



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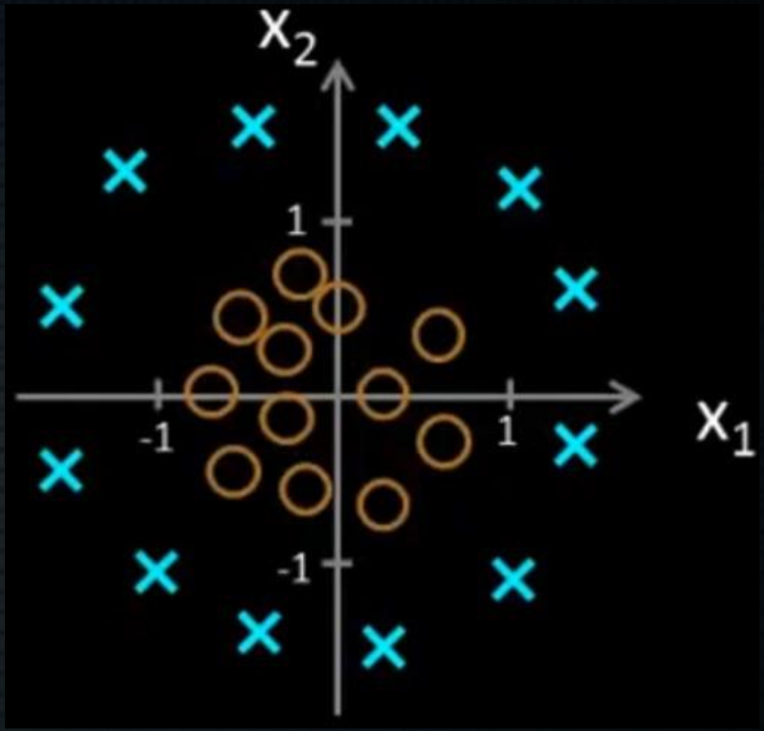
- Add two polynomial features

$$\theta=\begin{bmatrix}-1\\0\\0\\1\\1\end{bmatrix} \longrightarrow \begin{array}{c} y=1 \text{ if} \\ -1+x_1^2+x_2^2\geq 0 \end{array}$$



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$$\downarrow$$

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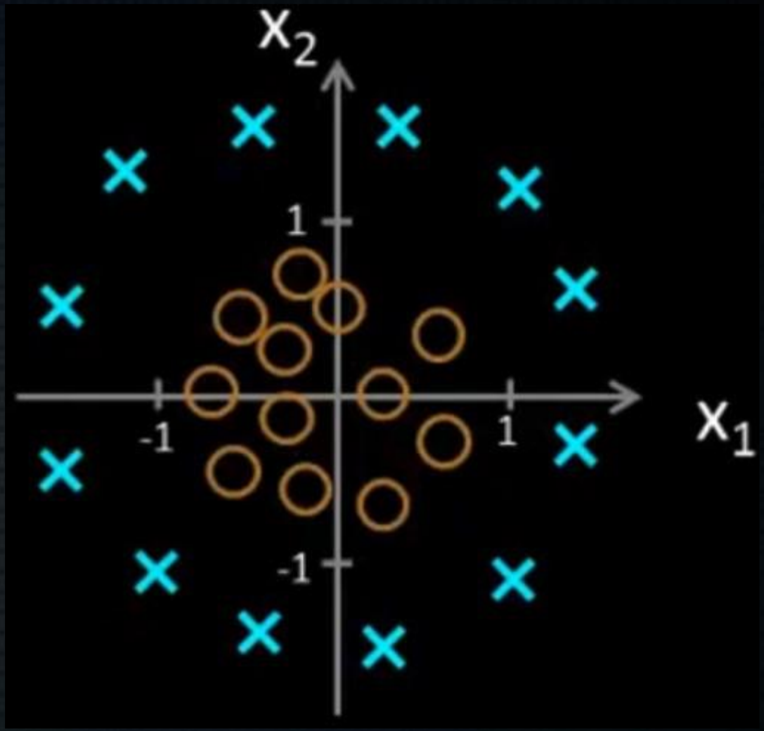

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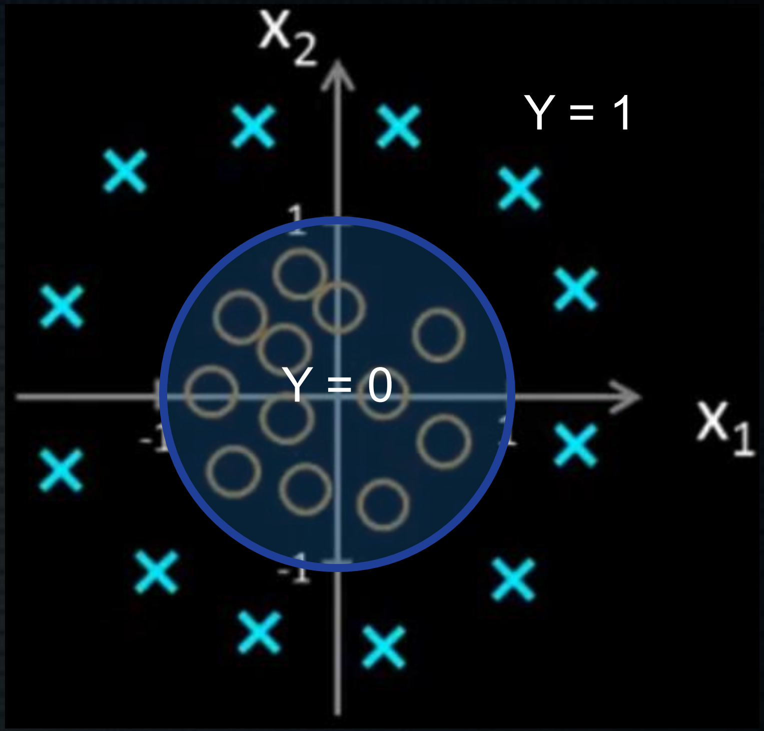
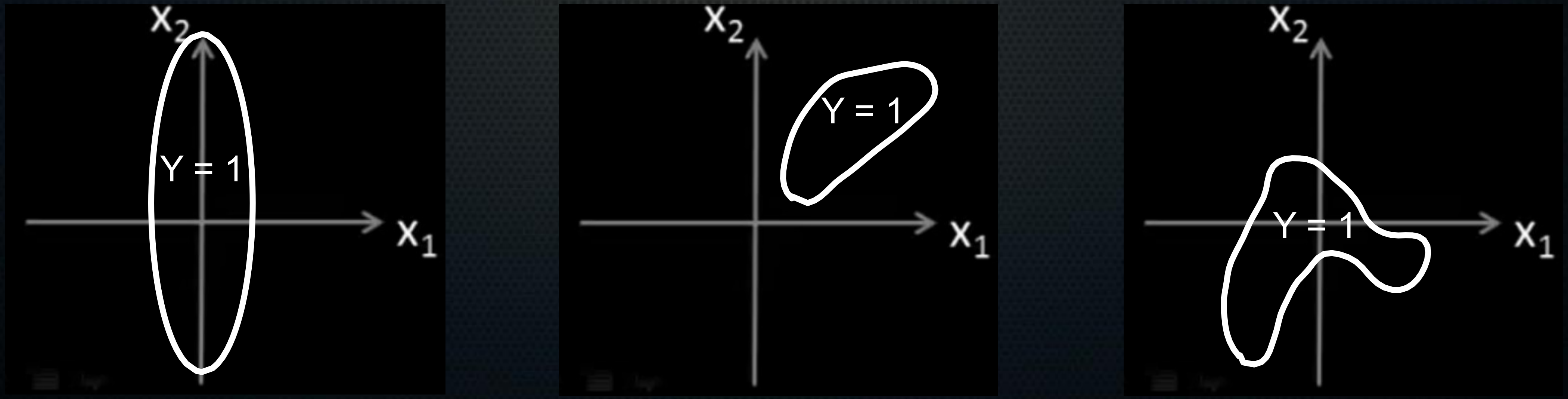


Non-linear decision boundary

- How does it look?
- If you add more and higher-order polynomial features, you can get complex boundaries:



So how do we get theta‘s?

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- Before: $$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

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$$\updownarrow$$

$$J(\theta)=\frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}(h_\theta(x^{(i)})-y^{(i)})^2$$

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$$\mathrm{Cost}(x)=\frac{1}{2}(h_\theta(x)-y)^2$$

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- Before: $$J(\theta)=\frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)})-y^{(i)})^2$$

$$\updownarrow$$

$$J(\theta)=\frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}(h_\theta(x^{(i)})-y^{(i)})^2$$

$$\mathrm{Cost}(x)=\frac{1}{2}(h_\theta(x)-y)^2$$

$$J(\theta)=\frac{1}{m}\sum_{i=1}^{m}\mathrm{Cost}(x^i)$$

So how do we get theta's?

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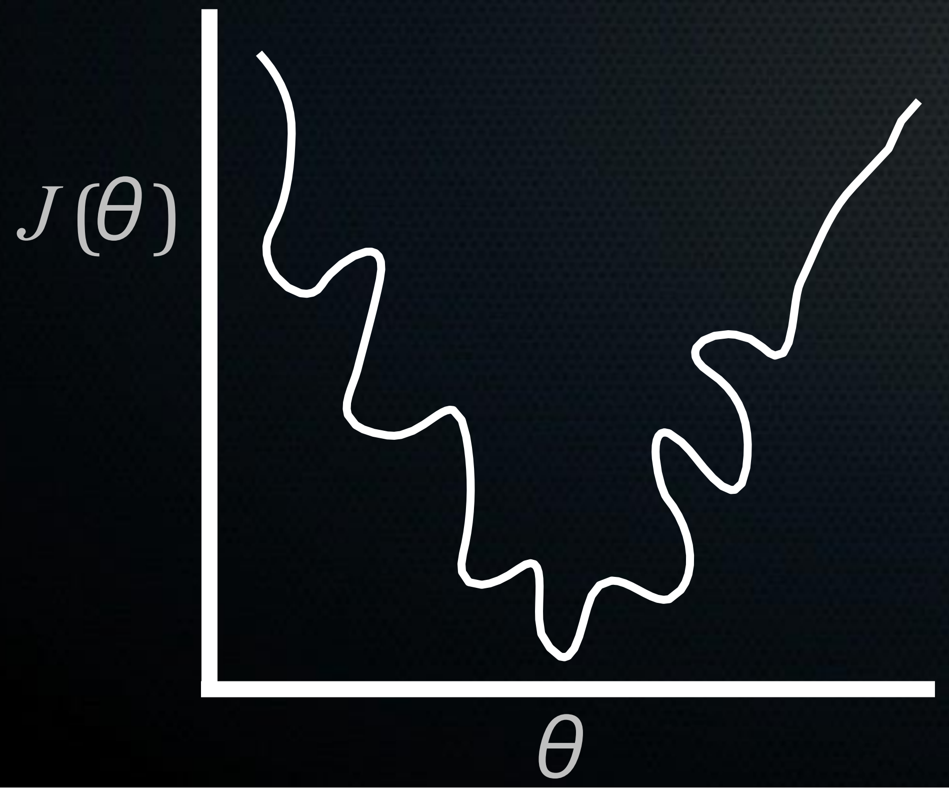
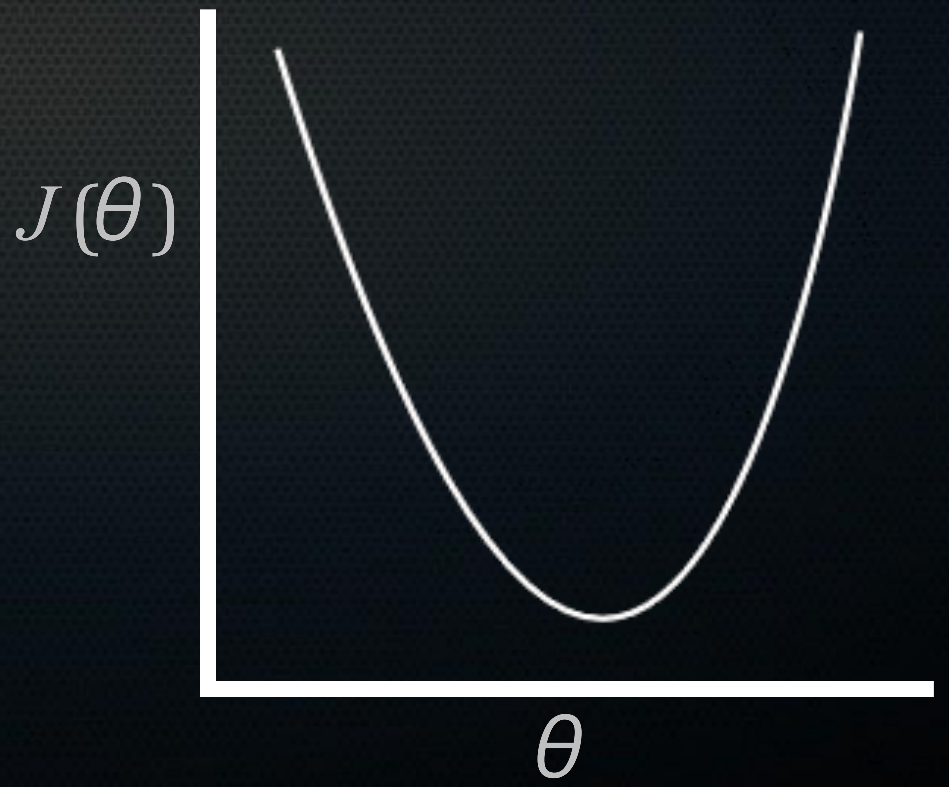


convex



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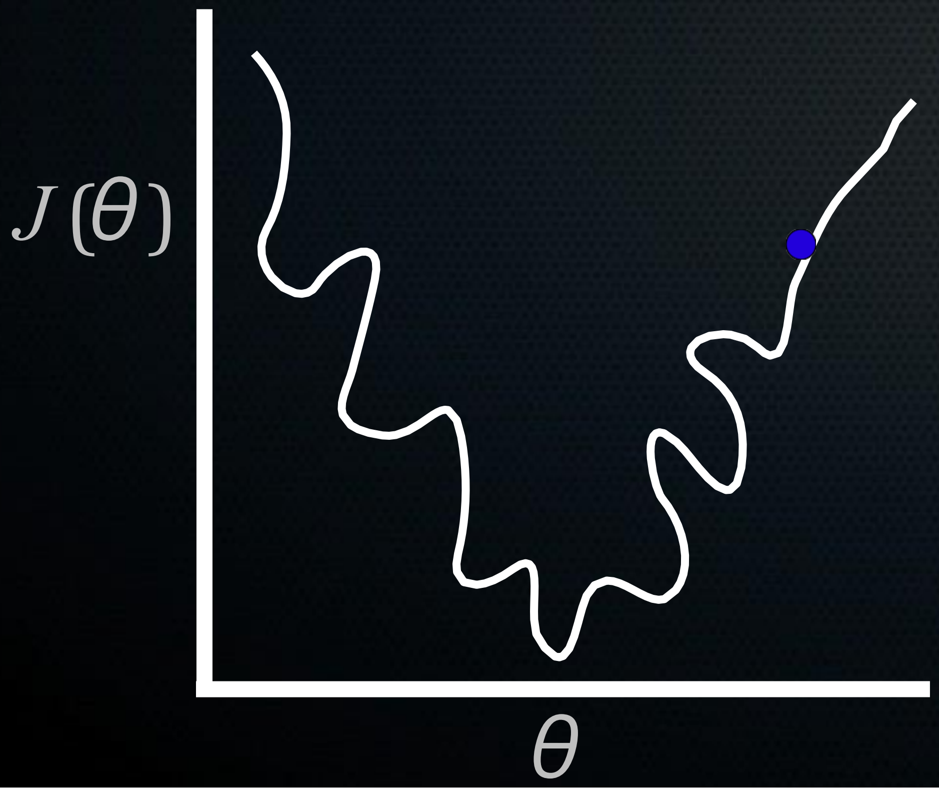

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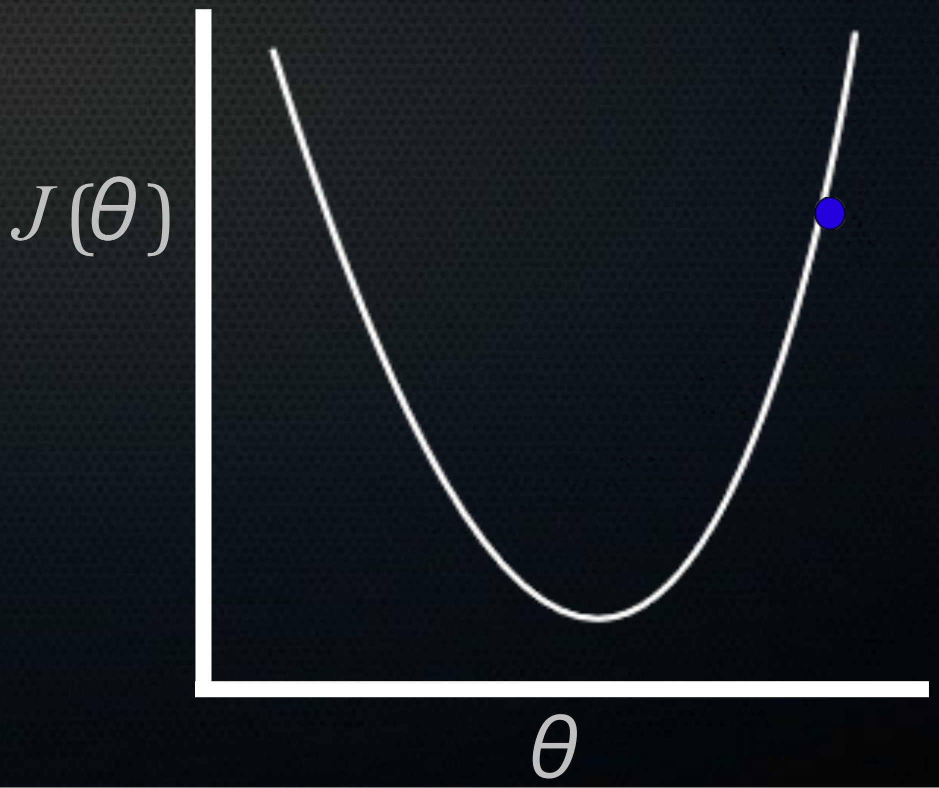

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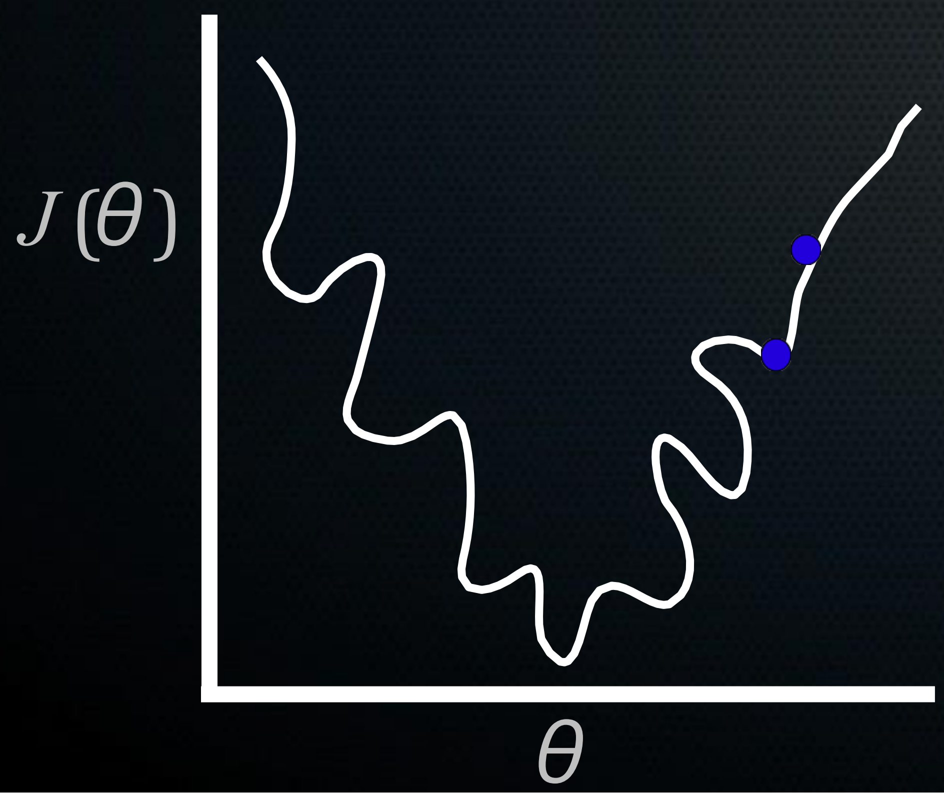

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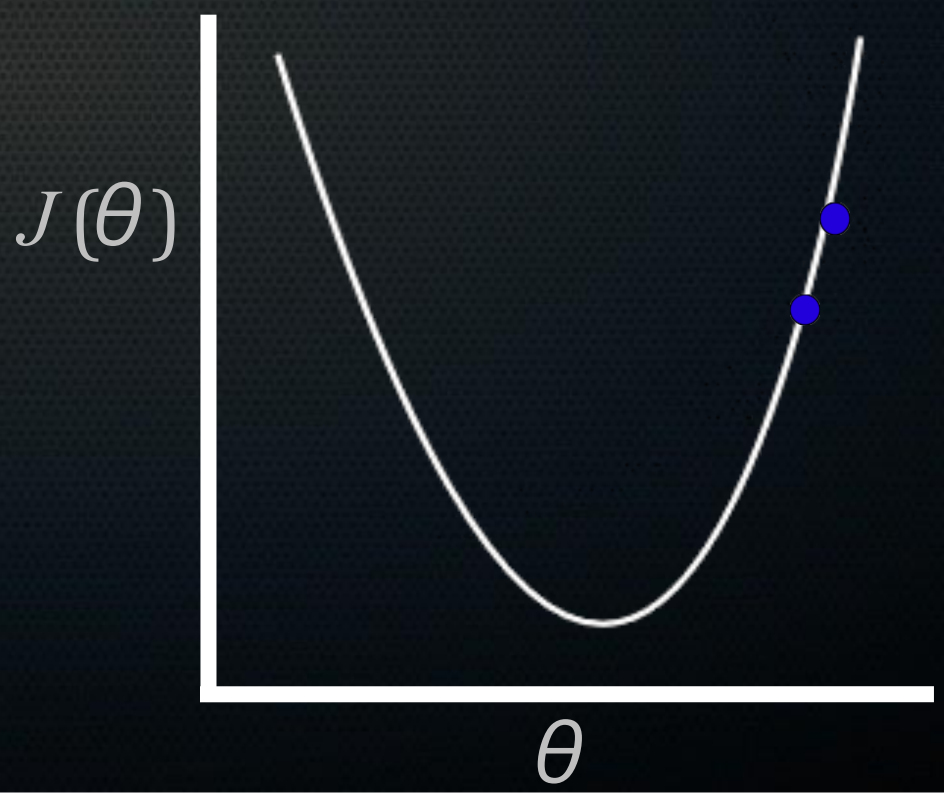

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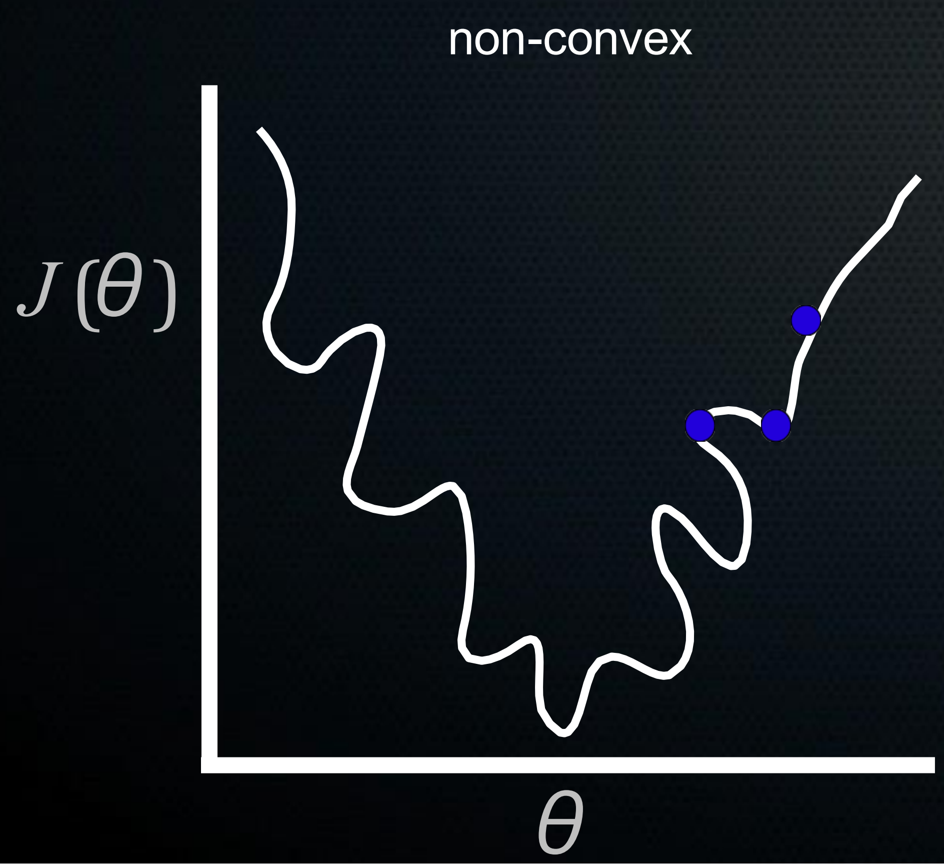

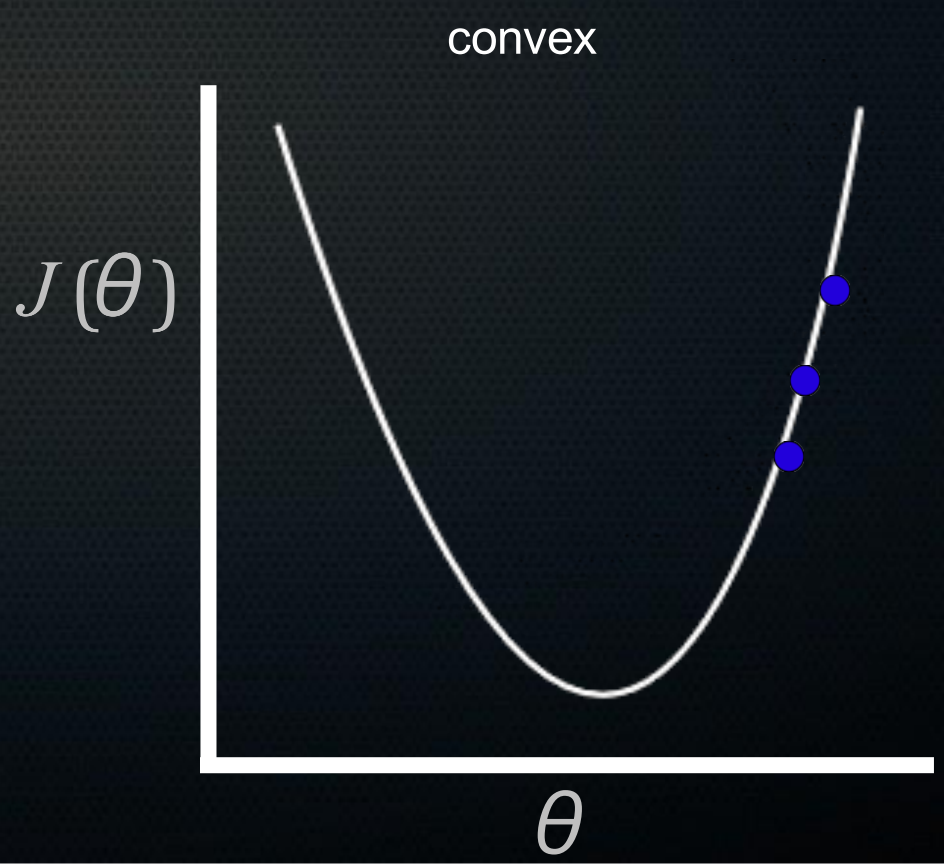

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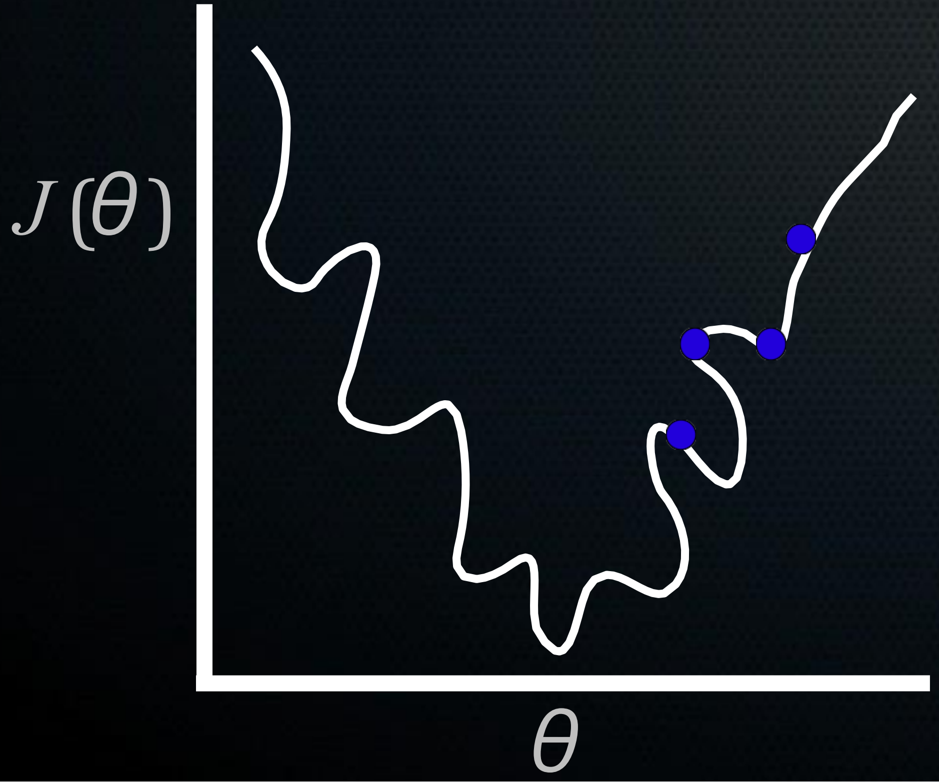

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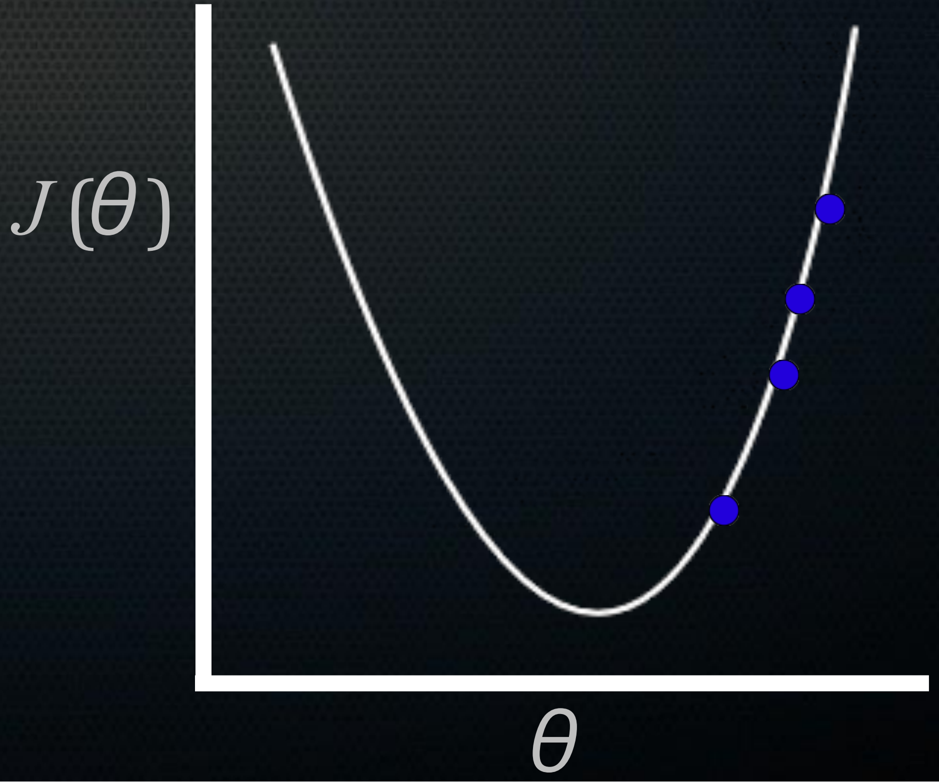

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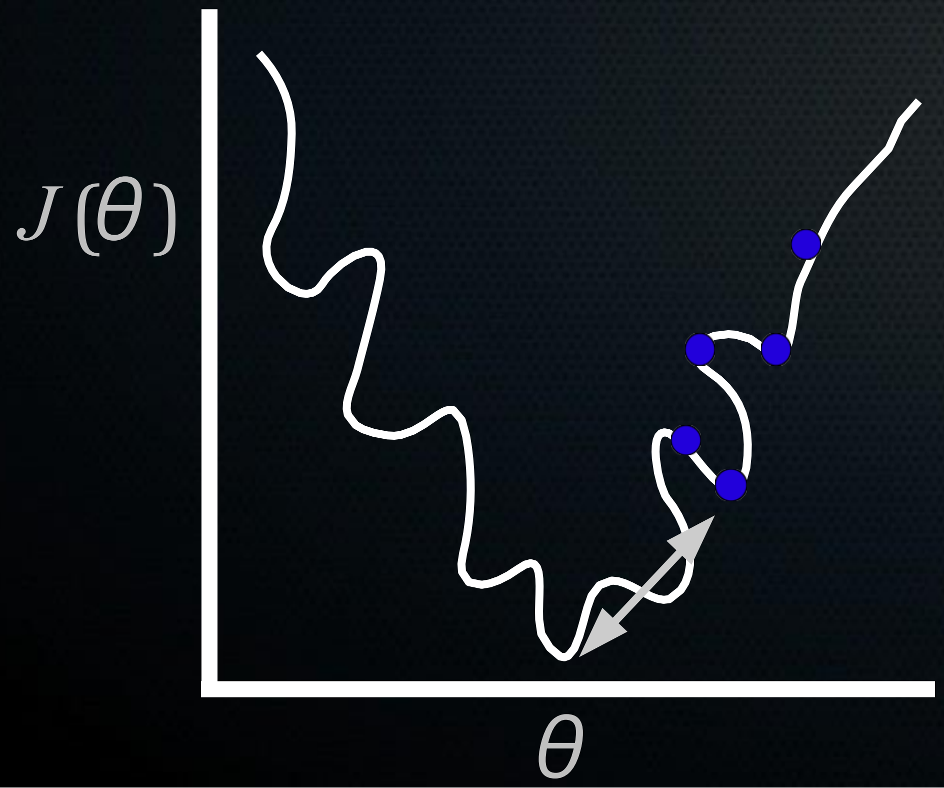

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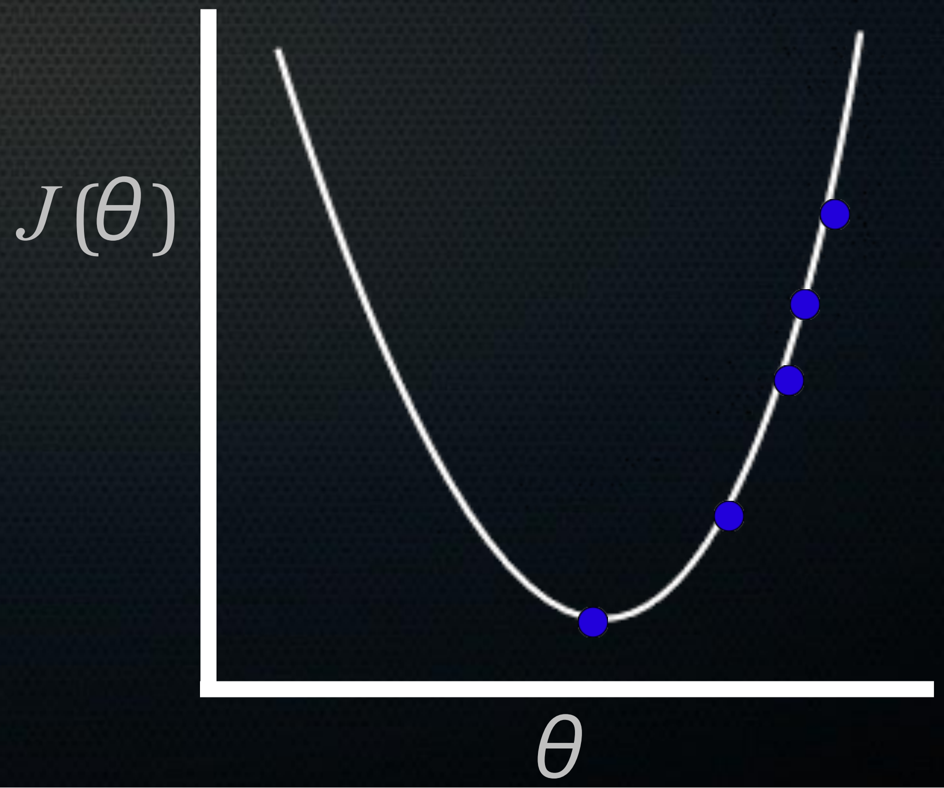

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- Need a cost function $J(\theta)=\frac{1}{m}\sum_{i=1}^{m}$ $\text{Cost}(x^i)$ $\cancel{\text{Cost}(x)=\frac{1}{2}(h_\theta(x)-y)^2}$
- What then?

$$\text{Cost}(x)=\begin{cases}-\log(h_\theta(x))\text{ if } y=1\\ -\log(1-h_\theta(x))\text{ if } y=0\end{cases}$$

What does this function look like?

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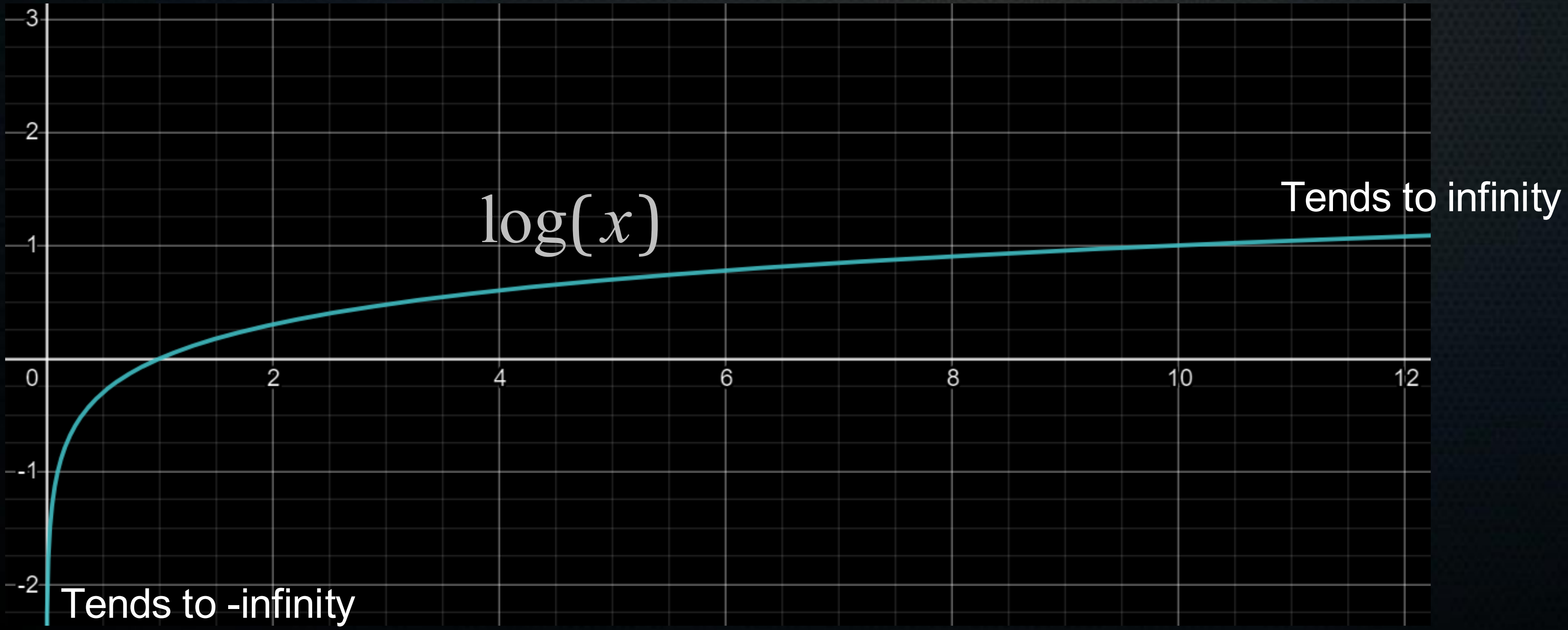

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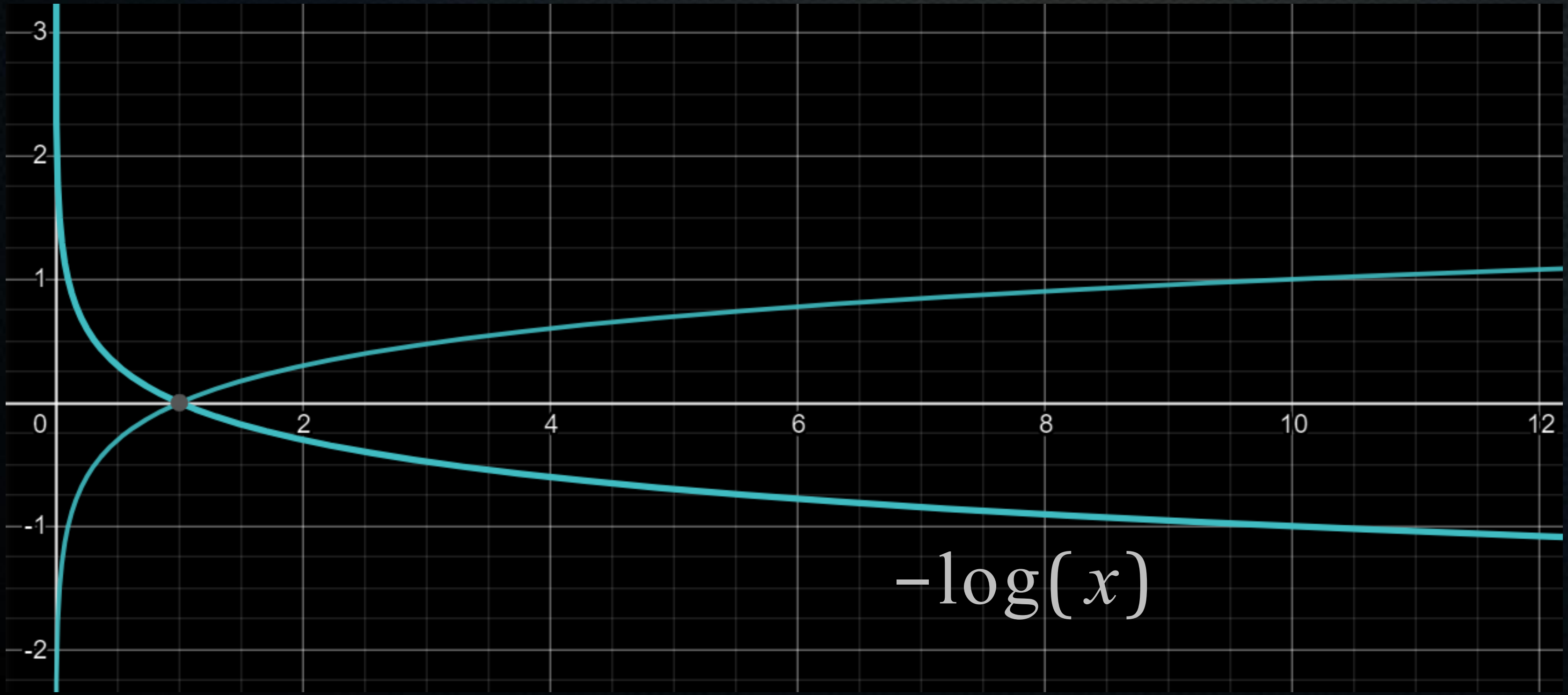

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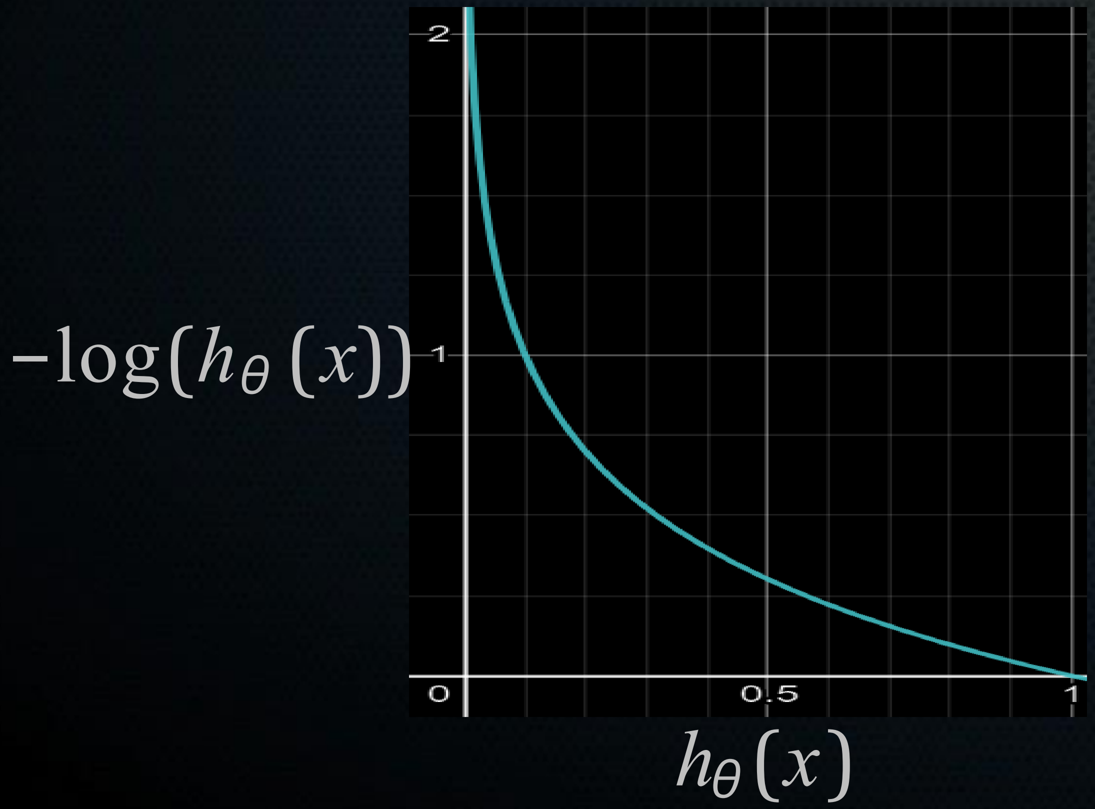

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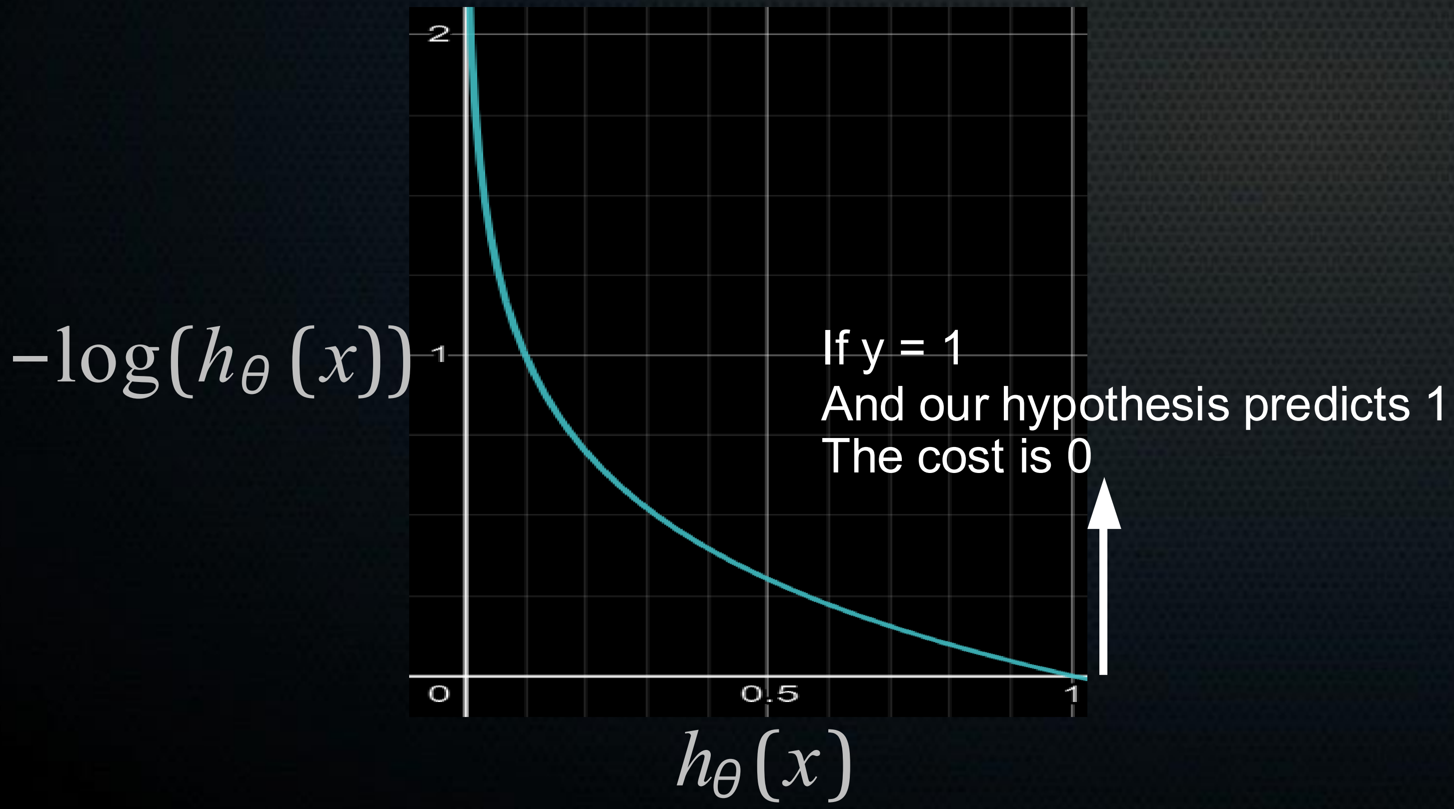

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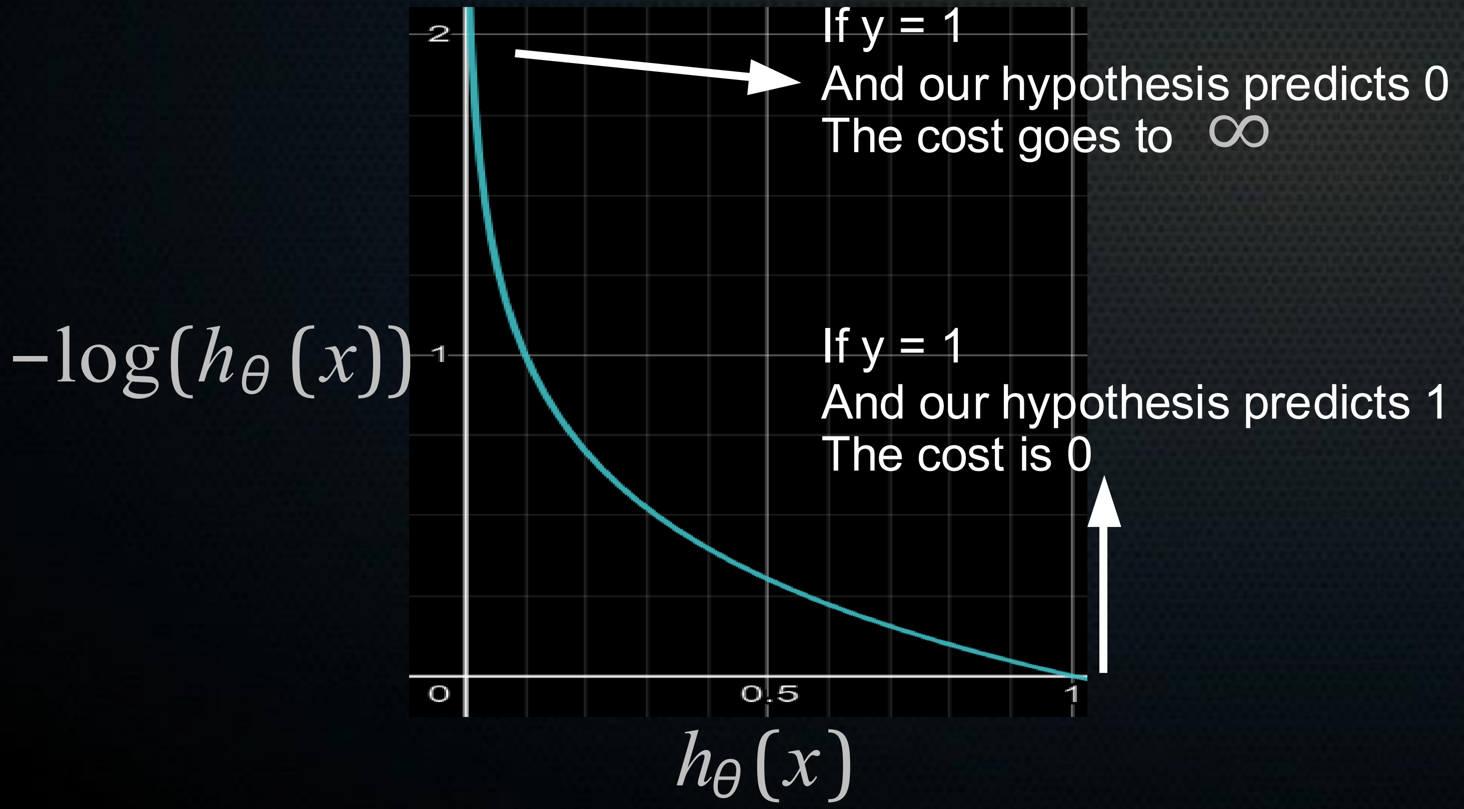

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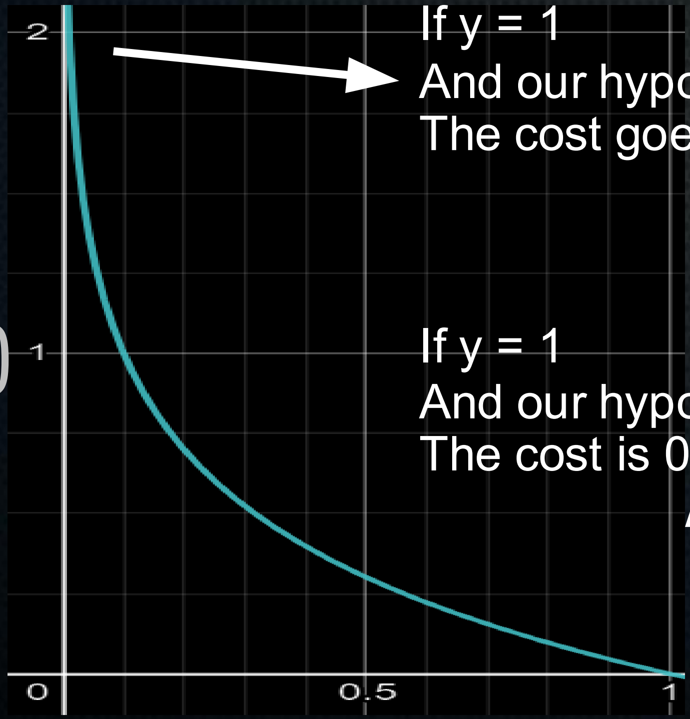

$h_\theta(x)$

If y = 1
And our hypothesis predicts 0
The cost goes to ∞

If we had
used MSE

$(0-1)^2 = 1$

Penalised far less!

If y = 1
And our hypothesis predicts 1
The cost is 0

What does this function look like?

$$\text{Cost}(x) = \begin{cases} -\log(h_\theta(x)) \text{ if } y=1 \\ \boxed{-\log(1-h_\theta(x)) \text{ if } y=0} \end{cases}$$

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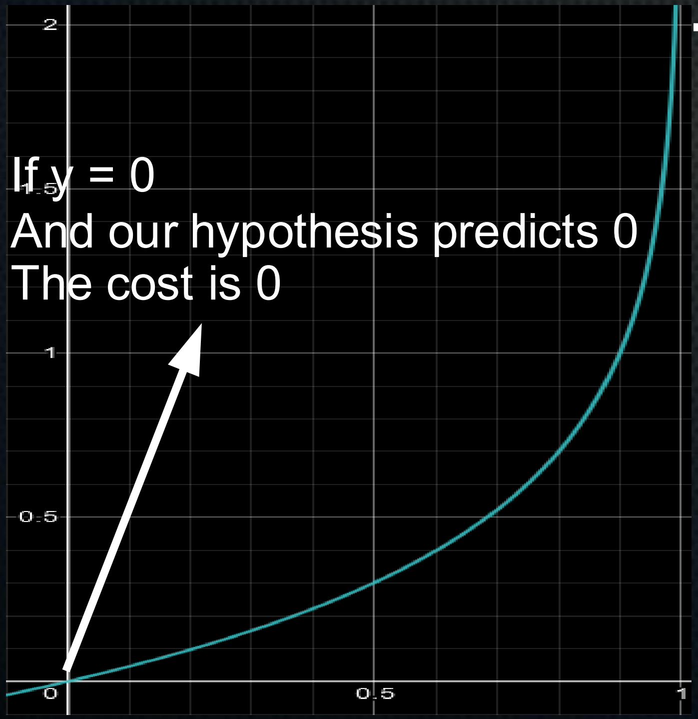

$h_\theta(x)$

If y = 0
And our hypothesis predicts 1
The cost goes to ∞

Simplified notation

$$\text{Cost}(x) = \begin{cases} -\log(h_\theta(x)) \text{ if } y=1 \\ -\log(1-h_\theta(x)) \text{ if } y=0 \end{cases}$$

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \text{Cost}(x^i)$$

$$\text{Cost}(x) = -y \cdot \log(h_\theta(x)) - (1-y) \cdot \log(1-h_\theta(x))$$

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$$\Big\downarrow\ y = 0$$

$$-0 \cdot \log(h_\theta(x)) - (1-0) \cdot \log(1-h_\theta(x))$$

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$$-0\cdot\log(h_\theta(x)) - (1-0)\cdot\log(1-h_\theta(x))$$

$$\boxed{-\log(1-h_\theta(x))}$$

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$y = 1$

$$-1 \cdot \log(h_\theta(x)) - (1-1) \cdot \log(1-h_\theta(x))$$

$$\boxed{-\log(h_\theta(x))}$$

Putting it all together

$$\mathrm{Cost}(x) = -y \cdot \log(h_\theta(x)) - (1-y) \cdot \log(1-h_\theta(x)) \qquad J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \mathrm{Cost}(x^i)$$

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} -y^{(i)} \cdot \log(h_\theta(x^{(i)})) - (1-y^{(i)}) \cdot \log(1-h_\theta(x^{(i)}))$$

Putting it all together

$$\mathrm{Cost}(x) = -y \cdot \log(h_\theta(x)) - (1-y) \cdot \log(1-h_\theta(x)) \qquad J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \mathrm{Cost}(x^i)$$

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$$J(\theta) = \boxed{-} \frac{1}{m}\sum_{i=1}^{m} y^{(i)} \cdot \log(h_\theta(x^{(i)})) + (1-y^{(i)}) \cdot \log(1-h_\theta(x^{(i)}))$$

Optimising the cost function

- Same form as for linear regression (only hypothesis function differs!)

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left(\left(\boxed{h_\theta(x^{(i)})} - y^{(i)} \right) \cdot x_j^{(i)} \right)$$

$$\theta_j := \theta_j - \frac{a}{m} \sum_{i=1}^{m} \left(\left(\boxed{h_\theta(x^{(i)})} - y^{(i)} \right) \cdot x_j^{(i)} \right)$$

$$\boxed{h_\theta(x^{(i)}) = \frac{1}{1 + e^{-\theta^T * x}}}$$

Derivation link

Summary

- By using the sigmoid function as a transformation of normal regression and interpreting the output as a chance of being 0 or 1 we can do classification.
- Only the form of our hypothesis function is different
- Need a different cost function: should be smooth, and give logical values for large errors.

Break for practical