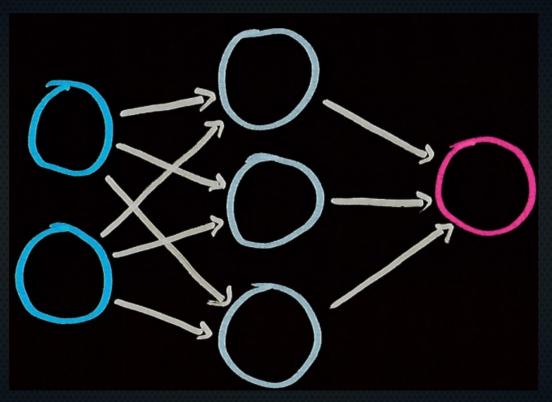
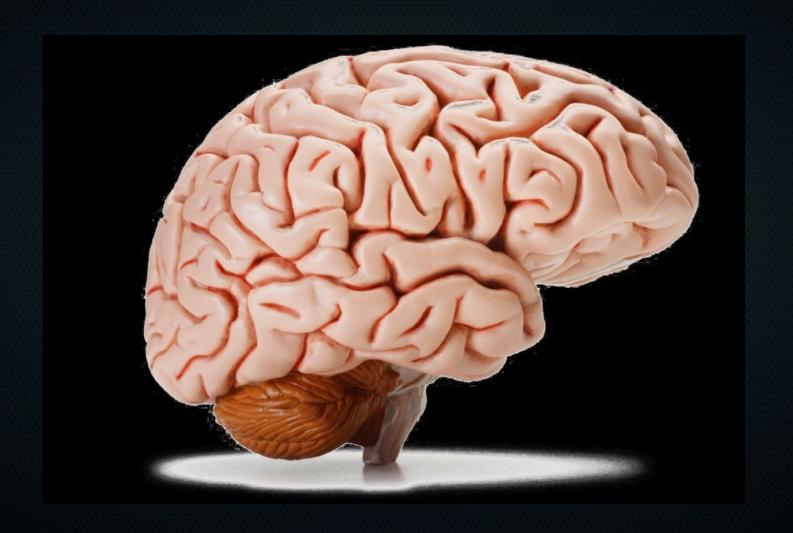
### Switching gears: neural networks

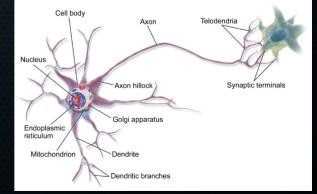


Source: https://thesharperdev.com/build-your-first-neural-network-part-2/



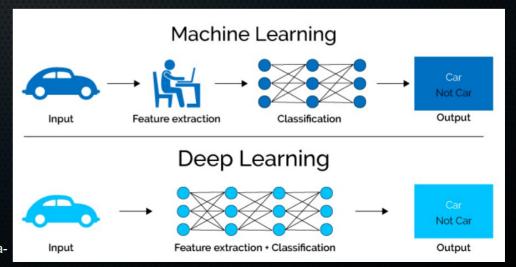
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(lines, edges) with later layers incorporating this information into more complex features about what is seen.



Source: https://en.wikipedia.org/wiki/Axon#/media/File:Bla usen 0657 MultipolarNeuron.png

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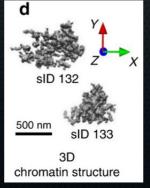


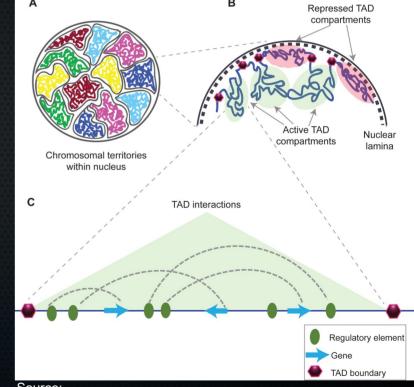
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 Example: in images, nearby pixels probably hold similar information, i.e. are involved in the same thing. Due to (long-range)
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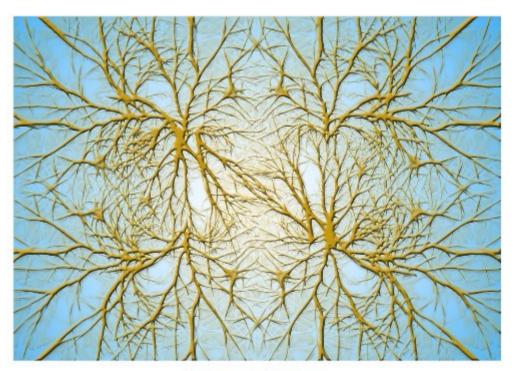
https://en.wikipedia.org/wiki/Topologically\_associating\_domain#/media/File:Structural\_organization\_of\_chromatin.png

 The mythical property of universal approximation. This says that neural networks can approximate any function with arbitrary accuracy, even with only 1 hidden layer (given enough neurons in it).

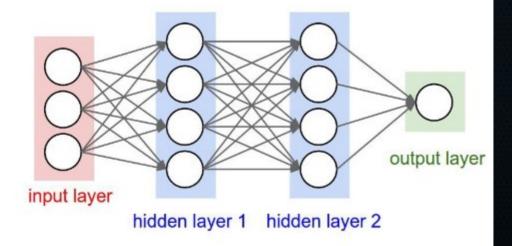
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### Like biology? No

#### Biological Neurons: Complex connectivity patterns



Neurons in a neural network: Organized into regular layers for computational efficiency



This image is CC0 Public Domain

### Like biology? No

#### **Biological Neurons:**

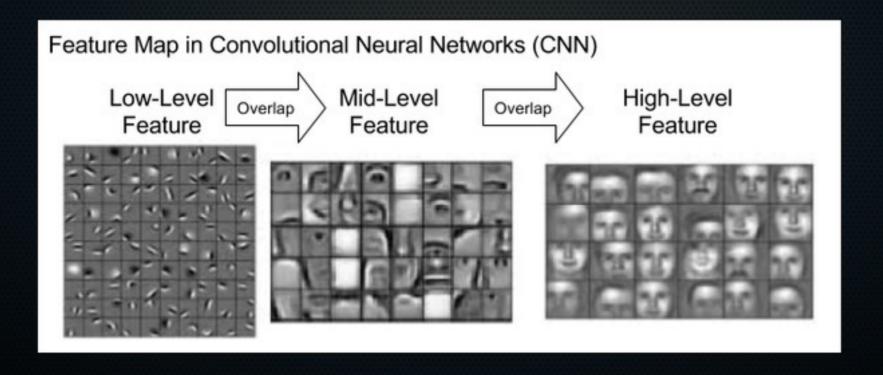
- Many different types
- Dendrites can perform complex non-linear computations
- Synapses are not a single weight but a complex non-linear dynamical system

Source: http://cs231n.stanford.edu/slides/2019/cs231n\_2019\_lecture04.pdf

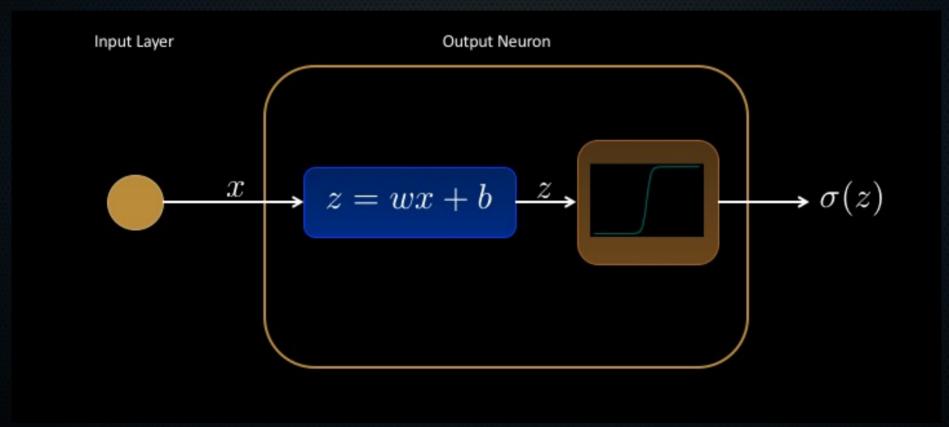
- Human brains ~a cool 86 billion neurons
- A neuron can have 400.000 dendrites
- Real brains vastly outclass their computational analogues

### Like biology? No

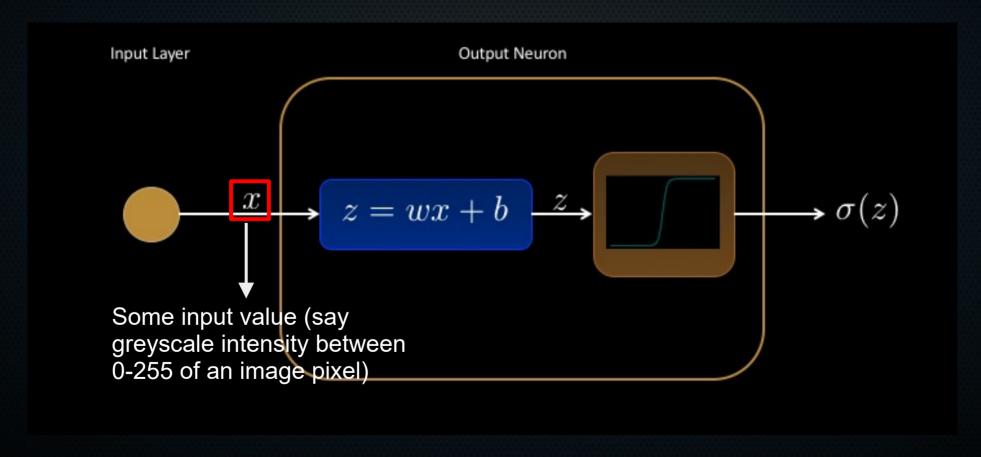
- Still extremely useful
- Parts of how they learn superficially resemble how we learn

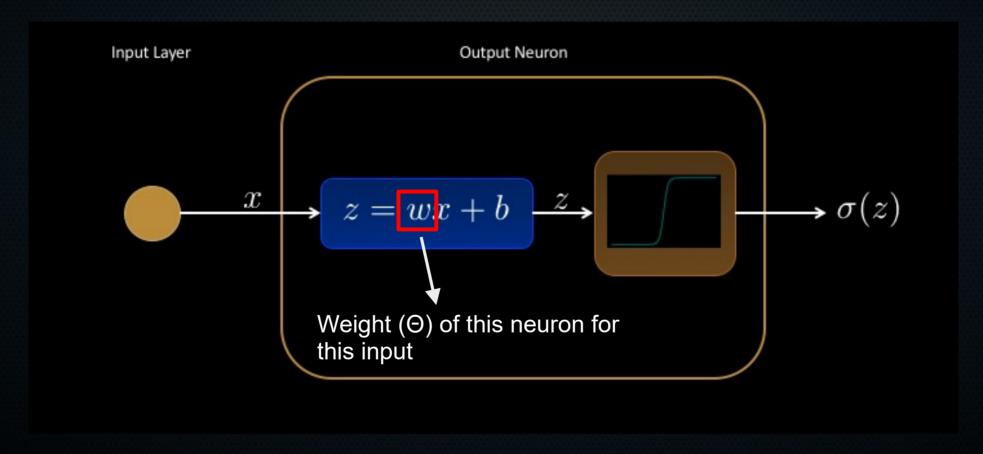


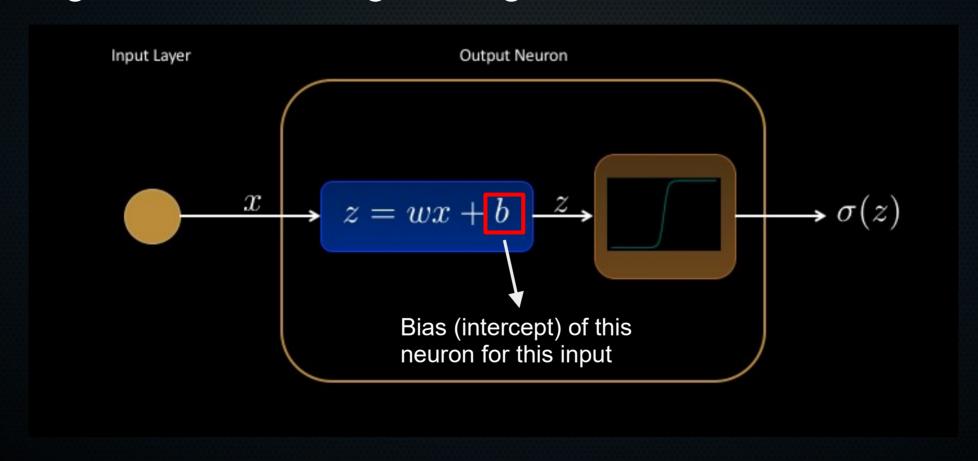
A single neuron is a logistic regressor!\*

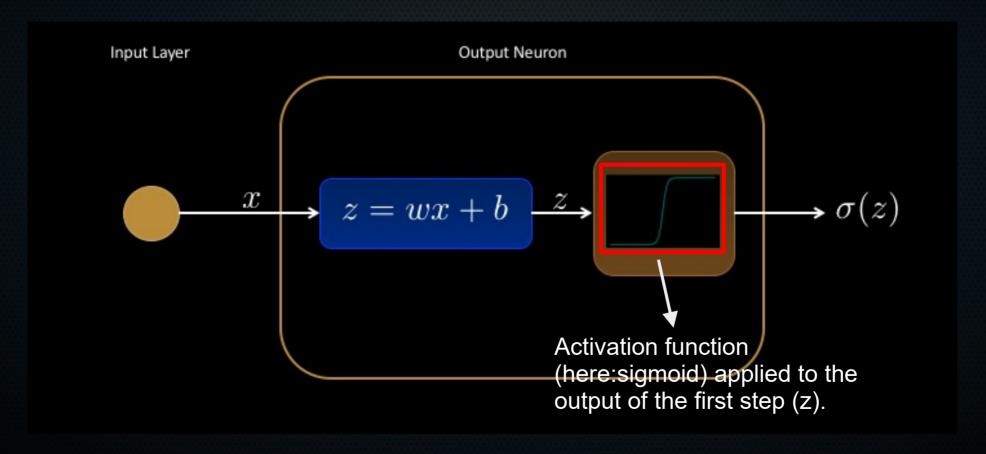


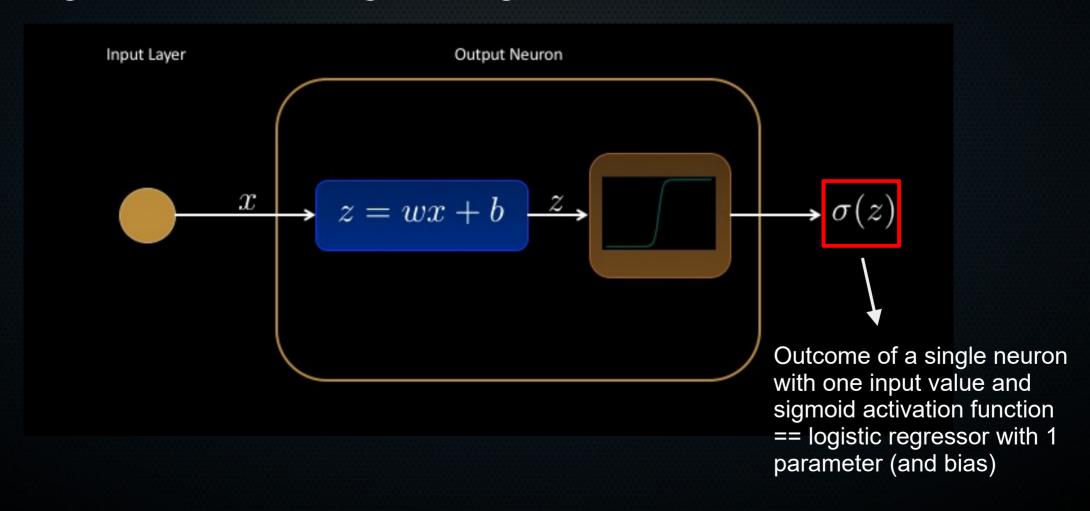
Source: https://thedatafrog.com/en/articles/logistic-regression/

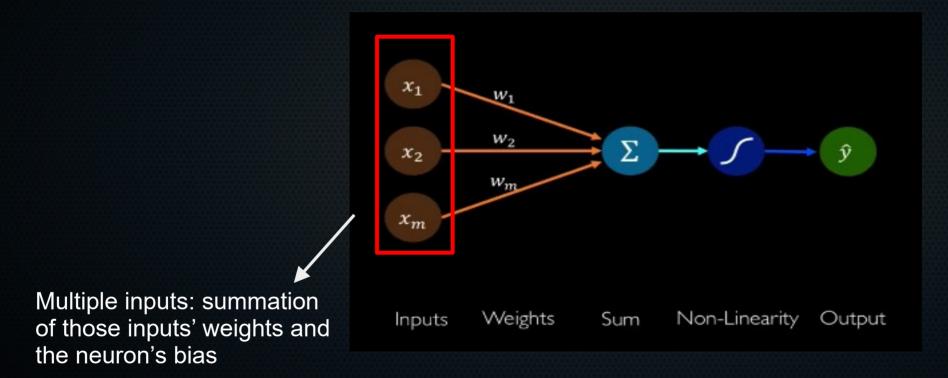


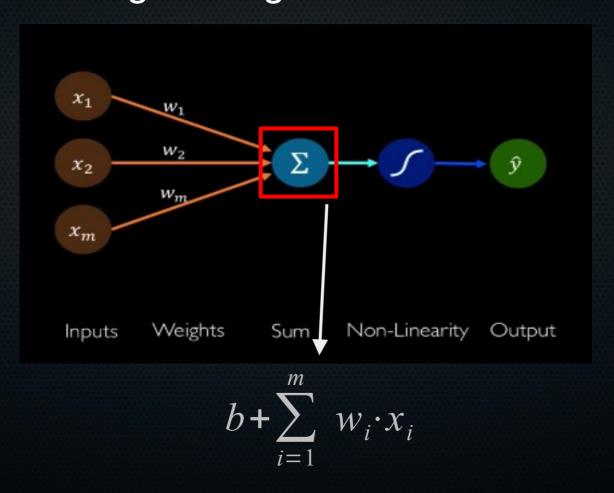


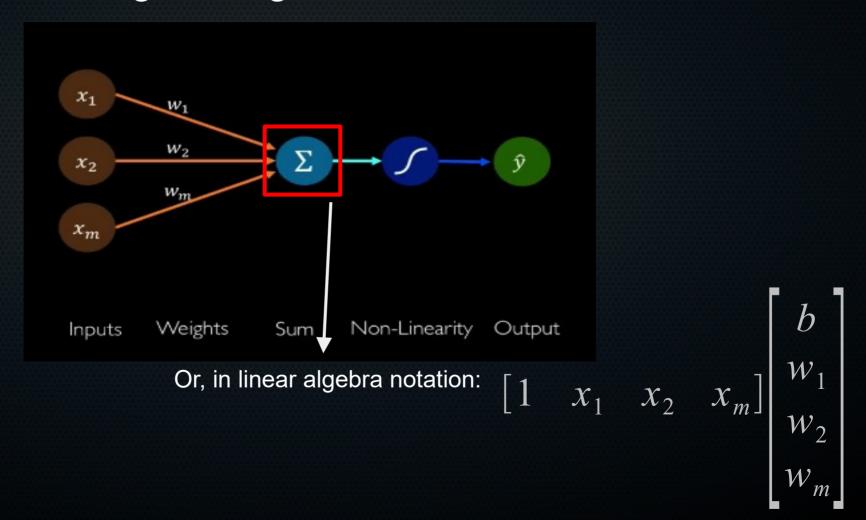


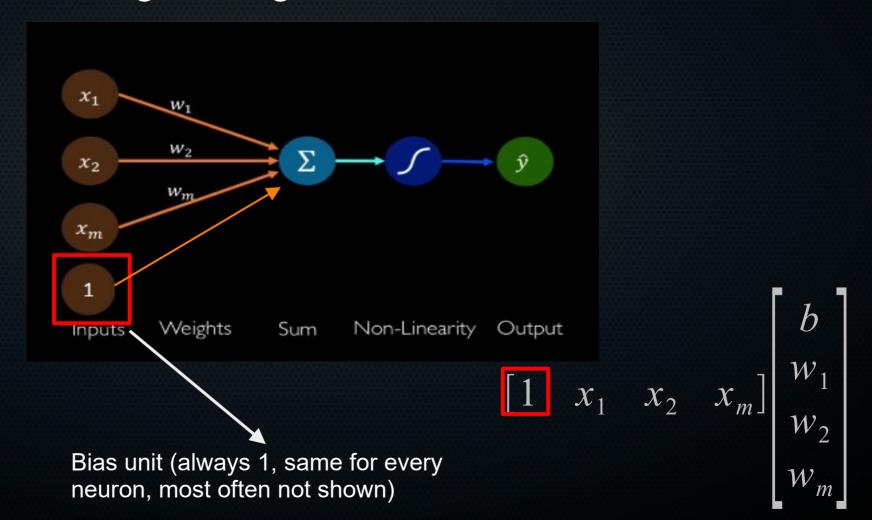


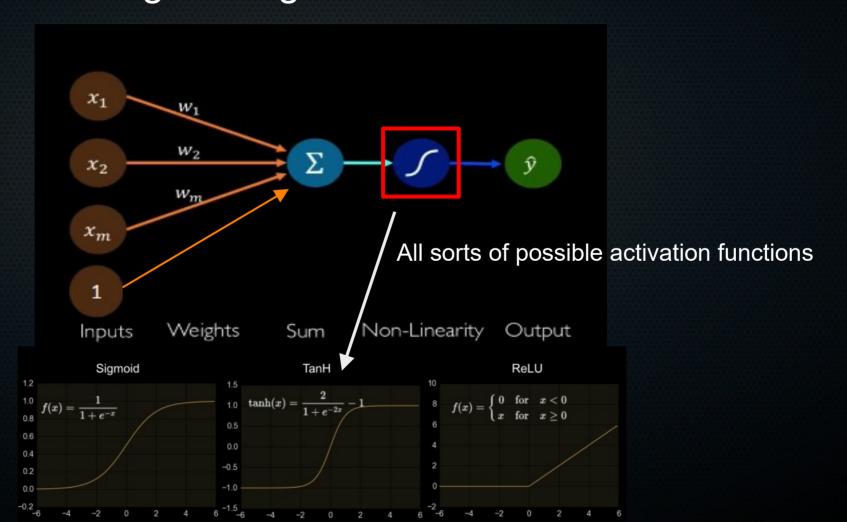




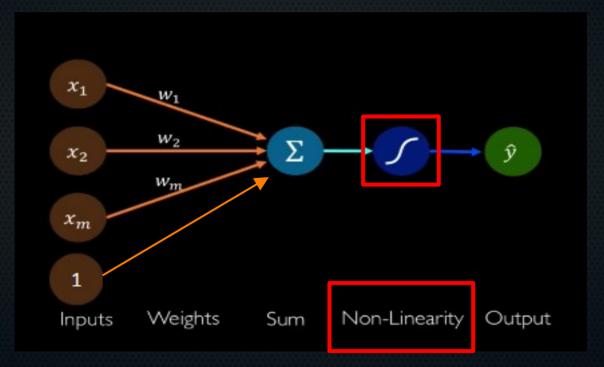






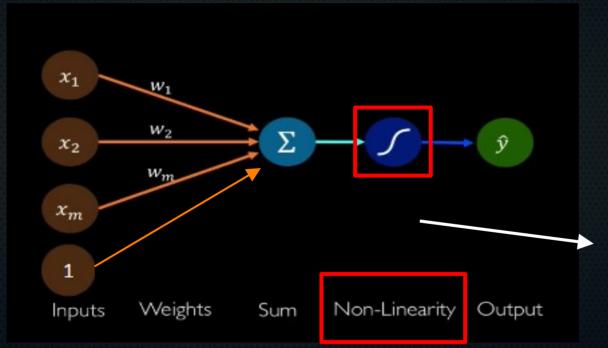


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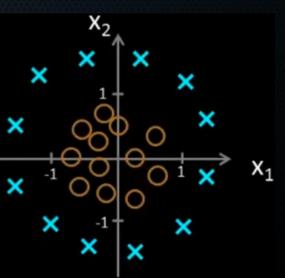


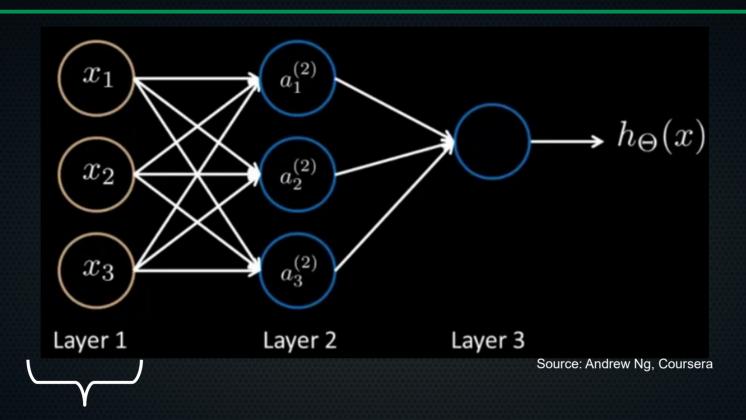
Why non-linearity?

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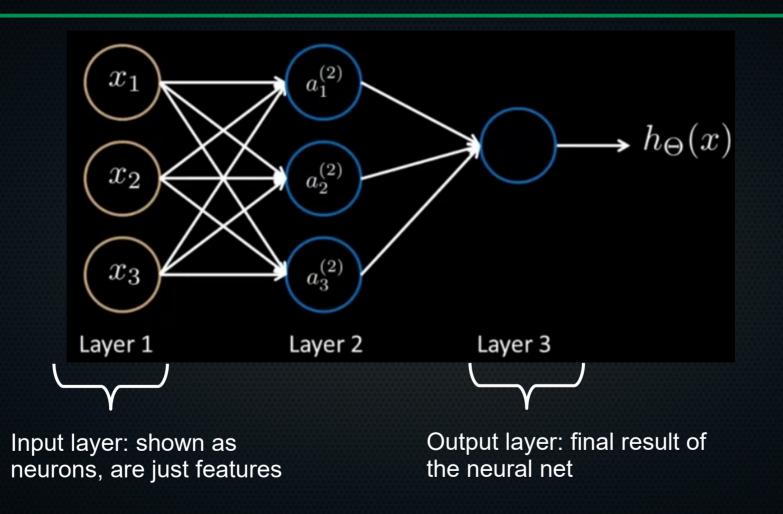


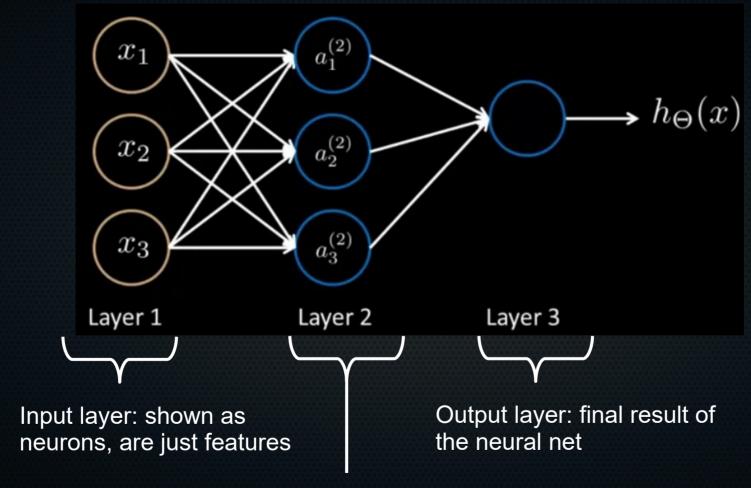
 Why non-linearity? → without them, a NN (no matter how deep) could only approximate linear functions



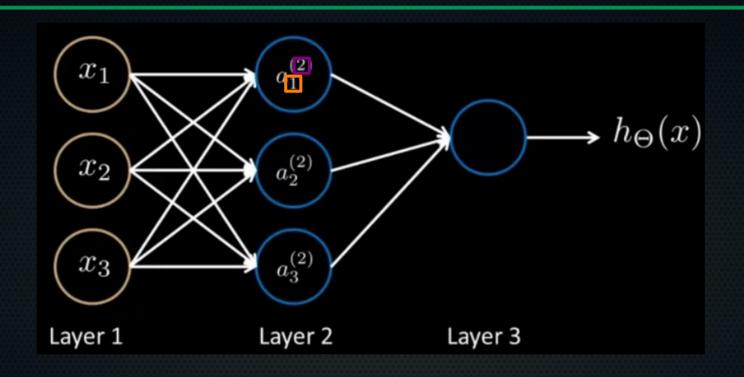


Input layer: shown as neurons, are just features

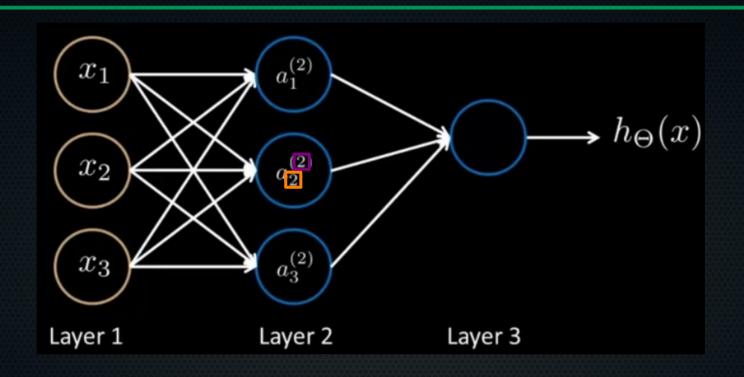




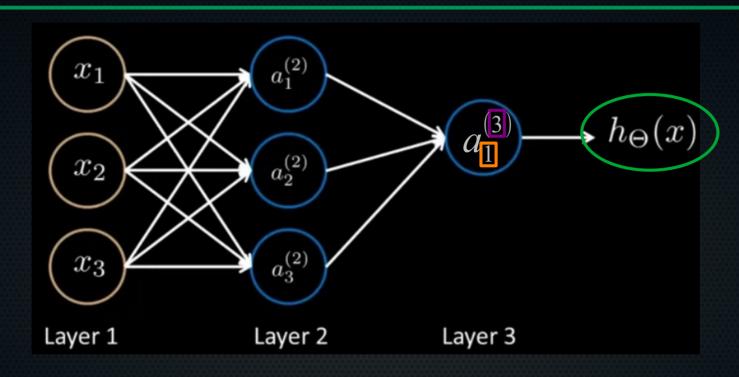
Hidden layer(s): intermediate layers whose outputs are not directly observed (hence hidden). Here: 1 HL. Facebook's DenseNet family of NNs had 121-264 HLs in 2016 (0.8-15.3 million parameters).



Activation of neuron 1 in the 2<sup>nd</sup> layer of the network.

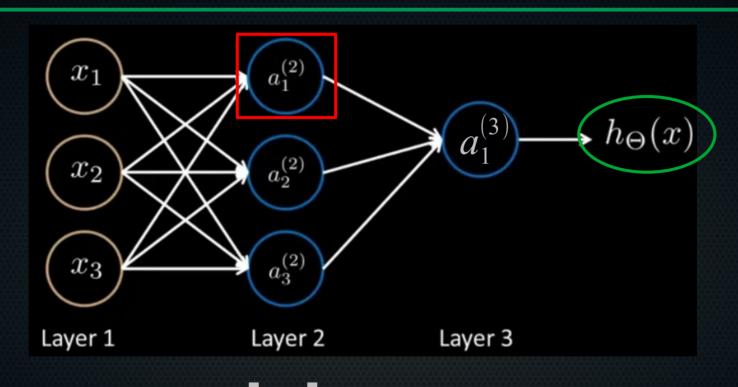


Activation of neuron 2 in the 2<sup>nd</sup> layer of the network.

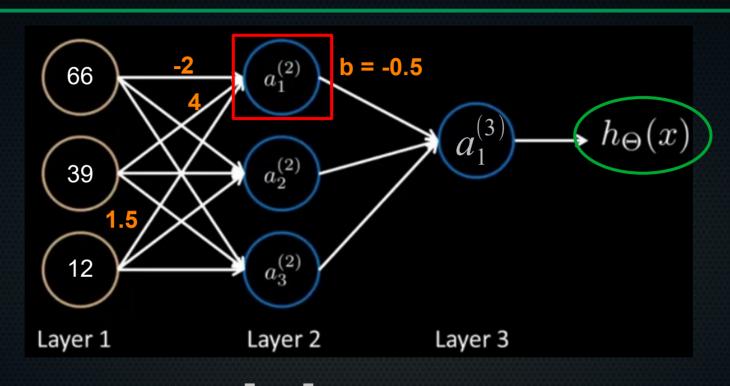


https://www.louisbouchard.ai/densenet-explained/

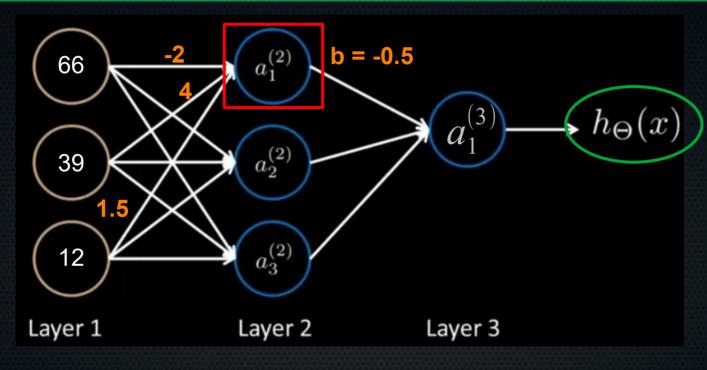
Activation of neuron 1 in the 3rd layer of the network.



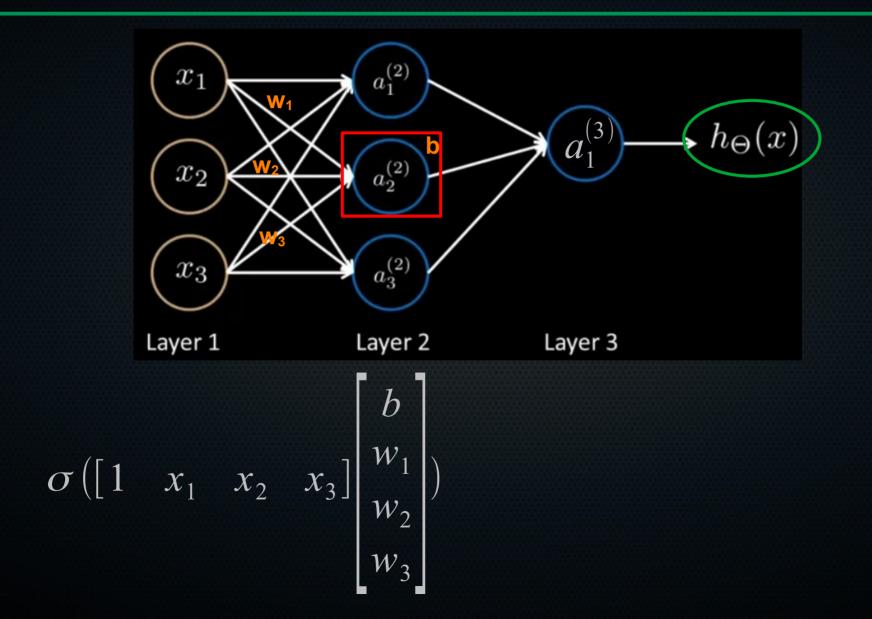
$$\sigma(\begin{bmatrix} 1 & x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} b \\ w_1 \\ w_2 \\ w_3 \end{bmatrix})$$

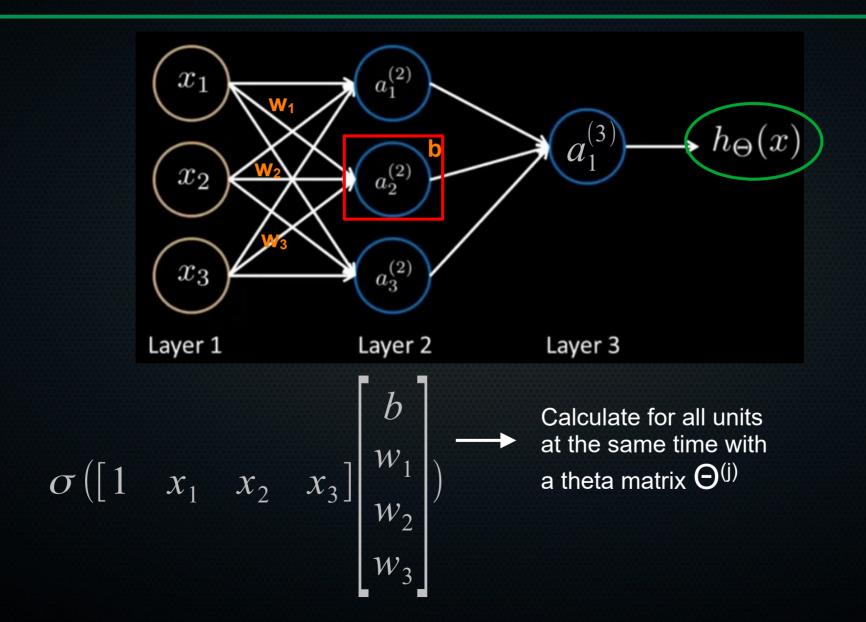


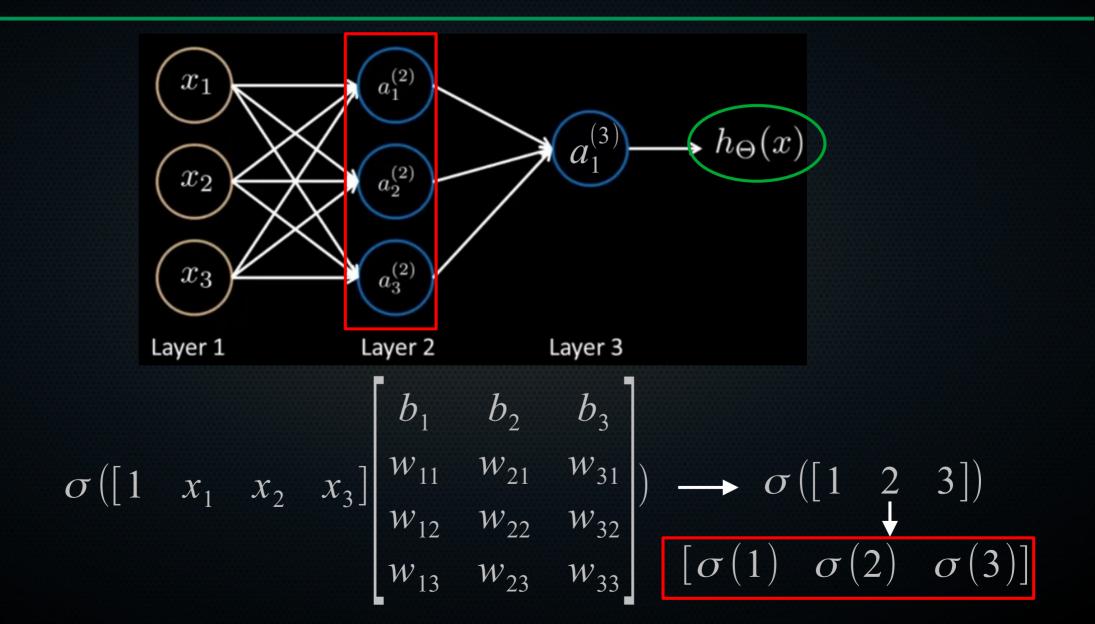
$$\sigma(\begin{bmatrix} 1 & x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} b \\ w_1 \\ w_2 \\ w_3 \end{bmatrix}) \rightarrow \sigma(\begin{bmatrix} 1 & 66 & 39 & 12 \end{bmatrix} \begin{bmatrix} -0.5 \\ -2 \\ 4 \\ 1.5 \end{bmatrix})$$

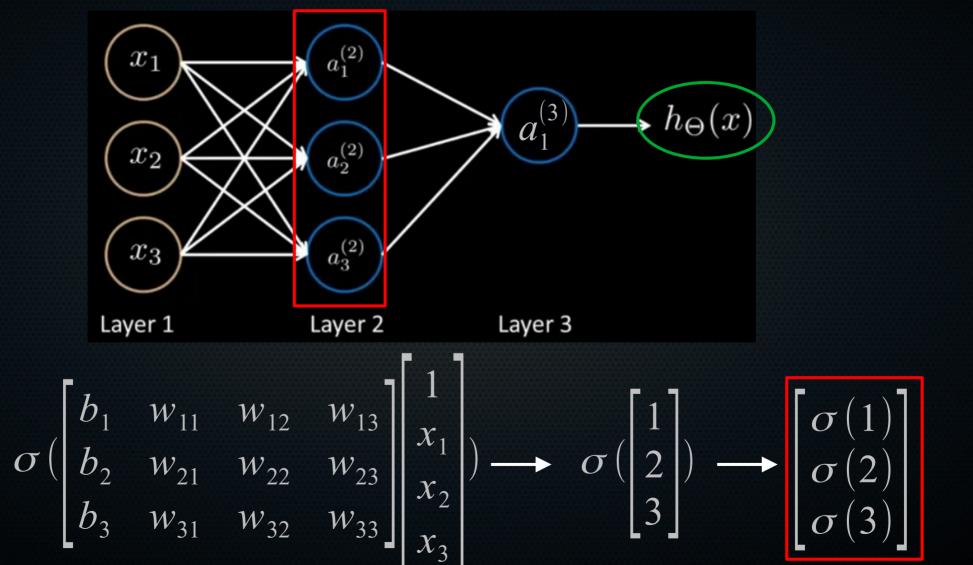


Layer 1 Layer 2 Layer 3
$$\sigma([1 \ 66 \ 39 \ 12] \begin{bmatrix} -0.5 \\ -2 \\ 4 \\ 1.5 \end{bmatrix}) \rightarrow \sigma(41.5) \rightarrow 0.999...$$

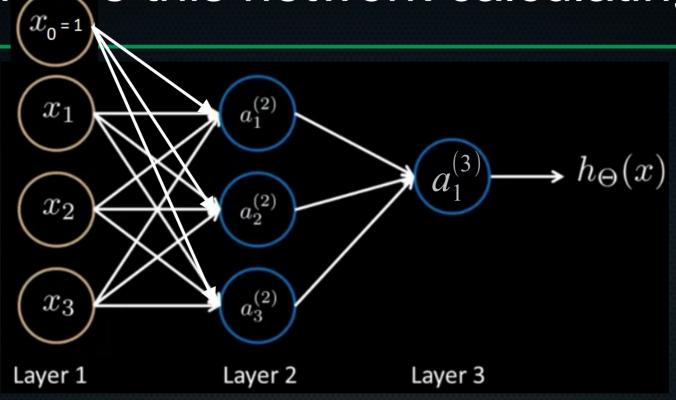








### What is this network calculating?



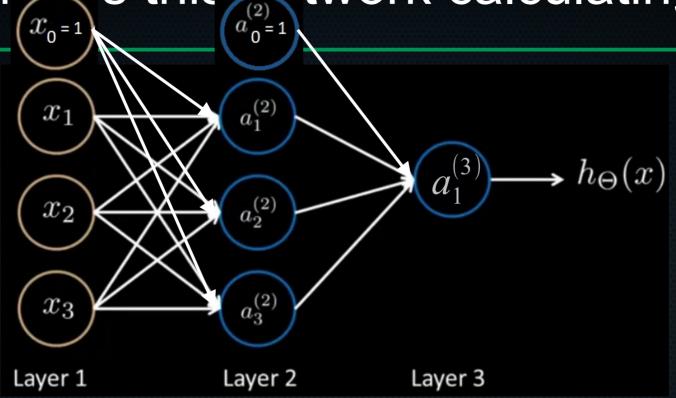
$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$

$$a_2^{(2)} = g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3)$$

$$a_3^{(2)} = g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3)$$

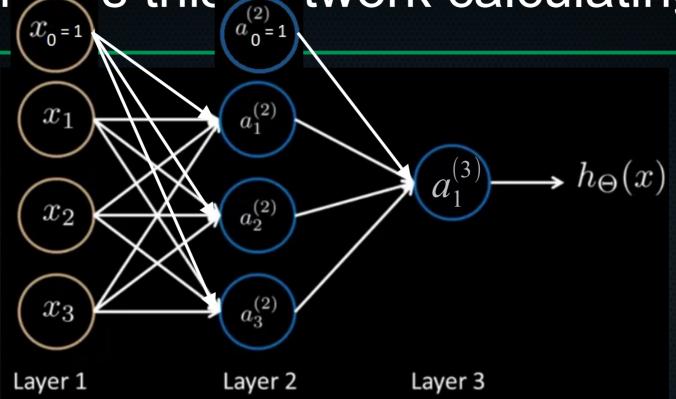
$$\Theta^{(1)}$$
 (layer 1 to layer 2)

What is this notwork calculating?  $x_0=1$ 



$$h_{\Theta}(x) = a_1^{(3)} = g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)}) \qquad \begin{bmatrix} b_1 & w_{11} & w_{12} & w_{13} \end{bmatrix}$$

What is this notwork calculating?



This calculation of the output of the network is called forward propagation

### How do we perform?

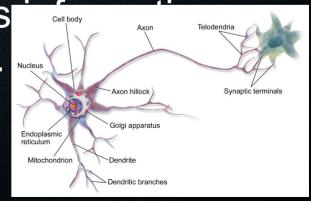
- Just like before, there is a cost function.
- But we will talk about that and its implementation tomorrow!

### How do we get parameters?

- Just like before, there is a cost function and a way to minimise this. But it's a bit more involved.
- To get parameters, we will use the principle of backpropagation. We'll get to that tomorrow.
- First, we need to discuss why we want to use neural networks at all!

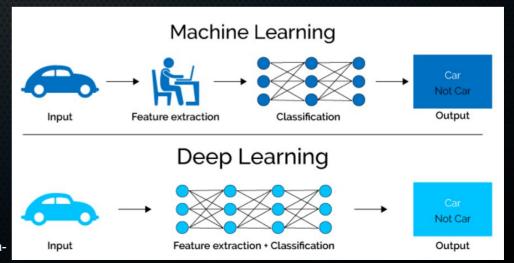
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Source: https://en.wikipedia.org/wiki/Axon#/media/File:Bla usen\_0657\_MultipolarNeuron.png

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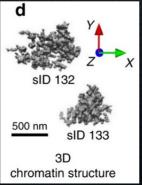


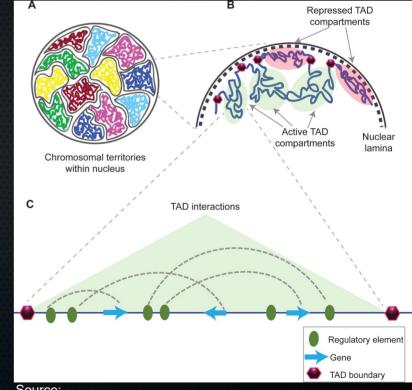
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### Recap so far

- Neurons in neural networks are not really like biological neurons, except superficially
- Neural networks can be thought of as hiërarchical sets of logistic regressors
- We essentially make earlier layers learn useful features for distinction on their own, and can use these best possible learned features for the classification by the final unit(s)
- Parsing an example through the network and getting the output is called forward propagation
- Universal approximation holds that, in principle, neural networks can learn any continuous function arbitrarily well

 Networks with a single hidden layer can approximate any function. Let's get some more intuition by building our own logic circuit.

- Networks with a single hidden layer can approximate any function. Let's get some more intuition by building our own logic circuit.
- Boolean operators:

	OT		AND			OR			XOR	
X	x '	X	У	xy	X	У	X+Y	X	У	<i>x</i> ⊕ <i>y</i>
0	1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	1	0	1	1
		1	0	0	1	0	1	1	0	1
		1	1	1	1	1	1	1	1	0

Networks with a single hidden layer can approximate any function. Let's get some more intuition by building our own logic circuit.

Boolean operators:

Only if two incoming connections are on, the output is on

A\_\_\_X

Source: https://www.electronicstutorial.net/digital-logic-gates/andgate/

N	ОТ		AND			OR			XOR	
X	x'	X	У	xy	X	У	x+y	X	У	<i>x</i> ⊕ <i>y</i>
0	1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	1	0	1	1
		1	0	0	1	0	1	1	0	1
		1	1	1	1	1	1	1	1	0

Only if either incoming connection

Networks with a single hidden layer can approximate any function. Let's get some more intuition by building our own logic circuit.

Boolean operators:

						O GILIP	at 10 011			
	OT		AND			OR			XOR	
X	x '	X	У	xy	X	У	X+Y	<i>X</i>	У	<i>x</i> ⊕ <i>y</i>
0	1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	1	0	1	1
		1	0	0	1	0	1	1	0	1
		1	1	1	1	1	1	1	1	0

is on, the output is on

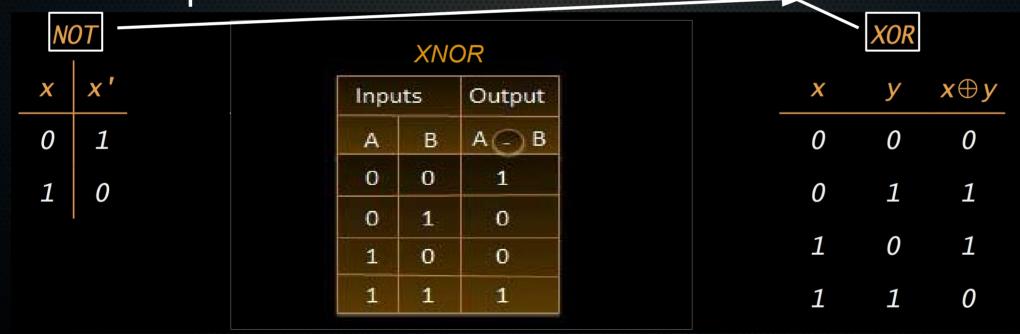
Source: https://introcs.cs.princeton.edu/java/71boolean/

6/introduction-to-xor-gate/

 Networks with a single hidden layer can approximate any function. Let's get some more intuition by building our own logic circuit.

Combine: XNOR

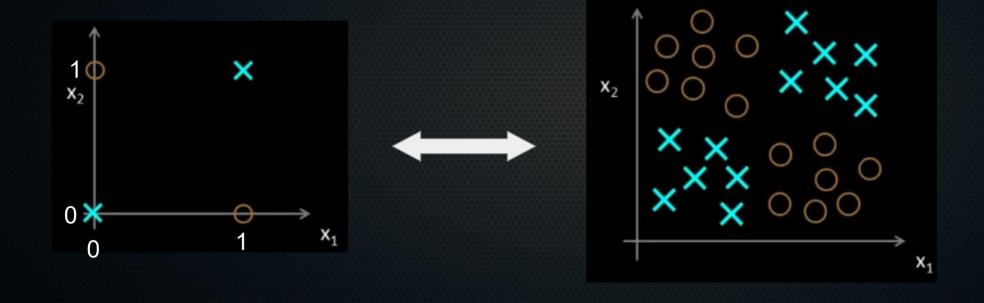
Boolean operators:



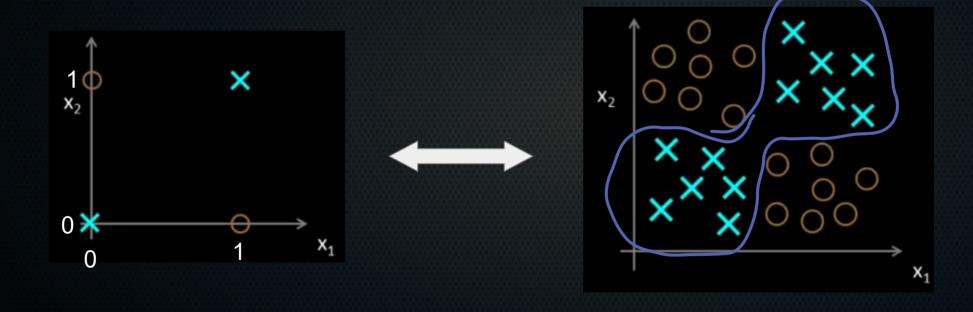
https://www.tutorialspoint.com/com

puter logical organization/logic ga

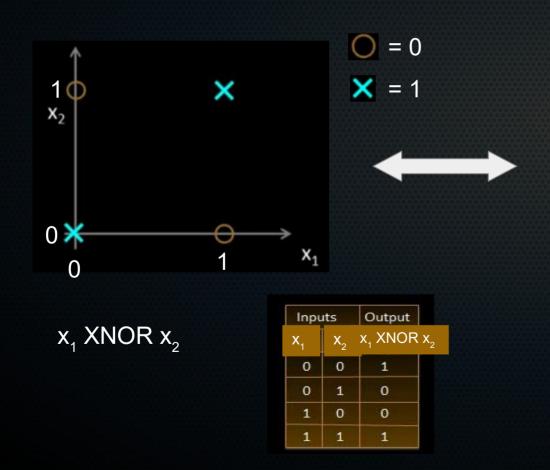
Motivating example:

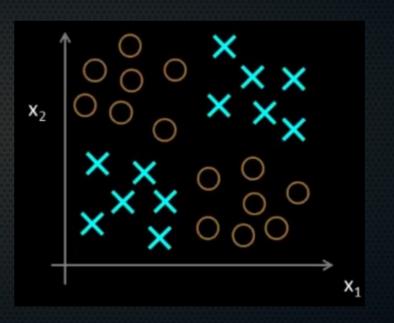


Motivating example:

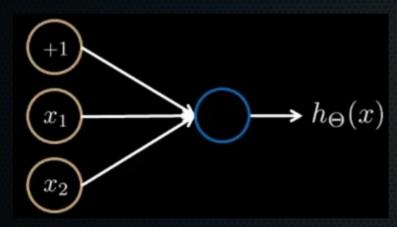


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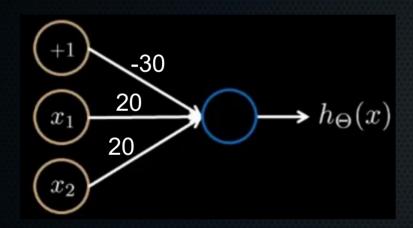




AND function → x<sub>1</sub> and x<sub>2</sub> can be either 0 or 1

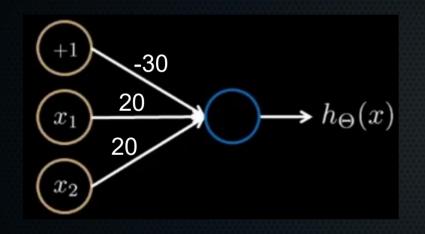


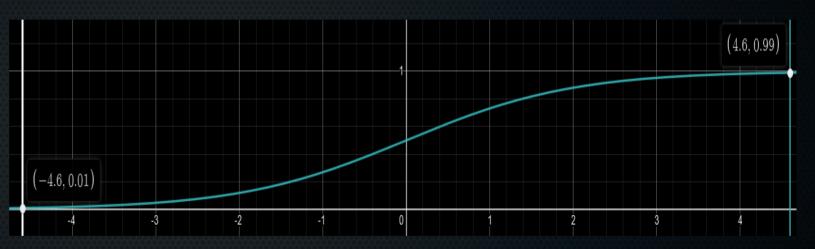
- AND function  $\rightarrow x_1$  and  $x_2$  can be either 0 or 1



$$h_{\theta}(x) = sigmoid(-30 + 20 \cdot x_1 + 20 \cdot x_2)$$
 $h_{\theta}(x) = \sigma([-30 \ 20 \ 20] \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix})$ 

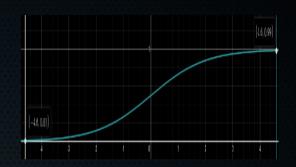
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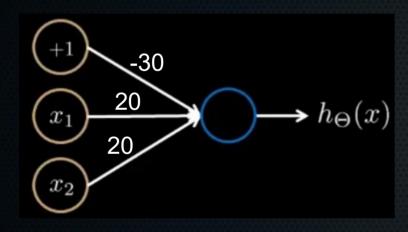




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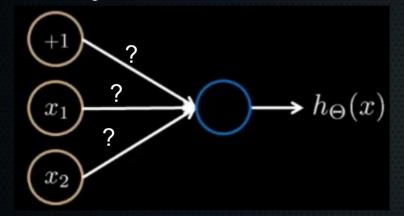




$$h_{\theta}(x) = sigmoid(-30 + 20 \cdot x_1 + 20 \cdot x_2)$$

$x_2$	$h_{\Theta}(x)$
0	g (-30) 20
1	g(-10) 20
0	g(-10) %0
1	9 (10) 21
	0 1 0

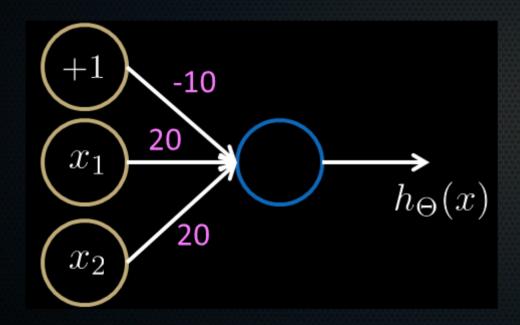
Over to you: make an OR function → try
it yourself and discuss with neighbours for 2 minutes

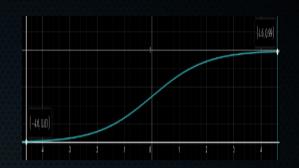


OR						
X	У	x+y				
0	0	0				
0	1	1				
1	0	1				
1	1	1				

$$h_{\theta}(x) = sigmoid(? + ? \cdot x_1 + ? \cdot x_2)$$

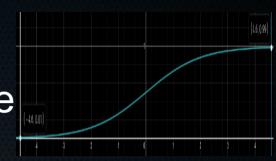
#### Answer

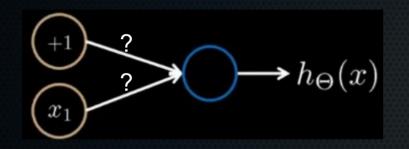




$x_1$	$x_2$	$h_{\Theta}(x)$
0	0	g(-10) ~= 0
0	1	g(-10+20) ~= 1
1	0	g(-10+20) ~= 1
1	1	g(-10 + 20 + 20) ~= 1

Over to you: make a NOT function → try
it yourself and discuss with neighbours for 2 minute





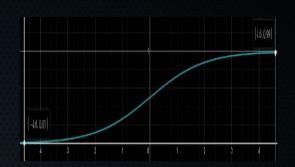
NOT							
X	x'						
0	1						
1	0						

$$h_{\theta}(x) = sigmoid(? + ? \cdot x_1)$$

#### Answer

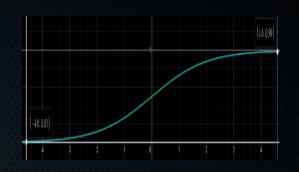


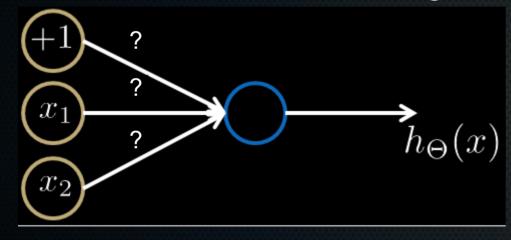
Just put a large negative weight in front of whatever you want to negate (enough to overcome the bias)



### Making simple functions ourselves: (NOT x<sub>1</sub>) AND (NOT x<sub>2</sub>)

 Make NOT x<sub>1</sub> AND NOT x<sub>2</sub> → try it yourself and discuss with neighbours for 2 minutes

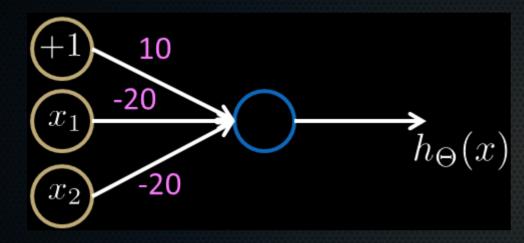




$$h_{\theta}(x) = sigmoid(? + ? \cdot x_1 + ? \cdot x_2)$$

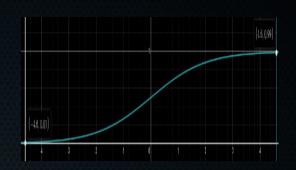
## Making simple functions ourselves: (NOT x<sub>1</sub>) AND (NOT x<sub>2</sub>)

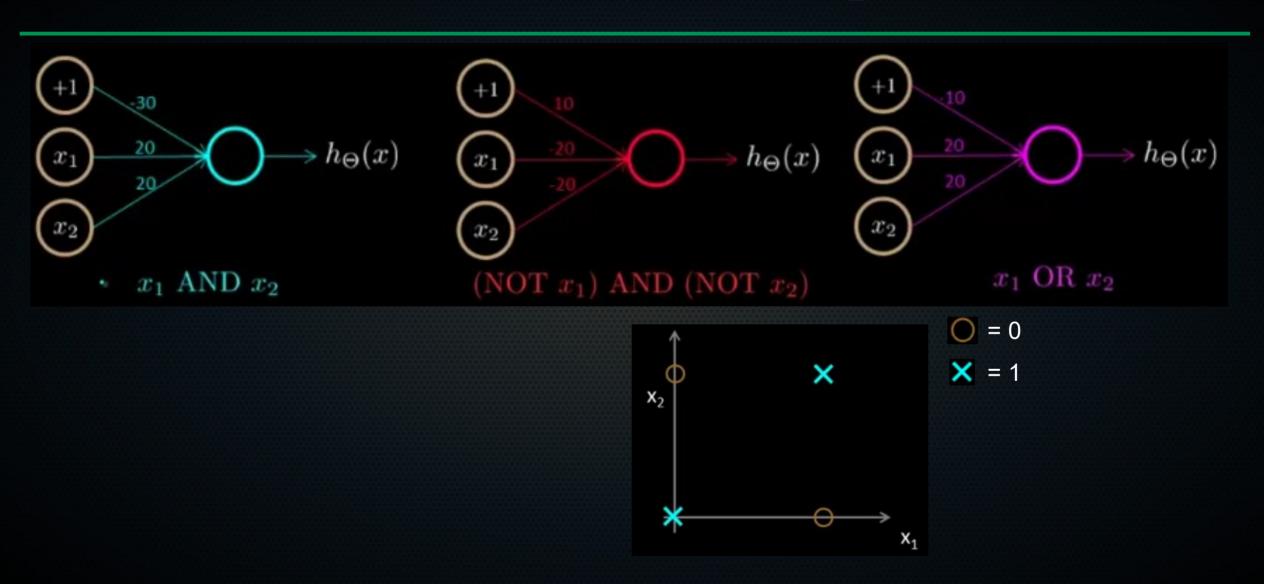
#### Answer

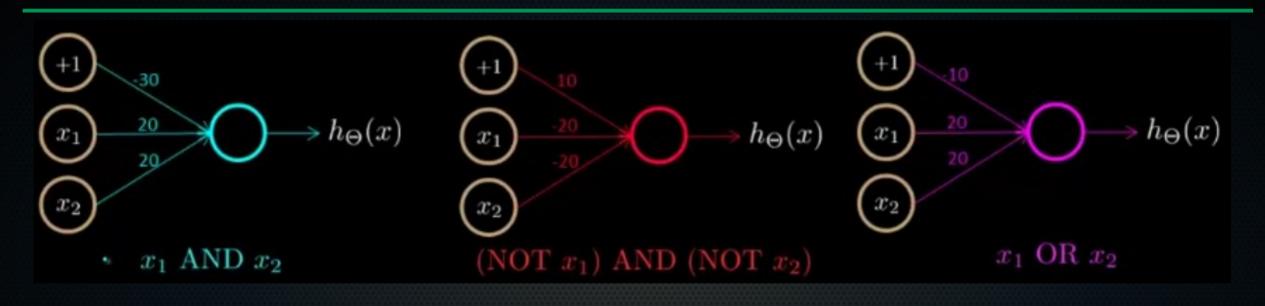


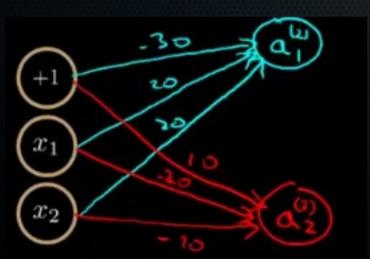
Just put a large negative weight in front of whatever you want to negate (enough to overcome the bias)

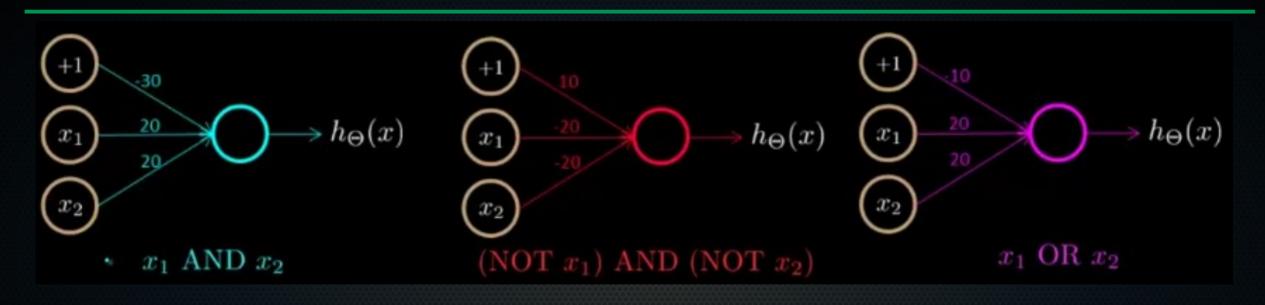
Only positive when  $x_1 = x_2 = 0$ 





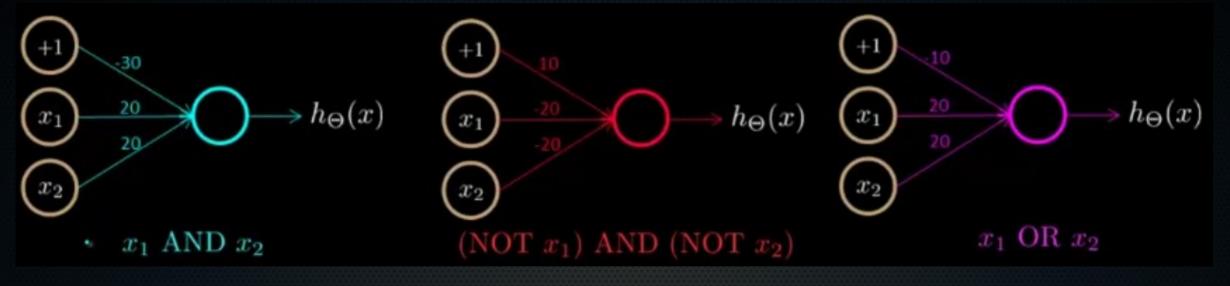








$x_1$	$x_2$	$a_1^{(2)}$	$a_2^{(2)}$	
0	0	Ö	1	t
0	1	0	0	
1	0	0	0	l
1	1	1	0	





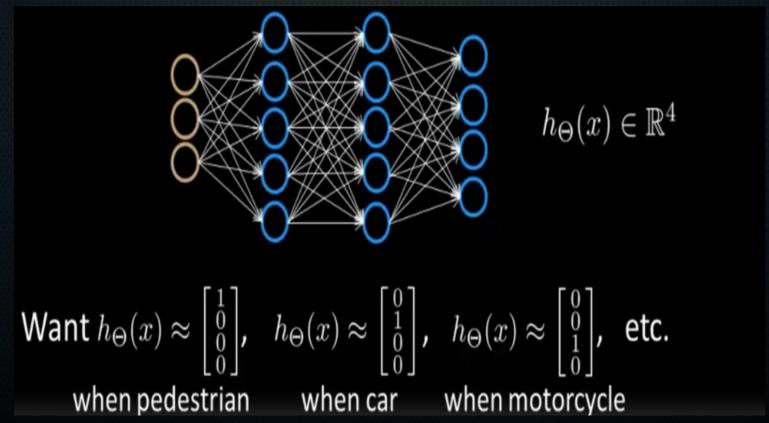
$x_1$	$x_2$	$a_1^{(2)}$	$a_2^{(2)}$	$h_{\Theta}(x)$
0	0	0	17	1 -
0	1	0	07	0
1	0	0	0	0
1	1	1	0	1
	_			

### Computing complex functions

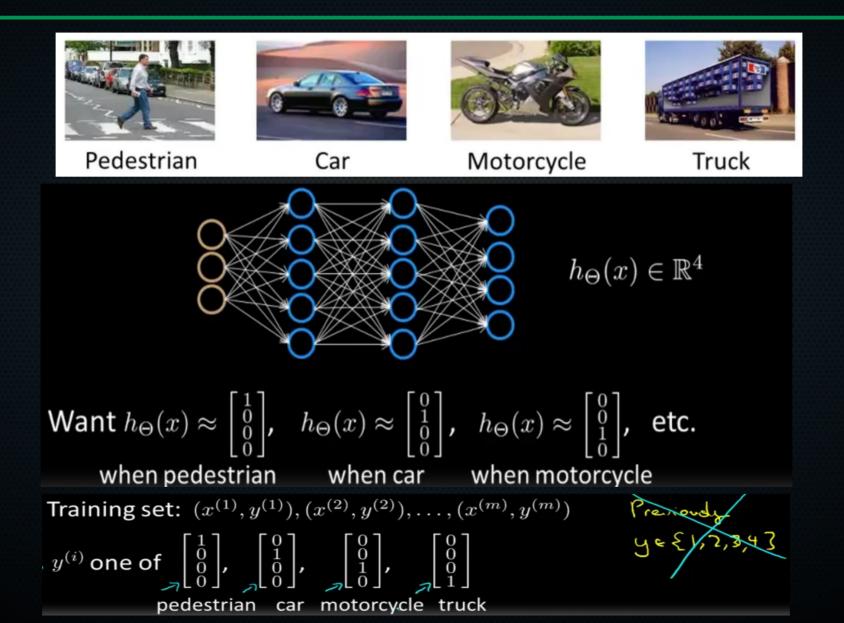
 This is an illustration of how neural networks work: earlier layers can compute simple functions like AND and OR. By combining those outputs, you can compute more complex functions.

### Multiclass classification in neural nets





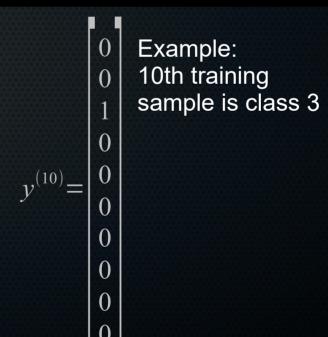
### Multiclass classification in neural nets



### Question to you

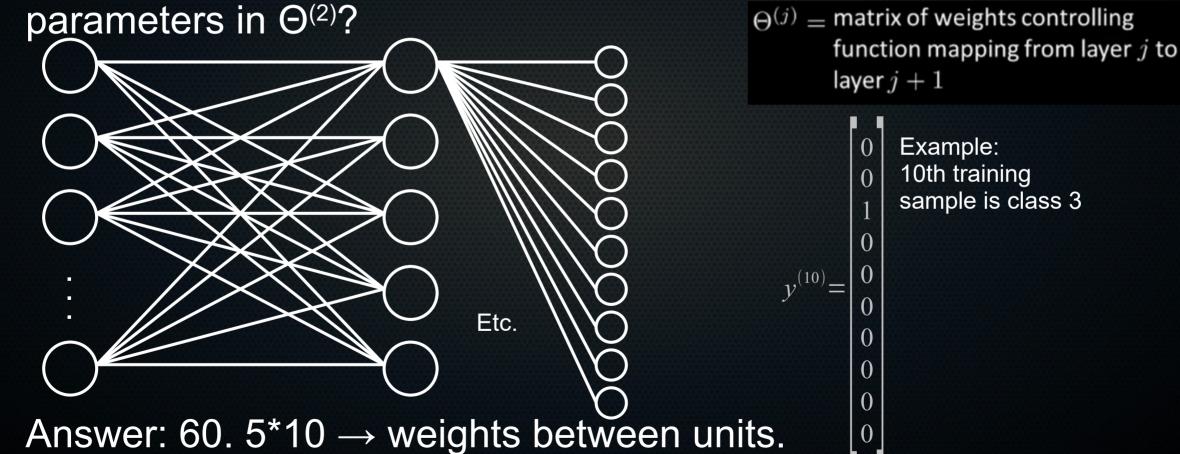
Say we have 10 classes and the following network, how many

parameters in  $\Theta^{(2)}$ ? Etc.  $\Theta^{(j)} = \text{matrix of weights controlling} \\ \text{function mapping from layer } j \text{ to} \\ \text{layer } j+1$ 



### Question to you

Say we have 10 classes and the following network, how many

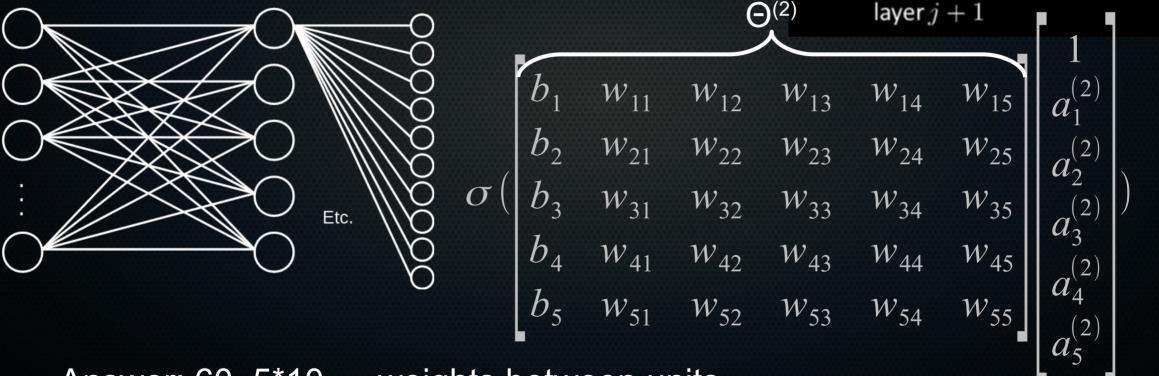


+ 10 → bias of each unit in output layer

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### Question to you

- Say we have 10 classes and the following network, how many parameters in  $\Theta^{(2)}$ ?  $\Theta^{(j)} = \text{matrix of weights controlling function mapping from layer } j \text{ to } j \text{$ 



- Answer: 60. 5\*10 → weights between units.
  - + 10 → bias of each unit in output layer

### Summary

- We can use individual neurons to calculate simple logic functions
- We can combine the outputs of single neurons to calculate more complex (logic) functions
- For multiclass classification, we simply make class a vector, where we strive for the real class to be ~1, and all other classes 0.

## Time for the afternoon practical!

