Today

- Recap yesterday
- Logistic regression: using regression tools for classification
- Neural network basics

Yesterday

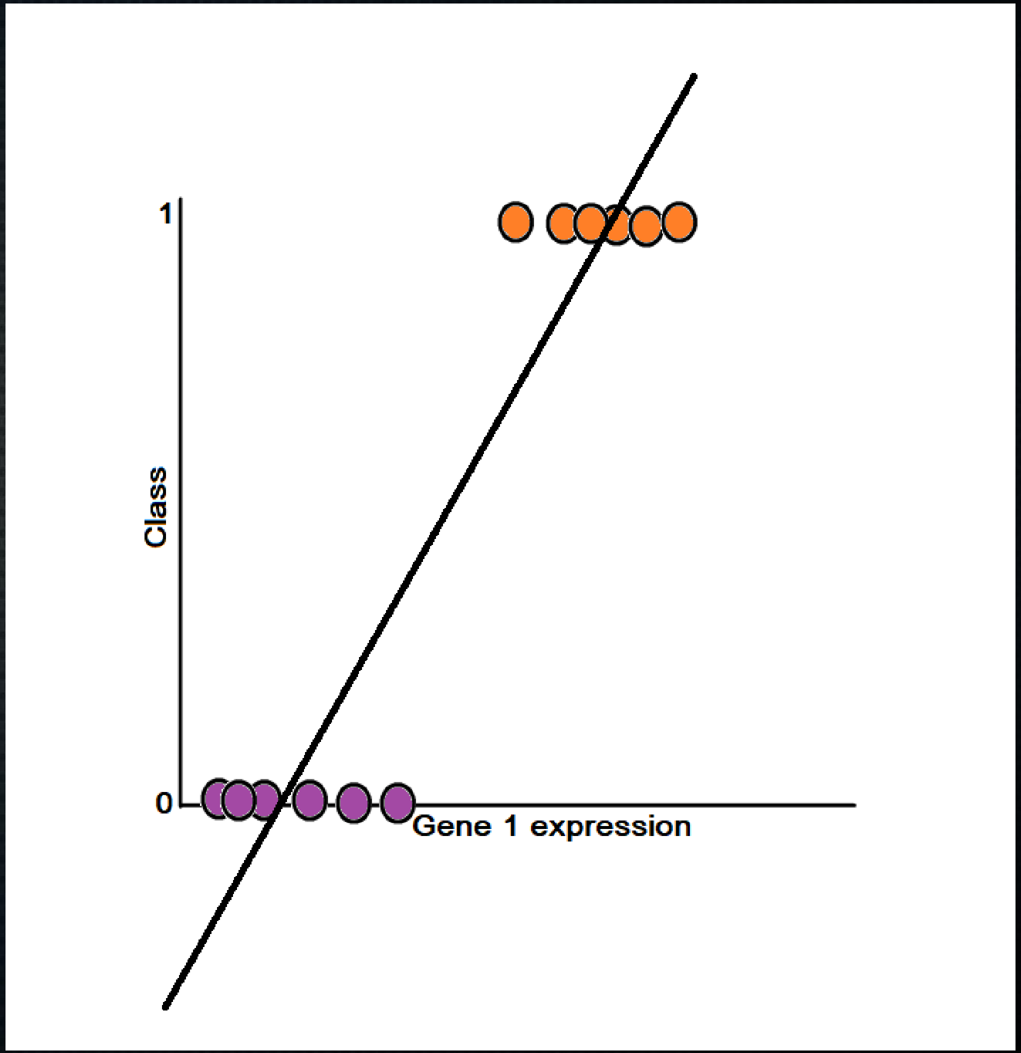
- Cost function: (differentiable) function that shows how wrong an estimate is for given parameters.
- Gradient descent: one common way to minimise the cost function automatically, i.e. to get optimal parameters
- Linear regression: very simple model that assumes that value to predict is linear combination of input features.
- Overfitting and underfitting, bias and variance: want our model to work wel for unseen data. Need just enough model freedom given the complexity of our problem. How:
  - Cross-validation to measure ability to generalise + get best hyperparameters
  - Use learning curves to diagnose bias vs. variance

Logistic regression

- Use regression-like framework for classification

Logistic regression

- Naïve idea:
  Train a linear regression. If Class >= 0.5, predict class 1. Otherwise, class 0.



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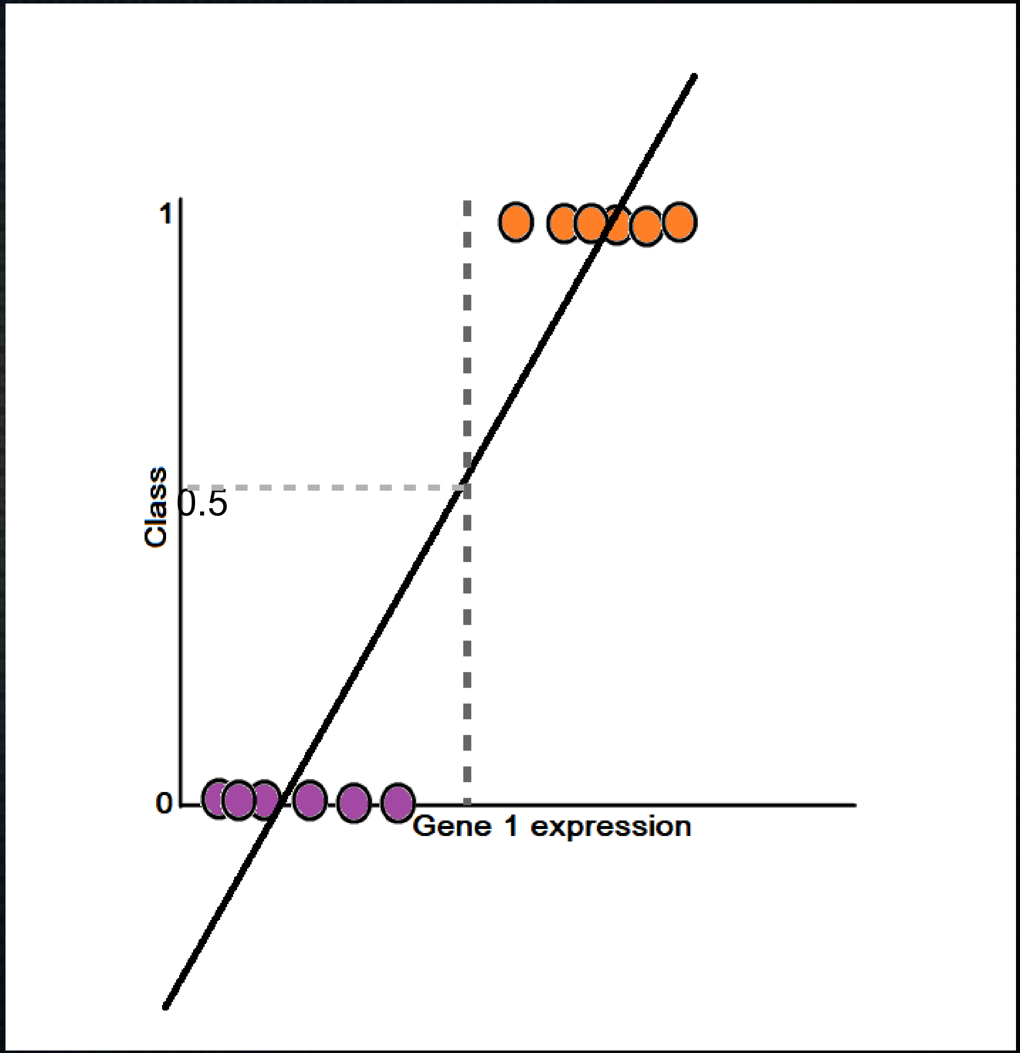

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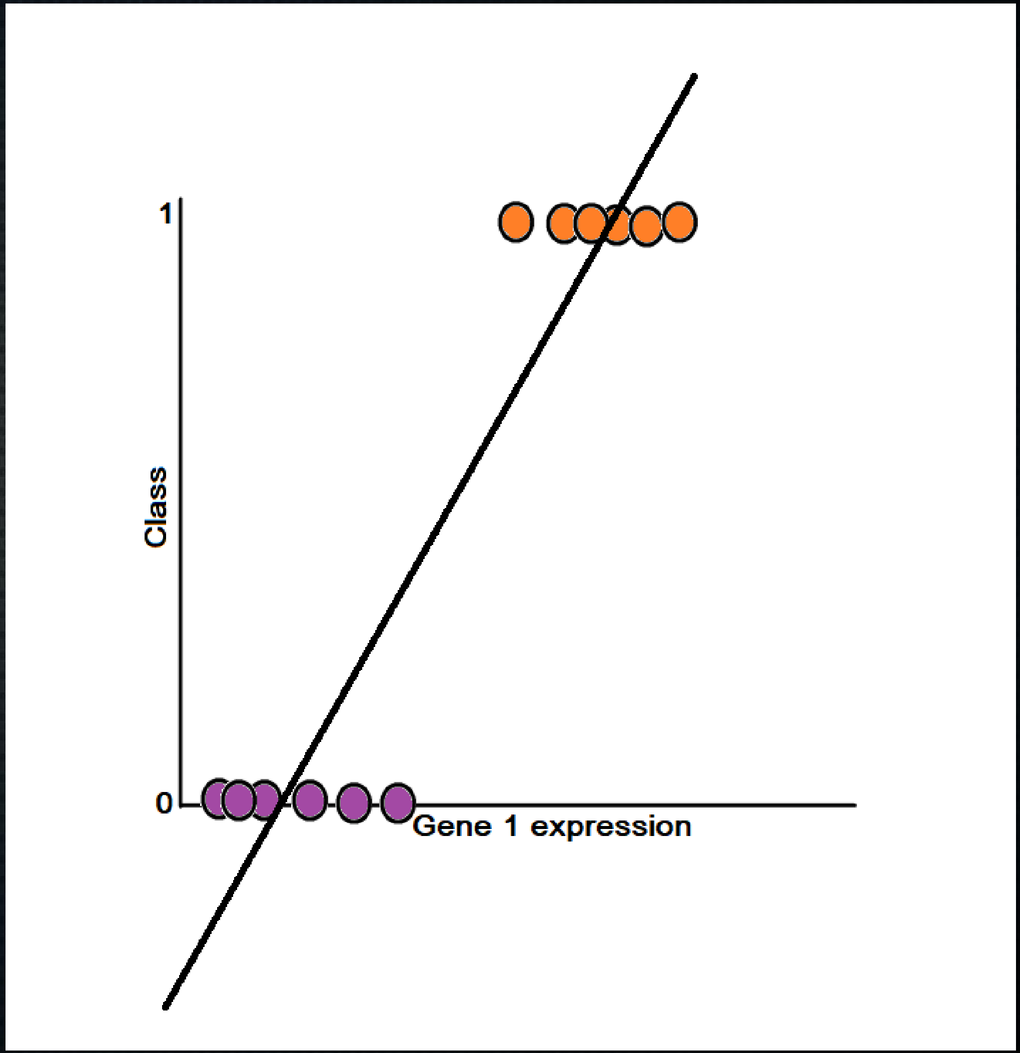

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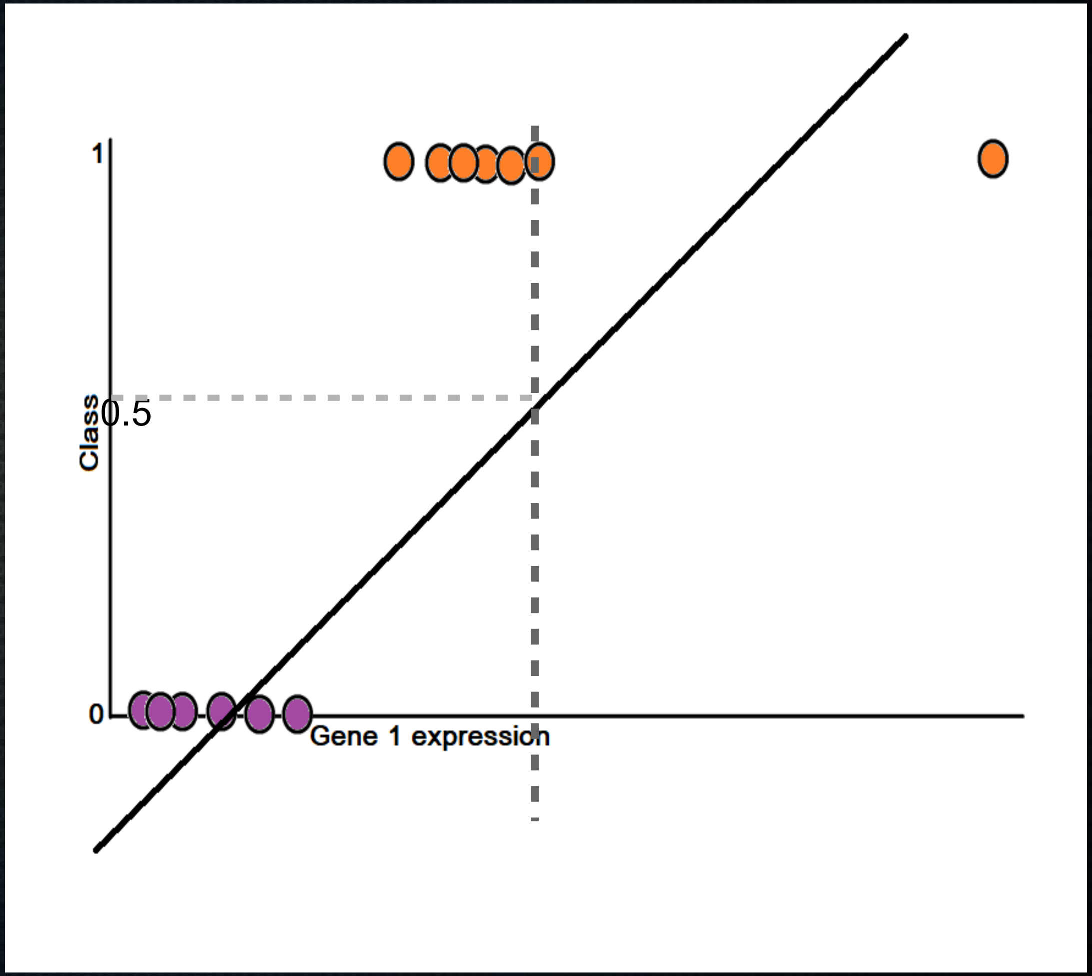

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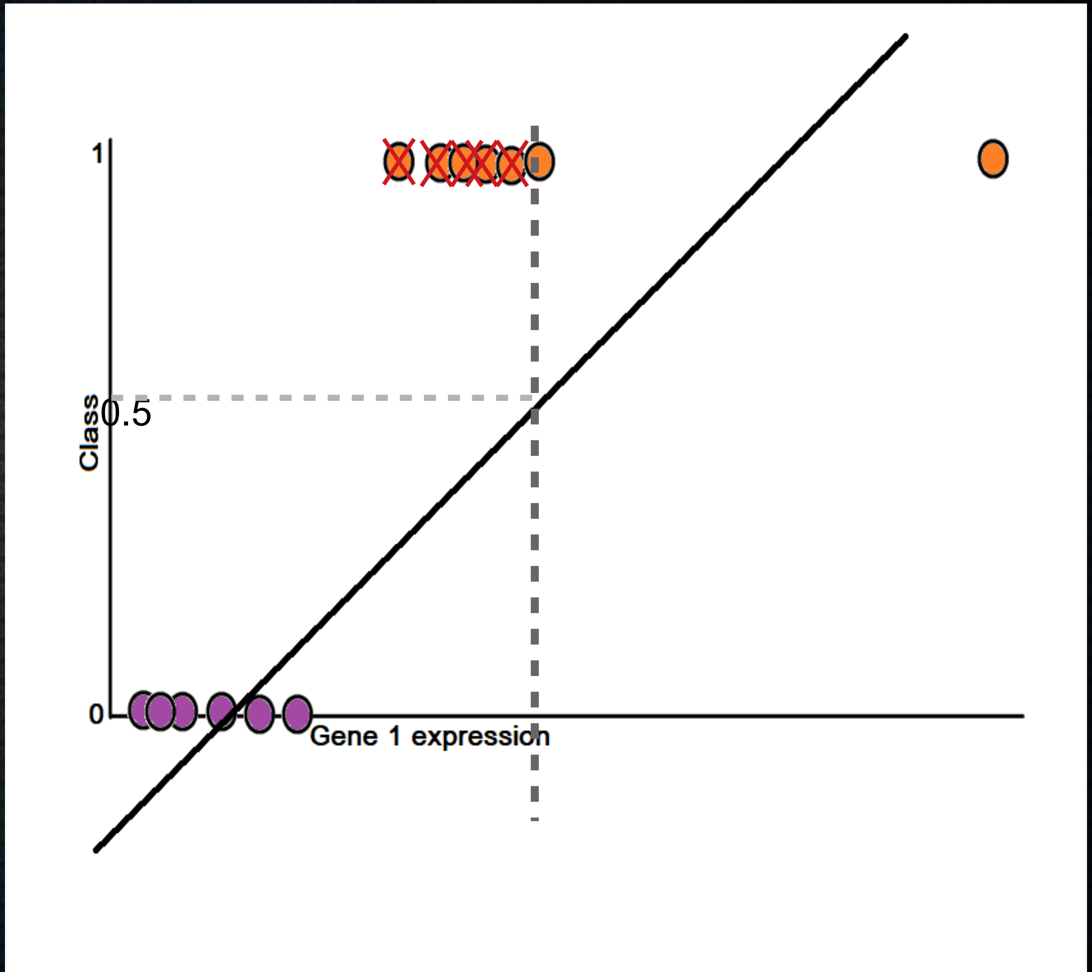
  get what is basically confirmation of hypothesis, but perform worse!



Logistic regression

- What we want:
  - Use the information that we only have two classes, 0 or 1.
  - Hypothesis function should output only numbers between 0 or 1.

Sigmoid or logistic function

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$$h_\theta(x) = \theta^T \cdot x \longrightarrow [0.5 \;\; 3 \;\; -1.5] \cdot \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix} = 0.5 \cdot 1 + 3 \cdot 3 - 1.5 \cdot 8 = -2.5$$

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$$z \rightarrow \infty, e^{-z} \rightarrow 0$$

$$z \rightarrow -\infty, e^{-z} \rightarrow \infty$$

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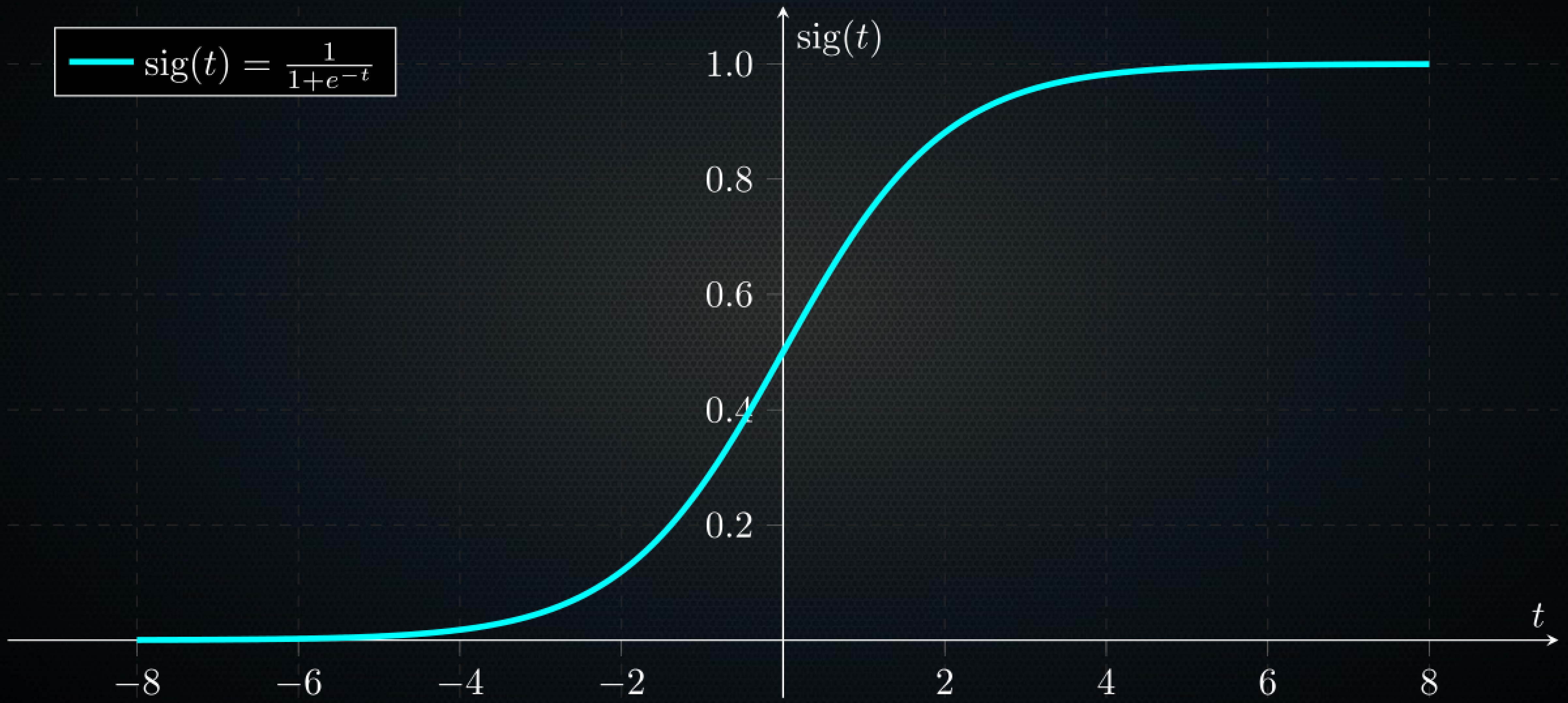

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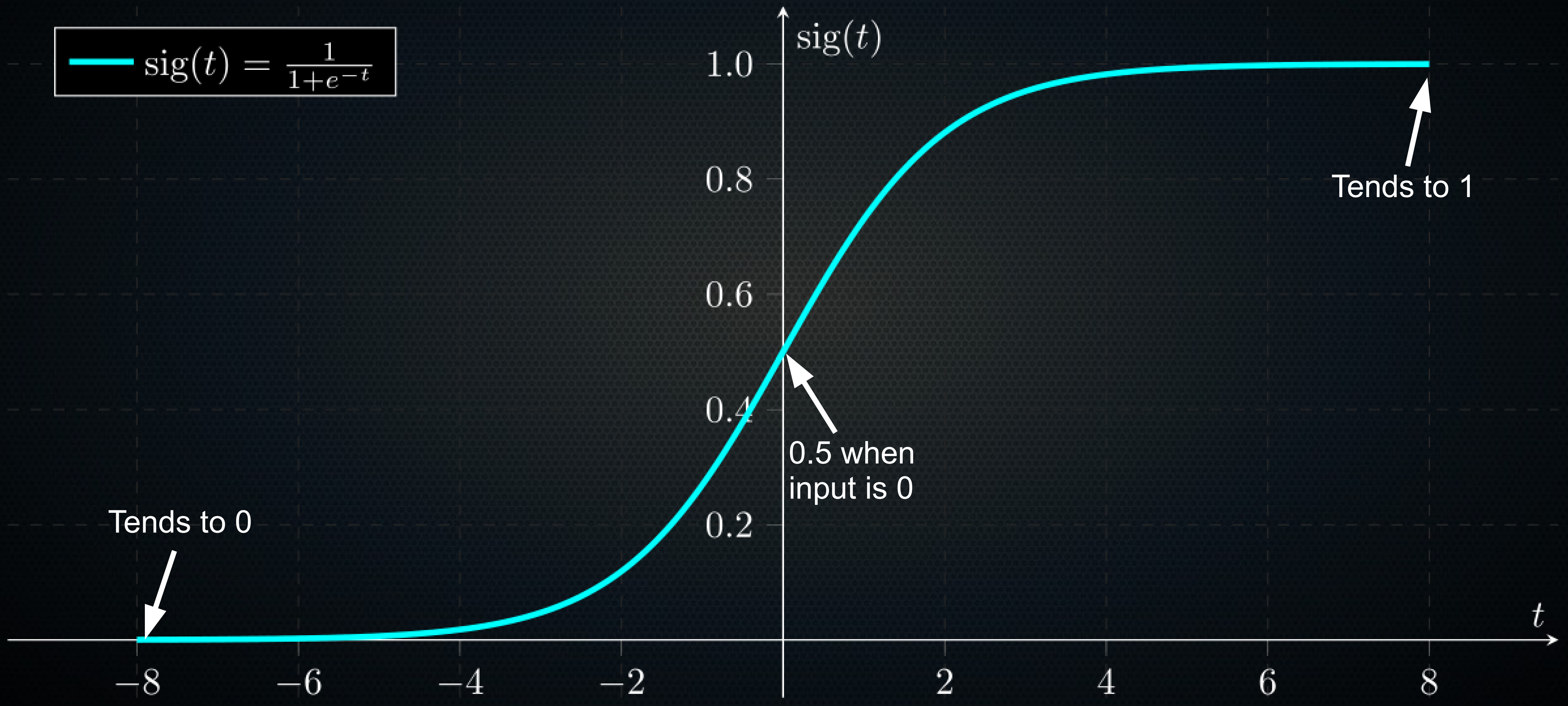

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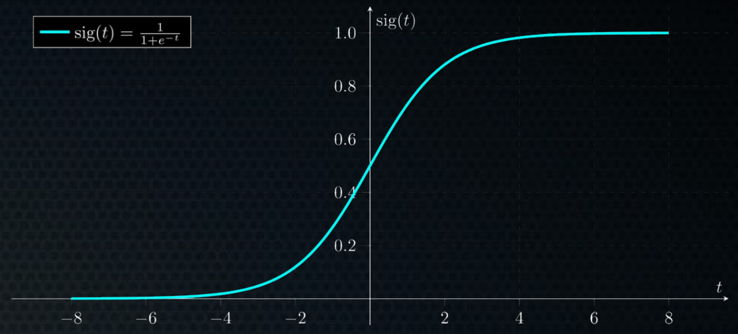

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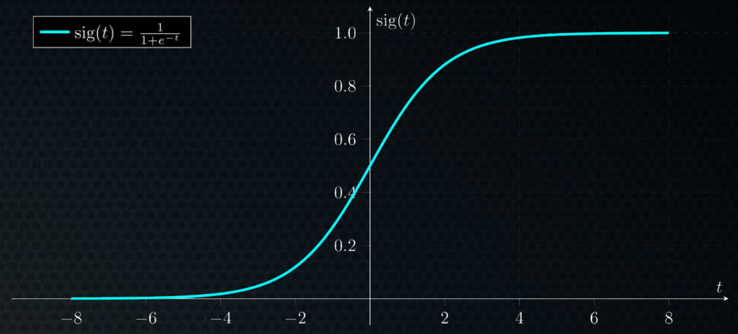

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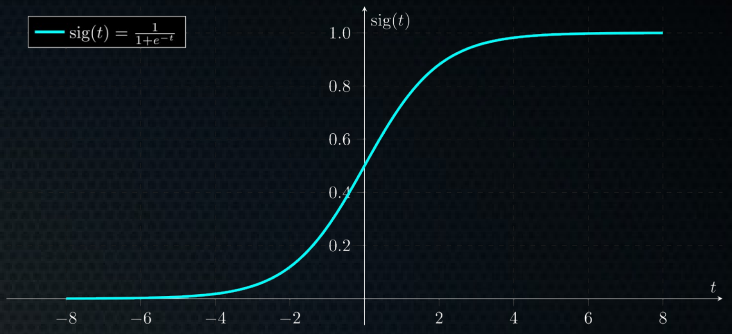

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$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{Tumor size} \\ \text{Neovascularisation level} \end{bmatrix}$$

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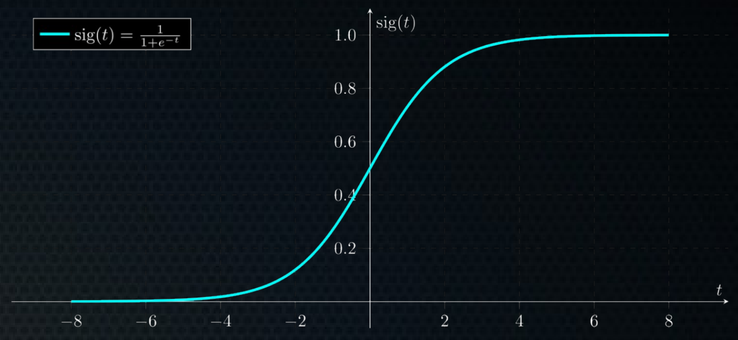

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100% - 80% → 20 % chance of being benign (class 0)

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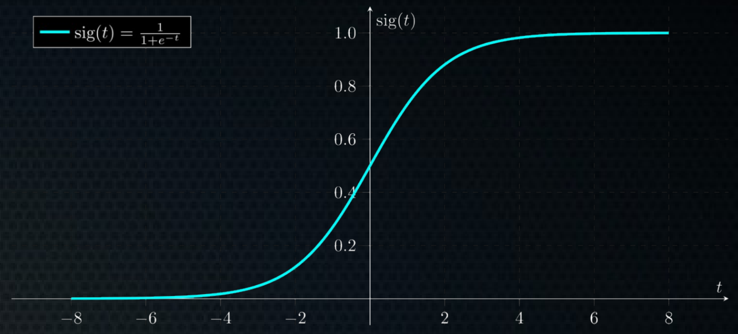

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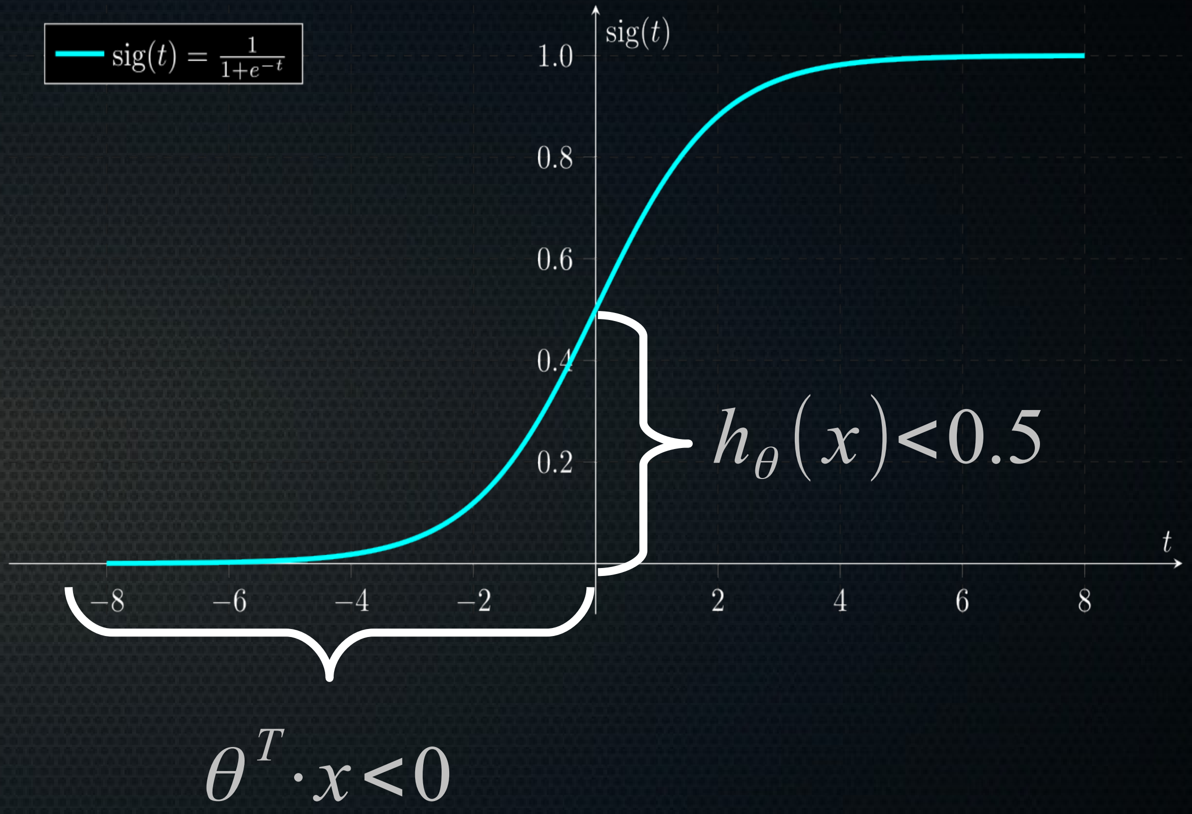
  - Formally:

$$h_\theta(x) = \frac{1}{1+e^{-(\theta^T \cdot x)}} = p(y=1|x;\theta)$$

$$p(y=0|x;\theta) = 1 - h_\theta(x)$$

Decision boundary

- Threshold:



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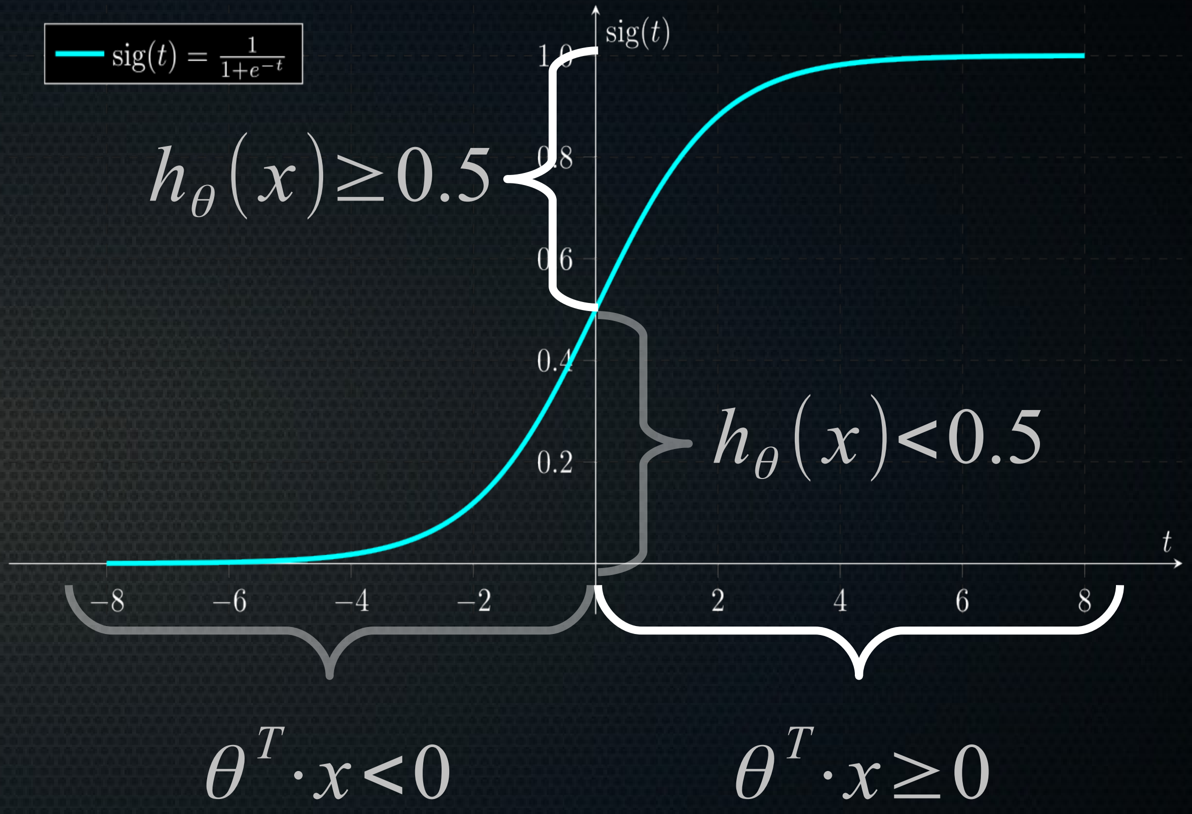

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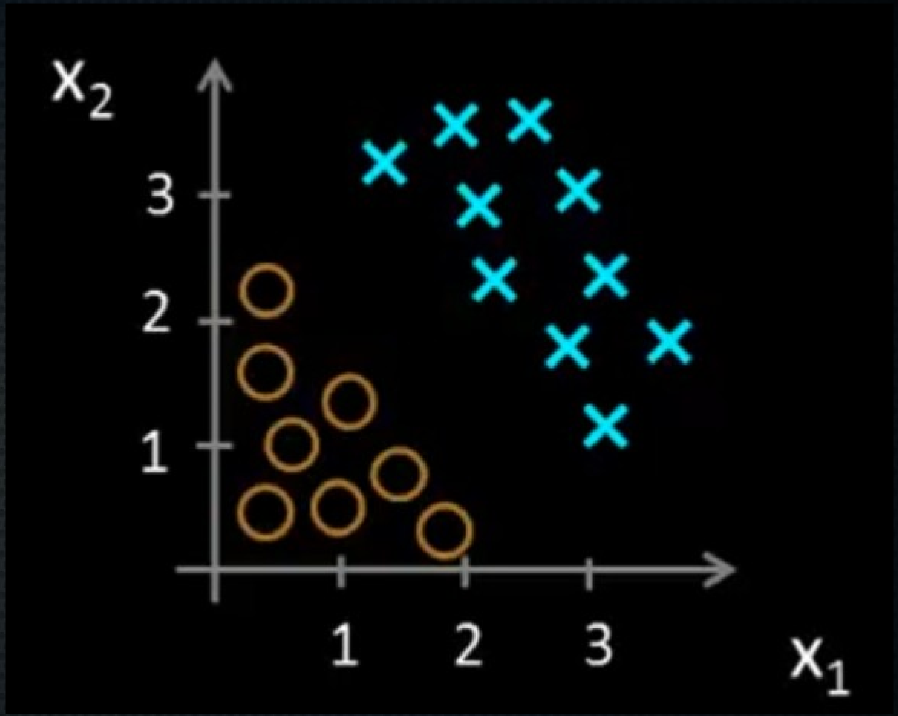

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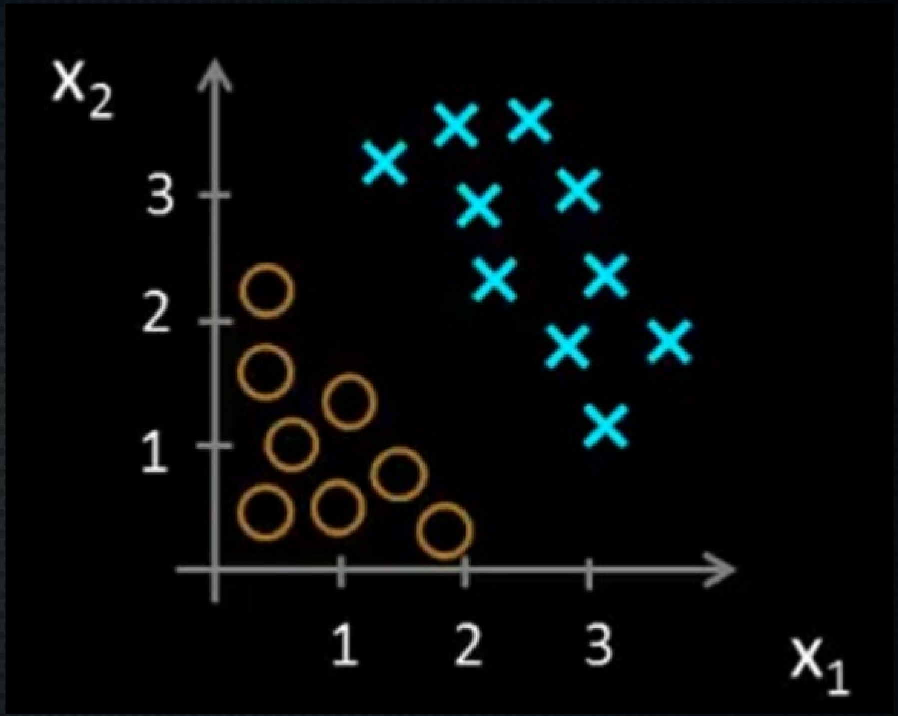

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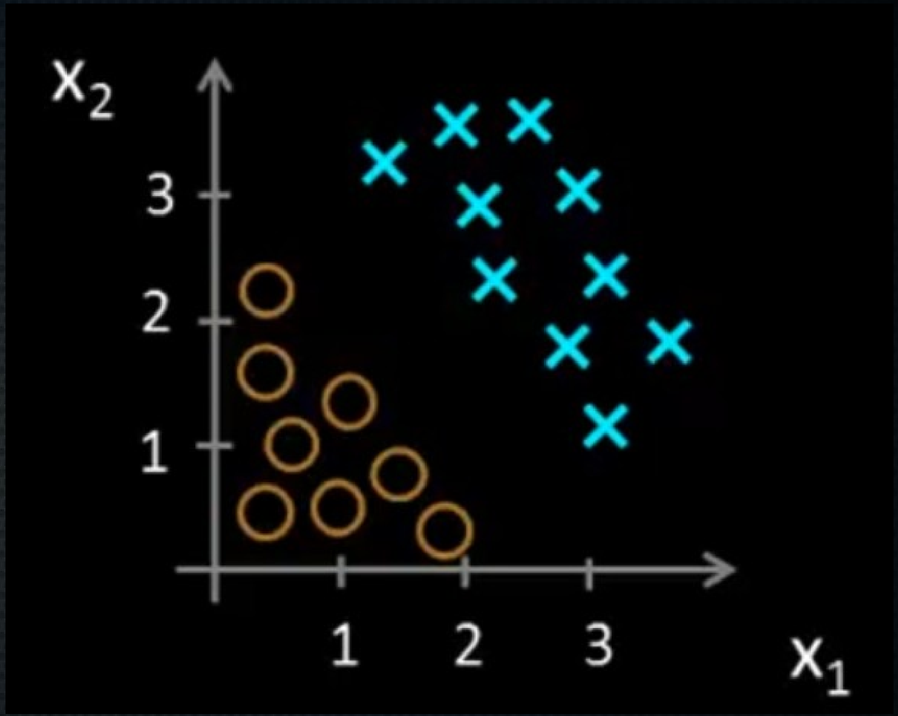

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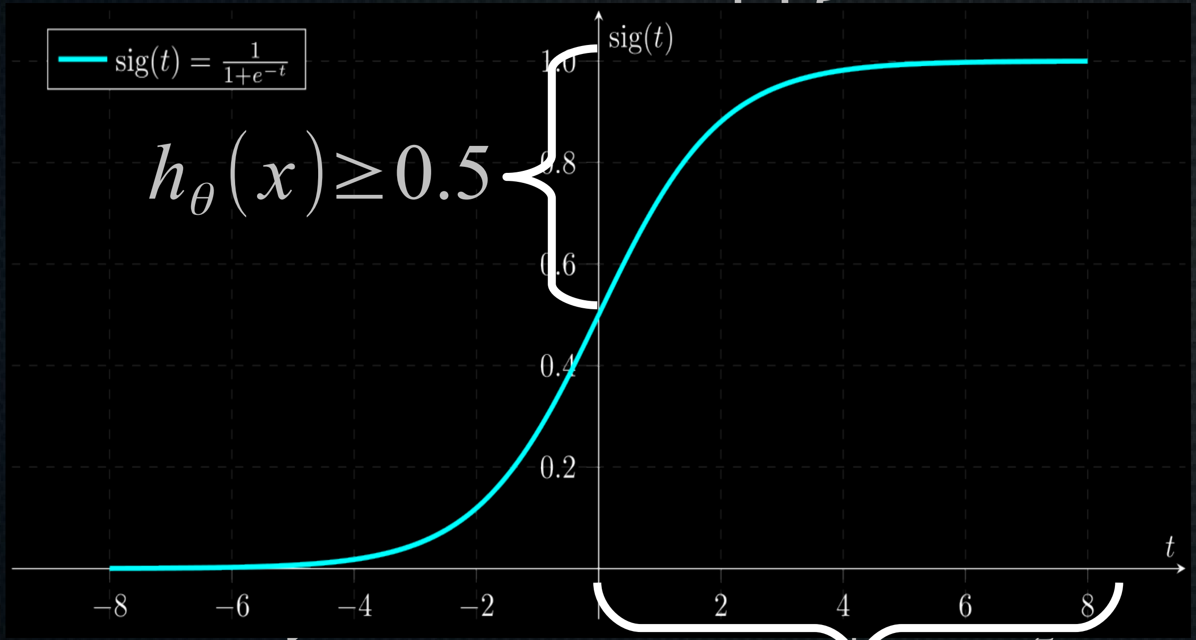
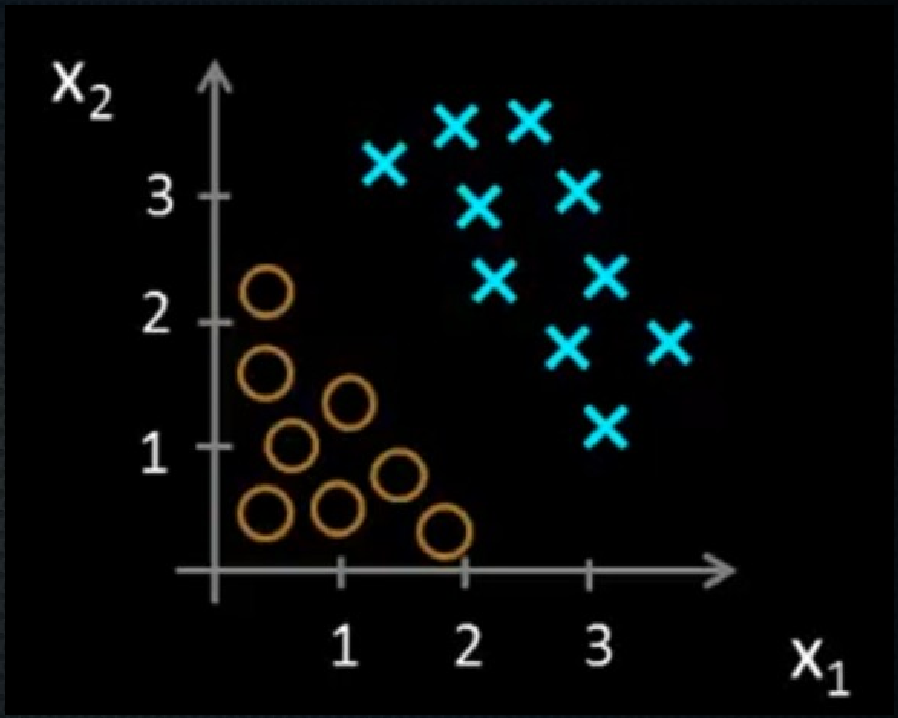


$$h_\theta(x) \geq 0.5$$

$$\theta^T \cdot x \geq 0$$



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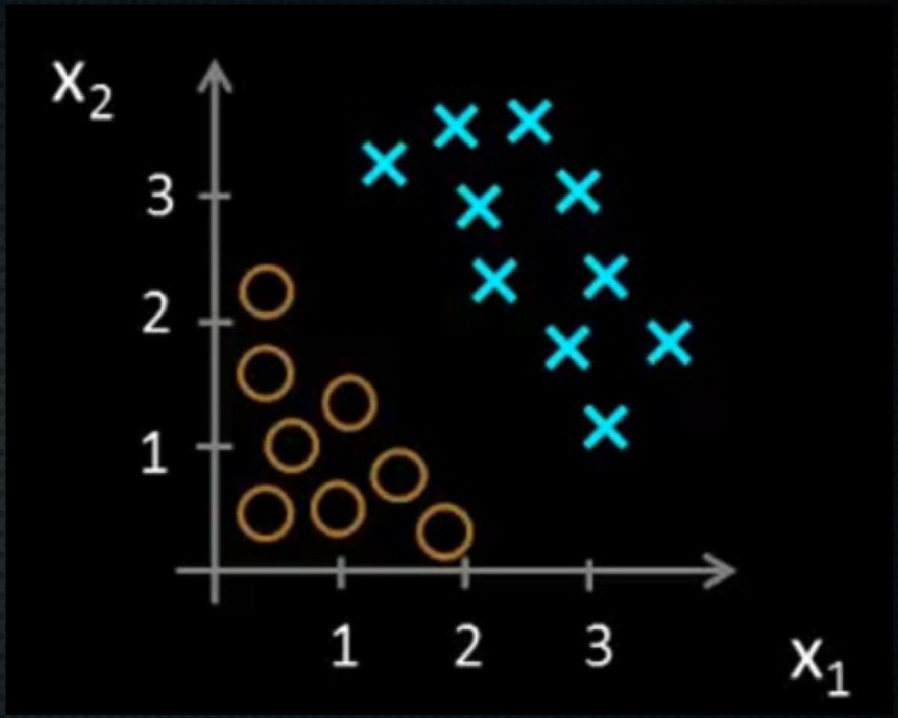

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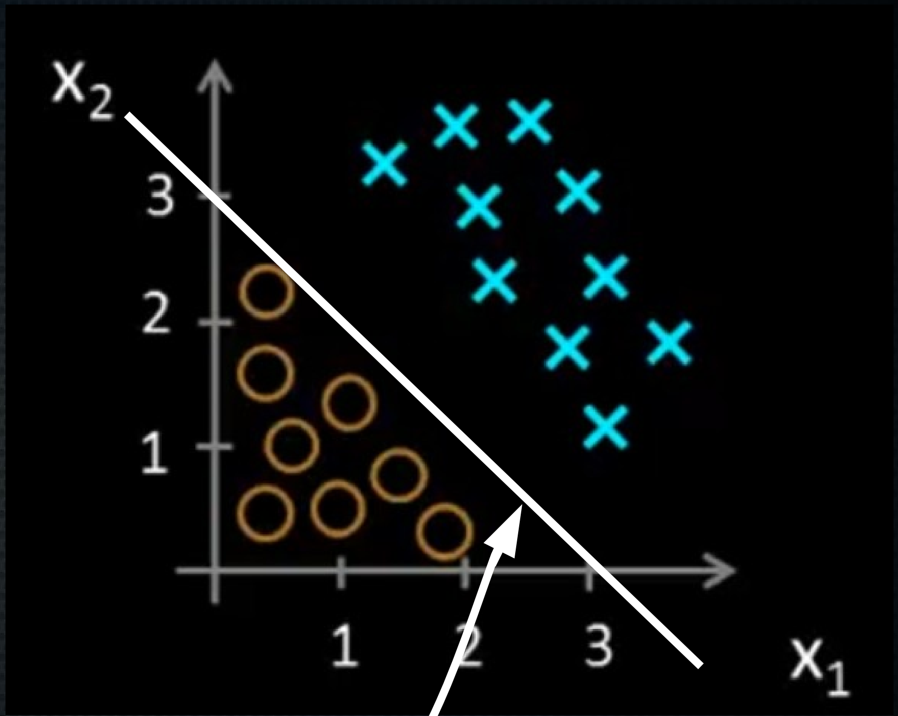

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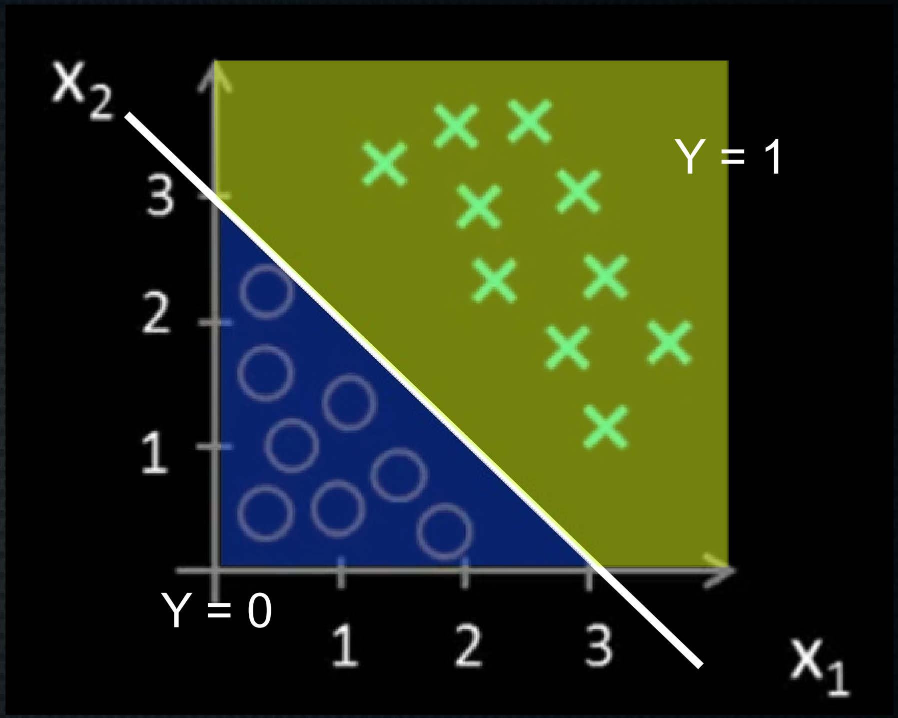

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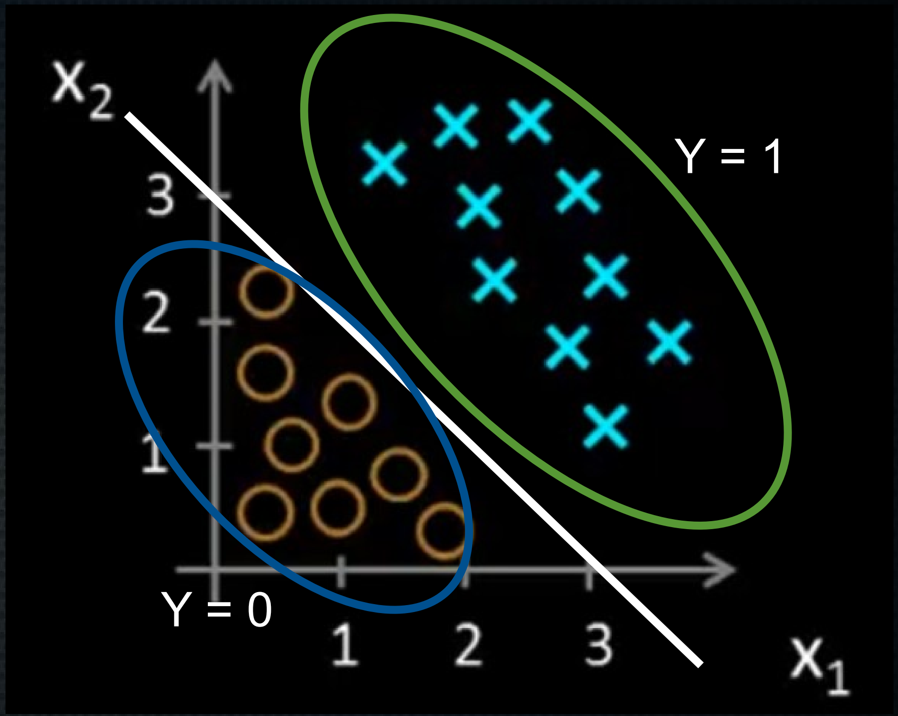

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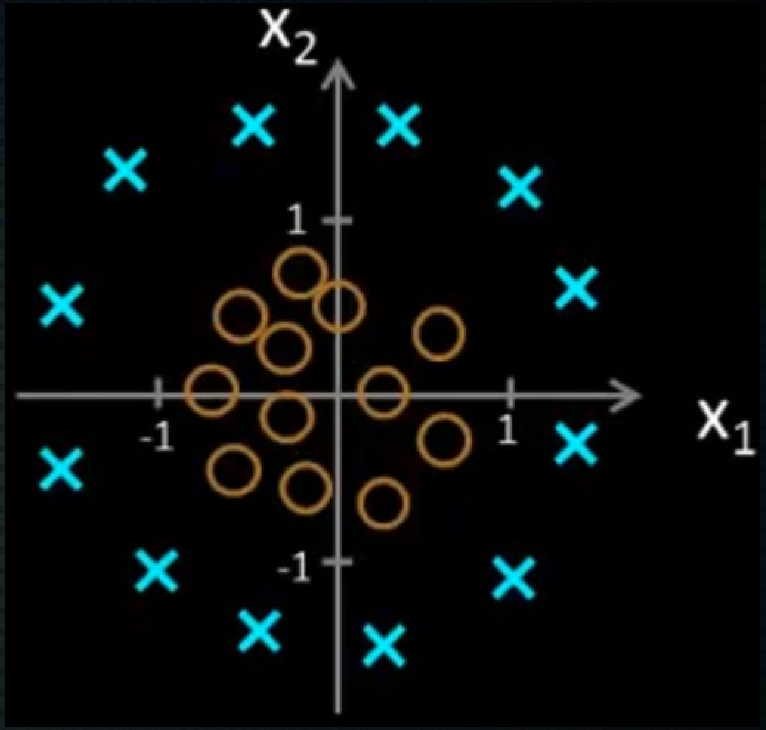

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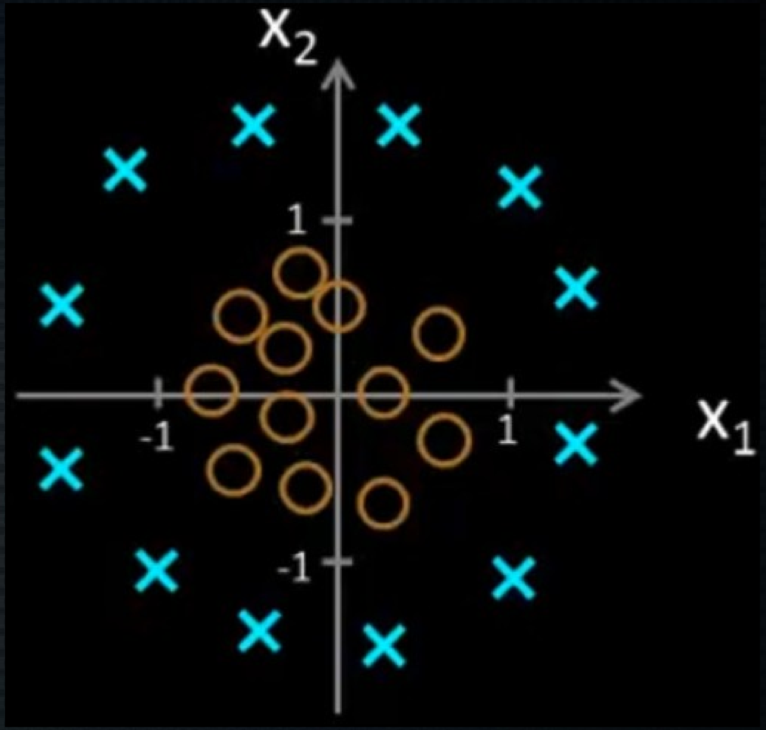

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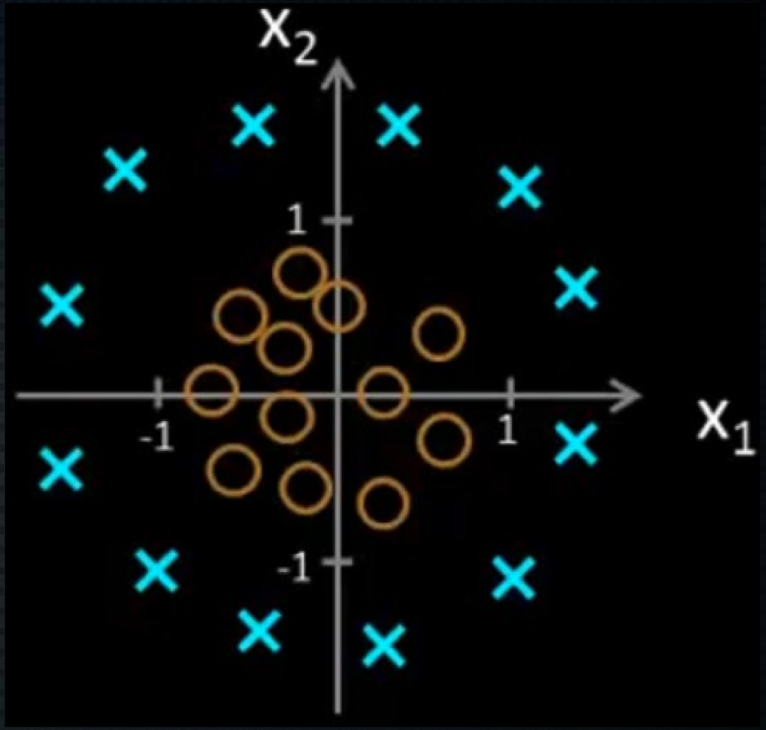

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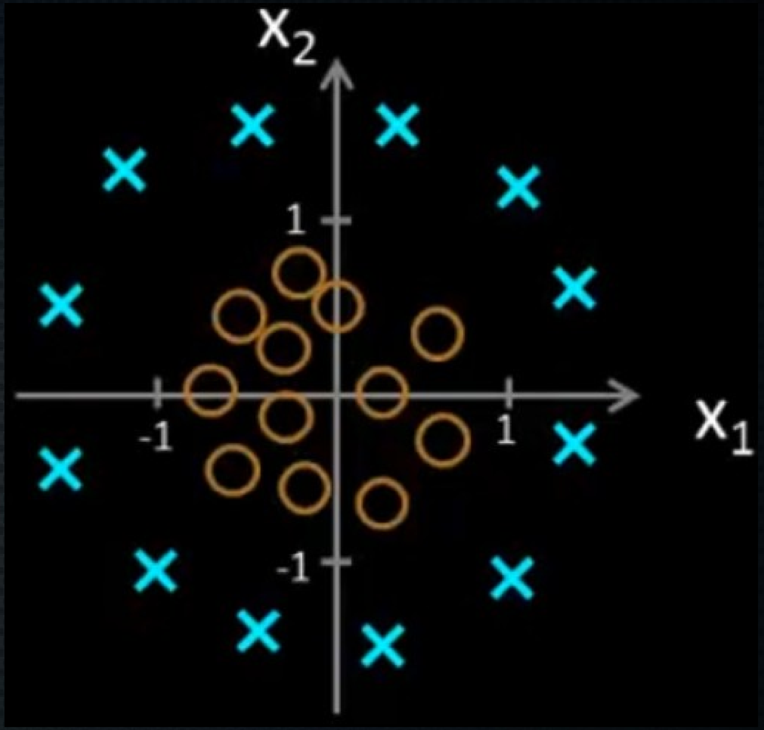

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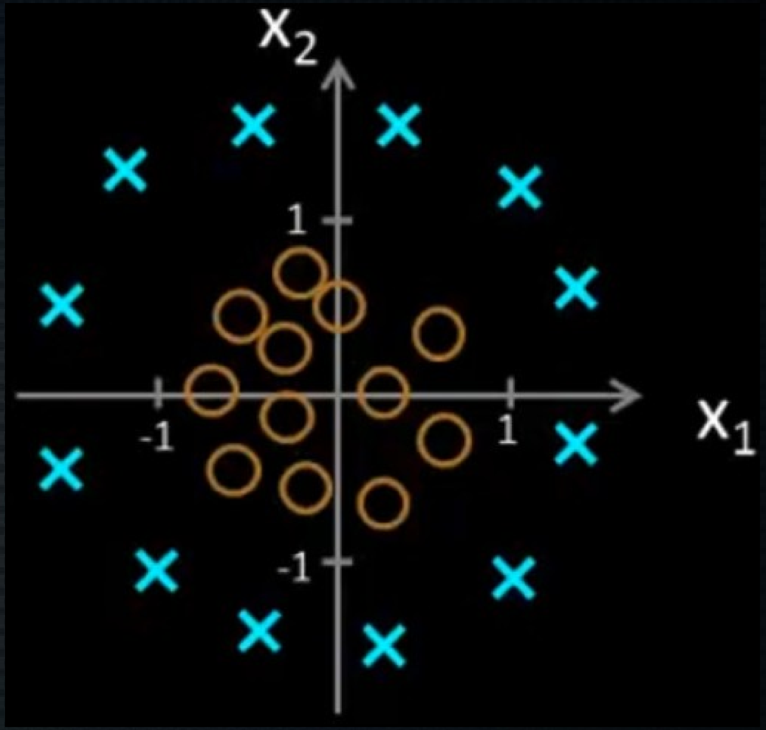

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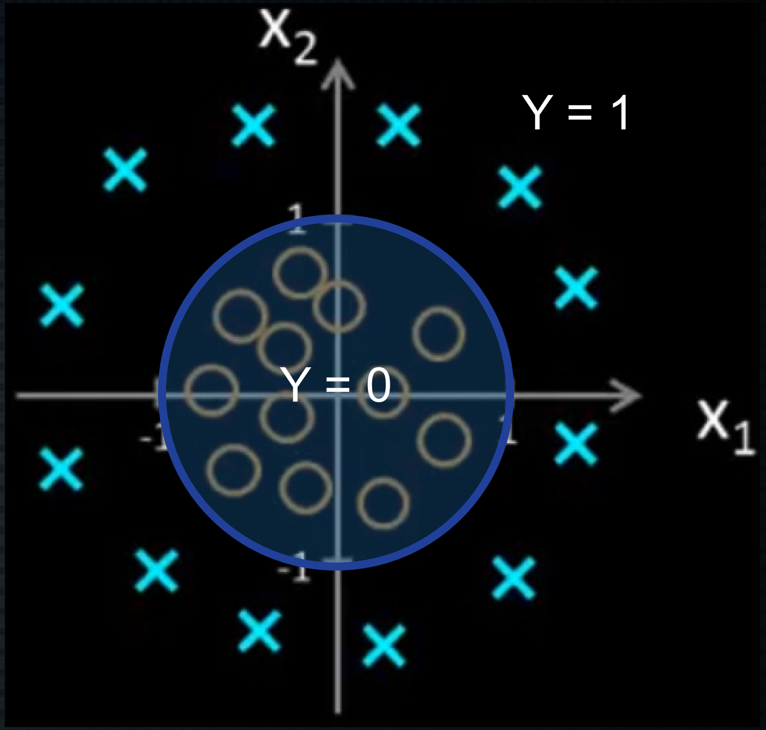
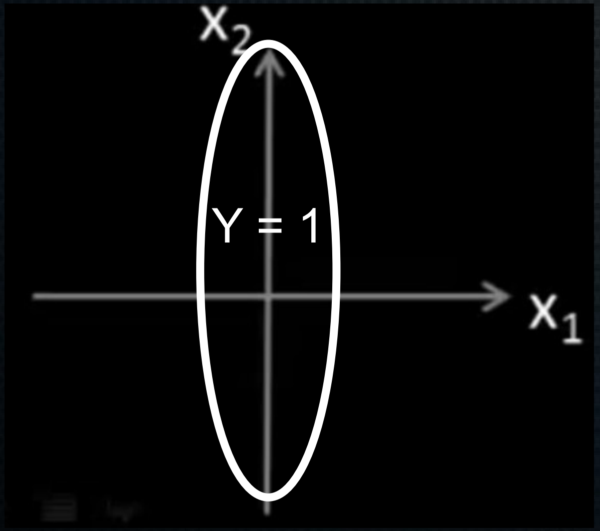
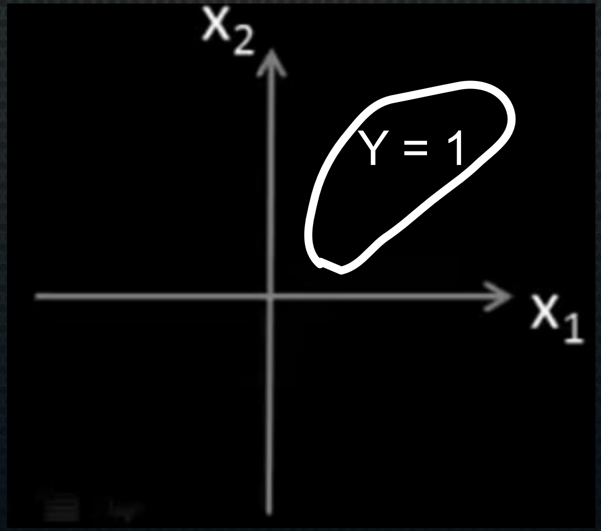
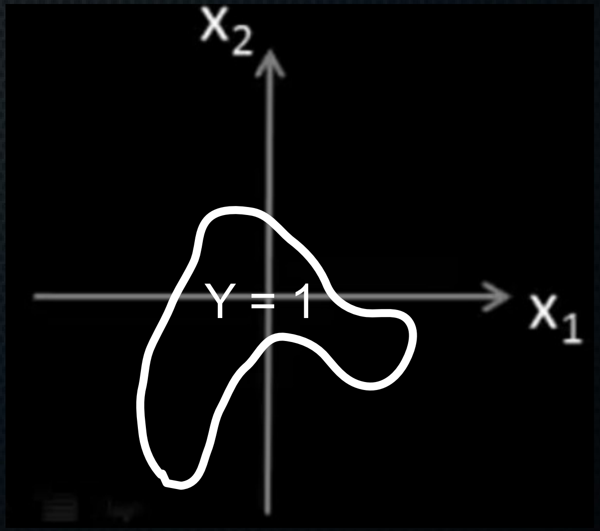


Non-linear decision boundary

- How does it look?
- If you add more and higher-order polynomial features, you can get complex boundaries:







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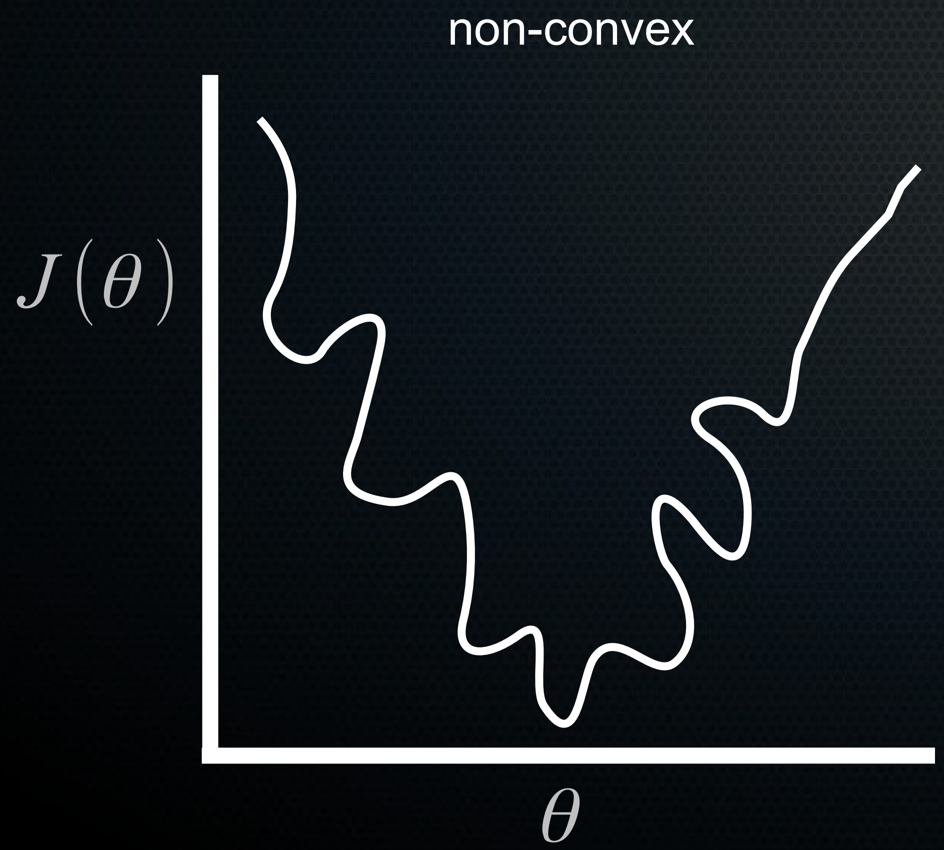

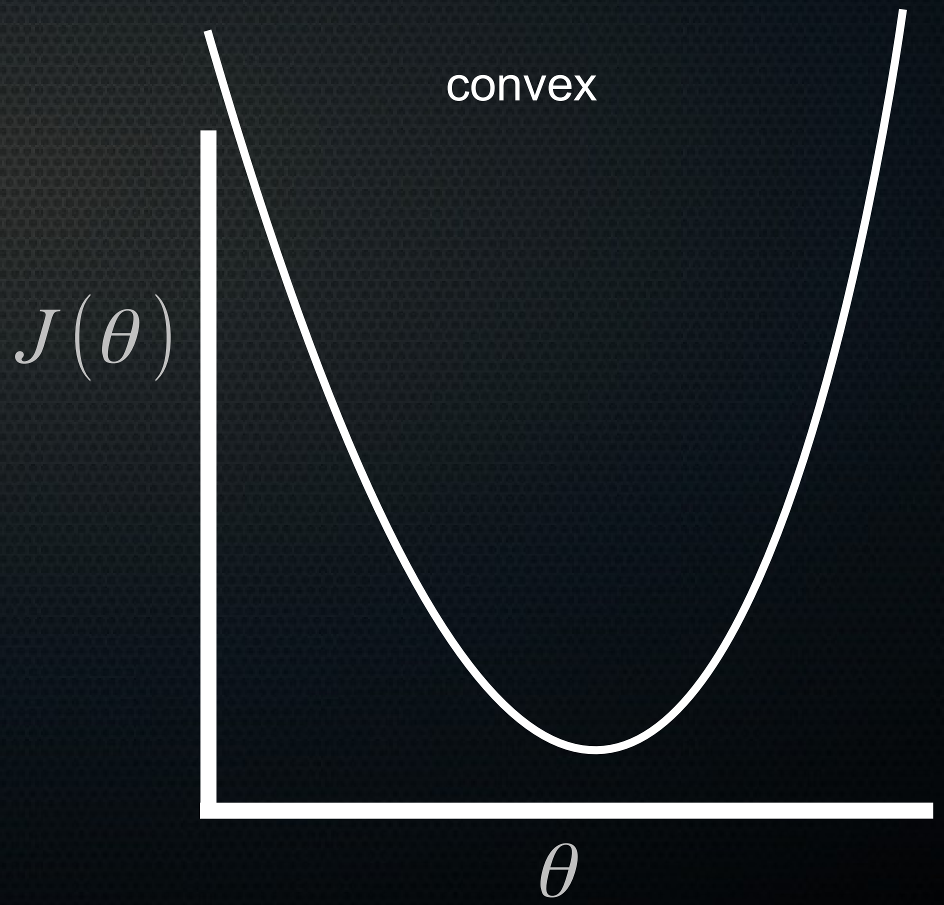

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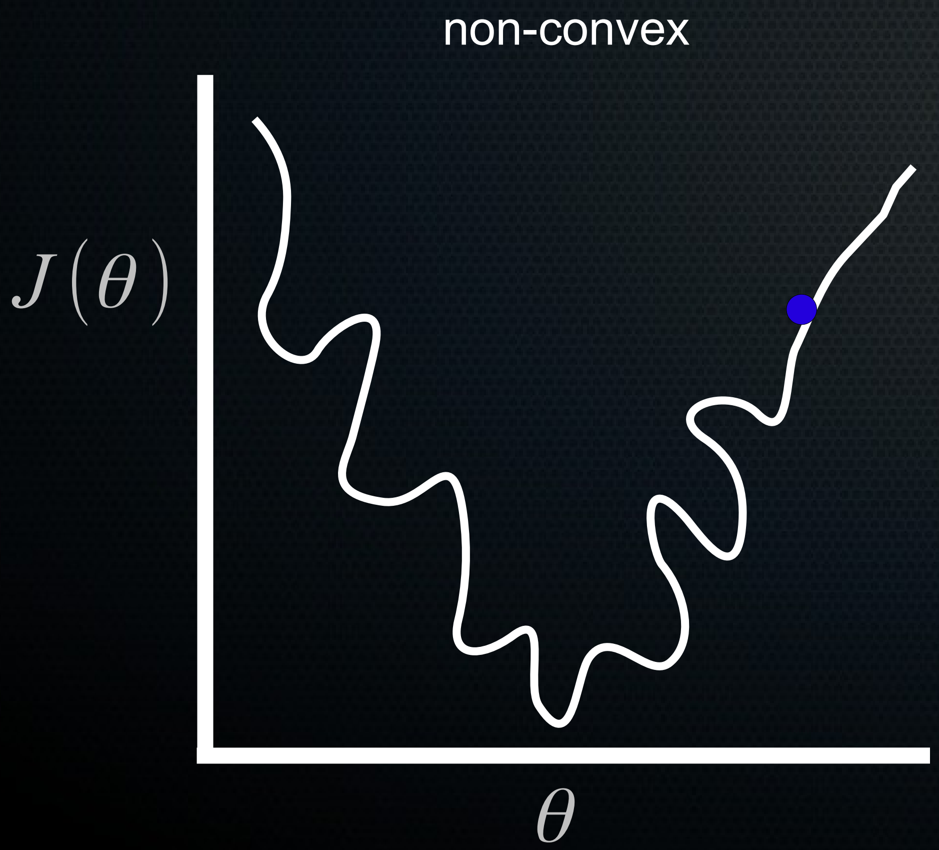

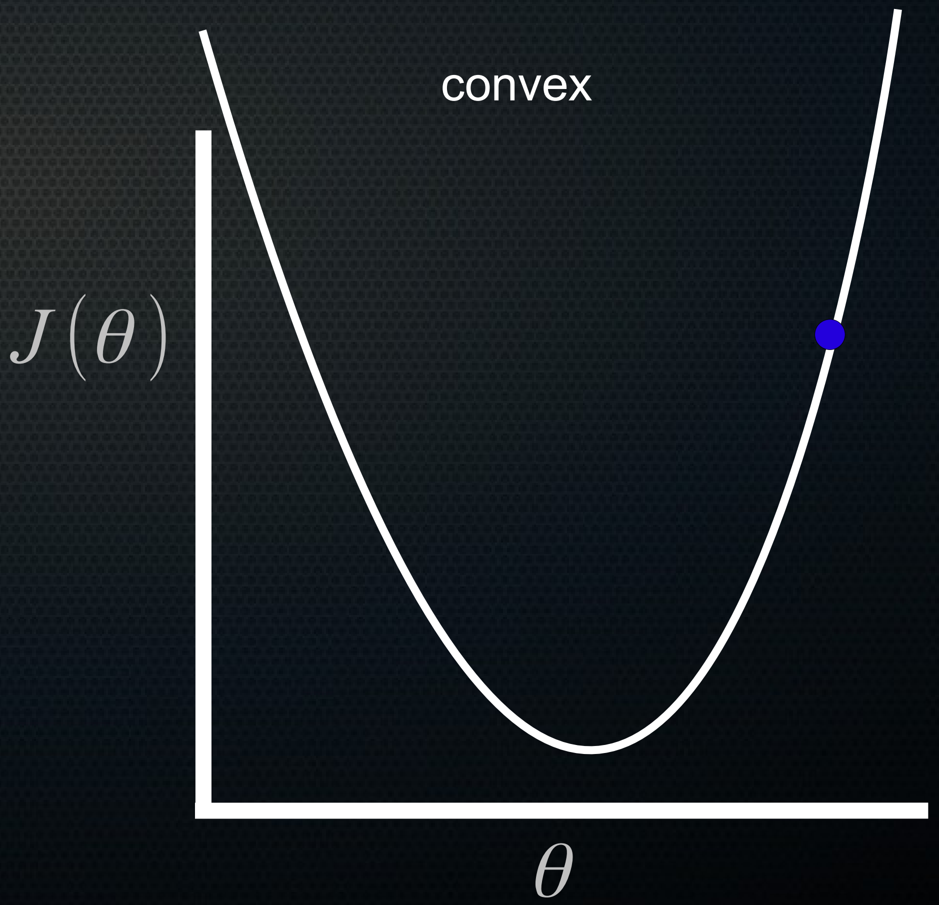

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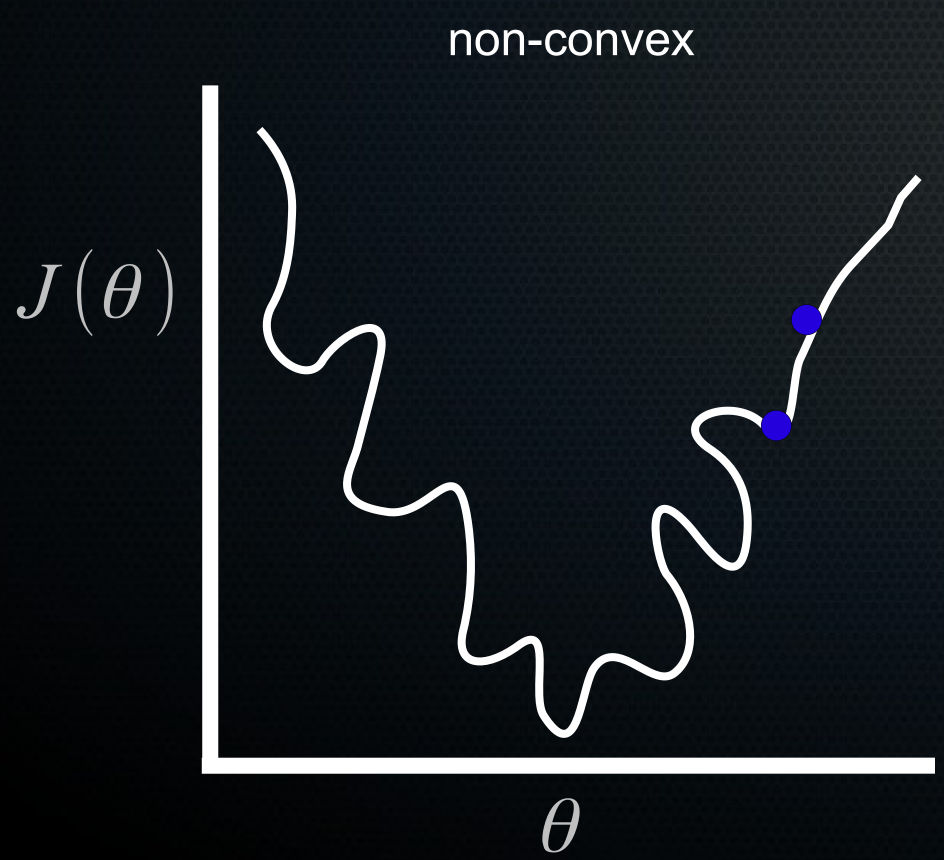
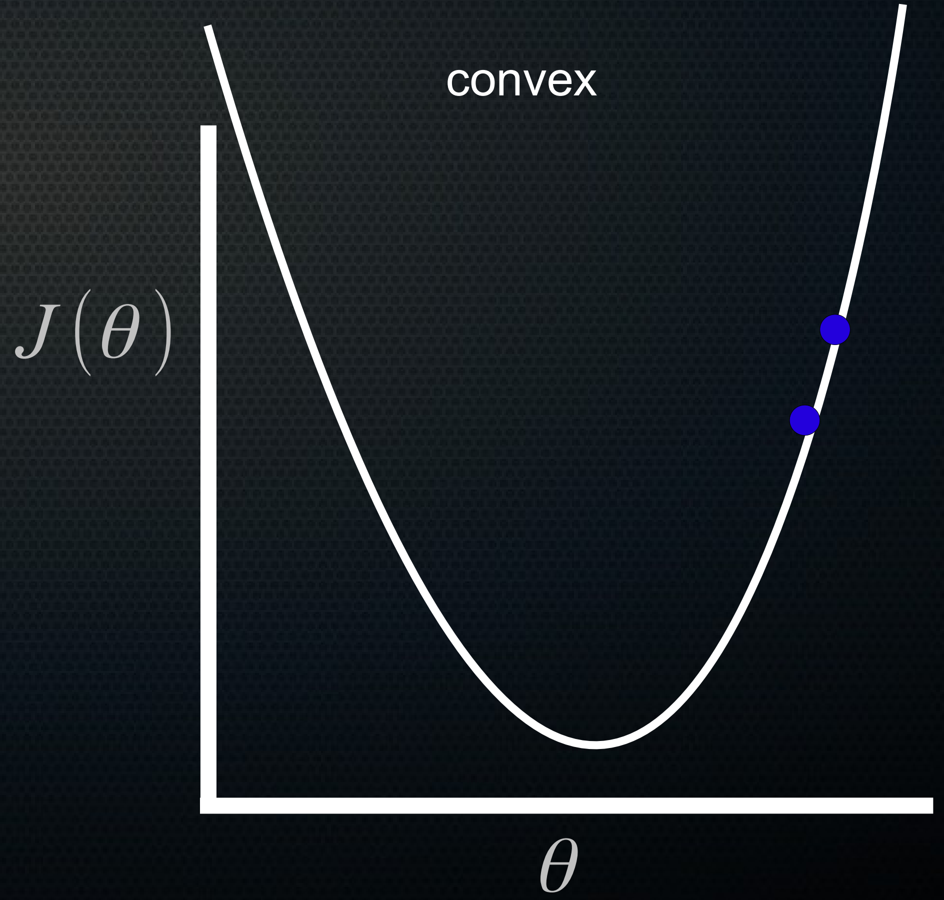
- Need a cost function $J(\theta)=\frac{1}{m}\sum_{i=1}^{m}\frac{1}{2}\mathrm{Cost}(x^i)$ $\mathrm{Cost}(x)=\frac{1}{2}(h_\theta(x)-y)^2$
- Why not MSE? → not convex





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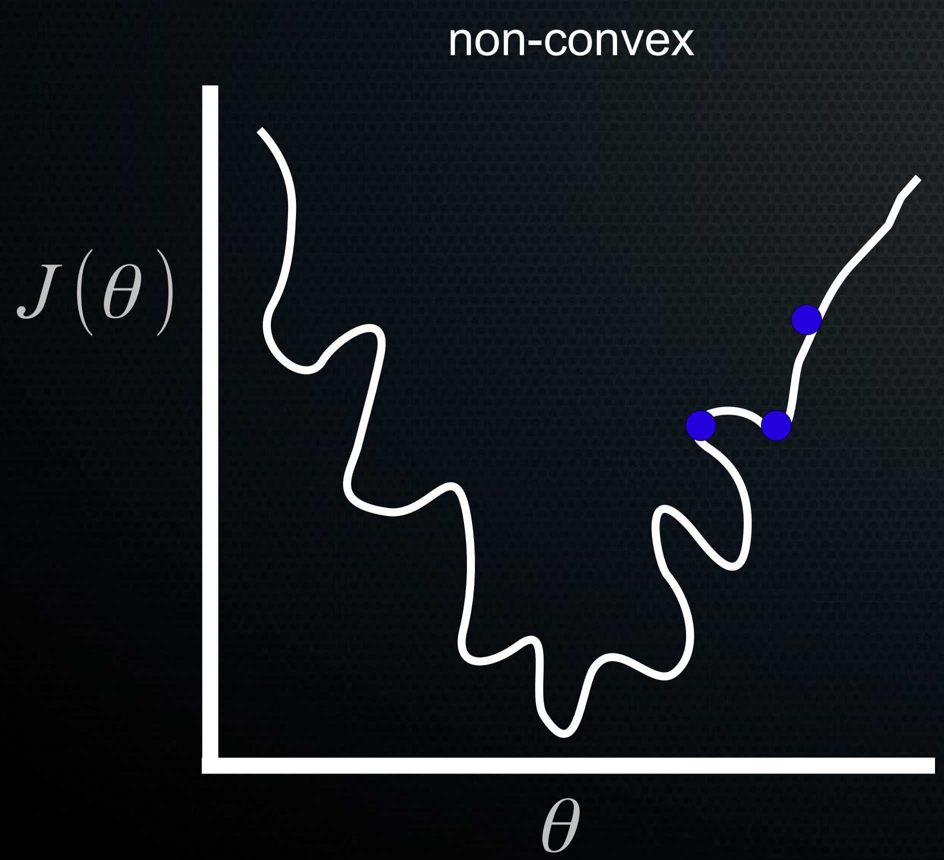

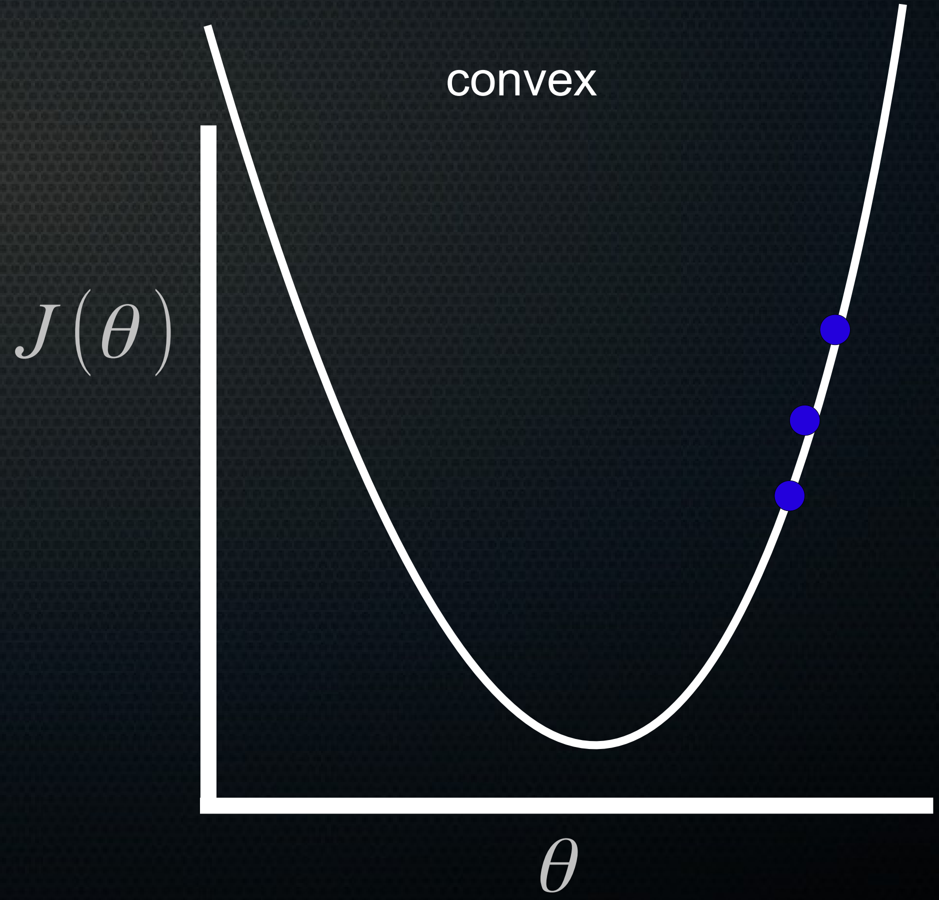

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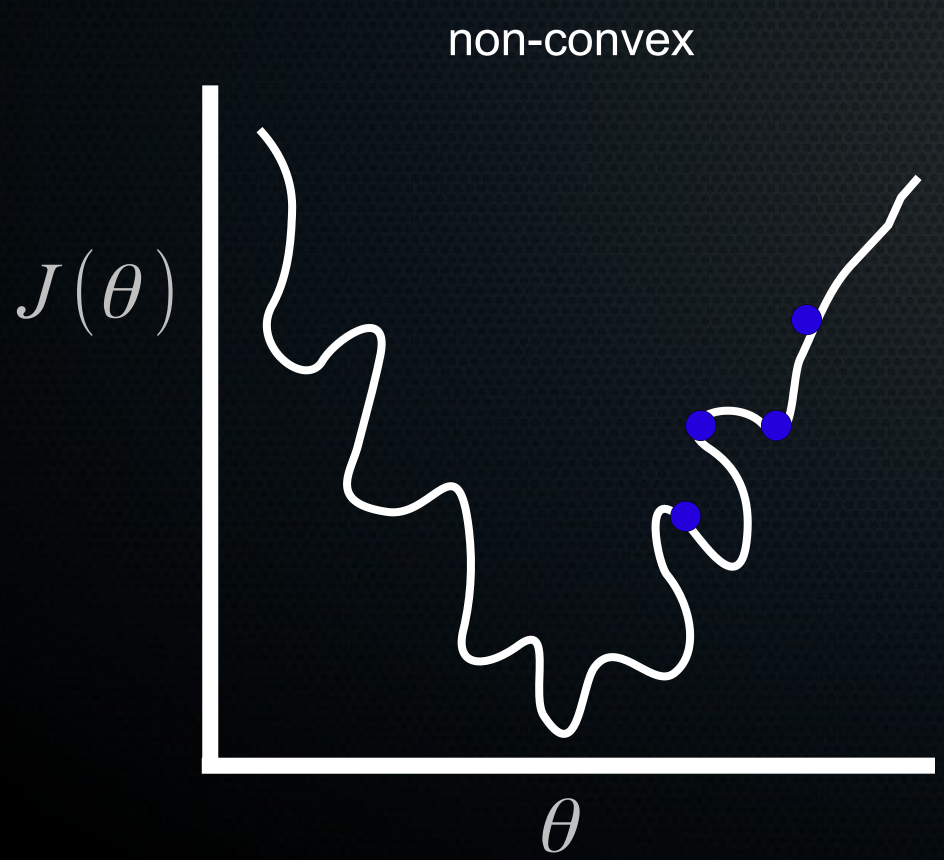

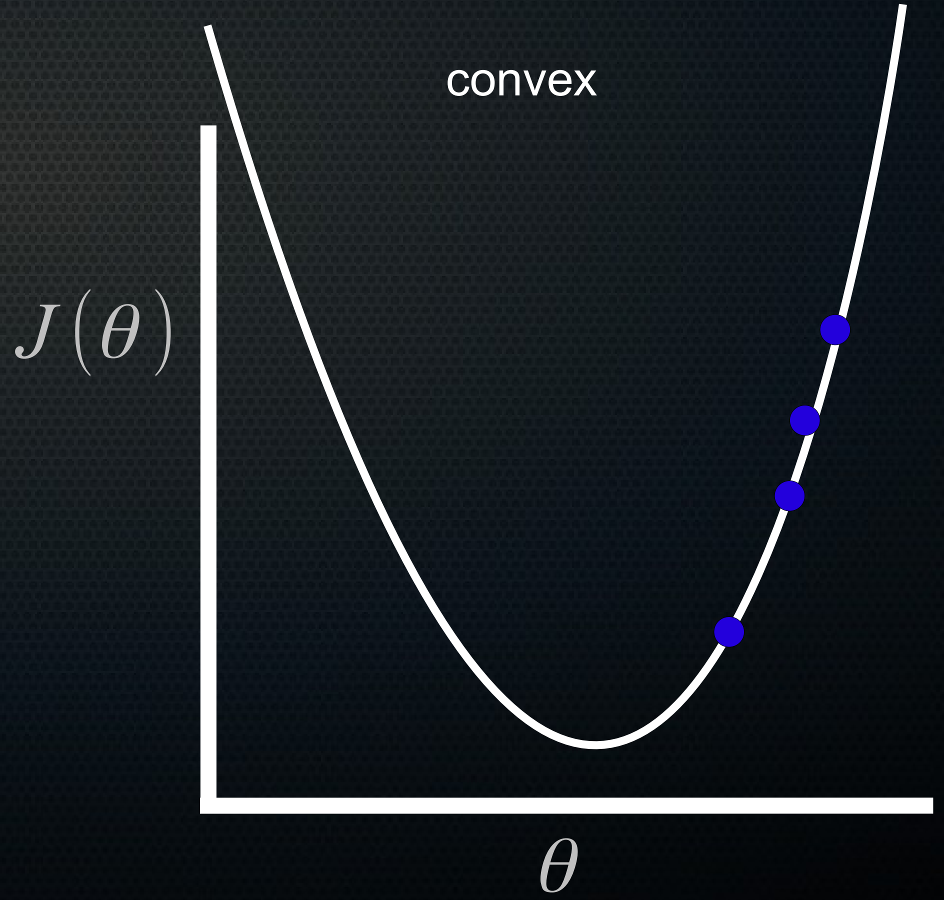

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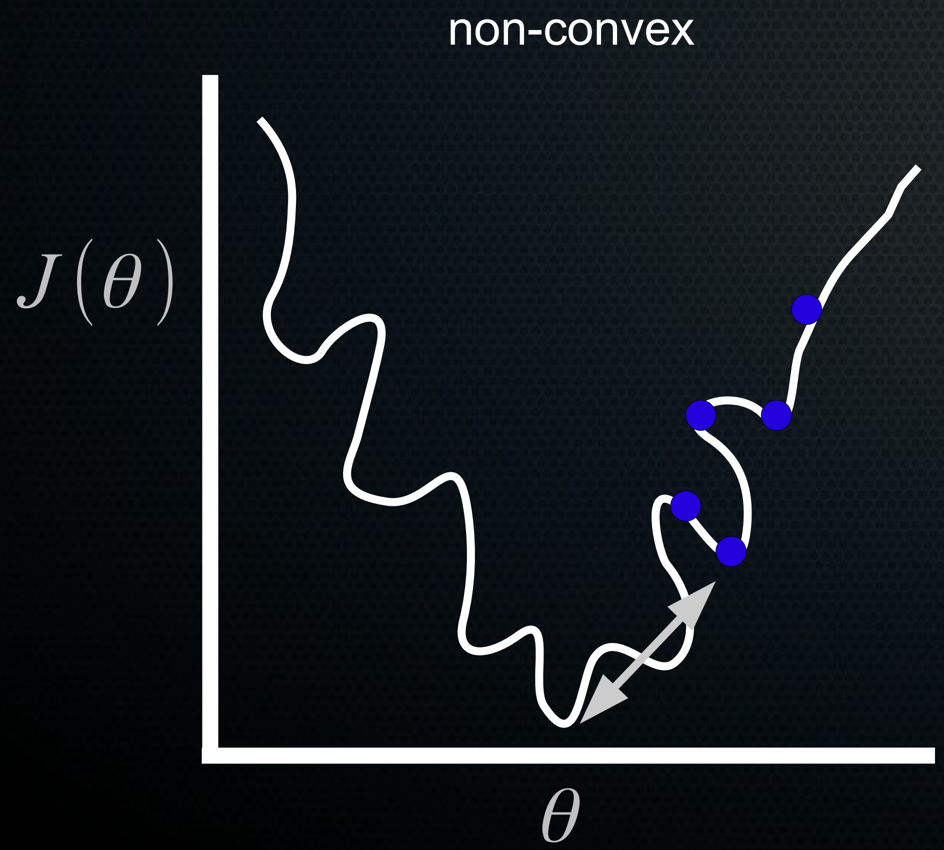

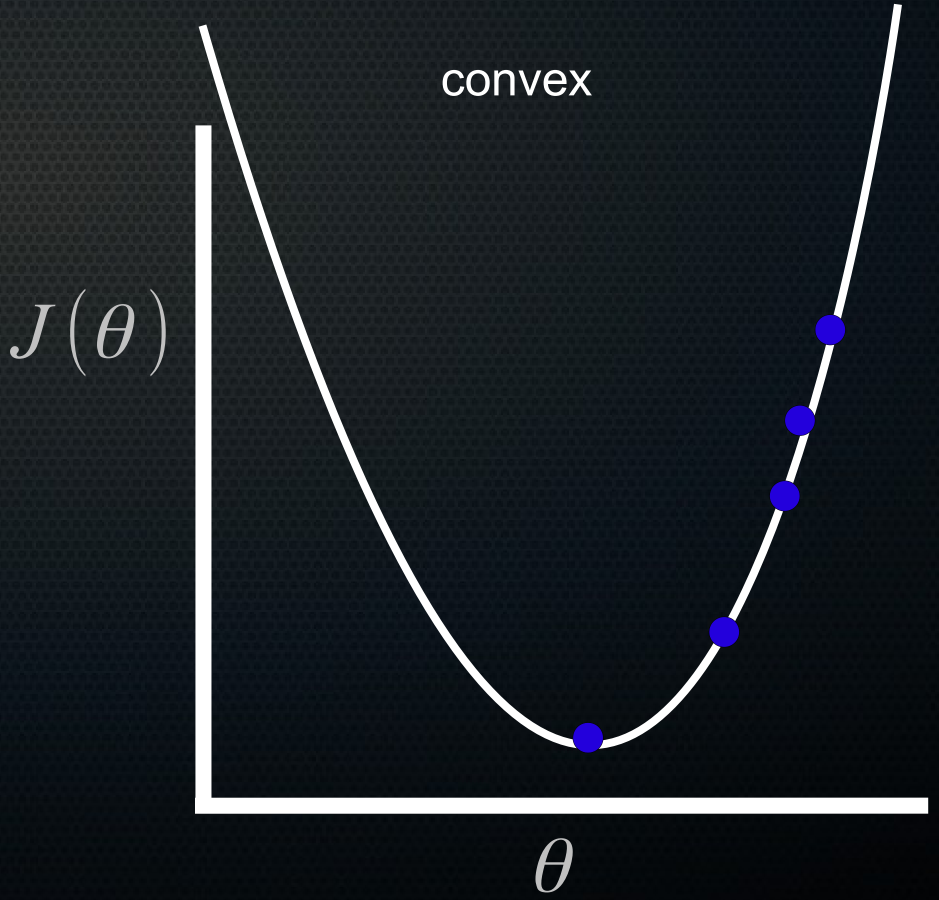

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- What then?

$$\mathrm{Cost}(x)=\begin{cases}-\log(h_\theta(x))\ \text{if } y=1\\ -\log(1-h_\theta(x))\ \text{if } y=0\end{cases}$$

What does this function look like?

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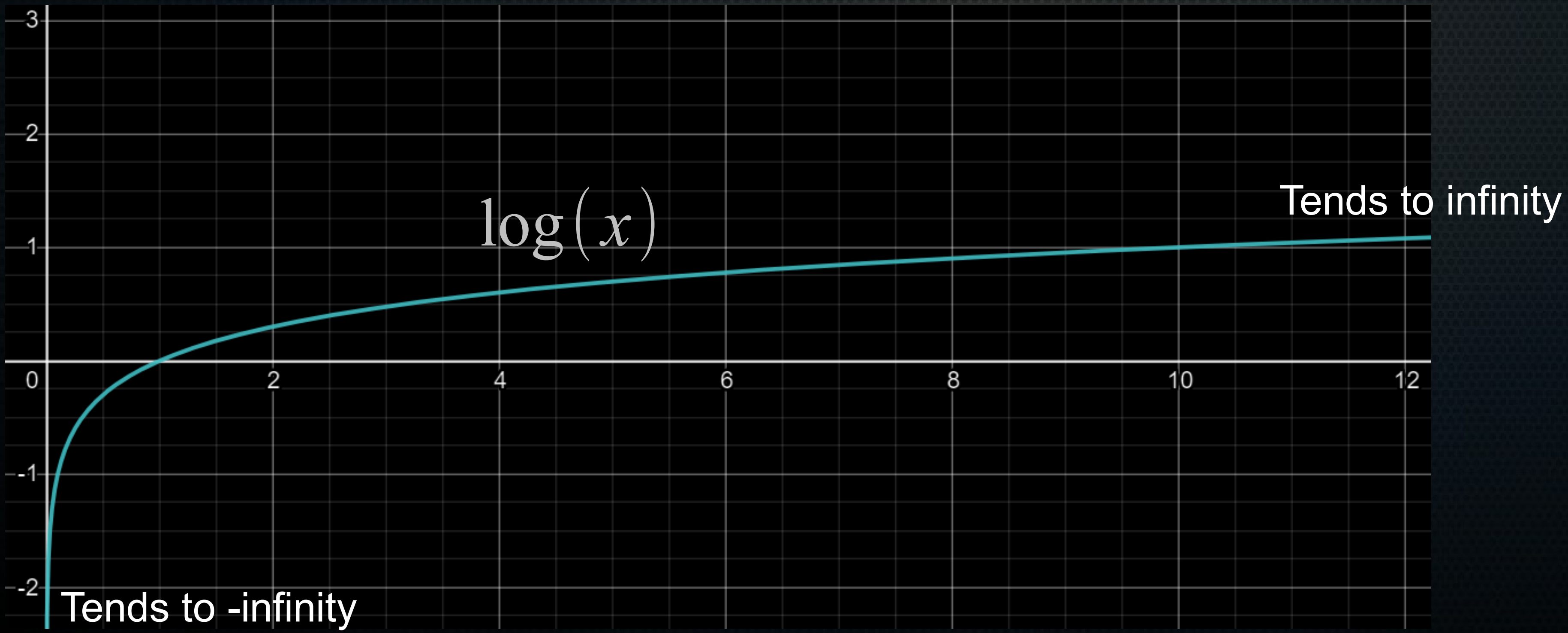

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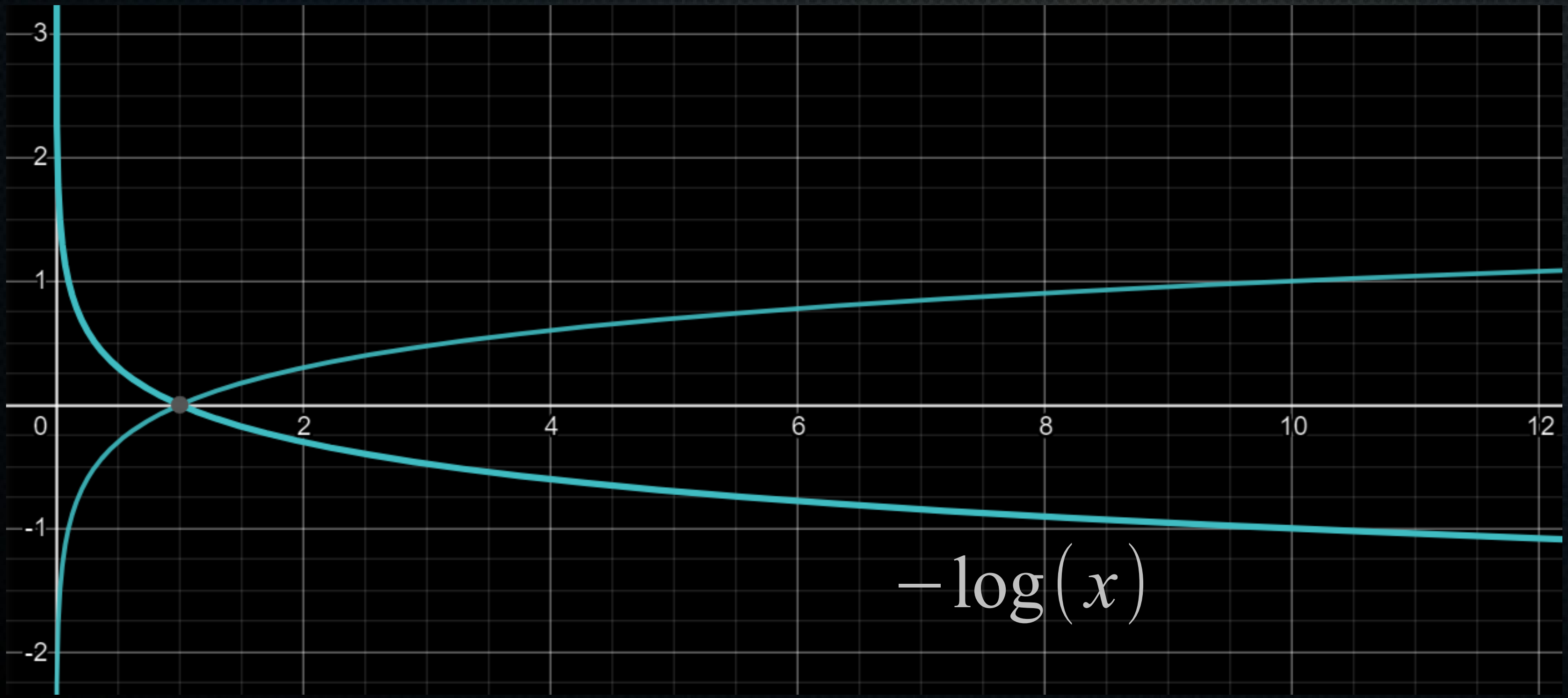

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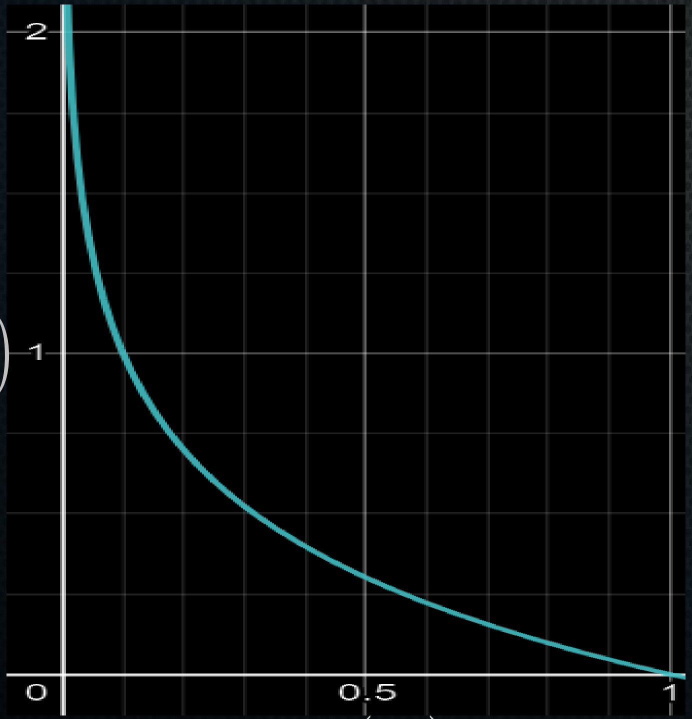

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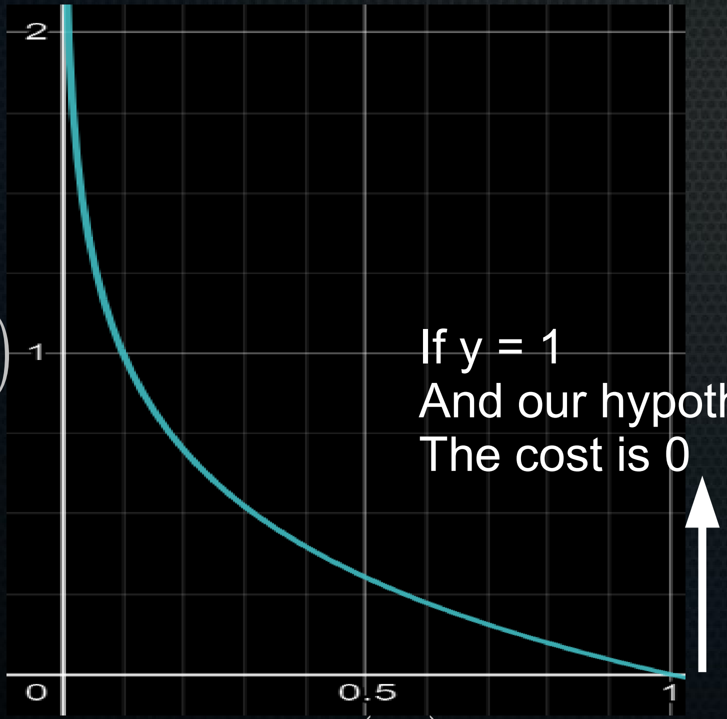

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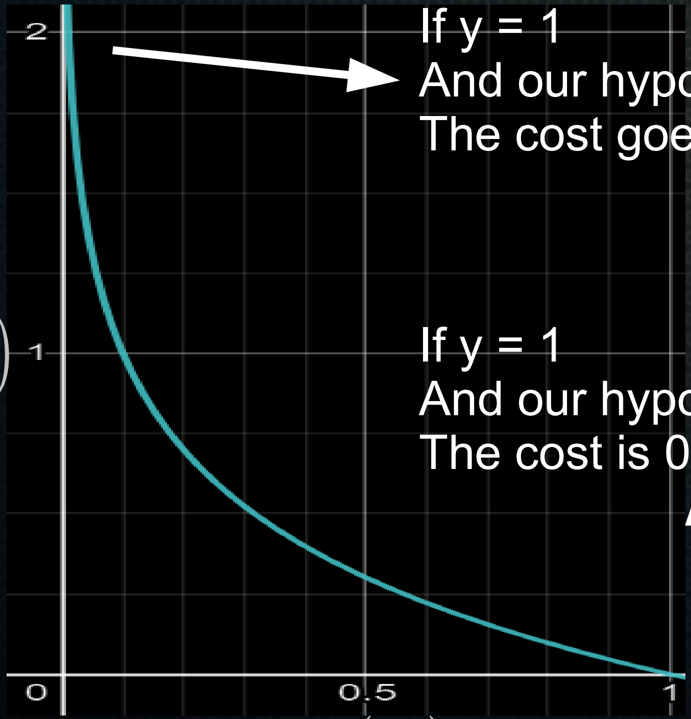

If y = 1
And our hypothesis predicts 0
The cost goes to ∞

If y = 1
And our hypothesis predicts 1
The cost is 0

$h_\theta(x)$

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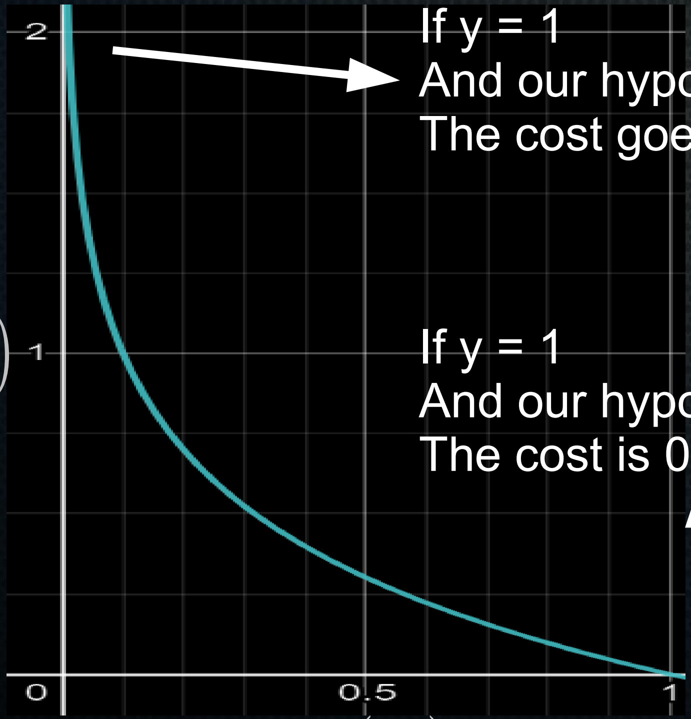

$h_\theta(x)$

If y = 1
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The cost goes to ∞

If y = 1
And our hypothesis predicts 1
The cost is 0

If we had used MSE

$$(0-1)^2 = 1$$

Penalised far less!

What does this function look like?

$$\mathrm{Cost}(x) = \begin{cases} -\log(h_\theta(x)) \text{ if } y=1 \\ \boxed{-\log(1-h_\theta(x)) \text{ if } y=0} \end{cases}$$

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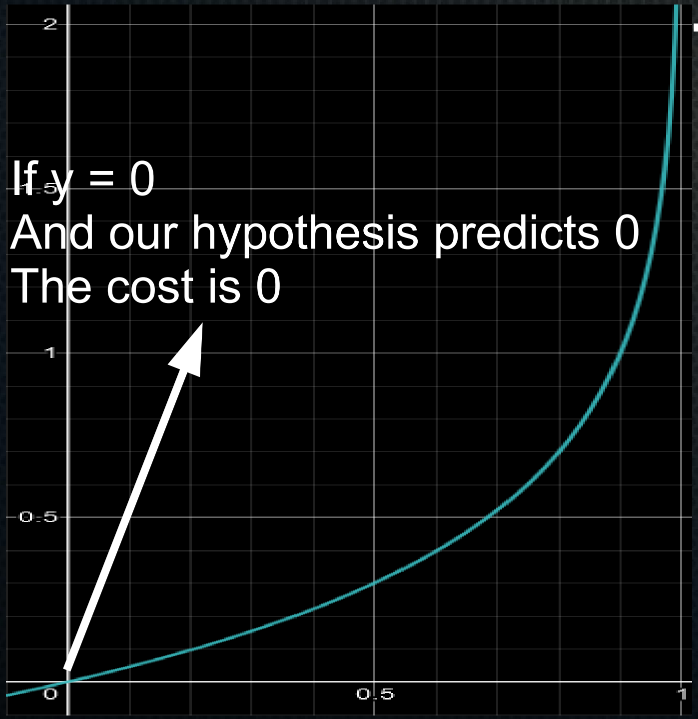

$h_\theta(x)$

If y = 0
And our hypothesis predicts 1
The cost goes to ∞

Simplified notation

$$\mathrm{Cost}(x) = \begin{cases} -\log(h_\theta(x)) \text{ if } y=1 \\ -\log(1-h_\theta(x)) \text{ if } y=0 \end{cases}$$

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \mathrm{Cost}(x^i)$$

$$\mathrm{Cost}(x) = -y \cdot \log(h_\theta(x)) - (1-y) \cdot \log(1-h_\theta(x))$$

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$$\Big\downarrow y = 0$$

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$$\boxed{-\log(1-h_\theta(x))}$$

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$y = 1$

$$-1\cdot\log(h_\theta(x))-(1-1)\cdot\log(1-h_\theta(x))$$

$$\boxed{-\log(h_\theta(x))}$$

Putting it all together

$$\mathrm{Cost}(x) = -y \cdot \log(h_\theta(x)) - (1-y) \cdot \log(1-h_\theta(x)) \qquad J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \mathrm{Cost}(x^i)$$

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} -y^{(i)} \cdot \log(h_\theta(x^{(i)})) - (1-y^{(i)}) \cdot \log(1-h_\theta(x^{(i)}))$$

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$$J(\theta) = \boxed{-} \frac{1}{m}\sum_{i=1}^{m} y^{(i)} \cdot \log(h_\theta(x^{(i)})) + (1-y^{(i)}) \cdot \log(1-h_\theta(x^{(i)}))$$

Optimising the cost function

- Same form as for linear regression (only hypothesis function differs!)

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left(\left(\boxed{h_\theta\left(x^{(i)}\right)} - y^{(i)} \right) \cdot x_j^{(i)} \right)$$

$$\theta_j := \theta_j - \frac{\alpha}{m} \sum_{i=1}^{m} \left(\left(\boxed{h_\theta\left(x^{(i)}\right)} - y^{(i)} \right) \cdot x_j^{(i)} \right)$$

$$\boxed{h_\theta\left(x^{(i)}\right) = \frac{1}{1+e^{-\theta^T * x}}}$$

Summary

- By using the sigmoid function as a transformation of normal regression and interpreting the output as a chance of being 0 or 1 we can do classification.
- Only the form of our hypothesis function is different
- Need a different cost function: should be smooth, and give logical values for large errors.

Break for practical