Remote Controlled Segway

Modelling and control of a motorized inverted pendulum

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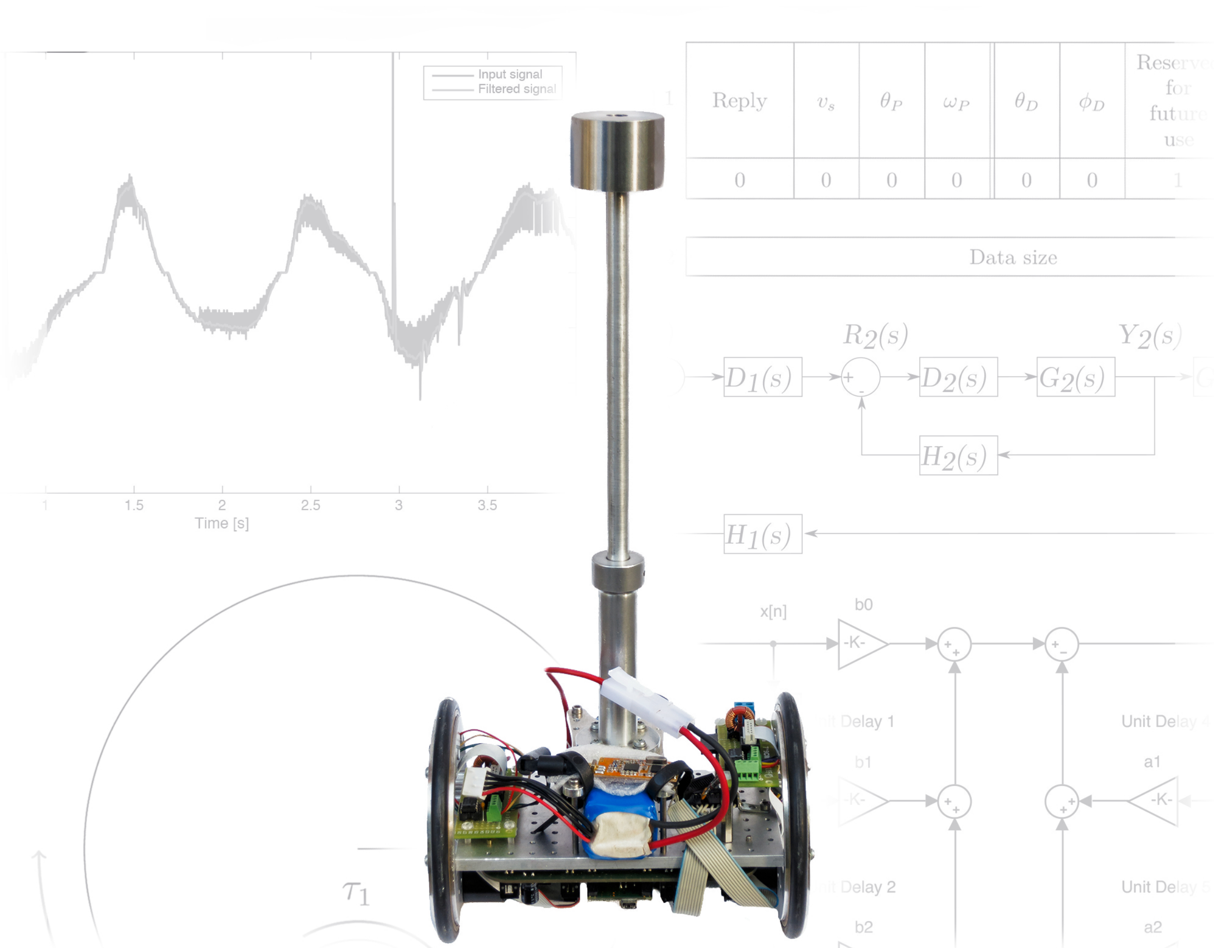

5th Semester
Group 15gr514
Electronics and IT
Fall 2015

AALBORG UNIVERSITY
STUDENT REPORT

This report is compiled in LaTeX, originally developed by Leslie Lamport, based on Donald Knuth's TeX. The main text is written in *Computer Modern* pt 11, designed by Donald Knuth. Flowcharts and diagrams are made using Microsoft Visio, Inkscape and Tikz, a TeXpackage for generating graphics.

AALBORG UNIVERSITY
STUDENT REPORT

Institute of Electronic Systems
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Title:
Flexible Equalizer for use in active speaker systems

Theme:
Digital Real-Time Signal Processing

Project Period:
6. Semester

Project Group:
16gr514

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Copies: 5

Page Numbers: xx

Date of Completion:
May 26, 2016

Abstract:

This is a box for an Abstract waiting to be written.

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Preface

This report has been carried out during spring of 2016 as a 6. semester Electronics and IT student bachelor project at Aalborg University by group 16grXXX. The project concerns the development of Somehting extremely awesome.

Aalborg University, May 27, 2016

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2 | Problem Analysis

2.1 Problem Statement

3 | Technical Analysis

3.1 Gate

3.2 Equalizer

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3.4 why a DSP?

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Appendix

A | Motor measurements

The purpose of these measurements is to determine the motor parameters. These parameters are:

- Motor resistance, R_a
- Motor inductance, L_a
- Generator konstant, K_e
- Motor & wheel friction, B
- Motor & wheel moment of inertia, J

These parameters can be seen in the electromechanical equivalent of a DC-motor, see Figure 3.1. Note that the measurements are performed on the motor with the gear and wheel attached, which are seen as a single mechanical system. Thus, all inertias and dampers measured in the following, are the total values of both the motor, gear and wheel. It is known that the gear ration from motor to shaft is $N_{ms} = 19$, while the gear ratio from shaft to wheel is $N_{sw} = \frac{25}{90} \approx 0.2778$

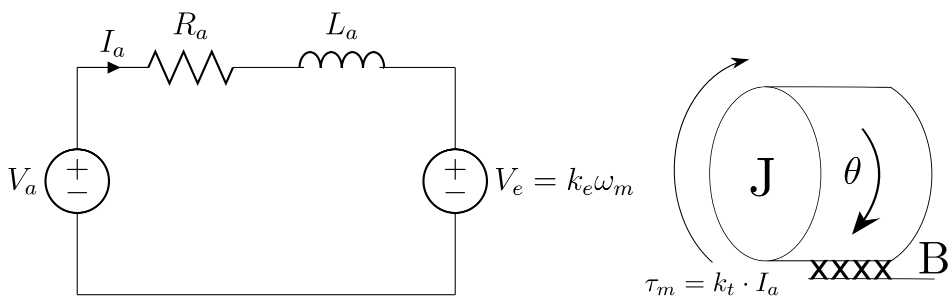

Figure 3.1: The electromechanical equivalent of a DC motor.

Equipment

For these measurements, the following equipments are used:

Name	**AAU No.**	**Type**
HAMEG HM7042-3	AAU60774	Power supply
Fluke 37	AAU33019	Multimeter
Fluke I30S	AAU78550	Amp-meter
Agilent DSO6034A	AAU64572	Oscilloscope
-	AAU08246	Analog Tachometer
Compact A2108	AAU77087	Tachometer

Table 3.1: Equipment used to determine motor parameters.

A.1 Motor resistance, R_a

Setup

The rotor is fixed, so the velocity is 0. In steady state, the motor current and the motor voltage are measured for current from 0 A to 0,9 A. The voltage is measured with a Fluke 37 (AAU33019) multimeter, and the current is measured using the power supply. The current was tried to be measured using a Fluke 37, but the series resistance in the Fluke might have affected the measurements too much, since the results seen were not realistic. Therefore, the multimeter measuring the current was removed. The current was measured with the power supply after verifying, that the measurements were consistent with the Fluke.

Results

Current [A]	**Voltage [V]**	**Resistance [Ω]**
0.00	0.00	-
0.046	0.493	10.7
0.10	0.981	9.81
0.20	1.518	7.59
0.30	2.00	6.93
0.35	2.50	7.15
0.47	3.06	6.52
0.52	3.52	6.77
0.61	4.05	6.64
0.68	4.54	6.68
0.75	5.07	6.76
0.82	5.50	6.75
0.91	6.02	6.65
	Average:	**6.76**

Table 3.2: Results from the resistance measurement.

Table 3.2 and Figure 3.2 show that the resistance goes towards linearity at higher currents. The fit for the trend line is $R^2 = 0.9956$, which means the trend line fits well. From linear regression the resistance is 6.74Ω.

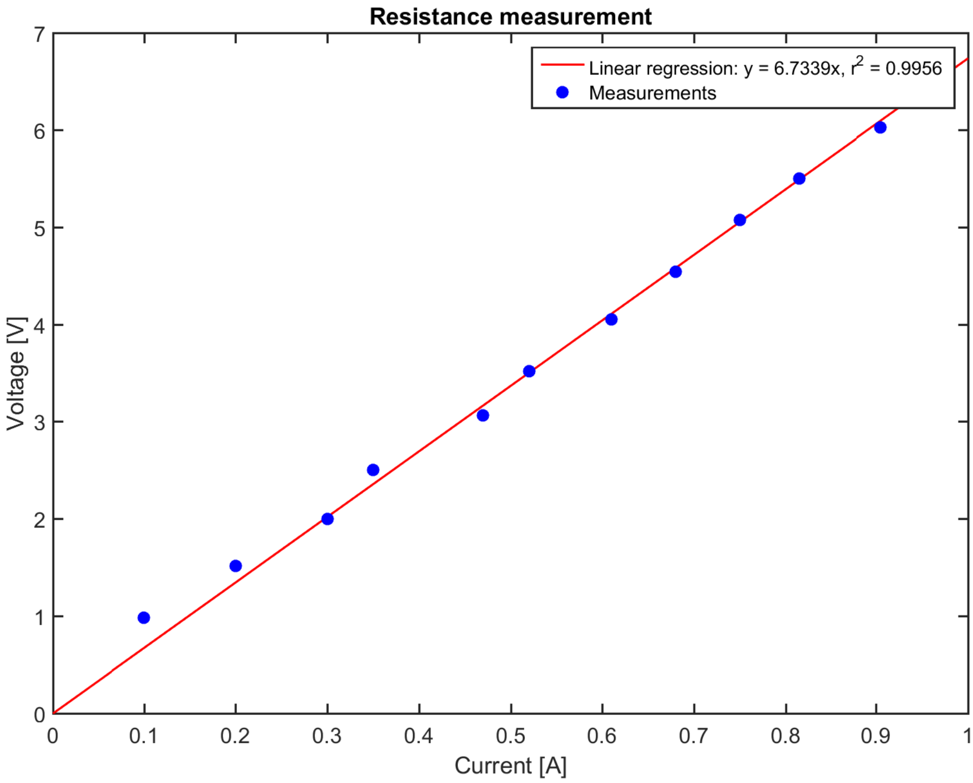

Figure 3.2: Linear regression of voltage-current measurements to determine the resistance.

The reason why the measurements are done in steady state is because in steady state, the model of the motor only consist of a resistor. The inductor and voltage generator are short circuited, making it easy to estimate the resistance.

A.2 Motor inductance, L_a

Setup

The rotor is fixed so the velocity is 0. The current response to a voltage input step is measured. The amp-meter is connected to the oscilloscope, so the current can be measured precisely over time. A voltage step is applied by the PWM drivers with a duty cycle at 200 out of 255.

Results

The current plotted as a function of time can be seen in Figure 3.3.

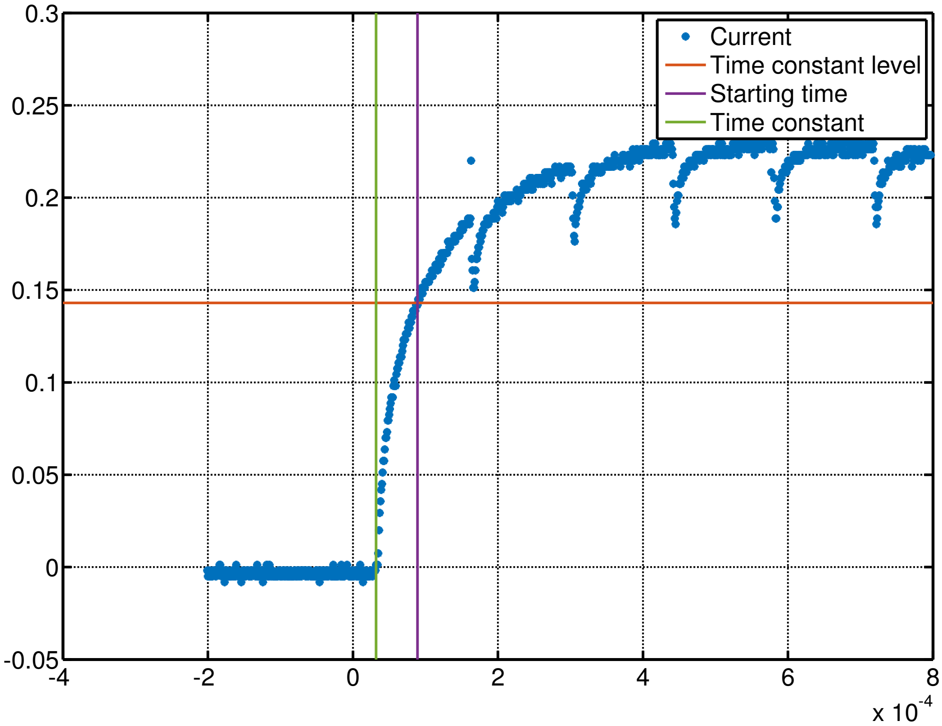

Figure 3.3: Plot of motor current as a function of time, used to find motor inductance.

The transfer function is of 1st order, because there is only one inductor and no capacitors in the motor equivalent circuit. The inductance is found using the time constant of a 1st order circuit. It is known that the time constant can be found as $\tau = \frac{L}{R}$, where $I(\tau) = 0.632 \cdot (I_{max} - I_{min})$. This can be seen in Figure 3.3 as the red line. It is estimated that the voltage step started at the green line. Using this, the time constant is found to be $\tau \approx 57\,\mu s$. Thus, the inductance is found:

$$L = R \cdot \tau = 6.74\,\Omega \cdot 57\,\mu s = 0.384\,\text{mH}$$

The inductance is 0.363 mH according to the datasheet, see [**?**].

A.3 Generator constant, K_e

Setup

In a number of steady state points, the motors voltage and the wheels velocity is measured, using the Fluke multimeter and the onboard encoders. A motor is attached to flexible shaft and to a secondary motor. The motor is fastened using a mounting arm then a current is applied to the secondary motor to make it turn.

Results

The results from the test can be seen in Table 3.3. In the table, the conversion from velocity v to angular velocity, ω_w, are found as:

$$\omega_w = \frac{\mathrm{v}}{r_w} \tag{3.1}$$

$$\omega_m = \frac{1}{N_{ms}} \cdot \frac{1}{N_{sw}} \cdot \omega_w \tag{3.2}$$

Where:

ω_w	is the angular velocity of the wheel	[rad/s]
v	is the translatoric velocity of the wheel	[m/s]
r_w	is the radius of the wheel	[m]
ω_m	is the angular velocity of the motor	[rad/s]
N_{ms}	is the gearing ratio from the motor to the shaft	[1]
N_{sw}	is the gearing ratio from the shaft to the wheel	[1]

Also, K_e can be found using the expression $K_e = \frac{U_a}{\omega}$. Note that the measured velocity has been divided with the gear ratios ($N_{ms} = \frac{1}{19} \approx 0.053$ and $N_{sw} = \frac{25}{90} \approx 0.277$), to obtain the angular velocity of the motor instead of the wheel, as it is the motors angular velocity, ω_m that is used.

$\mathbf{speed_w}$ **[m/s]**	$\boldsymbol{\omega_w}$ **[rad/s]**	$\boldsymbol{\omega_m}$ **[rad/s]**	**Voltage [V]**	$\boldsymbol{K_e}[\mathrm{V}/\frac{\mathrm{rad}}{\mathrm{s}}]$
0.34	5.81	397.5	4.25	0.0107
0.74	12.65	865.2	9.15	0.0106
0.46	7.86	537.8	5.70	0.0106
0.33	5.64	385.8	4.00	0.0104
0.22	3.76	257.2	2.70	0.0105

Table 3.3: Results from the test to determine the K_e factor.

Averaging the results in Table 3.3, the K_e factor is determined to be:

$$K_e = 10.5 \frac{\text{mV}}{\frac{\text{rad}}{s}}$$

A.4 Motor & wheel friction, B

Setup

In a number a steady state points, the motor current and motor velocity is measured. This is done using the ampmeter and the encoders.

Results

In steady state, i.e. constant angular velocity, the friction torque is $\tau_B = K_t \cdot i_a$. This is true since the resultant torque is 0 when the system is in steady state. Thus, all torque applied by the motor must be countered by an equal torque in opposite direction due to friction.

In Table 3.4 measurements of the angular velocity and the motor current i_a are listed.

ω_m **[rad/s]**	$\mathbf{i_a}$**[A]**
1216.00	0.083
1099.08	0.080
841.85	0.073
502.77	0.066
362.46	0.092
748.31	0.096
1052.31	0.100
1180.92	0.106

Table 3.4: The measurements of the angular velocity and I_a for determining the damper coefficient B.

The motor torque, and thus the damper torque τ_B, can be found using $\tau_B = K_t \cdot i_a$, where $K_t = K_e = 10.5 \frac{\text{mNm}}{A}$

Since it is known that the resultant torque of a damper can be found as $\tau_B = B \cdot \omega$, B can be found since both τ_B and ω are known. The results can be seen in Table 3.5.

$\boldsymbol{\omega_m[\frac{rad}{s}]}$	$\boldsymbol{\tau_B[Nm]}$	**B** $[\frac{\text{Nm}}{\frac{\text{rad}}{s}}]$
1216,00	$875,27 \cdot 10^{-6}$	$0,720 \cdot 10^{-6}$
1099,08	$843,63 \cdot 10^{-6}$	$0,768 \cdot 10^{-6}$
841,85	$769,82 \cdot 10^{-6}$	$0,914 \cdot 10^{-6}$
502,77	$696,00 \cdot 10^{-6}$	$1,384 \cdot 10^{-6}$
362,46	$970,18 \cdot 10^{-6}$	$2,677 \cdot 10^{-6}$
748,31	$1012,36 \cdot 10^{-6}$	$1,353 \cdot 10^{-6}$
1052,31	$1054,54 \cdot 10^{-6}$	$1,002 \cdot 10^{-6}$
1180,92	$1117,81 \cdot 10^{-6}$	$0,947 \cdot 10^{-6}$

Table 3.5: The friction torque and friction coefficient B.

Averaging the results for B in Table 3.5, B is found as:

$$B = 1.22 \frac{\mu\text{Nm}}{\frac{\text{rad}}{s}}$$

A.5 Coulomb friction, τ_c

Setup

In a number a steady state points, the motor current and applied voltage are measured. The voltage is applied by the Hameg power supply, the current is measured with the amp-meter, and the voltage with the multimeter. Then steps are made on the Hameg with approximately 0.1 V until 2.4 V from there the steps are 0.2 V to 5 V.

Results

It is expected that the current can be estimated by two straight lines, because of the coulomb friction. Until $Ia \cdot k_t$ equals the coulomb friction. The current is expected to have a steep slope, and when the motor starts rotating, the slope is expected to be less steep. This can be seen in Figure 3.4, where these slopes and offsets have been estimated using curve fitting.

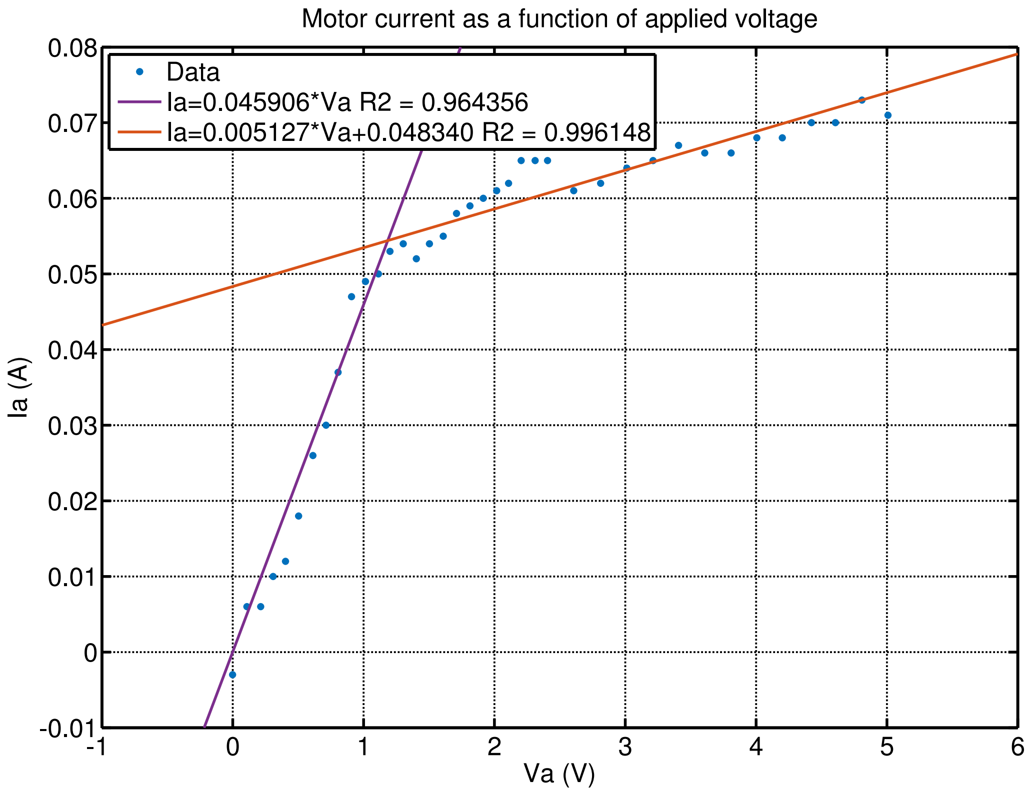

Figure 3.4: Motor current as a function of applied voltage with two curves fitted to the data area before and after wheel started turning.

The coulomb friction can be found based in Figure 3.4 by finding the current in the intersection between the two lines and then multiplying with k_t.

$$0.045906 \cdot Va = 0.005127 \cdot Va + 0.048340 \tag{3.3}$$
$$Va = 1.18541\,\mathrm{V} \tag{3.4}$$

The current in the intersection can be found by inserting Va in any of the equations.

$$Ia = 0.045906 \cdot 1.18541 \tag{3.5}$$
$$Ia = 0.054417\,\mathrm{A} \tag{3.6}$$

The coulomb friction can then be found as:

$$\tau_c = Ia \cdot k_t \tag{3.7}$$
$$\tau_c = 0.054417\,\mathrm{A} \cdot 0.0105 \frac{\mathrm{Nm}}{\mathrm{A}} \tag{3.8}$$
$$\tau_c = 571.4\mu\mathrm{Nm} \tag{3.9}$$

Note that during another version of this test, the maximum velocity was found to be 1.17 m/s when driven by the segway's PWM signal.

A.6 Moment of inertia, J

Setup

The motor is rotated with a fixed velocity, after which the motor is turned off i.e., making $i_a = 0$ by setting the duty cycle to 0. The motor velocity is measured with a interval of 5 ms. This is done by using the onboard encoders and a timer interrupt.

Results

Looking at a motor's kinematics, the mechanical equation can be expressed as shown in Equation 3.10

$$I \cdot \dot{\omega}(t) = k_t \cdot Ia(t) - B \cdot \omega(t) - \tau_c \tag{3.10}$$

If it is assumed that the motor is running at $\omega(0)$ and the motor then is terminated ($Ia = 0$), the differential equation can then be solved. The result can be seen in Equation 3.11

$$\omega(t) = -\frac{\tau_c}{B} + \left(\omega(0) + \frac{\tau_c}{B}\right) \cdot e^{-\frac{B}{I} \cdot t} \tag{3.11}$$

By knowing this, the output graph seen in Figure 3.5, shows that the rotational speed as a function of time can be interpreted. By inserting the previous found values, a fit has to be found manually, this is the purple graph in Figure 3.5. A better fit can be found if the damper is divided by 8 before fitting the inertia. This can be seen as the orange graph in Figure 3.5.

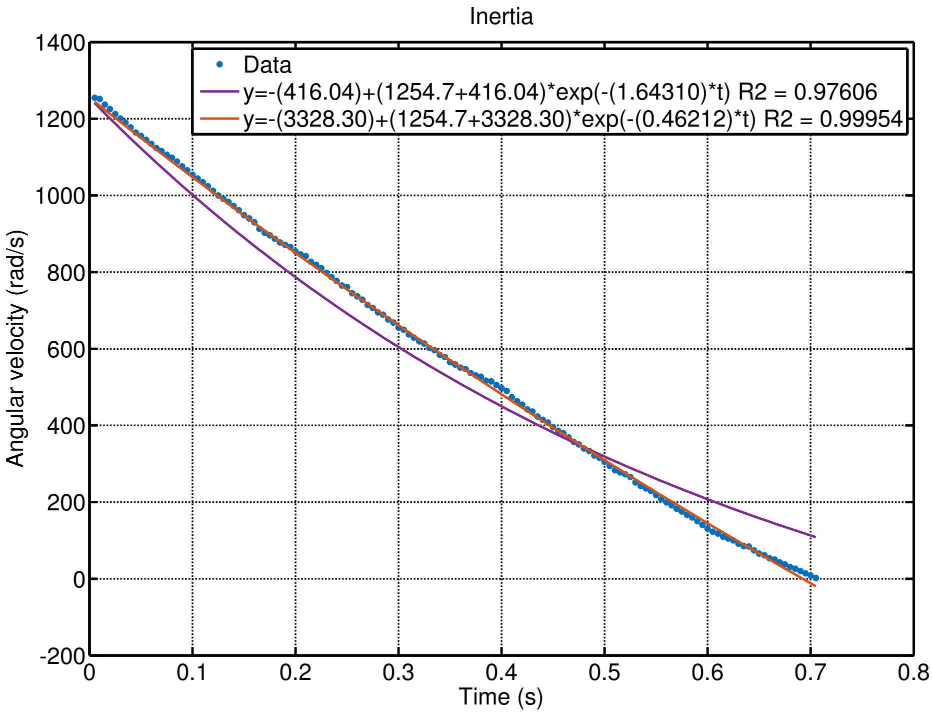

Figure 3.5: Motor velocity as a function of time, when the motor current i_a is set to 0 at time 0.

Because of this measurement, it is decided to divide the damper by a factor of 8 in the model. Based on this, the damper and inertia is found and can be seen in Table 3.6

Damper	$152.5 \cdot 10^{-9} \frac{\text{sNm}}{\text{rad}}$
Inertia	$330 \cdot 10^{-9} \frac{kg}{m^2}$

Table 3.6: The friction coefficient B, and Inertia J, to be used in model.