Journal of the Royal Statistical Society
Applied Statistics
Series C

*Appl. Statist.* (2020)
**69**, *Part* 5, *pp.* 1337–1356

# Markov switching modelling of shooting performance variability and teammate interactions in basketball

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[Received October 2018. Final revision July 2020]

**Summary.** In basketball, measures of individual player performance provide critical guidance for a broad spectrum of decisions related to training and game strategy. However, most studies on this topic focus on performance level measurement, neglecting other important factors, such as performance variability. Here we model shooting performance variability by using Markov switching models, assuming the existence of two alternating performance regimes related to the positive or negative synergies that specific combinations of players may create on the court. The main goal of this analysis is to investigate the relationships between each player's performance variability and team line-up composition by assuming shot-varying transition probabilities between regimes. Relationships between pairs of players are then visualized in a network graph, highlighting positive and negative interactions between teammates. On the basis of these interactions, we build a score for the line-ups, which we show correlates with the line-up's shooting performance. This confirms that interactions between teammates detected by the Markov switching model directly affect team performance, which is information that would be enormously useful to coaches when deciding which players should play together.

*Keywords*: Markov switching model; Performance analysis; Regime switching; Sport analytics;

## 1. Introduction

The management of a sport team requires data analysis, including 'big data' analytics. Statistical methods are essential support to coaches and technical experts when making a wide range of decisions. In basketball, these decisions are made both before the game, guiding training and playing strategies, and during the game, when fast and timely choices must be made. In the latter situation, deciding which players should be on the court at a given moment is particularly important. This choice depends on the state of play, the team's tactics, the opposing team's behaviour and other factors; however, it is also important to exploit potential synergies between teammates that enhance overall team performance.

Player performance analysis is a hot topic in the literature (see, for example, Page *et al.* (2007, 2013), Cooper *et al.* (2009), Sampaio *et al.* (2010), Piette *et al.* (2010), Fearnhead and Taylor (2011), Ozmen (2012), Erčulj and Štrumbelj (2015), Deshpande and Jensen (2016), Passos *et al.* (2016), Franks *et al.* (2016) and Zuccolotto and Manisera (2020)). Some studies deal with a broad concept of performance (accounting for offensive and defensive abilities, for example), whereas others focus only on shooting performance, also with reference to the so-called 'hot hand' effect (Gilovich *et al.*, 1985; Vergin, 2000; Koehler and Conley, 2003; Tversky and

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Gilovich, 2005; Arkes, 2010; Avugos *et al.*, 2013; Bar-Eli *et al.*, 2006) or the effect of high pressure game situations (Madden *et al.*, 1990, 1995; Goldman and Rao, 2012; Zuccolotto *et al.*, 2018). In this paper, we address shooting performance, with a special focus on performance variability.

More specifically, we analyse shooting performance from the dual perspectives of average and variability and then investigate its relationship to team line-up and team performance. This study is carried out with a dynamic approach. Data are treated as time series observed in the event time: differently from time series that commonly are sequences of observations observed in time at regular intervals, in the event time, each observation corresponds to an event (e.g. one shot), and events, which occur randomly in continuous realtime, may be irregularly spaced in time. We used a three-step procedure.

(a) For each shot made by a given player, we define a smoothed index of shooting performance, taking into account
   (i) whether the attempted shot is made,
   (ii) the difficulty of the shot and
   (iii) the player's shooting intensity.
(b) For each player, we analyse the index of shooting performance with Markov switching models, to detect the possible presence of different performance regimes. Simultaneously, we investigate performance relationships between this player and his teammates by assuming that the transition probabilities between regimes are dependent on the presence of given teammates on the court.
(c) We represent all significant relationships between players by means of a network graph and then score different line-ups, taking into account these relationships. Finally, we verify the correlation between this score and line-up performance, measured as the intensity of scored points. This last step moves the focus from pairs of players to overall team performance, to confirm that interactions between teammates that are detected by the Markov switching model directly affect team performance. Such insights could be used by the coach to decide which players should play together.

Alongside the methodological definition of our procedure, we present the results of a case-study. For each step, the description of our method will be immediately followed by its application to data, to understand better the theoretical framework. For the case-study, we use play-by-play data from the 82 games played by the National Basketball Association (NBA) team Golden State Warriors during the 2017–2018 regular season. Each single event (shot, assist, rebound, steal, foul, turnover, etc.) during the game is recorded, along with all relevant information (time, quarter, play length, players involved, distance from the basket, shot co-ordinates, etc.). For this case-study, we have selected, as units, only the shots and, as variables, those that describe each shot, including match, time elapsed since the beginning of the game, player making the shot, success (missed or made) and players on the court. In addition, we have generated a new variable describing, for each shot of a given player, the time elapsed since his previous shot or since he entered the court (if it is his first shot since entry). The play-by-play data set that we use is included in the R package `BasketballAnalyzeR` (`bdsports.unibs.it/basketballanalyzer/`; Sandri (2020), Sandri *et al.* (2020) and Manisera *et al.* (2019)) and has been kindly made available by BigDataBall (`www.bigdataball.com`): a data provider that leverages computer vision technologies to enrich and extend sports data sets with a number of unique metrics. Since its establishment, BigDataBall has supported many academic studies as a reliable source of validated and verified statistics for the NBA, Major League Baseball, the National Football League and the Women's National Basketball Association.

This paper is organized as follows: Section 2 discusses the literature that is relevant to our research question. Section 3 introduces the methods and presents results that are related to the first step of our procedure. Sections 4 and 5 are devoted to the second and third steps of our procedure respectively. Section 6 presents our conclusions.

## 2. Background: performance variability and teamwork assessment

Human performance evaluation is an important issue in several different fields of study, with the most prevalent conceptualization being typical performance. Psychological studies have identified typical performance as just one aspect of overall performance. Sackett *et al.* (1988) and DuBois *et al.* (1993) identified the concept of maximal performance, defined as what an individual can do, paired with typical performance definition as what an individual will do. At the same time, other research has focused on variability in performance over time, arguing that fluctuations of performance should not be treated as random noise but instead as a phenomenon that is worthy of separate analysis (Lecerf *et al.*, 2004), particularly intraindividual differences in variability (Rabbitt *et al.*, 2001). Barnes and Morgeson (2007) applied the above-mentioned studies (conceived in a general behavioural analysis context) to sports management. They investigated how the three conceptualizations of performance (typical, maximal and variable) affect remuneration levels of NBA players, finding strong support for the hypothesis that performance variability is negatively related to remuneration. Following the arguments that were put forward by these studies, we focus on the shooting performance variability of basketball players.

The idea of performance variability should not be confused with the so-called momentum effect, although it is not a completely unrelated concept. The term psychological momentum is used to describe changes in performance based on a recent success or failure. Adler (1981) defined psychological momentum, which can be positive or negative according to whether it refers to success or failure, as the tendency of an effect to be followed by a similar effect. Examples of momentum are represented in sports jargon by terms such as 'hot' and 'cold' streaks, the 'hot hand' in basketball shooting, and 'batting slumps' in baseball. Although many models and measures of psychological momentum have been proposed, and it has been demonstrated that athletes believe in the concept of momentum, evidence of such an effect has proved elusive (Vergin, 2000; Bar-Eli *et al.*, 2006; Avugos *et al.*, 2013). Gilovich *et al.* (1985) studied the hot hand effect in basketball and explained it as an unfounded belief in a law of small numbers, later confirmed by other studies (Koehler and Conley, 2003; Tversky and Gilovich, 2005). Similar conclusions have been reached for several sports, such as baseball (Albright, 1993), tennis (Silva *et al.*, 1988; Richardson *et al.*, 1988) and golf (Clark, 2005), although there is also evidence in favour of this phenomenon in sports such as bowling (Dorsey-Palmateer and Smith, 2004) and volleyball (Raab *et al.*, 2012).

In this study, we simply take note that 'hot' and 'cold' streaks occur, regardless of whether they can be explained by the concept of psychological momentum, and that their occurrence contributes to performance variability over time.

Performance variability may be due to other non-psychological factors, such as a player's physical condition and the presence of specific teammates on the court. In particular, our study of performance variability aims to identify cyclical patterns of shooting performance and to investigate whether they may be associated with the changes in the team's line-up.

In fact, overall team performance is not simply the sum of each player's performance; it is also the result of teamwork. Several studies have assessed teamwork from different perspectives. Fujimura and Sugihara (2005) constructed a player motion model based on a generalized

Voronoi diagram that divides space into dominant regions and Metulini *et al.* (2018) proposed a model based on surface areas. Players' motion was analysed by using motion charts by Metulini *et al.* (2017a, b) and Metulini (2017), who proposed a cluster analysis to separate different games strategies. Vilar *et al.* (2012) showed that interactions between players may be analysed by ecological dynamics, explaining the formation of successful and unsuccessful patterns of play. Carron and Chelladurai (1981) identified the factors that are correlated with the athlete's perception of cohesiveness (between coach and athlete and between team and athlete), which they described as a multi-dimensional construct.

One noteworthy approach to teamwork assessment is the use of network methods. Warner *et al.* (2012) and Lusher *et al.* (2010) used social network analysis to investigate how team cohesion and individual relationships impact team dynamics. Passos *et al.* (2011) revealed that the number of network interactions between team members should differ between successful and unsuccessful performance outcomes. Clemente *et al.* (2014) applied a set of network metrics to characterize the co-operation between teammates in a football team.

We hypothesize that specific combinations of players on the court may create positive or negative synergies and highlight these relationships by scoring line-ups, based on a performance variability analysis obtained by using Markov switching models.

## 3. Step 1: definition of a shooting performance index

To measure shooting performance we consider two circumstances: the shooting intensity and the shooting efficiency (the extent to which shots succeed in scoring baskets).

For a given player $i$, let $\tilde{\phi}_{ij}$ be a measure of his shooting intensity at shot $j$:

$$\tilde{\phi}_{ij} = 1/t_{ij} \tag{1}$$

where $t_{ij}$ denotes the time elapsed since shot $j-1$ of the same player or since the moment that he entered the court (if $j$ is the first shot since entry). The shooting intensity is partly determined by the opponent's strength and by the pace of the game, so we remove the match effect and obtain the adjusted shooting intensity:

$$\phi_{ij} = \tilde{\phi}_{ij}/\phi^{(m_{ij})} \tag{2}$$

where $m_{ij}$ is the match in which player $i$ attempted shot $j$ and $\phi^{(m_{ij})} = S^{(m_{ij})}/T$, with $S^{(m_{ij})}$ being the total number of shots that were attempted by the whole team in match $m_{ij}$ and $T$ the duration of the match. A high or low value of $\phi_{ij}$ means that the $j$th shot of player $i$ has occurred respectively earlier or later than expected, based on the shooting intensity of that match.

Following Zuccolotto *et al.* (2018), a measure of shot efficiency for shot $j$ is given by

$$E_{ij} = x_{ij} - p_{ij} \tag{3}$$

where $x_{ij}$ denotes the indicator function assuming a value of 1 if shot $j$ of player $i$ scored a basket and 0 otherwise, and $p_{ij}$ is the scoring probability of the shot. $E_{ij}$ is positive if the shot scored a basket (the lower the scoring probability, the higher its value) and negative if it missed (and the higher the scoring probability, the higher its absolute value). Thus, a basket is worth more when the scoring probability is low, whereas, when a miss occurs, it is considered more detrimental when the scoring probability is high (Zuccolotto *et al.*, 2018). To estimate the scoring probability $p_{ij}$, we use the goal percentage statistics of match $m_{ij}$ (so, we remove the possible match effect), separately for two-point shots, 2P, three-point shots, 3P, and free throws, FT. The scoring probability $p_{ij}$ of shot $j$ of player $i$ for shot type $\nu$ could be denoted by $p_{ij}(\nu)$, where $\nu \in \{2P, 3P, FT\}$, because it is computed as the percentage of shots of type $\nu$ of player $i$

who scored a basket on the total number of attempts of type $\nu$ in match $m_{ij}$. For simplicity, we omit index $\nu$ where it is unnecessary.

A unique measure of shooting performance $\psi_{ij}$ is then obtained as

$$\psi_{ij} = \phi_{ij} E_{ij}. \tag{4}$$

The rationale behind formula (4) is that the shooting intensities can amplify or shrink the shot efficiencies: a moment when the player's shots tend to be highly efficient results in a better performance if the shooting intensity is high at that moment. For example, a three-point shot that scores a basket has a high $E_{ij}$; if it occurs earlier than expected according to the shooting intensity of the match (high $\phi_{ij}$), then the global shooting performance $\psi_{ij}$ is enhanced.

The measures $\phi_{ij}$, $E_{ij}$ and $\psi_{ij}$ are event level, meaning that they are computed for each shot and thus represent the dynamic pattern of shooting performance in the event time. On this point, it is worth considering some remarks. In general, studies of intraindividual variability tend to examine performance over preaggregated periods, comparing weeks, semesters or years. As noted by Diener and Larsen (1984), aggregating data in this fashion leads to more stable and consistent estimates than those based on disaggregated data but may also mask episodic variation in performance. Investigating performance variability at the event level will avoid masking effects (Barnes and Morgeson, 2007), but event level measures are often characterized by a high level of noise that may obscure structural relationships. For these reasons, we opt for the middle ground solution of averaging the measures $\psi_{ij}$ over short moving periods by means of Nadaraya–Watson kernel regression (Nadaraya, 1964; Watson, 1964). We denote by $\hat{\psi}_{ij}$ the corresponding smoothed estimates.

### 3.1. *Application of step 1 to data*

We now compute the performance index for Golden State Warriors players, using play-by-play data from the 2017–2018 NBA regular season, as described in Section 1. To obtain the smoothed estimates $\hat{\psi}_{ij}$, we use a Gaussian kernel and set the bandwidth for each player equal to the 15th percentile of his total shots per match, producing intra-match averages. To check the robustness of the procedure, we repeated the analysis by varying the bandwidth from the fifth to the 25th percentile and obtained very similar final outcomes, as shown in the web-based supplementary materials. The measures $\psi_{ij}$ and $\hat{\psi}_{ij}$ for player Kevin Durant's (KD's) shots are represented respectively by coloured points and a black curve in Fig. 1.

As clearly shown in Fig. 1, a side-effect of smoothing is that performance variations are shrunk. For this reason, we introduce an amplifying factor, which is described in Section 4.

## 4. Step 2: regime switching models of two-regime performance

Having calculated the smoothed performance measure, we now aim to determine

(a) whether the fluctuating pattern of $\hat{\psi}_{ij}$ is simply random or whether it represents alternating good and bad performance regimes over time, and
(b) whether good or bad performance regimes are associated with the presence on court of given teammates.

There is a wide body of literature on models describing regime switching dynamics. One important approach assumes that regime switching is determined by observable variables, usually in the form of thresholds or smooth transitions (Tong, 1983). In another approach, regimes are assumed to be driven by latent (unobservable) stochastic variables. Here, we opt for the second

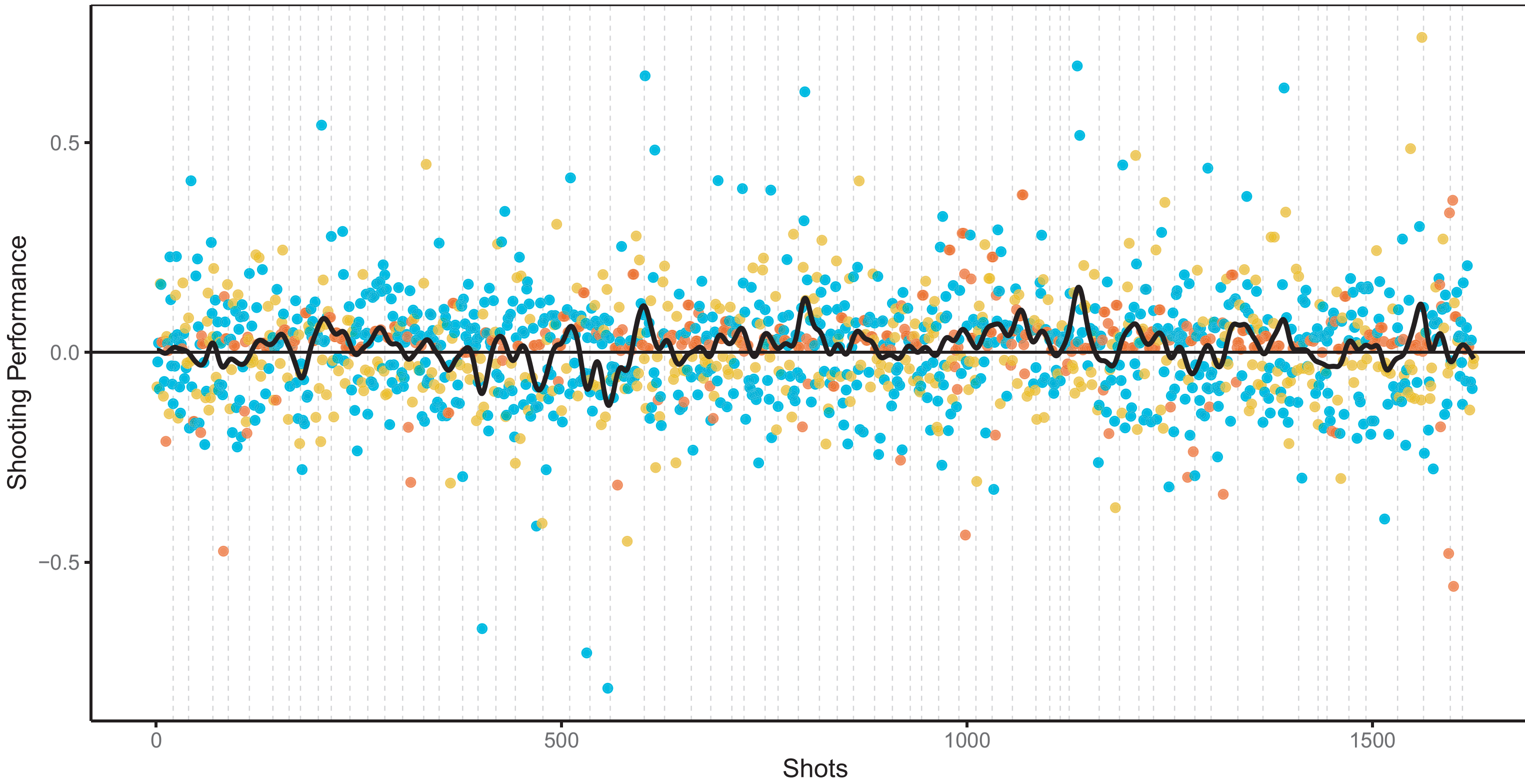


**Fig. 1.** Shooting performance measure $\psi_{ij}$ for KD during the 2017–2018 NBA regular season: ●, for two-point shots; ●, for three-point shots; ●, free throws; ——, the corresponding smoothed measure $\hat{\psi}_{ij}$

approach, with the assumption of a Markov structure for the latent variable describing regimes (Hamilton, 2008, 2010). The main advantages of this choice are that we do not need to identify *a priori* an observable variable that is responsible for regime switching and, more importantly, we can determine whether some specific variables affect the switching dynamics, by modelling the transition probabilities between regimes.

For these reasons, we utilize Markov switching models with time-varying transition probabilities (see Hamilton (2010)), mainly used in time series econometrics but fairly adaptable to this context, with the only difference that here we deal with event time instead of clock time. Another example of using Markov decision processes with dynamic transition probabilities in the context of basketball is presented in Sandholtz and Bornn (2020).

Assume that the performance of player $i$ switches between two different regimes, $B$ and $G$ ('bad' and 'good' respectively), where $B$ means that his or her performance is worsening, and $G$ means that it is improving. Improving or worsening performance is defined as performance that has increased or decreased from shot $j-1$ to shot $j$. High or low performance, in contrast, is defined according to the average performance level, so a period of high performance, for example, can comprise both improvement and worsening. We consider improving or worsening instead of high or low performance because it more closely reflects the immediate effect that a teammate entering the court might have on the player's performance.

To express performance variations (improvement or worsening), we consider the values

$$\hat{\psi}_{ij,\Delta}(k) = \mathrm{sgn}(\hat{\psi}_{ij} - \hat{\psi}_{i(j-1)})|\hat{\psi}_{ij} - \hat{\psi}_{i(j-1)}|^k, \tag{5}$$

where $\mathrm{sgn}(\cdot)$ denotes the sign function, and $k$ is an amplifying factor that is used to adjust the differences, to highlight the presence of different regimes better.

The amplifying factor $k$ is necessary because, as mentioned above, a side-effect of the smoothing that is carried out to compute the performance index $\hat{\psi}_{ij}$ is that the performance variations are shrunk. For each player, the choice of the optimal value $k_i^*$ for $k$ must be made according to the criterion of maximizing separation between the regimes, which will be explained in detail in Section 4.2.

### 4.1. Markov switching models with time-varying transition probabilities

In the simplest formulation of the model, let $R_{ij}$ be the (unobserved) random variable denoting the regime of player $i$'s performance when he attempted shot $j$. We denote by $\Psi_i^r$ the expected value of $\hat{\psi}_{ij,\Delta}(k_i^*)$ conditional on $R_{ij} = r$:

$$E_j\{\hat{\psi}_{ij,\Delta}(k_i^*)|R_{ij} = r\} = \Psi_i^r, \qquad r = B, G. \tag{6}$$

The probabilistic model describing the regime dynamics is assumed to be a two-state Markov chain (Baum *et al.*, 1970; Lindgren, 1978; Hamilton, 1989):

$$\Pr(R_{ij}|R_{i(j-1)}, R_{i(j-2)}, \ldots) = \Pr(R_{ij}|R_{i(j-1)}). \tag{7}$$

In the basic formulation of Markov switching models, probabilities (7) are constant over time. In detail, we denote by $\pi_{iBG} = \Pr(R_{ij} = G|R_{i(j-1)} = B)$ and $\pi_{iGG} = \Pr(R_{ij} = G|R_{i(j-1)} = G)$ the transition probabilities from regime $B$ and regime $G$ respectively, to regime $G$. Recalling that $\pi_{iBB} = \Pr(R_{ij} = B|R_{i(j-1)} = B) = 1 - \pi_{iBG}$ and $\pi_{iGB} = \Pr(R_{ij} = B|R_{i(j-1)} = G) = 1 - \pi_{iGG}$, the time constant transition matrix of the Markov chain for player $i$ is given by

$$\Pi_i = \begin{pmatrix} \pi_{iBB} & \pi_{iBG} \\ \pi_{iGB} & \pi_{iGG} \end{pmatrix} = \begin{pmatrix} 1 - \pi_{iBG} & \pi_{iBG} \\ 1 - \pi_{iGG} & \pi_{iGG} \end{pmatrix}. \tag{8}$$

We opt for a more complex model where the transition probabilities are time varying. We use the general term 'time varying' in this context to mean 'shot time varying'. Let $\pi_{ijBG}$ and $\pi_{ijGG}$ be the transition probabilities from regime $B$ and regime $G$ respectively, to regime $G$ for player $i$ at shot $j$. The transition matrix is then time varying, given by

$$\Pi_{ij} = \begin{pmatrix} 1-\pi_{ijBG} & \pi_{ijBG} \\ 1-\pi_{ijGG} & \pi_{ijGG} \end{pmatrix}. \tag{9}$$

To investigate the relationship between the performance variability of player $i$ and the presence on the court of $p$ teammates, we model time-varying transition probabilities by using logistic regression models:

$$\begin{aligned} \pi_{ijBG} &= \text{logit}(\beta_{0BG}^{(i)} + \beta_{1BG}^{(i)} X_{ij1} + \ldots + \beta_{pBG}^{(i)} X_{ijp}), \\ \pi_{ijGG} &= \text{logit}(\beta_{0GG}^{(i)} + \beta_{1GG}^{(i)} X_{ij1} + \ldots + \beta_{pGG}^{(i)} X_{ijp}) \end{aligned} \tag{10}$$

where $X_{ijh}$ is a dummy variable equal to 1 if player $h$ is on the court when player $i$ attempts shot $j$ and equal to 0 otherwise.

We assume the following Gaussian densities under the two regimes:

$$\hat{\psi}_{ij,\Delta}(k_i^*) | R_{ij} = r \sim \mathcal{N}\{\Psi_i^r, \sigma_i^2(r)\}, \qquad r = B, G. \tag{11}$$

where the parameter vector

$$\theta_i = (\Psi_i^B, \Psi_i^B, \sigma_i(B)^2, \sigma_i(B)^2, \beta_{0BG}^{(i)}, \beta_{1BG}^{(i)}, \ldots, \beta_{pBG}^{(i)}, \beta_{0GG}^{(i)}, \beta_{1GG}^{(i)}, \ldots, \beta_{pGG}^{(i)})'$$

is estimated via an expectation–maximization algorithm, as the regime is unobserved. The expectation–maximization algorithm also returns the so-called 'filtered' probabilities

$$\hat{\pi}_{ijr|j} = \Pr(R_{ij} = r | \Im_j, \theta_i) \tag{12}$$

where $\Im_j$ denotes the information that is available up to shot $j$.

### 4.2. Fine tuning of the amplifying factor k

After computing, for each player $i$, the smoothed performance index $\hat{\psi}_{ij}$, the first problem is choosing the optimal value $k_i^*$ for the amplifying factor $k$ in equation (5), allowing optimal regime separation, in terms of $\Psi_i^r$. For each player $i$, we fit the null model (6) with time constant transition probabilities to the values $\hat{\psi}_{ij,\Delta}(k)$ for various values of $k$. To assess the degree of separation between the two regimes for each value of $k$, we compute the differences

$$\Delta\Psi_i(k) = \text{sgn}(\Psi_i^G)\, |\Psi_i^G|^{1/k} - \text{sgn}(\Psi_i^B)\, |\Psi_i^B|^{1/k}. \tag{13}$$

The rationale behind formula (13) is that, since the values $\hat{\psi}_{ij,\Delta}(k)$ are obtained by raising the performance variations to the $k$th power, we need to compare the power means of order $k$ of the two regimes. The optimal value of $k$ for player $i$, $k_i^*$, is then

$$k_i^* = \arg\max_k \Delta\Psi_i(k). \tag{14}$$

### 4.3. Application of step 2 to data

In our case-study, parameter estimation via the expectation–maximization algorithm is carried out by using the R package `depmixS4`, version 1.4-0 (Visser and Speekenbrink, 2010; Zucchini *et al.*, 2017). To avoid instability in parameter estimation, we ran the estimation 500 times for each model and then chose the solution with the highest likelihood value. We analyse players who

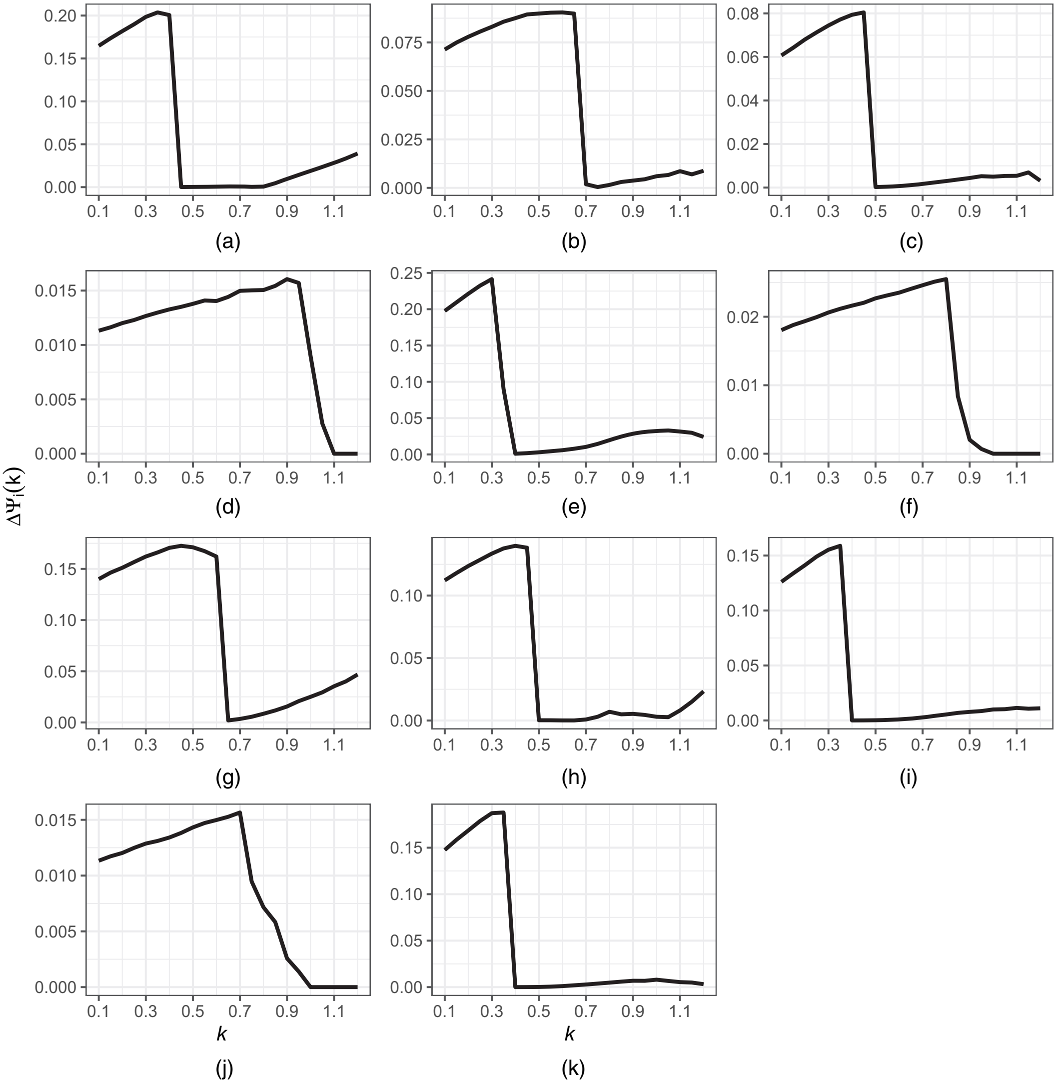


**Fig. 2.** Pattern of $\Delta\Psi_i(k)$ (on the vertical axis) *versus* $k$ (on the horizontal axis) for the 11 players (a) AI, (b) DW, (c) DG, (d) KD, (e) KL, (f) KT, (g) NY, (h) PMC, (i) SL, (j) SC and (k) ZP

played at least 900 min: Stephen Curry (SC) (1631 min; $i=1$), KD (2325 min; $i=2$), Draymond Green (DG) (2287 min; $i=3$), Andre Iguodala (AI) (1622 min; $i=4$), Shaun Livingston (SL) (1129 min; $i=5$), Kevon Looney (KL) (910 min; $i=6$), Patrick McCaw (PMC) (961 min; $i=7$), Zaza Pachulia (ZP) (972 min; $i=8$), Klay Thompson (KT) (2504 min; $i=9$), David West (DW) (999 min; $i=10$) and Nick Young (NY) (1393 min; $i=11$).

Firstly, we compute the optimal values $k_i^*$ for the 11 players. The relationship of $\Delta\Psi_i(k)$ *versus* $k$ for each player is represented in Fig. 2. The graphs show easily detectable peaks corresponding to the optimal values of $k$. The selection of optimal $k$ to some extent offsets the effects of different bandwidths. In other words, smoothing has the effect of shrinking the performance variations, and the choice of bandwidth affects the degree of shrinking, which is then neutralized by the optimal selection of $k$.

**Table 1.** Parameter estimates (with 95% confidence interval in parentheses) for non-null models†

| | *Results for regime B* | *Results for regime G* |
|---|---|---|
| *KD, i = 2* | | |
| Response parameter ($\Psi_2^r$) | *−0.012* (−0.013 to −0.011) | *0.014* (0.013 to 0.015) |
| Intercept ($\beta_{0BG}^{(2)}$ and $\beta_{0GG}^{(2)}$) | *−4.11* (−5.11 to −3.12) | *2.78* (2.54 to 3.01) |
| KT ($\beta_{9BG}^{(2)}$ and $\beta_{9GG}^{(2)}$) | *1.32* (0.28 to 2.36) | 0.013 (−0.090 to 0.116) |
| PMC ($\beta_{7BG}^{(2)}$ and $\beta_{7GG}^{(2)}$) | *1.05* (0.27 to 1.83) | −0.23 (−1.04 to 0.58) |
| *DG, i = 3* | | |
| Response parameter ($\Psi_3^r$) | *−0.24* (−0.26 to −0.21) | *0.24* (0.22 to 0.25) |
| Intercept ($\beta_{0BG}^{(3)}$ and $\beta_{0GG}^{(3)}$) | *−2.25* (−2.74 to −1.76) | *1.80* (1.40 to 2.19) |
| AI ($\beta_{4BG}^{(3)}$ and $\beta_{4GG}^{(3)}$) | *0.83* (0.15 to 1.51) | 0.27 (−0.43 to 0.98) |
| *KL, i = 6* | | |
| Response parameter ($\Psi_6^r$) | *−0.54* (−0.59 to −0.49) | *0.52* (0.48 to 0.56) |
| Intercept ($\beta_{0BG}^{(6)}$ and $\beta_{0GG}^{(6)}$) | *−0.72* (−1.30 to −0.15) | *0.65* (0.14 to 1.16) |
| DG ($\beta_{3BG}^{(6)}$ and $\beta_{3GG}^{(6)}$) | −0.45 (−1.36 to 0.45) | *1.17* (0.08 to 2.27) |
| DW ($\beta_{10BG}^{(6)}$ and $\beta_{10GG}^{(6)}$) | *2.45* (0.55 to 4.35) | −0.30 (−1.34 to 0.74) |
| *PMC, i = 7* | | |
| Response parameter ($\Psi_7^r$) | *−0.34* (−0.38 to −0.31) | *0.35* (0.31 to 0.38) |
| Intercept ($\beta_{0BG}^{(7)}$ and $\beta_{0GG}^{(7)}$) | −0.54 (−1.07 to 0.01) | *1.27* (0.67 to 1.88) |
| AI ($\beta_{4BG}^{(7)}$ and $\beta_{4GG}^{(7)}$) | 0.05 (−0.82 to 0.91) | *−1.03* (−1.96 to −0.09) |
| SL ($\beta_{5BG}^{(7)}$ and $\beta_{5GG}^{(7)}$) | −0.80 (−1.93 to 0.32) | *−1.19* (−2.18 to −0.20) |
| *NY, i = 11* | | |
| Response parameter ($\Psi_{11}^r$) | *−0.33* (−0.35 to −0.31) | *0.33* (0.31 to 0.36) |
| Intercept ($\beta_{0BG}^{(11)}$ and $\beta_{0GG}^{(11)}$) | *−0.82* (−1.12 to −0.52) | *0.88* (0.57 to 1.19) |
| KT ($\beta_{9BG}^{(11)}$ and $\beta_{9GG}^{(11)}$) | *−0.77* (−1.49 to −0.04) | −0.20 (−0.88 to 0.49) |

†Values in italics have confidence intervals that do not include 0.

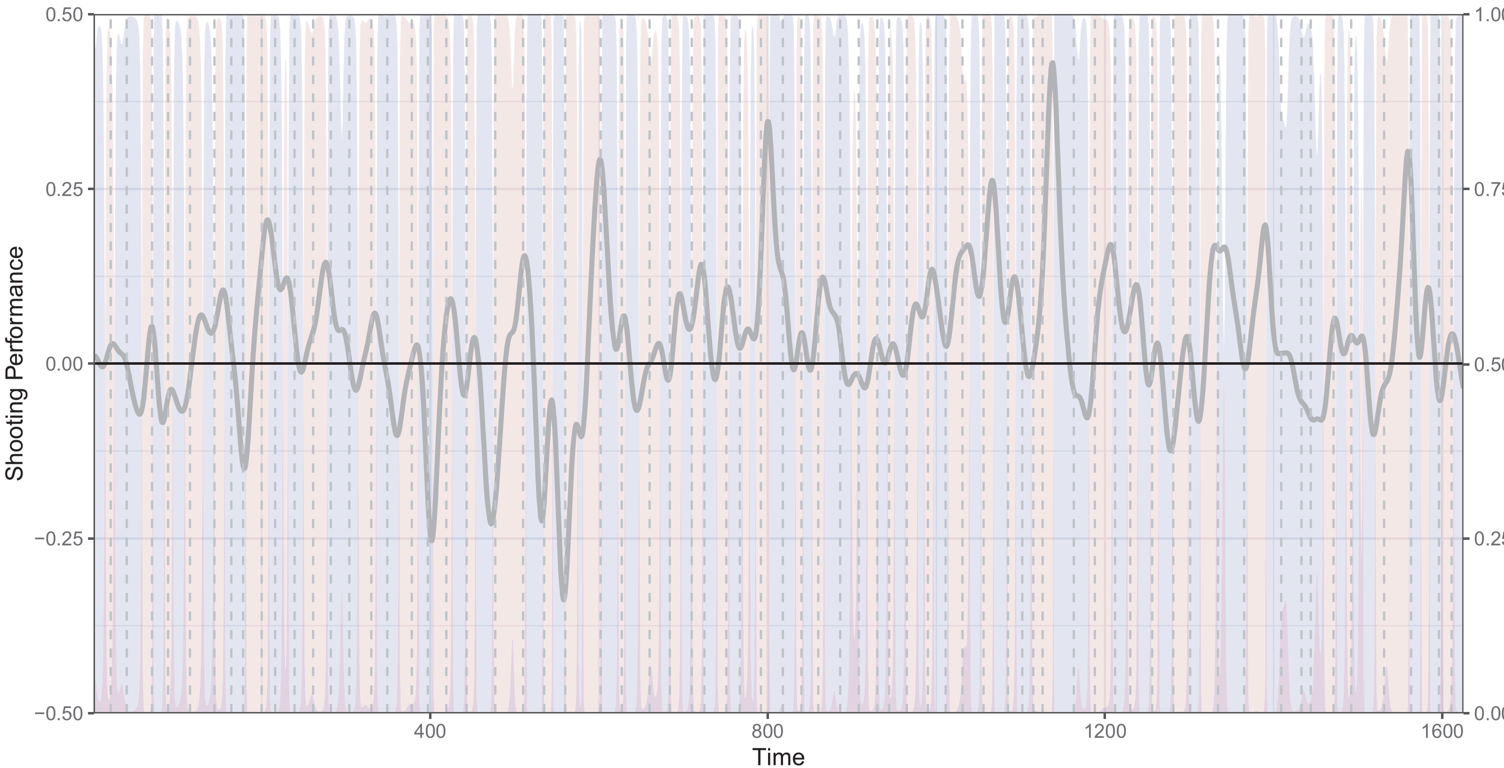


**Fig. 3.** Regime switching: measures $\hat{\psi}_{ij}$ (——) with filtered probabilities $\pi_{ijG|j}$ ( ) and $\pi_{ijB|j}$ ( ) for KD (player $i=2$) during the 2017–2018 NBA regular season

Once the optimal value for the amplifying factor $k$ has been selected, for each player $i$ we fit model (6) to the values $\hat{\psi}_{ij,\Delta}(k_i^*)$, with time-varying transition probabilities given by expressions (9) and (10), where the $p=10$ covariates indicate the presence or absence of the other 10 players on the court.

Considering the high number of covariates in expression (10), we have implemented a variable-selection procedure consisting of a forward stepwise regression based on the Akaike information criterion AIC. More specifically, starting from the null model with time constant transition probabilities, we have progressively added at each step the variable whose inclusion gives the highest improvement of the fit, according to the criterion of minimum AIC. The procedure is repeated until none of the remaining variables decrease AIC when added.

According to this criterion, non-null models were estimated for five players ($i=2$, 3, 6, 7, 11: KD, DG, KL, PMC, NY). According to the models, these five players' shooting performance is affected by the presence of one or more teammates on the court, whereas no such relationship was found for the other six players. Results are displayed in Table 1. For example, KD ($i=2$) exhibits a significant regime switching dynamic ($\Psi_2^B=-0.012$ and $\Psi_2^G=0.014$), and the final models of his transition probabilities after AIC variable selection are given by

$$\begin{aligned}\pi_{2jBG} &= \text{logit}(\beta_{0BG}^{(2)} + \beta_{9BG}^{(2)} X_{2j9} + \beta_{7BG}^{(2)} X_{2j7}),\\ \pi_{2jGG} &= \text{logit}(\beta_{0GG}^{(2)} + \beta_{9GG}^{(2)} X_{2j9} + \beta_{7GG}^{(2)} X_{2j7})\end{aligned} \tag{15}$$

meaning that the teammates affecting his transition probabilities are KT ($i=9$, the first teammate added to the model by the forward stepwise procedure) and PMC ($i=7$). After setting to 0 the parameters whose confidence interval includes 0, we obtain for KD

$$\pi_{2jBG} = \text{logit}(-4.11 + 1.32 X_{2j9} + 1.05 X_{2j7}), \tag{16}$$

which shows that KT and PMC have a positive influence on KD's probability of moving from regime $B$ to regime $G$. The probability of remaining in regime $G$ is constant (logit(2.78)). Overall, we conclude that KT and PMC have a positive influence on KD's shooting performance. Fig. 3 shows the estimated regime switching for KD.

## 5. Step 3: effects on team performance

In the third step, we consider the interactions between players that are described by the parameters $\beta_{1BG}^{(i)}, \ldots, \beta_{pBG}^{(i)}, \beta_{1GG}^{(i)}, \ldots, \beta_{pGG}^{(i)}$. The influence of player $h$ on the shooting performance of player $i$ is described by the parameters $\beta_{hBG}^{(i)}$ and $\beta_{hGG}^{(i)}$. We set to 0 those parameters whose covariate $X_{ijh}$ has not been included in the corresponding model by the variable-selection procedure and those parameters whose confidence interval overlaps 0. Finally, for each player $i$, we define the influence of teammate $h$ on his shooting performance as $\mathcal{I}_{ih} = \pm\max(|\beta_{hBG}^{(i)}|, |\beta_{hGG}^{(i)}|)$ (positive or negative according to the sign of the parameter determining its size), as summarized in Table 2.

We then generate a network plot as shown in Fig. 4, where the network nodes represent the players. The edge directed from player $h$ to player $i$ represents the influence of teammate $h$ on the shooting performance of player $i$. The edge thickness is proportional to $\mathcal{I}_{ih}$, and edges are coloured according to whether $\mathcal{I}_{ih}$ is negative (blue) or positive (red).

The main aim of step 3 is to validate the analysis by checking whether a relationship exists between the teammate interactions that we have identified and overall team performance, which is the ultimate goal of our analysis. In other words, we answer the following question: do line-ups composed of players with pairwise positive or negative influences perform respectively better

**Table 2.** Influence of player $h$ on the shooting performance of player $i$, $\mathcal{I}_{ih}$

| *Player i* | $\mathcal{I}_{i1}$ | $\mathcal{I}_{i2}$ | $\mathcal{I}_{i3}$ | $\mathcal{I}_{i4}$ | $\mathcal{I}_{i5}$ | $\mathcal{I}_{i6}$ | $\mathcal{I}_{i7}$ | $\mathcal{I}_{i8}$ | $\mathcal{I}_{i9}$ | $\mathcal{I}_{i10}$ | $\mathcal{I}_{i11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SC, $i=1$ | — | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KD, $i=2$ | 0 | — | 0 | 0 | 0 | 0 | 1.05 | 0 | 1.32 | 0 | 0 |
| DG, $i=3$ | 0 | 0 | — | 0.83 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| AI, $i=4$ | 0 | 0 | 0 | — | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SL, $i=5$ | 0 | 0 | 0 | 0 | — | 0 | 0 | 0 | 0 | 0 | 0 |
| KL, $i=6$ | 0 | 0 | 1.17 | 0 | 0 | — | 0 | 0 | 0 | 2.45 | 0 |
| PMC, $i=7$ | 0 | 0 | 0 | −1.03 | −1.19 | 0 | — | 0 | 0 | 0 | 0 |
| ZP, $i=8$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | — | 0 | 0 | 0 |
| KT, $i=9$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | — | 0 | 0 |
| DW, $i=10$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | — | 0 |
| NY, $i=11$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −0.77 | 0 | — |

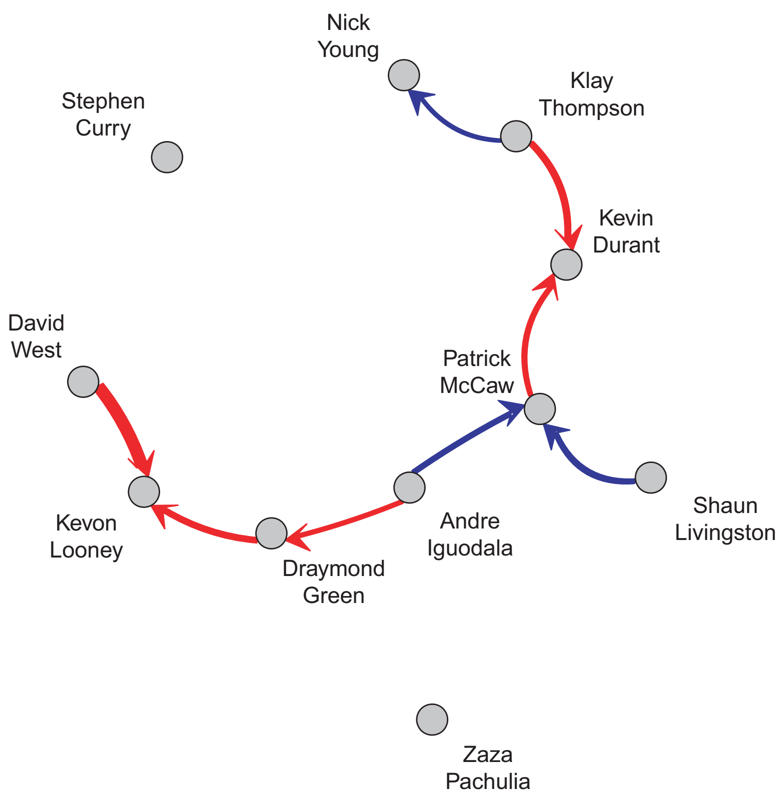


**Fig. 4.** Network showing the influence of teammates on players' shooting performance (edge thickness is proportional to $\mathcal{I}_{ih}$): ⟶ (blue), negative $\mathcal{I}_{ih}$; ⟶ (red), positive $\mathcal{I}_{ih}$

or worse? The answer is not straightforward, because we are shifting the focus from pairs of players to the whole team. We carry out the analysis both in sample (on the regular season data) and out of sample (on the play-off data).

To do that, we extract all line-ups composed of five players, from the set of the 11 who were analysed, who have played at least 15 min during the regular season and 6 min during the play-offs. To each line-up $q$, we assign a score $s_q$ given by the sum of the influences $\mathcal{I}_{ih}$ for the players belonging to the line-up:

$$s_q = \sum_{i,h \in q} \mathcal{I}_{ih}. \tag{17}$$

For example, for the line-up composed of SC, KD, DG, AI and KT, $s = 1.32 + 0.83 = 2.15$. The definition of this score is essentially heuristic: the idea is to consider all relationships within the line-up according to the model and then to investigate the relationship between this score and line-up performance.

The line-up performance can be measured by the intensity of scored points, ISP. Let $W$ be the random variable denoting the points that are scored in a given game second and $w_t$ ($w_t = 0, 1, 2, 3, 4$) the value that is assumed by $W$ at time (in seconds) $t$ ($t = 1, 2, \ldots, 2880$). Note that the points scored with free throws are assigned to the time of the corresponding foul.

Let $\tau_q$ be the set of seconds played by line-up $q$ during the regular season and $\#(\tau_q)$ the cardinality of this set.

We define ISP for line-up $q$ as

$$\text{ISP}_q = \frac{1}{\#(\tau_q)} \sum_{t \in \tau_q} w_t. \tag{18}$$

$\text{ISP}_q$ can be transformed to be interpreted with reference to a standard period of 48 min, into the corresponding index $\text{PTS}_q^{(48)} = \text{ISP}_q \times 2880$ (points made per 48 min).

In modelling the relationship between $\text{PTS}_q^{(48)}$ and $s_q$, we must take into account that there is a remarkable heterogeneity among line-ups in the variable $\text{PTS}_q^{(48)}$, due to the different shooting

**Table 3.** MOB parameters (in-sample data set: regular season)

| *Parameter* | *Results for line-ups with* $PTS_q^{(48)} \leqslant 97.21$ | *Results for line-ups with* $PTS_q^{(48)} > 97.21$ |
|---|---|---|
| $\alpha_0$ | 87.541 | 117.969 |
| $\alpha_1$ | 4.152 | 9.641 |
| | $p$-value 0.0231 | $p$-value 0.0238 |
| $R^2$ | 0.5895 | 0.4266 |

**Table 4.** MOB parameters (out-of-sample data set: play-offs)

| *Parameter* | *Results for line-ups with* $PTS_q^{(48)} \leqslant 107.83$ | *Results for line-ups with* $PTS_q^{(48)} > 107.83$ |
|---|---|---|
| $\alpha_0$ | 91.119 | 115.220 |
| $\alpha_1$ | 4.065 | 10.220 |
| | $p$-value 0.0938 | $p$-value 0.0238 |
| $R^2$ | 0.4551 | 0.4492 |

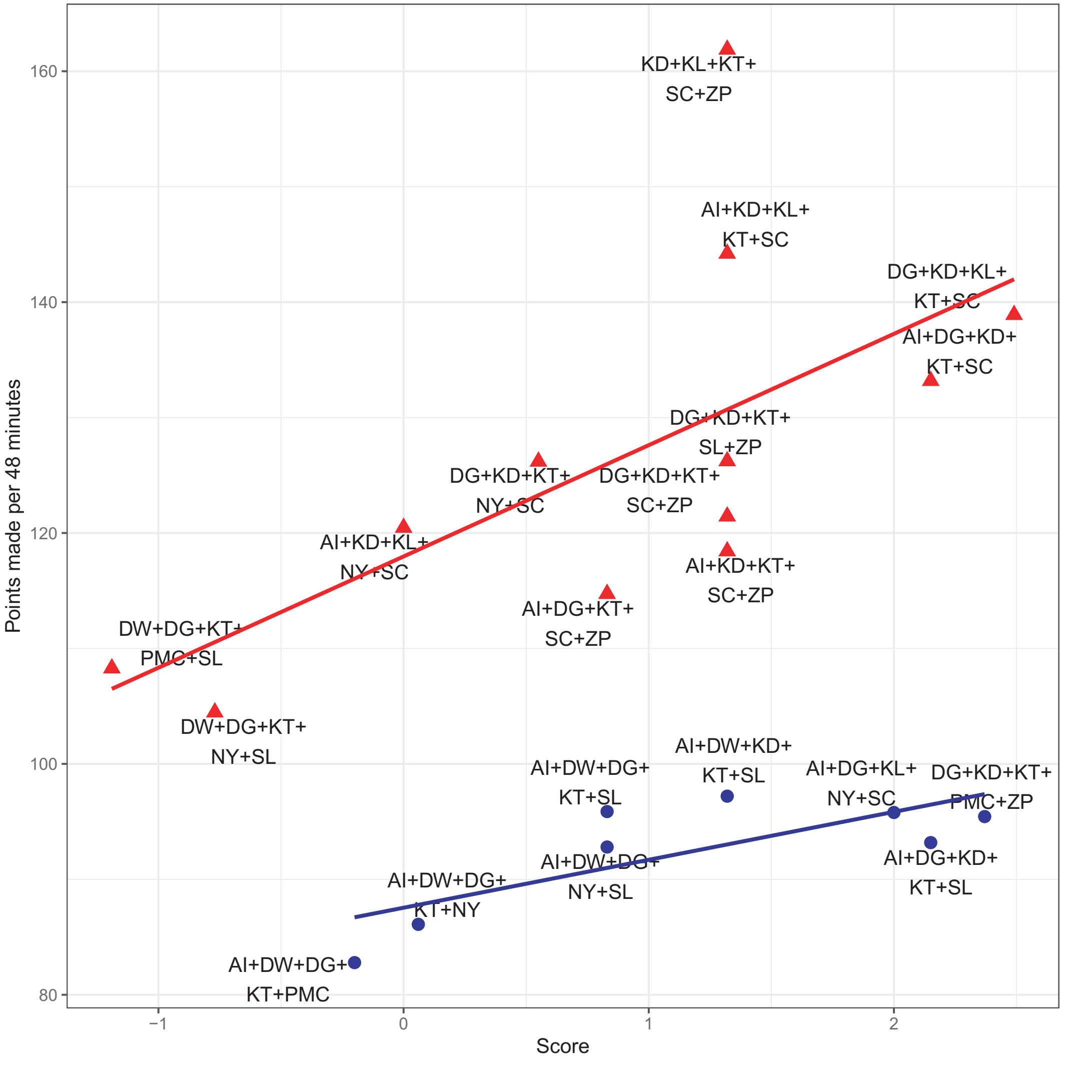


**Fig. 5.** Estimation of the relationship between line-up performance $\mathrm{PTS}_q^{(48)}$ and score $s_q$ with MOB linear models (in-sample data set: regular season): ●, ▲, line-ups (identified by initials of players' names) with performance below and above 97.21$\mathrm{PTS}_q^{(48)}$ respectively

abilities of the players in each line-up. We take into account this heterogeneity by using the model-based recursive partition algorithm MOB (Zeileis *et al.*, 2008) that yields a partitioned (or segmented) parametric model as follows.

(a) A parametric model is fitted to data (in our case-study, the linear model $\mathrm{PTS}_q^{(48)} = \alpha_0 + \alpha_1 s_q + \epsilon$).
(b) Parameter instability is tested over a set of partitioning variables (in our case-study, we test parameter instability only over variable $\mathrm{PTS}_q^{(48)}$).
(c) If parameter instability is confirmed, the data set is split according to the variable and to the cut point that is associated with the highest instability; two separate models are fitted to the subsets of data.

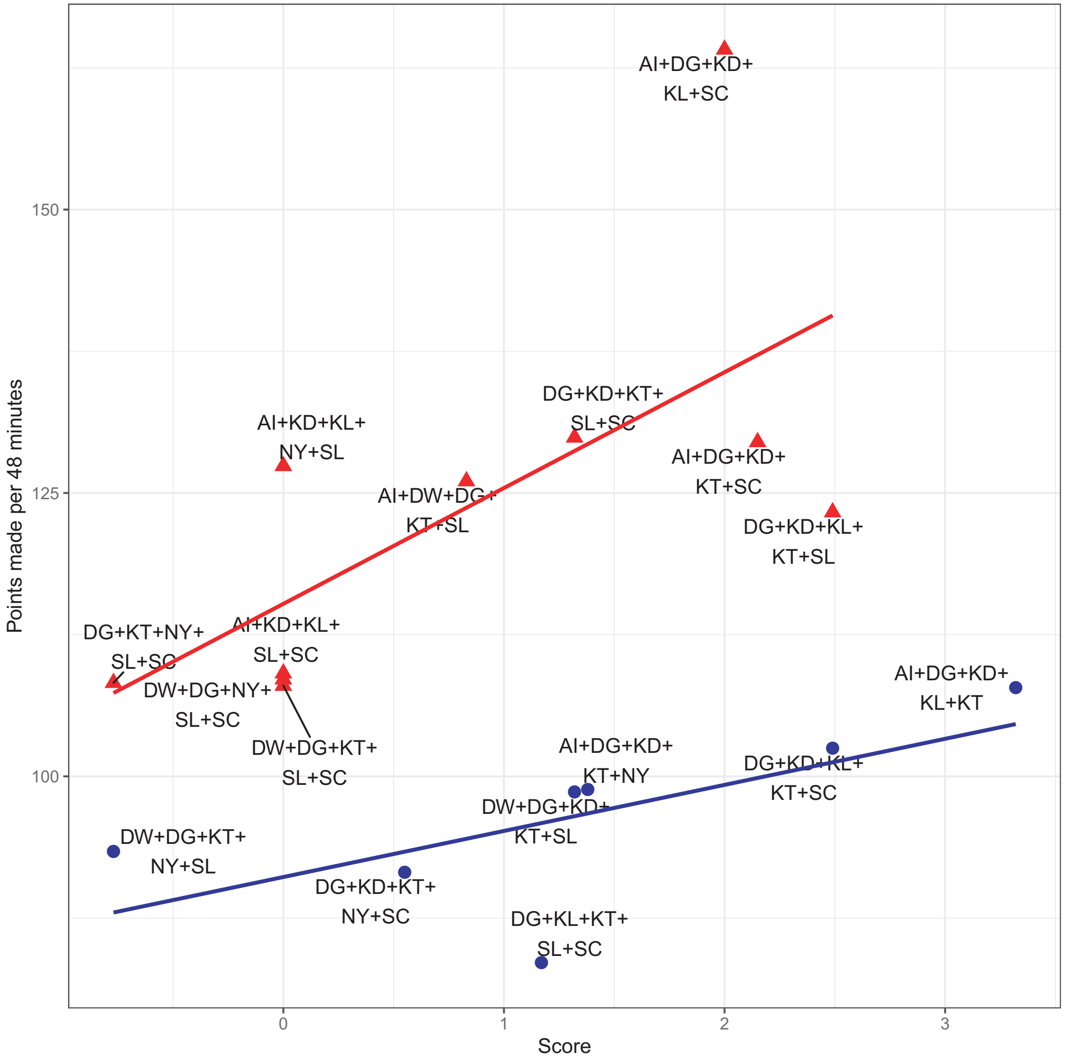


**Fig. 6.** Estimation of the relationship between line-up performance $\mathrm{PTS}_q^{(48)}$ and score $s_q$ with MOB linear models (out-of-sample data set: play-offs): ●, ▲, line-ups (identified by initials of players' names) with performance below and above 107.83 $\mathrm{PTS}_q^{(48)}$ respectively

(d) The procedure is repeated in the daughter nodes.

For both the in-sample and the out-of-sample data sets, we obtained a MOB model with one split, with cut points $\mathrm{PTS}_q^{(48)} = 97.21$ and $\mathrm{PTS}_q^{(48)} = 107.83$ respectively. This means that different linear models have been detected for line-ups with performance below and above these thresholds. The estimated parameters are displayed in Tables 3 and 4, whereas Figs 5 and 6 show scatter plots of the line-ups, with the interpolating lines corresponding to the estimated models.

The models confirm the existence of a significant relationship between the line-up performance $\mathrm{PTS}_q^{(48)}$ and the score $s_q$ computed according to Markov switching models, meaning that the estimated influences $\mathcal{I}_{ih}$ between pairs of players directly affect the team performance. This effect is detected in both in-sample and out-of-sample data, with very similar parameters (despite the huge differences between regular season and play-offs), demonstrating that this validation is robust and reliable.

## 6. Concluding remarks

Here, we have proposed a three-step procedure for modelling the dynamic pattern of basketball players' shooting performance, with a special focus on performance variability. Alongside a methodological definition of the procedure, we present a real data case-study developed by using play-by-play data for the Golden State Warriors during the 2017–2018 NBA regular season. Starting from the idea that player performance is naturally subject to improvement and worsening, we defined a shooting performance index and modelled its alternating dynamics by using Markov switching models with time-varying transition probabilities. The transition probabilities are assumed to be dependent on the presence on the court of specific teammates.

Several relevant issues are addressed in the development of this procedure.

(a) In the definition of the shooting performance index, we found it necessary to aggregate our data (expressed at the event level) over longer periods to obtain more stable and consistent estimates. At the same time, we had to avoid masking of episodic variations in performance. For these reasons, we chose to average measures over short moving periods by using Nadaraya–Watson kernel regression. To obtain intramatch averages for each player, we set the bandwidth equal to the 15th percentile of his number of shots per match. We also checked the robustness of estimated models under variations of this parameter: we repeated the analysis, varying the bandwidth from the fifth to the 25th percentile, and we obtained very similar results (see the web-based supplementary materials) compared with the final network displayed in Fig. 4.

(b) The smoothing of the performance index that was obtained by kernel regression also shrinks performance variations, so we introduced an amplifying factor $k$ and raised differences to the power of $k$. To optimize this parameter, we developed a procedure based on the distance between the power means of order $k$ of the two regimes, providing an optimal value $k_i^*$ for each player. The graphs in Fig. 2 representing the difference between the power means *versus* $k$ show easily detectable peaks, corresponding to the optimal values. The selection of the optimal $k$ can adjust shrunken performance variations obtained by using different bandwidths.

(c) The Markov switching model with time-varying transition probabilities has a complex structure, so parameter estimation via the expectation–maximization algorithm can be significantly improved by the proper choice of variables in the logistic model for the transition probabilities. For this reason, we developed a variable-selection procedure based on a forward stepwise regression. Starting from the null model with time constant transition probabilities, we progressively added one variable at each step, according to the criterion of minimum AIC and then we set to 0 the parameters whose confidence interval includes 0. This enabled us to obtain more parsimonious and stable models. This procedure is inspired by the idea that was presented in Holsclaw *et al.* (2017), who selected variables by means of a combination of two criteria, the Bayesian information criterion and predictive log-probability scores. In this context, the performance of alternative approaches for variable selection could be investigated. Some alternative approaches to variable selection for non-homogeneous hidden Markov models (for regressors on transition probabilities) are already present in the literature from a Bayesian perspective (see for example George and McCulloch (1993), Dellaportas *et al.* (2000), Meligkotsidou and Dellaportas (2011) and Spezia (2020)). Penalized methods could also be explored.

The estimated parameters have then been used to build a network graph visualizing the interactions between pairs of players, enabling us to identify player combinations having positive or negative synergies.

Finally, we checked whether these relationships between players effectively translate into better team performance, based on total points scored. To do that, we heuristically defined a score for line-ups, based on the relationships between teammates detected by Markov switching models, and we checked its relationship to line-up performance, measured by points scored. A significant direct relationship between score and line-up performance has been confirmed by the models estimated, both with in-sample and with out-of-sample data.

These results can be enormously useful to the coach, when deciding which players should (or should not) play together, using an easy-to-interpret network graph.

The main drawback of the procedure proposed is that the Markov switching model with time-varying transition probabilities cannot be fitted to short time series with few episodes of regime switching. So, it cannot be used early in the tournament, when data from only a few matches are available.

Another issue worth noting is that the procedure considers shooting performance and not other playing abilities, such as defensive skills. Future research could apply our Markov switching approach to a more comprehensive performance index, taking into account a wider set of game variables.

## Acknowledgements

The authors thank the Joint Editor, the Associate Editor and the reviewers for their valuable comments, which greatly improved the paper. Thanks go to Maurizio Rizzini and Samantha Sartori for helpful discussions and special thanks to Amy Volpert for her careful editing and proofreading.

This research was carried out in collaboration with the Big&Open Data Innovation Laboratory at the University of Brescia (project 'BDsports: Big Data analytics in sports'; `bdsports.unibs.it`).

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*Supporting information*

Additional 'supporting information' may be found in the on-line version of this article:

'Web-based supplementary materials for "Markov switching modelling of shooting performance variability and teammate interactions in basketball"'.