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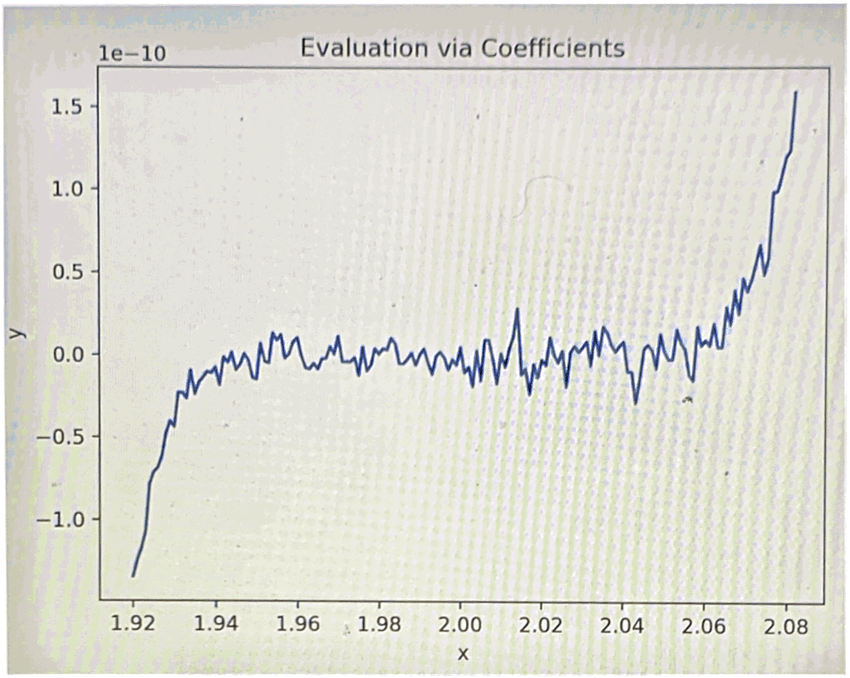
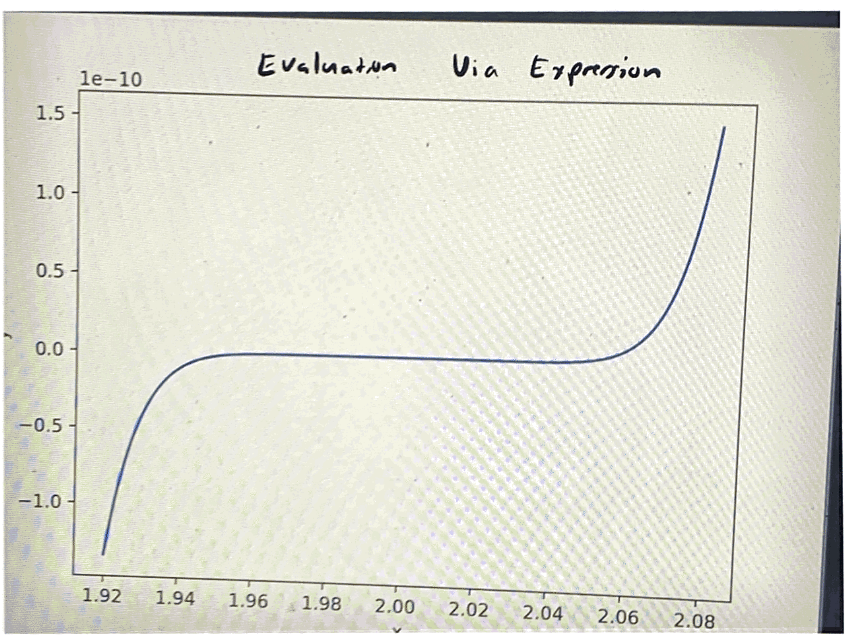
$P(x) = (x-2)^9 = x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512$

1i)



1ii)



1iii) The difference is the expression plot is smooth and the expanded plot is jagged because it is doing much more calculating with several high powers so it is less stable. The expression plot is correct.

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2i) $\sqrt{x+1} - 1 \quad x \simeq 0$

$$\sqrt{x+1} - 1 \left(\frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} \right)$$

$$\frac{x + 1 + \sqrt{x+1} - \sqrt{x+1} - 1}{\sqrt{x+1} + 1}$$

$$\frac{x}{\sqrt{x+1} + 1}$$ if x is viewed a 0 on a computer

$$\frac{\text{nearly } 0}{\sqrt{0+1} + 1}$$ ← doesn't cancel

2ii) $\sin(x) - \sin(y) \quad x \simeq y$

$$\sin(x) - \sin(y) \left(\frac{\sin(x) + \sin(y)}{\sin(x) + \sin(y)} \right)$$

$$\frac{\sin(x)^2 - \sin(x)\sin(y) + \sin(y)\sin(y) - \sin(y)^2}{\sin(x) + \sin(y)}$$

$$\frac{\sin(x)^2 - \sin(y)^2}{\sin(x) + \sin(y)}$$ difference of squares

$$\frac{\sin(x+y)\sin(x-y)}{\sin(x) + \sin(y)}$$ $\frac{\text{nearly } 0}{2\sin(x)}$ does not cancel

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2iii) $\frac{1-\cos(x)}{\sin(y)}$ $\quad x \simeq 0$

$$\frac{1-\cos(x)}{\sin(y)}\left(\frac{1+\cos(x)}{1+\cos(x)}\right)$$

$$\frac{1-\cos^2 x}{\sin(y)(1+\cos x)}$$

$\sin^2 y+\cos^2 x = 1$

$\sin^2 x = 1-\cos^2 x$

$$\frac{\sin^2 x}{\sin(y)(1+\cos x)}$$

$$\frac{\sin(x)}{1+\cos x} \qquad \frac{\text{nearly } 0}{1+1}$$

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3) $T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$

$c=0$

$P_2(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2}(x)^2$

$f'(x) = -(1+x+x^3)\sin x + \cos x\,(1+3x^2)$

$f''(x) = -x^3\cos(x) - 6x^2\sin(x) + 5x\cos(x) - 2\sin(x) - \cos(x)$

$P_2(x) = 1 + x - \frac{1}{2}x^2$

a) $P_2(.5) = 1 + .5 - .125$

$P_2(.5) = 1.375$

$|f(.5) - P_2(.5)| \leq .06$

$f''(.5) = -.47$

$\rightarrow R_2(.5) = \frac{f''(.5)}{2!}(.5)^2$

upper bound

Actual error 1.4 > .06 upper bound error

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b) $|f(x) - P_2(x)| \leq \frac{f''(x)}{2!}(x)^2$

c) $\int_0^1 P_2(x)\,dx$

$\int_0^1 (1 + x - \frac{1}{2}x^2)\,dx$

$x + \frac{x^2}{2} - \frac{1}{6}x^3 \Big|_0^1 \quad 1.\overline{33} - 0$

$\int_0^1 f(x)\,dx \approx \int_0^1 P_2(x)\,dx = 1.\overline{33}$ achel error .06

d) error $\approx R_2(1) = \frac{f''(1)}{2!}(1)^2$

error $= 2.55$

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4) $ax^2+bx+c=0$ $a=1$ $b=-56$ $c=1$

$x^2-56x+1$

$$r_{1,2} = \frac{-56 \pm \sqrt{3132}}{2} \quad \approx 55.964$$

$r_{1,2} = -28 \pm 27.982$

$r_1 = 55.982$

$r_2 = .018$

relative Error

$r_1 = -55.982$ $\left| \frac{r_1 - r_1^*}{r_1} \right| = 2.45 \times 10^{-6}$

$r_2 = -.01786$ $\left| \frac{r_2 - r_2^*}{r_1} \right| = 7.67 \times 10^{-3}$ ← bad root

(+)

b)

$$r_2^* = \frac{-b+\sqrt{b^2-4ac}}{2a}\left(\frac{-b-\sqrt{b^2-4ac}}{-b-\sqrt{b^2-4ac}}\right)$$

$$r_2^* = \frac{2c}{-b-\sqrt{b^2-4}}$$

$$r_2^* = \frac{2(1)}{-56-\sqrt{3132}}$$

$$r_2^* = -.01786$$

$$\left|\frac{r_a - r_2^*}{r_2}\right| = -2.45\times10^{-4} < 7.67\times10^{-3}$$

much better

5a)

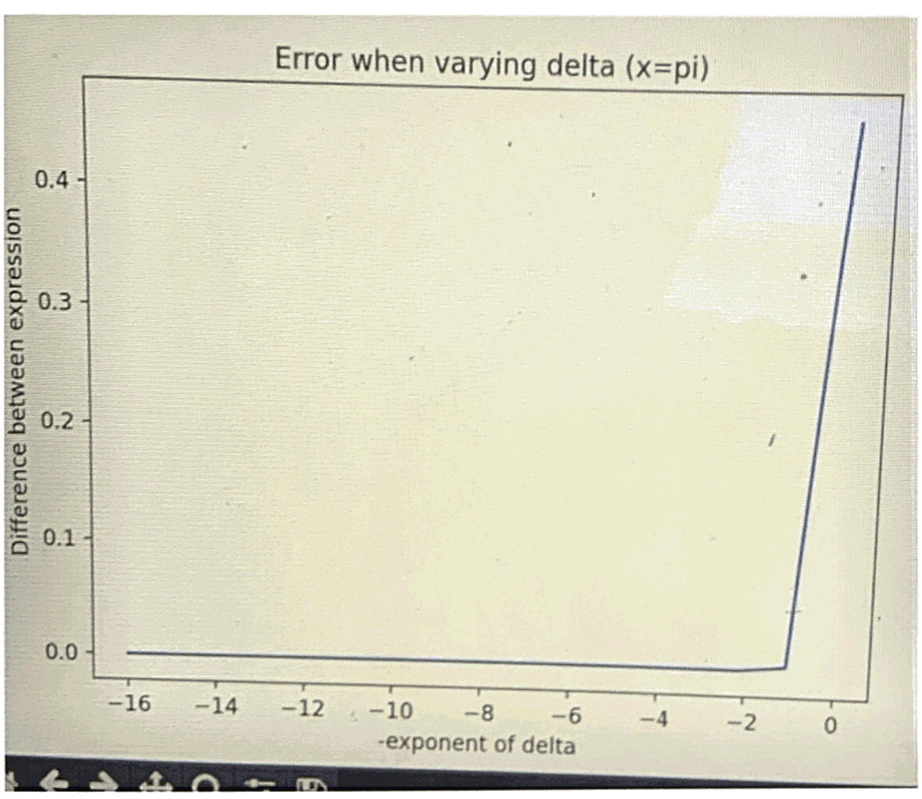
$\varepsilon_m = 10^{-2}$ $\quad x_1 = \frac{1}{3} \quad x_2 = \frac{2}{3}$

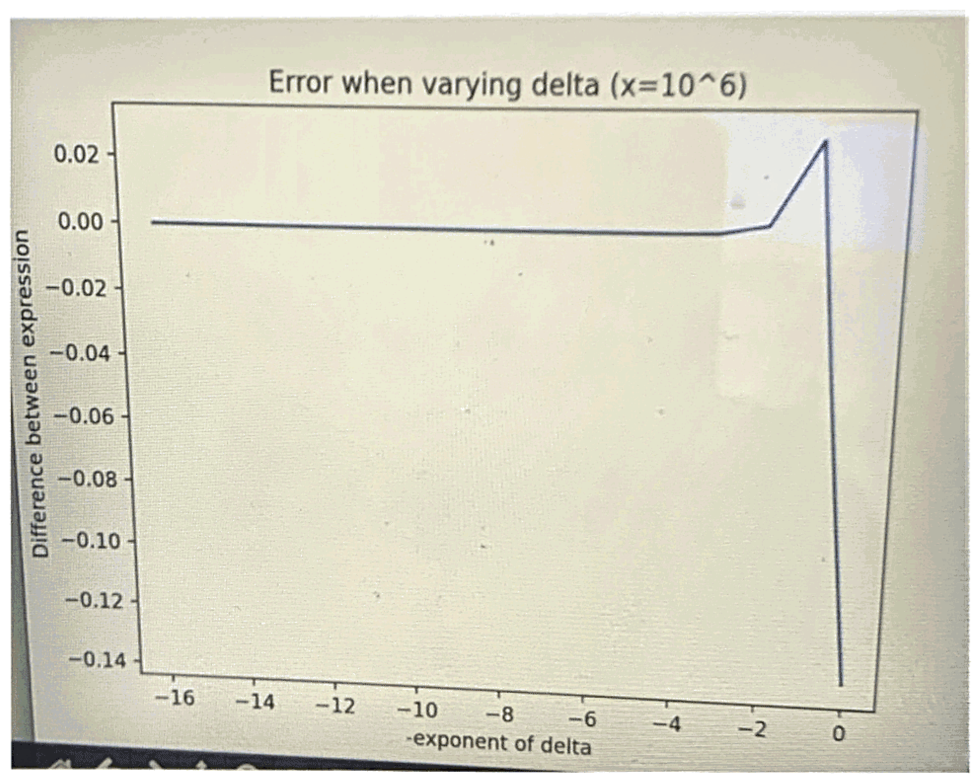
absolute error: $|(\tilde{x}_1 - x_1) - (\tilde{x}_2 - x_2)| = (\tilde{x}_1 - x_1)\,\varepsilon_m$

relative error: $\dfrac{|x - \tilde{x}|}{|x|} \leq \varepsilon_m$

5b) $\cos(x+\delta) - \cos(x)$

$\cos(x)\cos(\delta) - \sin(x)\sin(\delta) - \cos(x)$ $\quad x = \pi \quad x = 10^6$





5c) $T_2 = \delta \sin(x) + \dfrac{\delta^2}{2!}(-\cos x)$

This algorithm was chosen because it avoids subtracting two close #'s. the approximation is very similar to the other method of manipulating the expression.