

sCO2_Loop_Calcs

October 3, 2021

IMPORTANT Ensure you are utilizing 64-bit REFPROP with 64-bit python. If using the free version of REFPROP (MINI-REFPROP), please use 32-bit python and make changes to match the location where MINI-REFPROP is installed and make changes to the REFPROPFunctionLibrary function to read the REFPROP.DLL file.

Information on REFPROP and functions can be found here:
<https://buildmedia.readthedocs.org/media/pdf/refprop-docs/latest/refprop-docs.pdf>

0.0.1 IMPORT PACKAGES & FUNCTIONS

```
[1]: # Dictate the environment's location of REFPROP
import os
os.environ['RPPREFIX'] = r'C:/Program Files (x86)/REFPROP'

[2]: # Import the main class from the Python library
from ctREFPROP.ctREFPROP import REFPROPFunctionLibrary

# Imports from conda-installable packages
import pandas as pd

# Import numpy
import numpy as np

# Import matplotlib for plotting
import matplotlib.pyplot as plt

# Import Math for common values such as PI
import math

[3]: # Instantiate the library, and use the environment variable to explicitly state
    ↪ which path we want to use.
# As mentioned above, this will be changed to call the correct REFPROP
    ↪ functions to be used
# with MINI-REFPROP and 32-bit python.
# If using MINI-REFPROP and 32-bit python please make the following changes
# RP = REFPROPFunctionLibrary('C:/Program Files (x86)/MINI-REFPROP\REFPROP.
    ↪ DLL')
RP = REFPROPFunctionLibrary(os.environ['RPPREFIX'])
```

```
[4]: # This will call which root directory that will be used for the program.
RP.SETPATHdll(os.environ['RPPREFIX'])
```

```
[5]: # Get the unit system we want to use (Mass base SI gives units in
# K, Pa, kg, m, N, J, W, and s)
MASS_BASE_SI = RP.GETENUMdll(0, "MASS BASE SI").iEnum
```

0.0.2 sCO₂ Loop Calculations

System Parameters

```
[6]: # Tube inner diameter and outer diameter
Tube_OD = 0.50 # [inch]
Tube_Thick = 0.065 # [inch]
Tube_ID = Tube_OD - 2 * Tube_Thick # inch

Tube_OD = Tube_OD * 0.0254 # [Convert inches to meters]
Tube_ID = Tube_ID * 0.0254 # [Convert inches to meters]

# Mass flow rate of sCO2
m_dot = 0.2 # [kg/s]
```

Outlet of Compressor (State 1)

```
[7]: # Temperature will be compared at end of script
T1 = 60 # [C]
P1 = 2600 # [psia]

T1 = T1 + 273.15 # Convert C to Kelvin
P1 = P1 * 6894.8 # convert psia to Pa

print("Pressure at Outlet of Compressor =", P1/6894.8, "psia")
print("Temperature at Outlet of Compressor =" , (T1 - 273.15) * (9/5) + 32, "F")
```

Pressure at Outlet of Compressor = 2600.0 psia

Temperature at Outlet of Compressor = 140.0 F

```
[8]: # Obtain fluid properties from the pressure and temperature outlined above
State_1 = RP.REFPROPdll("CO2", "PT", "D;V;H;S;CP/CV;W;TCX;VIS;PRANDTL",
    ↳MASS_BASE_SI, 0, 0, P1, T1, [1.0])

# Outputs will be placed into data frame for organization
State_1 = pd.DataFrame(State_1.Output[0:9],
    index = ['Density [kg/m^3]', 'Volume [m^3/kg]', 'Enthalpy [J/kg]',
    ↳'Entropy [J/kg]',
    'CP/CV', 'Speed of Sound', 'Thermal Cond. [W/(mK)]',
    ↳'Viscosity [Pa-s]', 'Prandtl'],
    columns = ['State 1'])
```

```
# Display the data frame
State_1
```

```
[8]:
          State 1
Density [kg/m^3]      685.702611
Volume [m^3/kg]       0.001458
Enthalpy [J/kg]       331011.948444
Entropy [J/kg]        1374.687634
CP/CV                 2.953841
Speed of Sound        373.917781
Thermal Cond. [W/(mK)] 0.074005
Viscosity [Pa-s]       0.000055
Prandtl               2.058781
```

Pressure drop towards Heat Source

```
[9]: # Find Velocity, Reynolds Number, and darcy friction factor (assuming smooth
      ↳ pipe)
Velocity = 4 * m_dot * State_1.loc['Volume [m^3/kg]', 'State 1'] / (math.pi *
      ↳ Tube_ID**2)
Reynolds = State_1.loc['Density [kg/m^3]', 'State 1'] * Velocity * Tube_ID /
      ↳ State_1.loc['Viscosity [Pa-s]', 'State 1']
Darcy_f = (0.79 * math.log(Reynolds) - 1.64)**(-2)
```

```
[10]: # Using Estimated Length of Tubing connecting compressor and Heat Source,
      # find the amount of pressure drop caused by fanno flow
Length = 3.71475 # [meters]

# Force acted on the wall of tube
Force = math.pi * Tube_ID * Darcy_f * State_1.loc['Density [kg/m^3]', 'State 1']
      ↳ (Velocity**2) * Length / 8

# Dimensionless Friction factor
f_dim = 4 * Force / (P1 * math.pi * Tube_ID**2)

# Inlet Mach Number of length of tubing
Mach_inlet = Velocity / State_1.loc['Speed of Sound', 'State 1']

# Formulation used to calculate Pressure drop (found in Adv. Fluid Mechanics
      ↳ Textbook)
A_eq = ((Mach_inlet**2) * (1 + ((State_1.loc['CP/CV', 'State 1'] - 1) / 2) *
      ↳ (Mach_inlet**2))) \
      / ((1 + State_1.loc['CP/CV', 'State 1'] * (Mach_inlet**2) - f_dim)**2)

# Find the positive outcome to the biquadratic Mach number
Mach_outlet_1 = math.sqrt((-1 * (1 - 2 * A_eq * State_1.loc['CP/CV', 'State 1'])) \
```

```

        + ((1 - 2 * A_eq * (State_1.loc['CP/CV','State 1'] +
        ↳1))**0.5))\
        / ((State_1.loc['CP/CV','State 1'] - 1) - 2 * A_eq *
        ↳State_1.loc['CP/CV','State 1']**2))

# Find Outlet pressure caused by fanno flow (frictional loss)
P2 = P1 * (1 + State_1.loc['CP/CV','State 1'] * (Mach_inlet**2) - f_dim) / (1 +
        ↳State_1.loc['CP/CV','State 1']\
        *
        ↳(Mach_outlet_1**2))

print("Pressure at Inlet of Heat Source =", round(P2/6894.8 , 3), "psia")
print("Temperature at Inlet of Heat Source =" , (T1 - 273.15) * (9/5) + 32, "F")

```

Pressure at Inlet of Heat Source = 2595.418 psia
 Temperature at Inlet of Heat Source = 140.0 F

```

[11]: # Find the enthalpy at the inlet of the Heat Source
Enth_2 = (State_1.loc['Enthalpy [J/kg]','State 1'] * (1 + ((State_1.loc['CP/
        ↳CV','State 1'] - 1) / 2) * Mach_inlet**2)) / \
        (1 + ((State_1.loc['CP/CV','State 1'] - 1) / 2) *
        ↳(Mach_outlet_1**2))

Enth_2 # [J/kg]

```

[11]: 331011.80397706

Inlet of Heat Source (State 2)

```

[12]: #Pressure and temperature of fluid at inlet of Heat Source
P2 = P2

# Tube ID (using 1" OD Tubes)
Tube_HS_ID = .81 * 0.0254 # convert inches to meters

# Obtain fluid properties from the pressure and temperature outlined above
State_2 = RP.REFPROPdll("CO2","PH","T;D;V;S;CP/CV;W;TCX;VIS;PRANDTL",
        ↳MASS_BASE_SI,0,0,P2,Enth_2,[1.0])

# Outputs will be placed into data frame for organization
State_2 = pd.DataFrame(State_2.Output[0:9],
        index = ['Temperature [K]', 'Density [kg/m^3]', 'Volume [m^3/kg]',
        ↳'Entropy [J/kg]',
        'CP/CV', 'Speed of Sound', 'Thermal Cond. [W/(mK)]',
        ↳'Viscosity [Pa-s]', 'Prandtl'],
        columns = ['State 2'])

```

```
# Display the data frame
State_2
```

```
[12]: State 2
Temperature [K]      333.106709
Density [kg/m^3]     685.353389
Volume [m^3/kg]      0.001459
Entropy [J/kg]       1374.825537
CP/CV               2.957623
Speed of Sound       373.487733
Thermal Cond. [W/(mK)] 0.073968
Viscosity [Pa-s]     0.000055
Prandtl              2.061120
```

```
[13]: # Find Velocity, Reynolds Number, and darcy friction factor (assuming smooth
      ↪ pipe)
Velocity = 4 * m_dot * State_2.loc['Volume [m^3/kg]', 'State 2'] / (math.pi *
      ↪ Tube_HS_ID**2)
Reynolds = State_2.loc['Density [kg/m^3]', 'State 2'] * Velocity * Tube_HS_ID /
      ↪ State_2.loc['Viscosity [Pa-s]', 'State 2']
Darcy_f = (0.79 * math.log(Reynolds) - 1.64)**(-2)
```

```
[14]: # Using Estimated Length of Tubing used by Heat Source
      # find the amount of pressure drop caused by fanno flow and Heat addition
Length = 19.2024 # [meters]

# Force acted on the wall of tube
Force = math.pi * Tube_HS_ID * Darcy_f * State_2.loc['Density [kg/m^3]', 'State_
      ↪ 2'] * (Velocity**2) * Length / 8

# Dimensionless Friction factor
f_dim = 4 * Force / (P2 * math.pi * Tube_HS_ID**2)

# Heat Added into system (kW)
Q = 16

# Dimensionless heating factor
q_dim = Q * 1000 / (m_dot * Enth_2)

# Inlet Mach Number of length of tubing
Mach_inlet = Velocity / State_2.loc['Speed of Sound', 'State 2']

# Formulation used to calculate Pressure drop (found in Adv. Fluid Mechanics
      ↪ Textbook)
A_eq = ((Mach_inlet**2) * (1 + ((State_2.loc['CP/CV', 'State 2'] - 1) / 2) *
      ↪ (Mach_inlet**2) + q_dim)) \
      / ((1 + State_2.loc['CP/CV', 'State 2'] * (Mach_inlet**2) - f_dim)**2)
```

```

# Find the positive outcome to the biquadratic Mach number
Mach_outlet_1 = math.sqrt((-1 * (1 - 2 * A_eq * State_2.loc['CP/CV', 'State 2'])\
    + ((1 - 2 * A_eq * (State_2.loc['CP/CV', 'State 2'] + \
    ↪1))**0.5))\
    / ((State_2.loc['CP/CV', 'State 2'] - 1) - 2 * A_eq * \
    ↪State_2.loc['CP/CV', 'State 2']**2))

# Find Outlet pressure caused by fanno flow (frictional loss)
P3 = P2 * (1 + State_2.loc['CP/CV', 'State 2'] * (Mach_inlet**2) - f_dim) / (1 + \
    ↪State_2.loc['CP/CV', 'State 2']\
    ↪(Mach_outlet_1**2))

print("Pressure at Outlet of Heat Source =", round(P3/6894.8 , 3), "psia")

```

Pressure at Outlet of Heat Source = 2594.862 psia

```

[15]: # Find the enthalpy at the outlet of the Heat source
Enth_3 = (Enth_2 * (1 + ((State_2.loc['CP/CV', 'State 2'] - 1) / 2) * \
    ↪Mach_inlet**2 + q_dim)) / \
    (1 + ((State_2.loc['CP/CV', 'State 2'] - 1) / 2) * \
    ↪(Mach_outlet_1**2))

Enth_3 # [J/kg]

```

[15]: 411010.8332202174

Outlet of Heat Source (State 3)

```

[16]: # Using the new Pressure and enthalpy find the states of the fluid at the \
    ↪outlet of Heat Source

State_3 = RP.REFPROPdll("CO2", "PH", "T;D;V;S;CP/CV;W;TCX;VIS;PRANDTL", \
    ↪MASS_BASE_SI, 0, 0, P3, Enth_3, [1.0])

# Outputs will be placed into data frame for organization
State_3 = pd.DataFrame(State_3.Output[0:9],
    index = ['Temperature [K]', 'Density [kg/m^3]', 'Volume [m^3/kg]', \
    ↪'Entropy [J/kg]',
    ↪'CP/CV', 'Speed of Sound', 'Thermal Cond. [W/(mK)]', \
    ↪'Viscosity [Pa-s]', 'Prandtl'],
    columns = ['State 3'])

# Display the data frame
State_3

```

```
[16]:
```

	State 3
Temperature [K]	361.571914
Density [kg/m ³]	481.379685
Volume [m ³ /kg]	0.002077
Entropy [J/kg]	1605.371118
CP/CV	2.861185
Speed of Sound	299.342200
Thermal Cond. [W/(mK)]	0.055138
Viscosity [Pa-s]	0.000036
Prandtl	1.722122

```
[17]: # Add enthalpy to the data frame
State_2.loc['Enthalpy [J/kg]', 'State 2'] = Enth_2
State_3.loc['Enthalpy [J/kg]', 'State 3'] = Enth_3
```

```
[18]: # Find Velocity, Reynolds Number, and darcy friction factor (assuming smooth
      ↳ pipe)
Velocity = 4 * m_dot * State_3.loc['Volume [m^3/kg]', 'State 3'] / (math.pi *
      ↳ Tube_ID**2)
Reynolds = State_3.loc['Density [kg/m^3]', 'State 3'] * Velocity * Tube_ID /
      ↳ State_3.loc['Viscosity [Pa-s]', 'State 3']
Darcy_f = (0.79 * math.log(Reynolds) - 1.64)**(-2)
```

```
[19]: # Using Estimated Length of Tubing connecting Heat Source and Engine
      # find the amount of pressure drop caused by fanno flow
Length = 2.286 # [meters]

# Force acted on the wall of tube
Force = math.pi * Tube_ID * Darcy_f * State_3.loc['Density [kg/m^3]', 'State 3']
      ↳ * (Velocity**2) * Length / 8

# Dimensionless Friction factor
f_dim = 4 * Force / (P3 * math.pi * Tube_ID**2)

# Heat Added into system (kW)
Q = 0

# Dimensionless heating factor
q_dim = Q * 1000 / (m_dot * State_3.loc['Enthalpy [J/kg]', 'State 3'])

# Inlet Mach Number of length of tubing
Mach_inlet = Velocity / State_3.loc['Speed of Sound', 'State 3']

# Formulation used to calculate Pressure drop (found in Adv. Fluid Mechanics
      ↳ Textbook)
A_eq = ((Mach_inlet**2) * (1 + ((State_3.loc['CP/CV', 'State 3'] - 1) / 2) *
      ↳ (Mach_inlet**2) + q_dim)) \
```

```

        / ((1 + State_3.loc['CP/CV', 'State 3'] * (Mach_inlet**2) - f_dim)**2)

# Find the positive outcome to the biquadratic Mach number
Mach_outlet_1 = math.sqrt((-1 * (1 - 2 * A_eq * State_3.loc['CP/CV', 'State 3'])\
        + ((1 - 2 * A_eq * (State_3.loc['CP/CV', 'State 3'] + \
        →1))**0.5))\
        / ((State_3.loc['CP/CV', 'State 3'] - 1) - 2 * A_eq * \
        →State_3.loc['CP/CV', 'State 3']**2))

# Find Outlet pressure caused by fanno flow (frictional loss)
P4 = P3 * (1 + State_3.loc['CP/CV', 'State 3'] * (Mach_inlet**2) - f_dim) / (1 + \
        →State_3.loc['CP/CV', 'State 3']\
        * \
        →(Mach_outlet_1**2))

print("Pressure at Inlet of Engine =", round(P4/6894.8 , 3), "psia")

```

Pressure at Inlet of Engine = 2591.134 psia

```

[20]: # Find the enthalpy at the inlet of the Engine
Enth_4 = (State_3.loc['Enthalpy [J/kg]', 'State 3'] * (1 + ((State_3.loc['CP/
        →CV', 'State 3'] - 1) / 2) * Mach_inlet**2 + q_dim)) / \
        (1 + ((State_3.loc['CP/CV', 'State 3'] - 1) / 2) * \
        →(Mach_outlet_1**2))

Enth_4 # [J/kg]

```

[20]: 411010.39266619587

Check order of states before continuing

```

[21]: # Combine the data frames into one data frame for ease of use
sCO2_States = pd.concat([State_1, State_2, State_3], axis =1)

# Display the data frame to ensure proper layout
sCO2_States

```

```

[21]:

```

	State 1	State 2	State 3
Density [kg/m ³]	685.702611	685.353389	481.379685
Volume [m ³ /kg]	0.001458	0.001459	0.002077
Enthalpy [J/kg]	331011.948444	331011.803977	411010.833220
Entropy [J/kg]	1374.687634	1374.825537	1605.371118
CP/CV	2.953841	2.957623	2.861185
Speed of Sound	373.917781	373.487733	299.342200
Thermal Cond. [W/(mK)]	0.074005	0.073968	0.055138
Viscosity [Pa-s]	0.000055	0.000055	0.000036
Prandtl	2.058781	2.061120	1.722122
Temperature [K]	NaN	333.106709	361.571914


```
[22]: # Fill in the Missing data
sCO2_States.loc['Temperature [K]', 'State 1'] = T1
sCO2_States.loc['Pressure [Pa]', 'State 1'] = P1
sCO2_States.loc['Pressure [Pa]', 'State 2'] = P2
sCO2_States.loc['Pressure [Pa]', 'State 3'] = P3

# Display Data Frame
sCO2_States
```

```
[22]:
```

	State 1	State 2	State 3
Density [kg/m ³]	6.857026e+02	6.853534e+02	4.813797e+02
Volume [m ³ /kg]	1.458358e-03	1.459101e-03	2.077362e-03
Enthalpy [J/kg]	3.310119e+05	3.310118e+05	4.110108e+05
Entropy [J/kg]	1.374688e+03	1.374826e+03	1.605371e+03
CP/CV	2.953841e+00	2.957623e+00	2.861185e+00
Speed of Sound	3.739178e+02	3.734877e+02	2.993422e+02
Thermal Cond. [W/(mK)]	7.400547e-02	7.396752e-02	5.513816e-02
Viscosity [Pa-s]	5.533361e-05	5.528707e-05	3.622383e-05
Prandtl	2.058781e+00	2.061120e+00	1.722122e+00
Temperature [K]	3.331500e+02	3.331067e+02	3.615719e+02
Pressure [Pa]	1.792648e+07	1.789489e+07	1.789105e+07

```
[23]: # Reorder the Data Frame
sCO2_States = sCO2_States.reindex(["Pressure [Pa]", "Temperature [K]", 'Density_
→ [kg/m^3]', 'Volume [m^3/kg]', 'Enthalpy [J/kg]',
                                'Entropy [J/kg]', 'CP/CV', 'Speed of Sound', 'Thermal Cond.
→ [W/(mK)]', 'Viscosity [Pa-s]',
                                'Prandtl' ]) )
```

Inlet of Engine (State 4)

```
[24]: # Using the new Pressure and enthalpy find the states of the fluid at the Inlet_
→ of Engine

State_4 = RP.REFPROPdll("CO2", "PH", "T;D;V;S;CP/CV;W;TCX;VIS;PRANDTL",
→ MASS_BASE_SI, 0, 0, P4, Enth_4, [1.0])

# Outputs will be placed into data frame for organization
State_4 = pd.DataFrame(State_4.Output[0:9],
                        index = ['Temperature [K]', 'Density [kg/m^3]', 'Volume [m^3/kg]',
→ 'Entropy [J/kg]',
                                'CP/CV', 'Speed of Sound', 'Thermal Cond. [W/(mK)]',
→ 'Viscosity [Pa-s]', 'Prandtl'],
                        columns = ['State 4'])

# Display the data frame
State_4
```

```
[24]:
```

	State 4
Temperature [K]	361.501662
Density [kg/m ³]	480.973226
Volume [m ³ /kg]	0.002079
Entropy [J/kg]	1605.517639
CP/CV	2.863499
Speed of Sound	299.087675
Thermal Cond. [W/(mK)]	0.055106
Viscosity [Pa-s]	0.000036
Prandtl	1.723036

```
[25]: # Engine Parameters
mass_cylinder = State_4.loc['Density [kg/m^3]', 'State 4'] * .000308276
State_5_den = mass_cylinder / .000454574

# With Isentropic expansion
State_5_entr = State_4.loc['Entropy [J/kg]', 'State 4']
```

Outlet of Engine (State 5)

```
[26]: # Using the new density and entropy find the states of the fluid at the Outlet
      ↳ of Engine

State_5 = RP.REFPROPdll("CO2","DS","P;T;V;H;CP/CV;W;TCX;VIS;PRANDTL",
      ↳ MASS_BASE_SI,0,0,State_5_den,State_5_entr,[1.0])

# Outputs will be placed into data frame for organization
State_5 = pd.DataFrame(State_5.Output[0:9],
      index = ['Pressure [Pa]', 'Temperature [K]', 'Volume [m^3/kg]',
      ↳ 'Enthalpy [J/kg]',
      'CP/CV', 'Speed of Sound', 'Thermal Cond. [W/(mK)]',
      ↳ 'Viscosity [Pa-s]', 'Prandtl'],
      columns = ['State 5'])

# Display the data frame
State_5
```

```
[26]:
```

	State 5
Pressure [Pa]	8.417377e+06
Temperature [K]	3.137397e+02
Volume [m ³ /kg]	3.065801e-03
Enthalpy [J/kg]	3.878968e+05
CP/CV	6.565948e+00
Speed of Sound	2.017467e+02
Thermal Cond. [W/(mK)]	5.228222e-02
Viscosity [Pa-s]	2.415056e-05
Prandtl	3.291732e+00

```
[27]: # Combine the data frames into one data frame for ease of use
sCO2_States = pd.concat([sCO2_States, State_4, State_5], axis =1)

# Fill in the Missing data
sCO2_States.loc['Pressure [Pa]', 'State 4'] = P4
sCO2_States.loc['Enthalpy [J/kg]', 'State 4'] = Enth_4
sCO2_States.loc['Density [kg/m^3]', 'State 5'] = State_5_den
sCO2_States.loc['Entropy [J/kg]', 'State 5'] = State_5_entr

# Display the data frame
sCO2_States
```

```
[27]:
```

	State 1	State 2	State 3 \
Pressure [Pa]	1.792648e+07	1.789489e+07	1.789105e+07
Temperature [K]	3.331500e+02	3.331067e+02	3.615719e+02
Density [kg/m^3]	6.857026e+02	6.853534e+02	4.813797e+02
Volume [m^3/kg]	1.458358e-03	1.459101e-03	2.077362e-03
Enthalpy [J/kg]	3.310119e+05	3.310118e+05	4.110108e+05
Entropy [J/kg]	1.374688e+03	1.374826e+03	1.605371e+03
CP/CV	2.953841e+00	2.957623e+00	2.861185e+00
Speed of Sound	3.739178e+02	3.734877e+02	2.993422e+02
Thermal Cond. [W/(mK)]	7.400547e-02	7.396752e-02	5.513816e-02
Viscosity [Pa-s]	5.533361e-05	5.528707e-05	3.622383e-05
Prandtl	2.058781e+00	2.061120e+00	1.722122e+00

	State 4	State 5
Pressure [Pa]	1.786535e+07	8.417377e+06
Temperature [K]	3.615017e+02	3.137397e+02
Density [kg/m^3]	4.809732e+02	3.261790e+02
Volume [m^3/kg]	2.079118e-03	3.065801e-03
Enthalpy [J/kg]	4.110104e+05	3.878968e+05
Entropy [J/kg]	1.605518e+03	1.605518e+03
CP/CV	2.863499e+00	6.565948e+00
Speed of Sound	2.990877e+02	2.017467e+02
Thermal Cond. [W/(mK)]	5.510589e-02	5.228222e-02
Viscosity [Pa-s]	3.618962e-05	2.415056e-05
Prandtl	1.723036e+00	3.291732e+00

```
[28]: # Find Velocity, Reynolds Number, and darcy friction factor (assuming smooth
      ↳ pipe)
Velocity = 4 * m_dot * sCO2_States.loc['Volume [m^3/kg]', 'State 5'] / (math.pi *
      ↳ Tube_ID**2)
Reynolds = sCO2_States.loc['Density [kg/m^3]', 'State 5'] * Velocity * Tube_ID /
      ↳ sCO2_States.loc['Viscosity [Pa-s]', 'State 5']
Darcy_f = (0.79 * math.log(Reynolds) - 1.64)**(-2)
```

```

[29]: # Using Estimated Length of Tubing connecting Engine and Heat Exchanger
# find the amount of pressure drop caused by fanno flow
Length = 6.82 # [meters]

# Force acted on the wall of tube
Force = math.pi * Tube_ID * Darcy_f * sCO2_States.loc['Density [kg/m^3]', 'State 5'] * (Velocity**2) * Length / 8

# Dimensionless Friction factor
f_dim = 4 * Force / (sCO2_States.loc['Pressure [Pa]', 'State 5'] * math.pi * Tube_ID**2)

# Heat Added into system (kW)
Q = 0

# Dimensionless heating factor
q_dim = Q * 1000 / (m_dot * sCO2_States.loc['Enthalpy [J/kg]', 'State 5'])

# Inlet Mach Number of length of tubing
Mach_inlet = Velocity / sCO2_States.loc['Speed of Sound', 'State 5']

# Formulation used to calculate Pressure drop (found in Adv. Fluid Mechanics Textbook)
A_eq = ((Mach_inlet**2) * (1 + ((sCO2_States.loc['CP/CV', 'State 5'] - 1) / 2) * ((Mach_inlet**2) + q_dim)) \
        / ((1 + sCO2_States.loc['CP/CV', 'State 5'] * (Mach_inlet**2) - f_dim)**2))

# Find the positive outcome to the biquadratic Mach number
Mach_outlet_1 = math.sqrt((-1 * (1 - 2 * A_eq * sCO2_States.loc['CP/CV', 'State 5']) \
                          + ((1 - 2 * A_eq * (sCO2_States.loc['CP/CV', 'State 5'] + 1))**0.5)) \
                          / ((sCO2_States.loc['CP/CV', 'State 5'] - 1) - 2 * A_eq * sCO2_States.loc['CP/CV', 'State 5']**2))

# Find Outlet pressure caused by fanno flow (frictional loss)
P6 = sCO2_States.loc['Pressure [Pa]', 'State 5'] * (1 + sCO2_States.loc['CP/CV', 'State 5'] * (Mach_inlet**2) - f_dim) / (1 + sCO2_States.loc['CP/CV', 'State 5']) * (Mach_outlet_1**2)

print("Pressure at Inlet of Heat Exchanger =", round(P6/6894.8 , 3), "psia")

```

Pressure at Inlet of Heat Exchanger = 1205.343 psia

```
[30]: # Find the enthalpy at the inlet of the Heat Exchanger
Enth_6 = (sCO2_States.loc['Enthalpy [J/kg]', 'State 5'] * (1 + ((sCO2_States.
    ↳loc['CP/CV', 'State 5'] - 1) / 2) * Mach_inlet**2 + q_dim)) / \
    (1 + ((sCO2_States.loc['CP/CV', 'State 5'] - 1) / 2) *
    ↳(Mach_outlet_1**2))

Enth_6 # [J/kg]
```

[30]: 387843.81241681956

Inlet of Heat Exchanger (State 6)

```
[31]: # Using the new Pressure and enthalpy find the states of the fluid at the Inlet
    ↳of Heat Exchanger

State_6 = RP.REFPROPdll("CO2", "PH", "T;D;V;S;CP/CV;W;TCX;VIS;PRANDTL",
    ↳MASS_BASE_SI, 0, 0, P6, Enth_6, [1.0])

# Outputs will be placed into data frame for organization
State_6 = pd.DataFrame(State_6.Output[0:9],
    index = ['Temperature [K]', 'Density [kg/m^3]', 'Volume [m^3/kg]',
    ↳'Entropy [J/kg]',
    'CP/CV', 'Speed of Sound', 'Thermal Cond. [W/(mK)]',
    ↳'Viscosity [Pa-s]', 'Prandtl'],
    columns = ['State 6'])

# Display the data frame
State_6
```

```
[31]:
```

	State 6
Temperature [K]	312.969473
Density [kg/m^3]	322.837925
Volume [m^3/kg]	0.003098
Entropy [J/kg]	1606.398556
CP/CV	6.677678
Speed of Sound	200.585930
Thermal Cond. [W/(mK)]	0.052346
Viscosity [Pa-s]	0.000024
Prandtl	3.336801

```
[32]: # Add enthalpy and Pressure to Data frame
State_6.loc['Enthalpy [J/kg]', 'State 6'] = Enth_6
State_6.loc['Pressure [Pa]', 'State 6'] = P6

# Combine the data frames into one data frame for ease of use
sCO2_States = pd.concat([sCO2_States, State_6], axis = 1)

# Display the data frame
```

sCO2_States

```
[32]:
```

	State 1	State 2	State 3 \
Pressure [Pa]	1.792648e+07	1.789489e+07	1.789105e+07
Temperature [K]	3.331500e+02	3.331067e+02	3.615719e+02
Density [kg/m ³]	6.857026e+02	6.853534e+02	4.813797e+02
Volume [m ³ /kg]	1.458358e-03	1.459101e-03	2.077362e-03
Enthalpy [J/kg]	3.310119e+05	3.310118e+05	4.110108e+05
Entropy [J/kg]	1.374688e+03	1.374826e+03	1.605371e+03
CP/CV	2.953841e+00	2.957623e+00	2.861185e+00
Speed of Sound	3.739178e+02	3.734877e+02	2.993422e+02
Thermal Cond. [W/(mK)]	7.400547e-02	7.396752e-02	5.513816e-02
Viscosity [Pa-s]	5.533361e-05	5.528707e-05	3.622383e-05
Prandtl	2.058781e+00	2.061120e+00	1.722122e+00

	State 4	State 5	State 6
Pressure [Pa]	1.786535e+07	8.417377e+06	8.310597e+06
Temperature [K]	3.615017e+02	3.137397e+02	3.129695e+02
Density [kg/m ³]	4.809732e+02	3.261790e+02	3.228379e+02
Volume [m ³ /kg]	2.079118e-03	3.065801e-03	3.097530e-03
Enthalpy [J/kg]	4.110104e+05	3.878968e+05	3.878438e+05
Entropy [J/kg]	1.605518e+03	1.605518e+03	1.606399e+03
CP/CV	2.863499e+00	6.565948e+00	6.677678e+00
Speed of Sound	2.990877e+02	2.017467e+02	2.005859e+02
Thermal Cond. [W/(mK)]	5.510589e-02	5.228222e-02	5.234597e-02
Viscosity [Pa-s]	3.618962e-05	2.415056e-05	2.394502e-05
Prandtl	1.723036e+00	3.291732e+00	3.336801e+00

```
[33]: # Find Velocity, Reynolds Number, and darcy friction factor (assuming smooth
      ↪pipe)
Velocity = 4 * m_dot * sCO2_States.loc['Volume [m^3/kg]', 'State 6'] / (math.pi *
      ↪Tube_ID**2)
Reynolds = sCO2_States.loc['Density [kg/m^3]', 'State 6'] * Velocity * Tube_ID /
      ↪sCO2_States.loc['Viscosity [Pa-s]', 'State 6']
Darcy_f = (0.79 * math.log(Reynolds) - 1.64)**(-2)
```

```
[34]: # Using Estimated Length of Tubing used for Heat Exchanger
      # find the amount of pressure drop caused by fanno flow & Heat loss
Length = 11.582 # [meters]

# Force acted on the wall of tube
Force = math.pi * Tube_ID * Darcy_f * sCO2_States.loc['Density [kg/m^3]', 'State
      ↪6'] * (Velocity**2) * Length / 8

# Dimensionless Friction factor
f_dim = 4 * Force / (sCO2_States.loc['Pressure [Pa]', 'State 6'] * math.pi *
      ↪Tube_ID**2)
```

```

# Heat into system (kW)
Q = -6.272

# Dimensionless heating factor
q_dim = Q * 1000 / (m_dot * sCO2_States.loc['Enthalpy [J/kg]', 'State 6'])

# Inlet Mach Number of length of tubing
Mach_inlet = Velocity / sCO2_States.loc['Speed of Sound', 'State 6']

# Formulation used to calculate Pressure drop (found in Adv. Fluid Mechanics
↳Textbook)
A_eq = ((Mach_inlet**2) * (1 + ((sCO2_States.loc['CP/CV', 'State 6'] - 1) / 2) *
↳(Mach_inlet**2) + q_dim)) \
      / ((1 + sCO2_States.loc['CP/CV', 'State 6'] * (Mach_inlet**2) -
↳f_dim)**2)

# Find the positive outcome to the biquadratic Mach number
Mach_outlet_1 = math.sqrt((-1 * (1 - 2 * A_eq * sCO2_States.loc['CP/CV', 'State
↳6'])\
                        + ((1 - 2 * A_eq * (sCO2_States.loc['CP/CV', 'State
↳6'] + 1))**0.5))\
                        / ((sCO2_States.loc['CP/CV', 'State 6'] - 1) - 2 * A_eq
↳* sCO2_States.loc['CP/CV', 'State 6']**2))

# Find Outlet pressure caused by fanno flow (frictional loss)
P7 = sCO2_States.loc['Pressure [Pa]', 'State 6'] * (1 + sCO2_States.loc['CP/
↳CV', 'State 6'] * (Mach_inlet**2) - f_dim) / (1 + sCO2_States.loc['CP/
↳CV', 'State 6'])\
                                          *
↳(Mach_outlet_1**2))

print("Pressure at Outlet of Heat Exchanger =", round(P7/6894.8 , 3), "psia")

```

Pressure at Outlet of Heat Exchanger = 1180.109 psia

Outlet of Heat Exchanger (State 7)

```

[35]: # Using the new Pressure and Temperature find the properties of the fluid at
↳the Outlet of Heat Exchanger
T7 = 36 + 273.15 # [K] will be controlled by cooling loop of Heat Exchanger

State_7 = RP.REFPROPdll("CO2", "PT", "D;V;H;S;CP/CV;W;TCX;VIS;PRANDTL",
↳MASS_BASE_SI, 0, 0, P7, T7, [1.0])

# Outputs will be placed into data frame for organization
State_7 = pd.DataFrame(State_7.Output[0:9],

```

```

        index = ['Density [kg/m^3]', 'Volume [m^3/kg]', 'Enthalpy [J/kg]',
        → 'Entropy [J/kg]',
                'CP/CV', 'Speed of Sound', 'Thermal Cond. [W/(mK)]',
        → 'Viscosity [Pa-s]', 'Prandtl'],
        columns = ['State 7'])

# Display the data frame
State_7

```

```

[35]:
          State 7
Density [kg/m^3]    410.242639
Volume [m^3/kg]     0.002438
Enthalpy [J/kg]     356323.446412
Entropy [J/kg]      1506.524735
CP/CV              17.593695
Speed of Sound      185.233364
Thermal Cond. [W/(mK)] 0.078137
Viscosity [Pa-s]    0.000029
Prandtl            8.213117

```

```

[36]: # Add Pressure and Temperature to Data frame
State_7.loc['Pressure [Pa]', 'State 7'] = P7
State_7.loc['Temperature [K]', 'State 7'] = T7

# Combine the data frames into one data frame for ease of use
sCO2_States = pd.concat([sCO2_States, State_7], axis = 1)

# Display the data frame
sCO2_States

```

```

[36]:
          State 1      State 2      State 3 \
Pressure [Pa]    1.792648e+07  1.789489e+07  1.789105e+07
Temperature [K]    3.331500e+02  3.331067e+02  3.615719e+02
Density [kg/m^3]    6.857026e+02  6.853534e+02  4.813797e+02
Volume [m^3/kg]    1.458358e-03  1.459101e-03  2.077362e-03
Enthalpy [J/kg]    3.310119e+05  3.310118e+05  4.110108e+05
Entropy [J/kg]    1.374688e+03  1.374826e+03  1.605371e+03
CP/CV             2.953841e+00  2.957623e+00  2.861185e+00
Speed of Sound     3.739178e+02  3.734877e+02  2.993422e+02
Thermal Cond. [W/(mK)] 7.400547e-02  7.396752e-02  5.513816e-02
Viscosity [Pa-s]    5.533361e-05  5.528707e-05  3.622383e-05
Prandtl           2.058781e+00  2.061120e+00  1.722122e+00

          State 4      State 5      State 6      State 7
Pressure [Pa]    1.786535e+07  8.417377e+06  8.310597e+06  8.136615e+06
Temperature [K]    3.615017e+02  3.137397e+02  3.129695e+02  3.091500e+02
Density [kg/m^3]    4.809732e+02  3.261790e+02  3.228379e+02  4.102426e+02

```


Volume [m ³ /kg]	2.079118e-03	3.065801e-03	3.097530e-03	2.437582e-03
Enthalpy [J/kg]	4.110104e+05	3.878968e+05	3.878438e+05	3.563234e+05
Entropy [J/kg]	1.605518e+03	1.605518e+03	1.606399e+03	1.506525e+03
CP/CV	2.863499e+00	6.565948e+00	6.677678e+00	1.759369e+01
Speed of Sound	2.990877e+02	2.017467e+02	2.005859e+02	1.852334e+02
Thermal Cond. [W/(mK)]	5.510589e-02	5.228222e-02	5.234597e-02	7.813652e-02
Viscosity [Pa-s]	3.618962e-05	2.415056e-05	2.394502e-05	2.864207e-05
Prandtl	1.723036e+00	3.291732e+00	3.336801e+00	8.213117e+00

```
[37]: # Find Velocity, Reynolds Number, and darcy friction factor (assuming smooth
      ↪ pipe)
Velocity = 4 * m_dot * sCO2_States.loc['Volume [m^3/kg]', 'State 7'] / (math.pi *
      ↪ Tube_ID**2)
Reynolds = sCO2_States.loc['Density [kg/m^3]', 'State 7'] * Velocity * Tube_ID /
      ↪ sCO2_States.loc['Viscosity [Pa-s]', 'State 7']
Darcy_f = (0.79 * math.log(Reynolds) - 1.64)**(-2)
```

```
[38]: # Using Estimated Length of Tubing connecting Heat Exchanger and Compressor
      ↪ # find the amount of pressure drop caused by fanno flow
Length = 2 # [meters]

# Force acted on the wall of tube
Force = math.pi * Tube_ID * Darcy_f * sCO2_States.loc['Density [kg/m^3]', 'State
      ↪ 7'] * (Velocity**2) * Length / 8

# Dimensionless Friction factor
f_dim = 4 * Force / (sCO2_States.loc['Pressure [Pa]', 'State 7'] * math.pi *
      ↪ Tube_ID**2)

# Inlet Mach Number of length of tubing
Mach_inlet = Velocity / sCO2_States.loc['Speed of Sound', 'State 7']

# Formulation used to calculate Pressure drop (found in Adv. Fluid Mechanics
      ↪ Textbook)
A_eq = ((Mach_inlet**2) * (1 + ((sCO2_States.loc['CP/CV', 'State 7'] - 1) / 2) *
      ↪ (Mach_inlet**2))) \
      / ((1 + sCO2_States.loc['CP/CV', 'State 7'] * (Mach_inlet**2) -
      ↪ f_dim)**2)

# Find the positive outcome to the biquadratic Mach number
Mach_outlet_1 = math.sqrt((-1 * (1 - 2 * A_eq * sCO2_States.loc['CP/CV', 'State
      ↪ 7'])) \
      + ((1 - 2 * A_eq * (sCO2_States.loc['CP/CV', 'State
      ↪ 7'] + 1))**0.5)) \
      / ((sCO2_States.loc['CP/CV', 'State 7'] - 1) - 2 * A_eq
      ↪ * sCO2_States.loc['CP/CV', 'State 7']**2))
```

```
# Find Outlet pressure caused by fanno flow (frictional loss)
P8 = (sCO2_States.loc['Pressure [Pa]', 'State 7']) * (1 + sCO2_States.loc['CP/
→CV', 'State 7'] * (Mach_inlet**2) - f_dim) / (1 + sCO2_States.loc['CP/
→CV', 'State 7'] * (Mach_outlet_1**2))

print("Pressure at Inlet of Compressor =", round(P8/6894.8 , 3), "psia")
```

Pressure at Inlet of Compressor = 1176.345 psia

```
[39]: # Find the enthalpy at the inlet of the Compressor
Enth_8 = (sCO2_States.loc['Enthalpy [J/kg]', 'State 7'] * (1 + ((sCO2_States.
→loc['CP/CV', 'State 7'] - 1) / 2) * Mach_inlet**2)) / \
        (1 + ((sCO2_States.loc['CP/CV', 'State 7'] - 1) / 2) * \
→(Mach_outlet_1**2))

Enth_8 # [J/kg]
```

[39]: 356296.8110545878

Inlet of Compressor (State 8)

```
[40]: # Using the new Pressure and Enthalpy find the properties of the fluid at the
→Inlet of Compressor

State_8 = RP.REFPROPdll("CO2", "PH", "T;D;V;S;CP/CV;W;TCX;VIS;PRANDTL", \
→MASS_BASE_SI, 0, 0, P8, Enth_8, [1.0])

# Outputs will be placed into data frame for organization
State_8 = pd.DataFrame(State_8.Output[0:9],
        index = ['Temperature [K]', 'Density [kg/m^3]', 'Volume [m^3/kg]', \
→'Entropy [J/kg]',
        'CP/CV', 'Speed of Sound', 'Thermal Cond. [W/(mK)]', \
→'Viscosity [Pa-s]', 'Prandtl'],
        columns = ['State 8'])

# Display the data frame
State_8
```

```
[40]:
```

	State 8
Temperature [K]	308.987945
Density [kg/m^3]	409.358511
Volume [m^3/kg]	0.002443
Entropy [J/kg]	1506.643437
CP/CV	17.989866
Speed of Sound	184.720305
Thermal Cond. [W/(mK)]	0.078657
Viscosity [Pa-s]	0.000029

Prandtl

8.357031

```
[41]: # Add Pressure and Enthalpy to Data frame
State_8.loc['Pressure [Pa]', 'State 8'] = P8
State_8.loc['Enthalpy [J/kg]', 'State 8'] = Enth_8

# Combine the data frames into one data frame for ease of use
sCO2_States = pd.concat([sCO2_States, State_8], axis =1)

# Display the data frame
sCO2_States
```

```
[41]:
```

	State 1	State 2	State 3 \
Pressure [Pa]	1.792648e+07	1.789489e+07	1.789105e+07
Temperature [K]	3.331500e+02	3.331067e+02	3.615719e+02
Density [kg/m ³]	6.857026e+02	6.853534e+02	4.813797e+02
Volume [m ³ /kg]	1.458358e-03	1.459101e-03	2.077362e-03
Enthalpy [J/kg]	3.310119e+05	3.310118e+05	4.110108e+05
Entropy [J/kg]	1.374688e+03	1.374826e+03	1.605371e+03
CP/CV	2.953841e+00	2.957623e+00	2.861185e+00
Speed of Sound	3.739178e+02	3.734877e+02	2.993422e+02
Thermal Cond. [W/(mK)]	7.400547e-02	7.396752e-02	5.513816e-02
Viscosity [Pa-s]	5.533361e-05	5.528707e-05	3.622383e-05
Prandtl	2.058781e+00	2.061120e+00	1.722122e+00

	State 4	State 5	State 6 \
Pressure [Pa]	1.786535e+07	8.417377e+06	8.310597e+06
Temperature [K]	3.615017e+02	3.137397e+02	3.129695e+02
Density [kg/m ³]	4.809732e+02	3.261790e+02	3.228379e+02
Volume [m ³ /kg]	2.079118e-03	3.065801e-03	3.097530e-03
Enthalpy [J/kg]	4.110104e+05	3.878968e+05	3.878438e+05
Entropy [J/kg]	1.605518e+03	1.605518e+03	1.606399e+03
CP/CV	2.863499e+00	6.565948e+00	6.677678e+00
Speed of Sound	2.990877e+02	2.017467e+02	2.005859e+02
Thermal Cond. [W/(mK)]	5.510589e-02	5.228222e-02	5.234597e-02
Viscosity [Pa-s]	3.618962e-05	2.415056e-05	2.394502e-05
Prandtl	1.723036e+00	3.291732e+00	3.336801e+00

	State 7	State 8
Pressure [Pa]	8.136615e+06	8.110666e+06
Temperature [K]	3.091500e+02	3.089879e+02
Density [kg/m ³]	4.102426e+02	4.093585e+02
Volume [m ³ /kg]	2.437582e-03	2.442846e-03
Enthalpy [J/kg]	3.563234e+05	3.562968e+05
Entropy [J/kg]	1.506525e+03	1.506643e+03
CP/CV	1.759369e+01	1.798987e+01
Speed of Sound	1.852334e+02	1.847203e+02

Thermal Cond. [W/(mK)]	7.813652e-02	7.865692e-02
Viscosity [Pa-s]	2.864207e-05	2.857717e-05
Prandtl	8.213117e+00	8.357031e+00