

The Capacitated Lot Sizing Model

Sets

- T = the planning horizon; (index $t = 0, 1, \dots, n$)

Parameters

- d_t = the demand forecast at time t ;
- c_t = the unit production or purchasing cost at time t ;
- h_t = the unit inventory cost at time t ;
- K_t = the fixed setup or ordering cost at time t ;
- C_t = the maximum feasible lot size (capacity) at time t ;

Variables

- I_t = inventory level at the end of period t ;
- q_t = quantity to be produced or ordered during period t ;

$$y_t = \begin{cases} 1 & \text{if units of the product are manufactured / ordered in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$\min \sum_{t=1}^n K_t \cdot y_t + c_t \cdot q_t + h_t \cdot I_t \quad (1)$$

s.t

$$I_t = 0 \quad t = 0 \text{ and } t = n \quad (2)$$

$$q_t + I_{t-1} = d_t + I_t \quad \forall t \in T \setminus \{0\} \quad (3)$$

$$q_t \leq C_t \cdot y_t \quad \forall t \in T \setminus \{0\} \quad (4)$$

$$q_t \geq 0 \quad \forall t \in T \setminus \{0\} \quad (5)$$

$$I_t \geq 0 \quad \forall t \in T \setminus \{0\} \quad (6)$$

$$y_t \in \{0, 1\} \quad \forall t \in T \setminus \{0\} \quad (7)$$

The objective function (1) represents the total management costs, including the production (and/or purchasing), inventory and setup or ordering costs.

Conditions (2) impose that inventory levels at the beginning and the end of the planning horizon are equal to zero.

Constraints (3) reproduce the demand satisfaction and inventory balance constraint for each period.

Constraints (4)-(5) allow a positive production (constrained between 0 and a value C_t) in period t if and only if the setup variable is equal to 1.

In particular, the problem turns out to be uncapacitated for large values of C_t :

$$C_t \geq \sum_{t \in T} d_t$$

Constraints (6)-(7) express the non-negativity and binary restrictions on I_t and y_t the variables. As known, model (1)-(7) has $\theta(n)$ constraints in $\theta(n)$ variables.