Heterogeneous Fleet Composition Problem

Parameters:

- m = number of vehicles type; *(index k)*
- n = number of periods of the time horizon; *(index t)*
- v_{kt} = number of vehicles of type k required at time $t \in \{1, ..., n_k\}$;
- v_k^{max} = maximum number of vehicles of type k required during the time horizon; $\forall k = 1, \cdots, m$

$$v_k^{max} = \max_{\forall t=1,\cdots,n}(v_{kt}); \quad \forall k = 1, \cdots, m$$

- c_F^k = Fixed cost related to vehicle type k; $\forall k = 1, \cdots, m$
- c_V^k = Variable cost related to vehicle type k; $\forall k = 1, \cdots, m$
- c_H^k = Hiring cost related to vehicle type k; $\forall k = 1, \cdots, m$

Variables:

- v_k= number of vehicles of type k owned in the fleet; $\forall k = 1, \cdots, m$

- x_t^k $\quad \forall k = 1, \cdots, m; \quad \forall t = 1, \cdots, n$

$$x_t^k = \begin{cases} 1 & \text{if} \quad v_{kt} \leq v_k \\ 0 & \text{if} \quad v_{kt} > v_k \end{cases}$$

- $y_t^k = x_t^k \cdot v_k \quad \forall k = 1, \cdots, m; \quad \forall t = 1, \cdots, n_k$

Assumption:

We assume that the following preliminary condition is satisfied:

$$c_F^k + c_V^k \leq c_H^k \qquad \forall k = 1, \cdots, m$$

Math Formulation

$$min \cdot \sum_{k=1}^{m} nc_F^k \cdot v_k + \sum_{k=1}^{m}\sum_{t=1}^{n} c_V^k \cdot v_{kt} \cdot x_t^k + \sum_{k=1}^{m}\sum_{t=1}^{n} c_V^k v_k - c_V^k \cdot y_t^k + \sum_{k=1}^{m}\sum_{t=1}^{n} c_H^k \cdot (v_{kt} - v_{kt} \cdot x_t^k - v_k + y_t^k) \tag{1}$$

s.t

$$v_k \geq v_{kt} - v_k^{max} \cdot (1 - x_t^k) \quad \forall k = 1, \cdots, m; \quad \forall t = 1, \cdots, n \tag{2}$$

$$v_k \leq v_{kt} + v_k^{max} \cdot x_t^k \quad \forall k = 1, \cdots, m; \quad \forall t = 1, \cdots, n \tag{3}$$

$$y_t^k \leq v_{kt} \quad \forall k = 1, \cdots, m; \quad \forall t = 1, \cdots, n \tag{4}$$

$$y_t^k \leq v_k^{max} x_t^k \quad \forall k = 1, \cdots, m; \quad \forall t = 1, \cdots, n \tag{5}$$

$$y_t^k \geq v_k - v_k^{max} \cdot (1 - x_t^k) \quad \forall k = 1, \cdots, m; \quad \forall t = 1, \cdots, n \tag{6}$$

$$v_k \in \{0, \cdots, v_k^{max}\} \quad \forall k = 1, \cdots, m \tag{7}$$

$$y_t^k \in \{0, \cdots, v_k^{max}\} \quad \forall k = 1, \cdots, m; \quad \forall t = 1, \cdots, n \tag{8}$$

$$x_t^k \in \{0, 1\} \quad \forall k = 1, \cdots, m; \quad \forall t = 1, \cdots, n \tag{9}$$

Non linearized objective function:

$$min \cdot \sum_{k=1}^{m} nc_F^k \cdot v_k + \sum_{k=1}^{m}\sum_{t=1}^{n} c_V^k \cdot v_{kt} \cdot x_t^k + \sum_{k=1}^{m}\sum_{t=1}^{n} c_V^k \cdot v_k \cdot (1 - x_t^k) + \sum_{k=1}^{m}\sum_{t=1}^{n} c_H^k \cdot (v_{kt} - v_k)(1 - x_t^k)$$