Homogeneous Fleet Composition

Parameters

- n = time horizon's number of periods;
- v = number of owned vehicles;
- v_t = required number of vehicles at time t; $\quad t = 1, \cdots, n$
- c_F = fixed cost per time period of an owned vehicle;
- c_V = variable cost per time period of an owned vehicle;
- c_H = cost per time period of hiring a vehicle such that $c_F + c_V < c_H$;

Variables

$$
x_t = \left\{ \begin{array}{ll} 1 & \text{if} \quad v_t \leq v \\ 0 & \text{if} \quad v_t > v \end{array} \right.
$$

$$
y_t = v \cdot x_t = \left\{ \begin{array}{ll} v & \text{if} \quad x_t = 1 \\ 0 & \text{if} \quad x_t = 0 \end{array} \right.
$$

Model

$$
min \ n \cdot c_F \cdot v + c_V \cdot \left(\sum_{t=1}^{n} v_t x_t + \sum_{t=1}^{n} v(1 - x_t) \right) + c_H \cdot \left(\sum_{t=1}^{n} (v_t - v)(1 - x_t) \right) \quad (1)
$$

$$
\text{s.v}
$$

$$
v \geq v_t - \overline{v} \cdot (1 - x_t) \qquad t = 1, \cdots, n \qquad (2)
$$

$$
v \leq v_t + \overline{v} \cdot x_t \qquad t = 1, \cdots, n \qquad (3)
$$

$$
v \in \{0, \cdots, \overline{v}\} \qquad (4)
$$

$$
x_t \in \{0, 1\} \qquad t = 1, \cdots, n \qquad (5)
$$

$$
y_t \leq v \qquad t = 1, \cdots, n \qquad (6)
$$

$$
y_t \leq \overline{v} \cdot x_t \qquad t = 1, \cdots, n \qquad (7)
$$

$$y_t \geq v - \overline{v} \cdot (1 - x_t) \quad t = 1, \cdots, n \tag{8}$$

$$y_t \in \{0, \cdots, \overline{v}\} \qquad t = 1, \cdots, n \tag{9}$$

Objective function:

$$min\ n \cdot c_F \cdot v + c_V \cdot \left[\sum_{t=1}^{n} (v_t x_t) + \sum_{t=1}^{n} (v - y_t) \right] + c_H \cdot \left(\sum_{t=1}^{n} v_t - v_t \cdot x_t - v + y_t \right) \tag{10}$$

$$min\ n \cdot c_F \cdot v + c_V \cdot \left[\sum_{t=1}^{n} (v_t x_t) + (nv) - \sum_{t=1}^{n} y_t \right] + c_H \cdot \left[\sum_{t=1}^{n} v_t - \sum_{t=1}^{n} (v_t x_t) - (nv) + \sum_{t=1}^{n} y_t \right] \tag{11}$$