Math Formulation

Sets

- P = set of products (*index* i)
- T = set of periods in time horizion (*index* t)
- J = set of vehicles (*index* j)

Parameters

- D_{it} = Demand of product i at time t;
- R_i = Quantity of product i that can be loaded in a track;
- Y_i = Initial stock level of product i;
- K_j = Capacity of truck j;
- M = Very large positive number;
- C^α = Set up cost for a truck;
- C^δ = Penalty costs for a truck's loss of capacity;
- C^λ = Cost of unbalanced stock;
- LB_{it} = Minimum level for each product i in each time period t;
- UB_{it} = Maximum level for each product i in each time period t;

Variables

- y_{it} = Stock level of product i at time period t; $\forall i \in I,\ t \in T$
- v_{ijt} = Quantity of product i on truck j at time period t; $\forall i \in I,\ j \in J,\ t \in T \setminus \{0\}$
- ε_{jt} = Number of different variants on truck j at time period t; $\forall j \in J,\ t \in T \setminus \{0\}$
- λ_t = Level of balanced stock of all the products at time period t; $\forall t \in T \setminus \{0\}$

$$-\alpha_{jt} = \begin{cases} 1 & \text{if truck } j \text{ is used at time period } t;\ \forall j \in J,\ t \in T \setminus \{0\} \\ 0 & \text{otherwiese} \end{cases}$$

$$-\delta_{ijt} = \begin{cases} 1 & \text{if one product } i \text{ is on truck } j \text{ at time period } t;\ \forall i \in I,\ j \in J,\ t \in T \setminus \{0\} \\ 0 & \text{otherwiese} \end{cases}$$

$$min \sum_{t\in T\setminus\{0\}} \sum_{j\in J} \alpha_{jt} \cdot C^{\alpha} + \sum_{t\in T\setminus\{0\}} \sum_{j\in J} C^{\delta} \cdot (\varepsilon_{jt} - 1) + \sum_{t\in T\setminus\{0\}} \lambda_t \cdot C^{\lambda} \tag{1}$$

$$y_{i0} = Y_i \qquad \forall i \in I \tag{2}$$

$$y_{it} = y_{i,t-1} - D_{it} + \sum_{j\in J} v_{ijt} \qquad \forall i \in I,\, t \in T \setminus \{0\} \tag{3}$$

$$LB_{it} \leq y_{it} \leq UB_{it} \qquad \forall i \in I,\, t \in T \setminus \{0\} \tag{4}$$

$$(1 - \lambda_t) \leq \frac{UB_{it} - y_{it}}{UB_{it} - LB_{it}} \qquad \forall i \in I,\, t \in T \setminus \{0\} \tag{5}$$

$$v_{ijt} \leq M \cdot \delta_{ijt} \qquad \forall i \in I,\, j \in J,\, t \in T \setminus \{0\} \tag{6}$$

$$\sum_{i\in I} \delta_{ijt} \leq \varepsilon_{jt} \qquad \forall j \in J,\, t \in T \setminus \{0\} \tag{7}$$

$$\varepsilon_{jt} \geq 1 \qquad \forall j \in J,\, t \in T \setminus \{0\} \tag{8}$$

$$K_j \cdot \alpha_{jt} - (\varepsilon_{jt} - 1) = \sum_{i\in I} \frac{v_{ijt}}{R_i} \qquad \forall j \in J,\, t \in T \setminus \{0\} \tag{9}$$

$$\lambda_t \in [0, 1] \qquad \forall t \in T \setminus \{0\} \tag{10}$$

$$y_{it} \geq 0 \text{ and integer} \qquad \forall j \in J,\, t \in T \tag{11}$$

$$v_{ijt} \geq 0 \text{ and integer} \qquad \forall i \in I, j \in J,\, t \in T \setminus \{0\} \tag{12}$$

$$\varepsilon_{jt} \geq 0 \text{ and integer} \qquad \forall j \in J,\, t \in T \setminus \{0\} \tag{13}$$

$$\delta_{ijt} \in \{0, 1\} \qquad \forall i \in I,\, j \in J,\, t \in T \setminus \{0\} \tag{14}$$

$$\alpha_{jt} \in \{0, 1\} \qquad \forall j \in J,\, t \in T \setminus \{0\} \tag{15}$$

The linear objective function (1) consists in minimizing total supply costs.
The initial inventory levels of products are known (2).
Continuity constraints (3) apply to the model.
The stock level reached at the end of a period must be above a minimum level without exceeding a maximum level (4).
Balancing stock levels is determined as a percentage according to the values of the stock level limits (5).
By Constraint (6), we know if product i is on truck j at time period t .
Constraints (7)-(8) determine the number of variants loaded and the penalties associated with each truck used.
Constraint (9), it is assumed that a truck's capacity in racks less its capacity penalty is equal to the racks loaded onto a truck.
Constraints (10)-(15) concern variables' domain.