Multy Item Capacitated Lot Sizing Problem (CLSP)

Sets

- T = the planning horizon; (index $t = 0, 1, ..., n$)
- J = set of items; (index $j = 1, ..., m$)

Parameters

- d_{tj} = the demand forecast at time t for item j;

- c_{tj} =the unit production or purchasing cost at time t for item j;

- h_{tj} = the unit inventory cost at time t for item j;

- K_j = the fixed setup or ordering cost for item j;

- C_{tj} =the maximum feasible lot size (capacity) at time t for item j;

Variables

- I_{tj} = inventory level at the end of period t for item j;

- q_{tj} =quantity to be produced or ordered during period t for item j;

$$y_{tj} = \begin{cases} 1 & \text{if units of the item } j \text{ are manufactured / ordered in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$min \sum_{t=1}^{n} \sum_{j=1}^{m} K_j \cdot y_{tj} + c_{tj} \cdot q_{tj} + h_{tj} \cdot I_{tj} \tag{1}$$

s.t

$$I_{tj} = 0 \qquad t = 0 \text{ and } t = n\,;\ \forall j \in J \tag{2}$$

$$q_{tj} + I_{t-1,j} = d_{tj} + I_{tj} \qquad \forall t \in T \backslash \{0\}\ ;\ \forall j \in J \tag{3}$$

$$q_{tj} \leq C_{tj} \cdot y_{tj} \qquad \forall t \in T \backslash \{0\}\ ;\ \forall j \in J \tag{4}$$

$$q_{tj} \geq 0 \qquad \forall t \in T \setminus \{0\} \; ; \; \forall j \in J \tag{5}$$

$$I_{tj} \geq 0 \qquad \forall t \in T \setminus \{0\} \; ; \; \forall j \in J \tag{6}$$

$$y_{tj} \in \{0,1\} \qquad \forall t \in T \setminus \{0\} \; ; \; \forall j \in J \tag{7}$$

The objective function (1) represents the total management costs, including the production (and/or purchasing), inventory and setup or ordering costs.

Conditions (2) impose that inventory levels at the beginning and the end of the planning horizon are equal to zero.

Constraints (3) reproduce the demand satisfaction and inventory balance constraint for each period.

Constraints (4)-(5) allow a positive production (constrained between 0 and a value C_{it}) in period t if and only if the setup variable is equal to 1.