Asymmetric Traveling Salesman Problem with Time Windows (ATSPTW)

Sets

Let $G = (V, A)$ be a complete graph where:
-$V = \{0, 1, \ldots, n\}$ = set of the nodes;
-$A = \{(i, j) : i \neq j; i, j \in V\}$ = set of undirected arcs;

Parameters:

-u_i = upper bound of a specified time window for node i ($u_i \geq 0$); $\forall i \in V$
-t_{ij} = the shortest time required to travel from node i to node j; $\forall (i, j) \in A$
Every node $i \in V$ represents a node with an associated time window:
$TW_i = [l_i, u_i]$, where l_i is the *release time* and u_i the *deadline* of node $i \in V$.
Each arc $(i, j) \in A$ has an associated a travel time $t_{ij} \geq 0$.
We assume they also have a distance d_{ij} representing a distance from node i to node j, hence travel time $t_{ij} = \dfrac{d_{ij}}{v}$ $\forall (i, j) \in A$, where v is a speed.
In order to make the model easier, if a service time is needed at a node i, this time will be included in the travel time t_{ij}'s.
TSPTW is named as symmetric if $t_{ij} = t_{ji}$ and asymmetric if $t_{ij} \neq t_{ji}$.

Variables:

-t_i = time instant in which node i is visited; $\forall i \in V$
-t_{n+1} = time at which tour is completed;

$$-y_{ij} = \begin{cases} 1 & \text{if the traveler goes from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases}$$

Math Formulation

$$min\ t_{n+1} \tag{1}$$

$$t_i \geq t_{0i} \cdot y_{0i} \qquad i = 1, 2, ..., n \tag{2}$$

$$t_i - t_j + t_{ij} \leq M_{ij} \cdot (1 - y_{ij}) \qquad \forall i, j = 1, 2, ..., n : i \neq j \tag{3}$$

$$\sum_{i=0 \,:\, i \neq j}^{n} y_{ij} = 1 \qquad j = 1, 2, ..., n \tag{4}$$

$$\sum_{j=0 \,:\, j \neq i}^{n} y_{ij} = 1 \qquad i = 1, 2, ..., n \tag{5}$$

$$t_i + t_{i0} \leq t_{n+1} \qquad i = 1, 2, ..., n \tag{6}$$

$$l_i \leq t_i \leq u_i \qquad i = 1, 2, ..., n \tag{7}$$

$$\sum_{i=1}^{n} y_{i0} = 1 \tag{8}$$

$$\sum_{j=1}^{n} y_{0j} = 1 \tag{9}$$

$$y_{ij} \in \{0, 1\} \qquad \forall (i, j) \in A \tag{10}$$

$$t_i \geq 0 \qquad \forall i = 0, 1, ..., n + 1 \tag{11}$$

where $M_{ij} = u_i - l_j + t_{ij}$, so (3) becomes:

$$t_i - t_j + (u_i - l_j + t_{ij}) \cdot y_{ij} \leq u_i - l_j \qquad \forall i, j = 1, 2, ..., n : i \neq j$$