Time Constrained Symmetric Traveling Salesman Problem (STSPTW)

Sets

Let $G = (V, A)$ be a complete graph where:
-$V = \{0, 1, \ldots, n\}$ = set of the nodes;
-$A = \{(i, j) : i \neq j; i, j \in V\}$ = set of undirected arcs;

Parameters

-l_i = lower bound of a specified time window for node i ($l_i \geq 0$); $\forall i \in V$
-u_i = upper bound of a specified time window for node i ($u_i \geq 0$); $\forall i \in V$
-t_{ij} = the shortest time required to travel from node i to node j; $\forall (i, j) \in A$
Every node $i \in V$ represents a node with an associated time window:
$TW_i = [l_i, u_i]$, where l_i is the *release time* and u_i the *deadline* of node $i \in V$.
Each arc $(i, j) \in A$ has an associated a travel time $t_{ij} \geq 0$.
We assume they also have a distance d_{ij} representing a distance from node i to node j, hence travel time $t_{ij} = \dfrac{d_{ij}}{v}$ $\forall (i, j) \in A$, where v is a speed.
In order to make the model easier, if a service time is needed at a node i, this time will be included in the travel time t_{ij}'s.
TSPTW is named as symmetric if $t_{ij} = t_{ji}$ and asymmetric if $t_{ij} \neq t_{ji}$.

Additional restrictions

For example, if the node j must be visited just after i, it is sufficient to add the constraints $t_i + t_{ij} = t_j$ to the model. If j must be visited before i then we add the constraints $t_j \leq t_i + t_{ij}$ to the model.

Variables

-t_i = time instant in which node i is visited; $\forall i \in V$
-t_{n+1} = time at which tour is completed;
-y_{ij} = binary variables defined as follows $\forall (i, j) \in A$:

$$y_{ij} = \begin{cases} 1 & \text{if node } j \text{ follows node } i \\ 0 & \text{otherwise} \end{cases}$$

$$min\ t_{n+1} - t_0 \tag{1}$$

$$t_i - t_0 \geq t_{0i} \qquad i = 1, 2, ..., n \tag{2}$$

$$t_{n+1} - t_i \geq t_{i0} \qquad i = 1, 2, ..., n \tag{3}$$

$$t_i - t_j \geq t_{ij} - M_{ij} \cdot y_{ij} \qquad \forall i, j = 0, 1, ..., n : i \neq j \tag{4}$$

$$t_j - t_i \geq t_{ij} - (1 - y_{ij}) \cdot M_{ij} \qquad \forall i, j = 0, 1, ..., n : i \neq j \tag{5}$$

$$l_i \leq t_i \leq u_i \qquad i = 1, 2, ..., n \tag{6}$$

$$t_0 = 0 \tag{7}$$

$$y_{ij} \in \{0, 1\} \qquad \forall i, j = 0, 1, ..., n : i \neq j \tag{8}$$

$$t_i \geq 0 \qquad \forall i = 0, 1, ..., n + 1 \tag{9}$$

where $M_{ij} = u_i - l_j + t_{ij}$.