Two-Echelon Multicommodity Location Model

Sets

- I = set of production plants; (*index* i)
- J = set of potential Distribution Centers (DCs); (*index* j)
- R = set of demand nodes; (*index* r)
- K = set of homogeneous commodities; (*index* k)

Parameters

- p = maximum number of DCs that can be opened;
- c_{ijr}^k = unit transportation cost of commodity $k \in K$ from plant node $i \in I$ to demand node $r \in R$ across DC $j \in J$;
- d_r^k = demand of commodity $k \in K$ from demand node $r \in R$;
- p_i^k = maximum quantity of commodity $k \in K$ that can be manufactured by plant $i \in I$;
- q_j^- = minimum activity level of potential DC $j \in J$;
- q_j^+ = maximum activity level of potential DC $j \in J$;
- f_j = fixed cost of potential DC $j \in J$;
- g_j = marginal cost of potential DC $j \in J$;

Variables

$$z_j = \begin{cases} 1 & \text{if DC } j \in J \text{ is opened} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{jr} = \begin{cases} 1 & \text{if demand node } r \in R \text{ is assigned to DC } j \in J \\ 0 & \text{otherwise} \end{cases}$$

- s_{ijr}^k = amount of commodity $k \in K$ transported from plant node $i \in I$ to demand node $r \in R$ across DC $j \in J$;

The following feasibility condition must hold:

$$\sum_{i \in I} p_i^k \geq \sum_{r \in R} d_r^k \quad k \in K$$

TEMC Mathematical Formulation

$$\sum_{i \in I} \sum_{j \in J} \sum_{r \in R} \sum_{k \in K} c_{ijr}^k \cdot s_{ijr}^k + \sum_{j \in J} \left(f_j \cdot z_j + g_j \cdot \sum_{r \in R} \sum_{k \in K} d_r^k \cdot y_{jr} \right)$$

$$\sum_{j \in J} \sum_{r \in R} s_{ijr}^k \leq p_i^k \quad i \in I,\, k \in K \tag{1}$$

$$\sum_{i \in I} s_{ijr}^k = d_r^k \cdot y_{jr} \quad j \in J,\, r \in R,\, k \in K \tag{2}$$

$$\sum_{j \in J} y_{jr} = 1 \quad r \in R \tag{3}$$

$$q_j^- \cdot z_j \leq \sum_{r \in R} \sum_{k \in K} d_r^k \cdot y_{jr} \leq q_j^+ \cdot z_j \quad j \in J \tag{4}$$

$$\sum_{j \in J} z_j = p \tag{5}$$

$$z_j \in \{0, 1\} \quad j \in J \tag{6}$$

$$y_{jr} \in \{0, 1\} \quad j \in J,\, r \in R \tag{7}$$

$$s_{ijr}^k \geq 0 \quad i \in I,\, j \in J,\, r \in R,\, k \in K \tag{8}$$

Demand Allocation Problem

If a set $\overline{z}$ $j \in J$ and $\overline{y}_{jr}$, $j \in J$, $r \in R$ of feasible values is available, you just have to solve the following LP problem in order to determine the optimal demand allocation:

$$\sum_{i \in I} \sum_{j \in J} \sum_{r \in R} \sum_{k \in K} c_{ijr}^k \cdot s_{ijr}^k$$

$$\sum_{j \in J} \sum_{r \in R} s_{ijr}^k \leq p_i^k \quad i \in I,\, k \in K$$

$$\sum_{i \in I} s_{ijr}^k = d_r^k \cdot \overline{y}_{jr} \quad j \in J,\, r \in R,\, k \in K$$

$$s_{ijr}^k \geq 0 \quad i \in I,\, j \in J,\, r \in R,\, k \in K$$