Week 5: Linear Regression

Video 1: Introduction to Linear Regression

EPI202 – Epidemiologic Methods II
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Key Concepts

- Advantages of regression models
- Linear regression
- Log odds scale
- Logistic regression

Advantages of Regression Models

- Generally more efficient than stratification-based methods when data are sparse
- Modelling of a continuous outcomes
- Specify continuous and categorical exposures, confounders and modifiers
- Specify interactions to model effect modification
- Model nonlinear relationships between exposure and outcome and other covariates

Regression Models

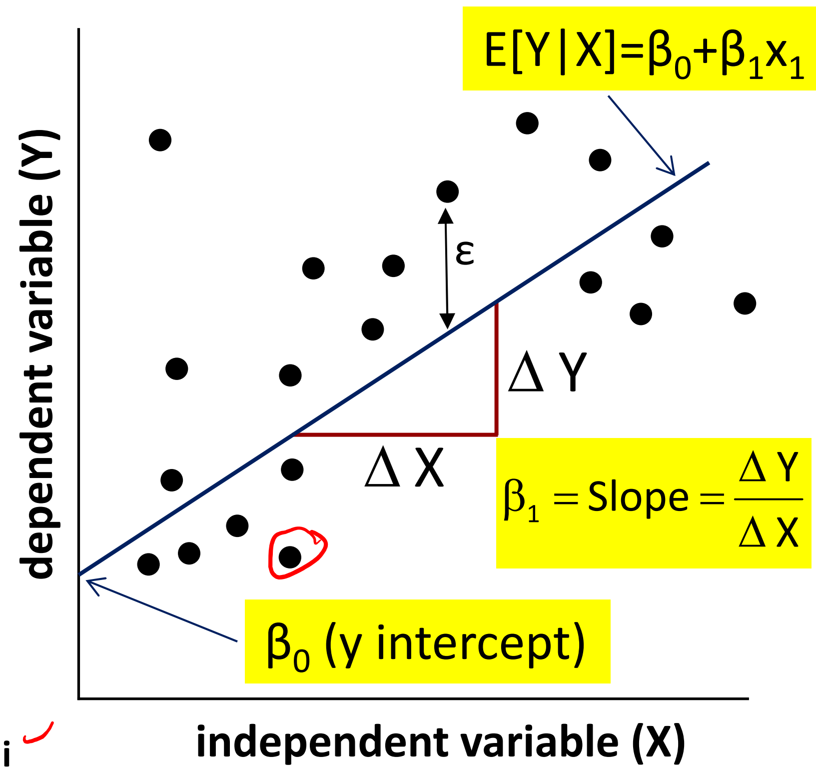
y	=	$\beta_0+\beta_1 x_1+\ldots\beta_i x_i+\varepsilon$
Dependent		Independent
Predicted		Predictor variables
Response variable		Explanatory variables
Outcome variable		Covariables

Simple Linear Regression

- Predict a continuous dependent (outcome) variable y from a continuous independent (exposure) variable x

- Simple linear regression fits a straight line to the data using the least squares method.

- Regression line: $\mathbf{E[Y|X]=\beta_0+\beta_1 x_1}$
  - Often presented as y = mx + b where
    - b=y-intercept
    - m=slope=$\Delta y/\Delta x$ (rise/run)



- Individual predicted value: $\mathbf{Y_i=\beta_0+\beta_1 x_{1i}+\varepsilon_i}$
  - β_0 =y-intercept (where the line crosses the Y-axis)
  - β_1 =slope=$\Delta y/\Delta x$ =average change in y when x changes by one unit
  - X_1 is a known constant
  - ε, the error, is an observation's deviation from the conditional mean, N(0, σ2)

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Video 2: Linear Regression Example

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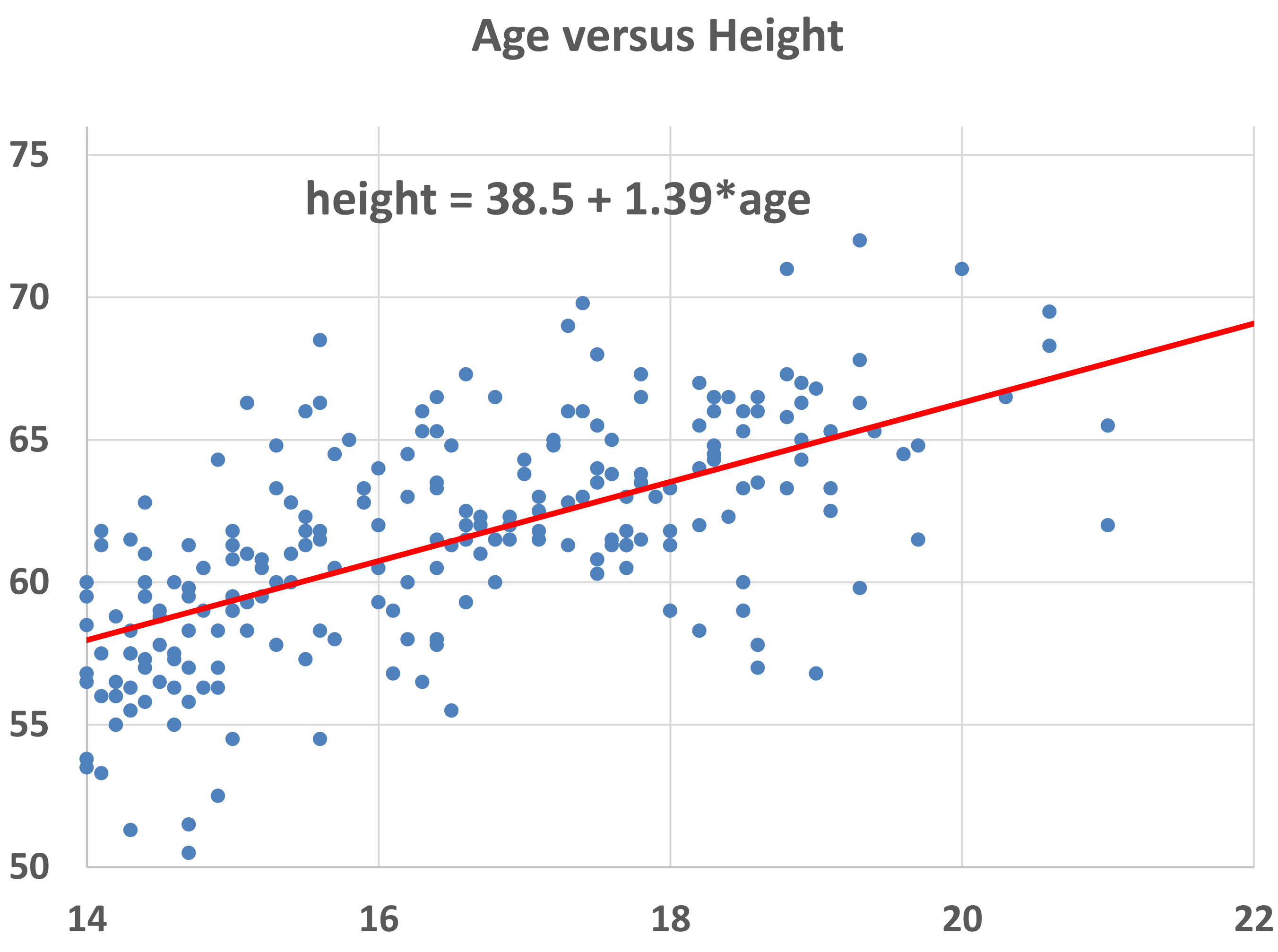
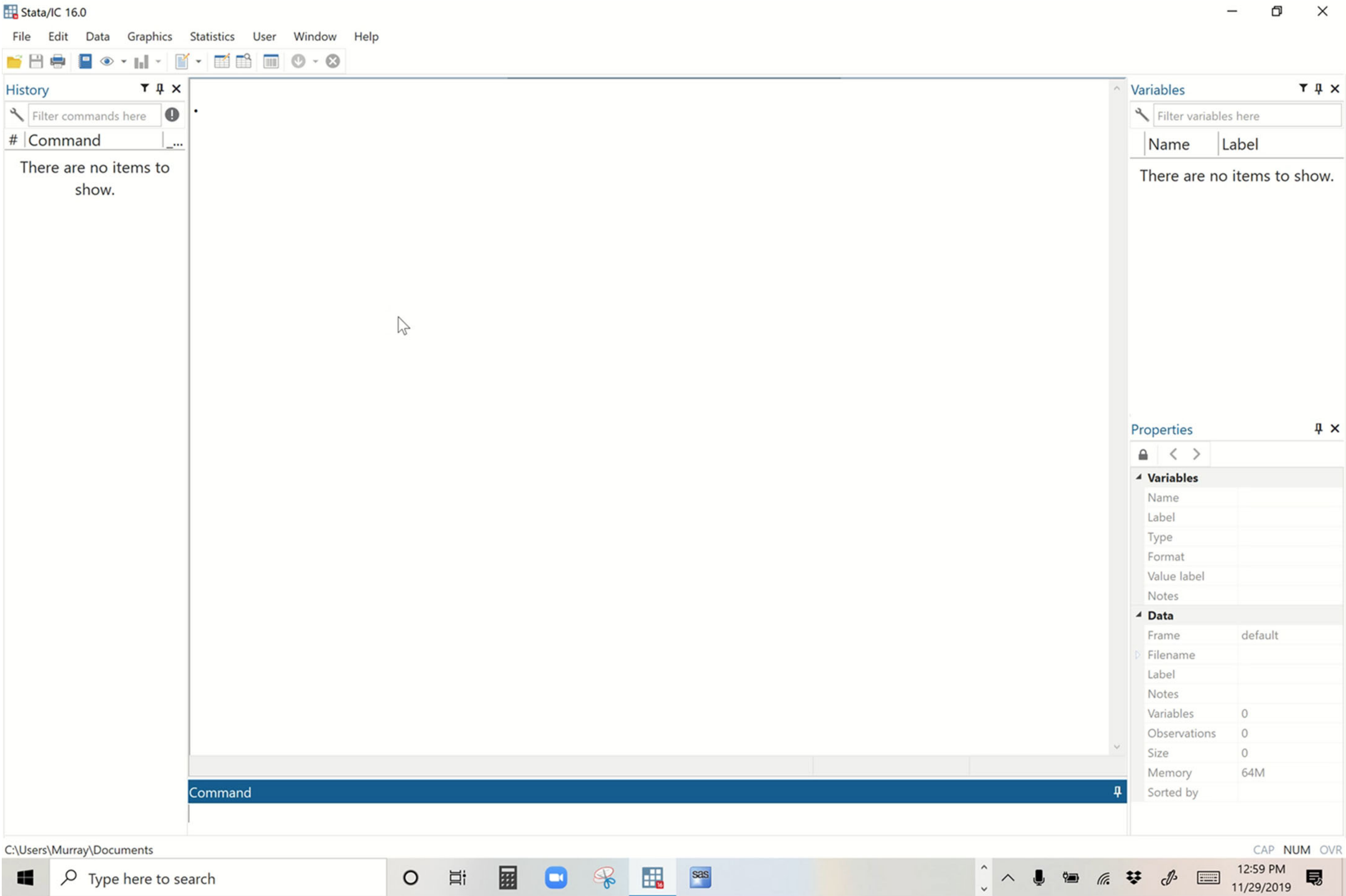
Linear Regression



Linear Regression Example

- Download the HeightWeight dataset from the Regression I module on Canvas
  - The dataset is available in multiple formats
    - CSV
    - Excel
    - R
    - SAS
    - Stata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History
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Command
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Variables
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Observations
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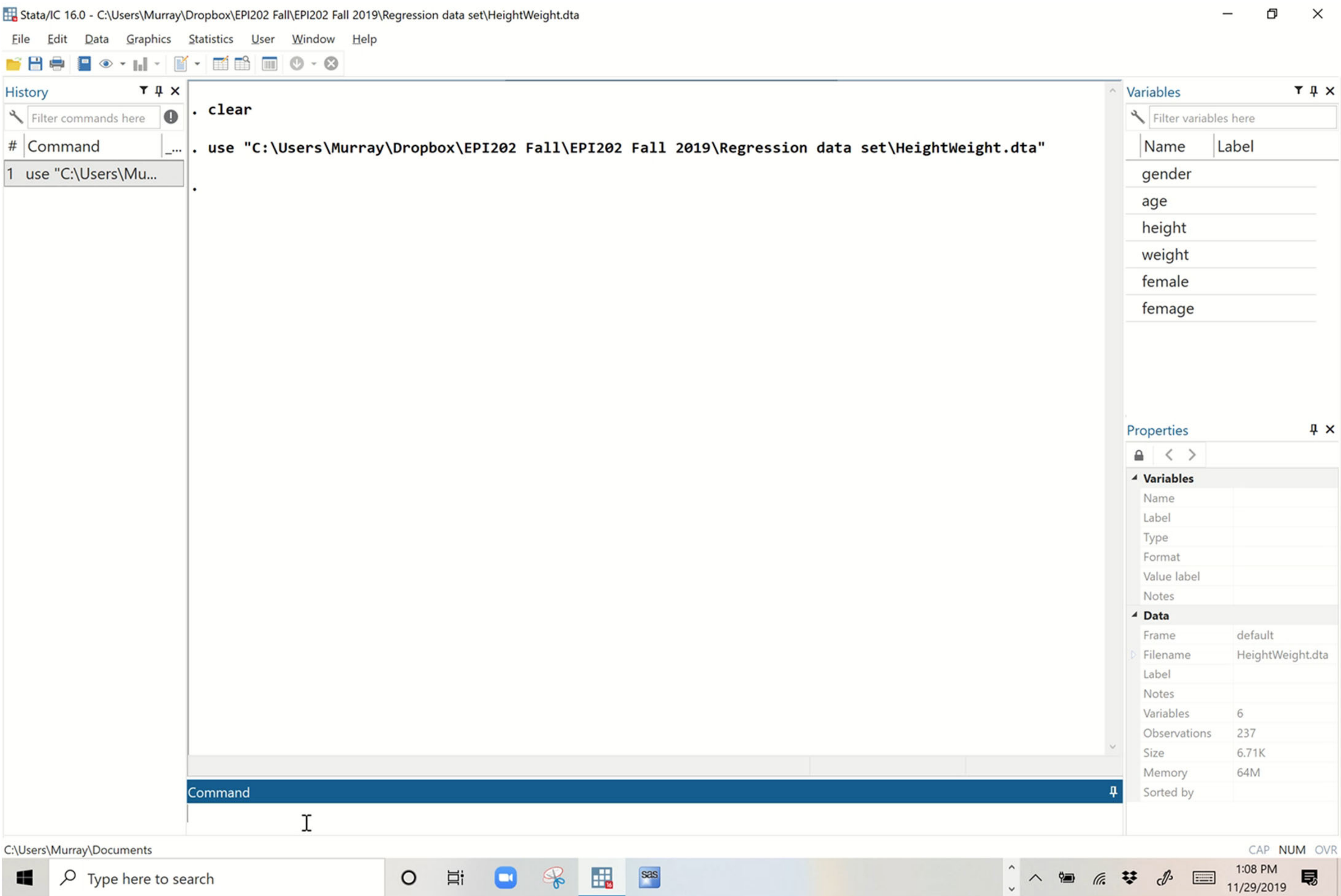
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History
Filter commands here
Command
1 use "C:\Users\Mu...
. clear
. use "C:\Users\Murray\Dropbox\EPI202 Fall\EPI202 Fall 2019\Regression data set\HeightWeight.dta"
.
Variables
Filter variables here
Name Label
gender
age
height
weight
female
femage
Properties
Variables
Name
Label
Type
Format
Value label
Notes
Data
Frame default
Filename HeightWeight.dta
Label
Notes
Variables 6
Observations 237
Size 6.71K
Memory 64M
Sorted by
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height	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age	1.457297	.1107962	13.15	0.000	1.239011	1.675583
_cons	37.42901	1.828191	20.47	0.000	33.82719	41.03083

Modified

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Video 3: Linear Regression Adjusted for Covariates

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height	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age	1.457297	.1107962	13.15	0.000	1.239011	1.675583
_cons	37.42901	1.828191	20.47	0.000	33.82719	41.03083

$$Y = \beta_0 + \beta_1 X_1 + \varepsilon$$

$$E(\text{height} \mid \text{Age}) = \beta_0 + \beta_1(\text{age})$$

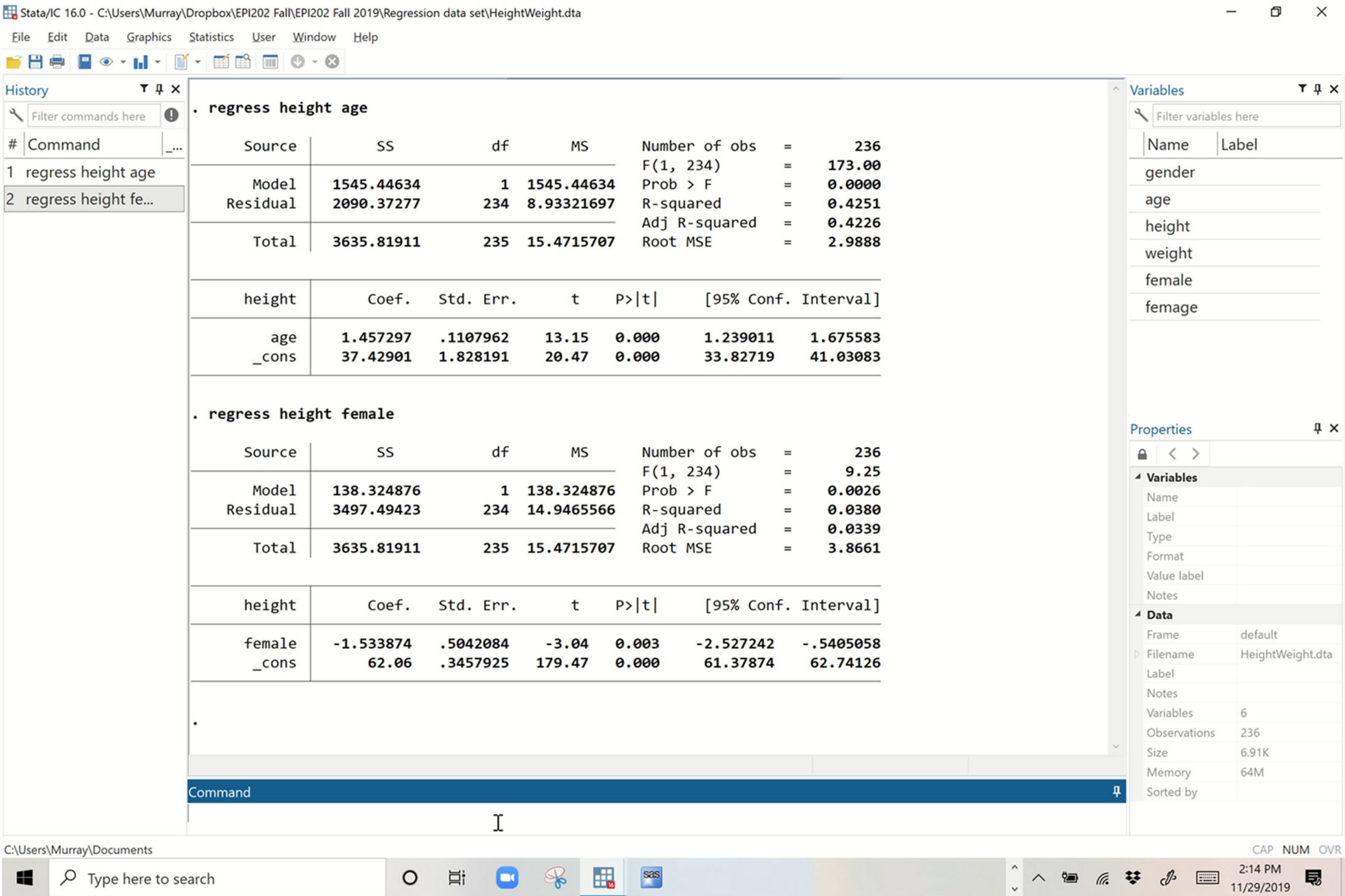
$$E(\text{height} \mid \text{Age}) = 37.43 + 1.46\,(\text{age})$$

$$E(\text{height} \mid 15) = 37.43 + 1.46\,(15)$$

female Age height

female Age height

height	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
female	-1.533874	.5042084	-3.04	0.003	-2.527242	-.5405058
_cons	62.06	.3457925	179.47	0.000	61.37874	62.74126




```
. regress height age

      Source |       SS           df       MS      Number of obs   =       236
-------------+----------------------------------   F(1, 234)       =    173.00
       Model |  1545.44634         1  1545.44634   Prob > F        =    0.0000
    Residual |  2090.37277       234  8.93321697   R-squared       =    0.4251
-------------+----------------------------------   Adj R-squared   =    0.4226
       Total |  3635.81911       235  15.4715707   Root MSE        =    2.9888

------------------------------------------------------------------------------
      height |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   1.457297   .1107962    13.15   0.000     1.239011    1.675583
       _cons |   37.42901   1.828191    20.47   0.000     33.82719    41.03083
------------------------------------------------------------------------------

. regress height female

      Source |       SS           df       MS      Number of obs   =       236
-------------+----------------------------------   F(1, 234)       =      9.25
       Model |  138.324876         1  138.324876   Prob > F        =    0.0026
    Residual |  3497.49423       234  14.9465566   R-squared       =    0.0380
-------------+----------------------------------   Adj R-squared   =    0.0339
       Total |  3635.81911       235  15.4715707   Root MSE        =    3.8661

------------------------------------------------------------------------------
      height |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -1.533874   .5042084    -3.04   0.003    -2.527242   -.5405058
       _cons |      62.06   .3457925   179.47   0.000     61.37874    62.74126
------------------------------------------------------------------------------

.
```

height	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age	1.465677	.1068379	13.72	0.000	1.255185	1.676169
female	-1.627297	.3758752	-4.33	0.000	-2.367845	-.8867485
_cons	38.0569	1.768544	21.52	0.000	34.57252	41.54128

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Video 4: Linear Regression with Interaction Terms to Account for Effect Measure Modification

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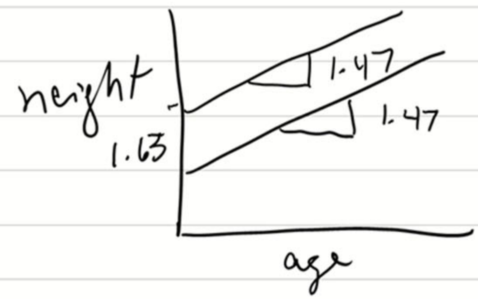
$$\text{height} = 38.06 + 1.47(\text{age}) - 1.63(\text{female})$$



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File Edit Data Graphics Statistics User Window Help

History

#	Command
1	regress height age
2	regress height fe...
3	regress height ag...

```
. regress height female

      Source |       SS           df       MS      Number of obs   =       236
-------------+----------------------------------   F(1, 234)       =      9.25
       Model |  138.324876         1  138.324876   Prob > F        =    0.0026
    Residual |  3497.49423       234  14.9465566   R-squared       =    0.0380
-------------+----------------------------------   Adj R-squared   =    0.0339
       Total |  3635.81911       235  15.4715707   Root MSE        =    3.8661

------------------------------------------------------------------------------
      height |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -1.533874   .5042084    -3.04   0.003    -2.527242   -.5405058
       _cons |      62.06   .3457925   179.47   0.000     61.37874    62.74126
------------------------------------------------------------------------------

. regress height age female

      Source |       SS           df       MS      Number of obs   =       236
-------------+----------------------------------   F(2, 233)       =    102.43
       Model |  1701.08304         2  850.541519   Prob > F        =    0.0000
    Residual |  1934.73607       233  8.30358829   R-squared       =    0.4679
-------------+----------------------------------   Adj R-squared   =    0.4633
       Total |  3635.81911       235  15.4715707   Root MSE        =    2.8816

------------------------------------------------------------------------------
      height |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   1.465677   .1068379    13.72   0.000     1.255185    1.676169
      female |  -1.627297   .3758752    -4.33   0.000    -2.367845   -.8867485
       _cons |    38.0569   1.768544    21.52   0.000     34.57252    41.54128
------------------------------------------------------------------------------

.
```

Variables

Name	Label
gender	
age	
height	
weight	
female	
femage	

Properties

Variables	
Name	
Label	
Type	
Format	
Value label	
Notes	
Data	
Frame	default
Filename	HeightWeight.dta
Label	
Notes	
Variables	6
Observations	236
Size	6.91K
Memory	64M
Sorted by	

Command

C:\Users\Murray\Documents

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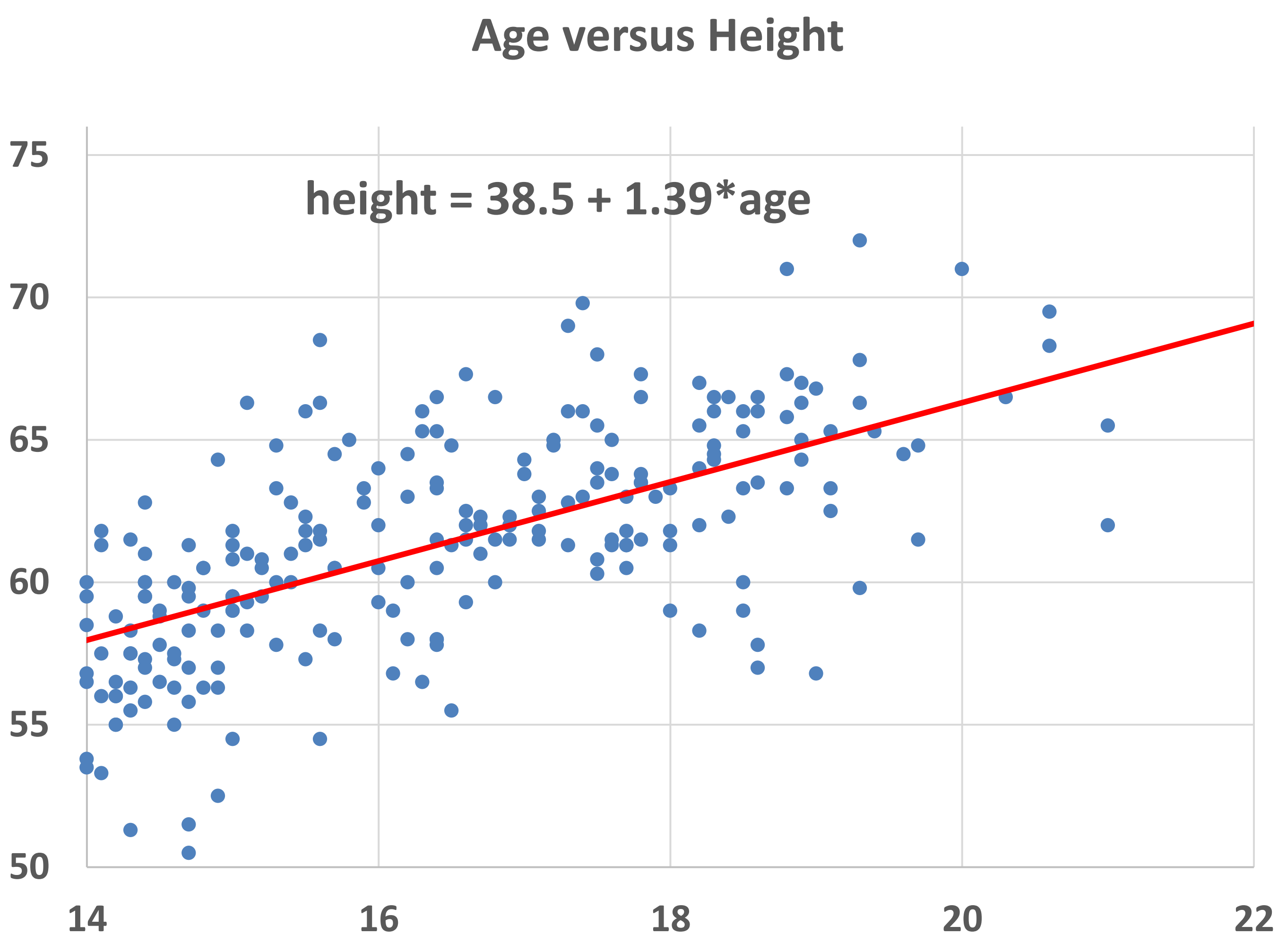
height	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age	1.918139	.1449452	13.23	0.000	1.632562	2.203716
female	13.31463	3.393413	3.92	0.000	6.628783	20.00047
femage	-.9106005	.2056254	-4.43	0.000	-1.315732	-.5054687
_cons	30.64702	2.386661	12.84	0.000	25.94473	35.34932

$$height = \beta_0 + \beta_1(age) + \beta_2(female) + \beta_3(age * female)$$

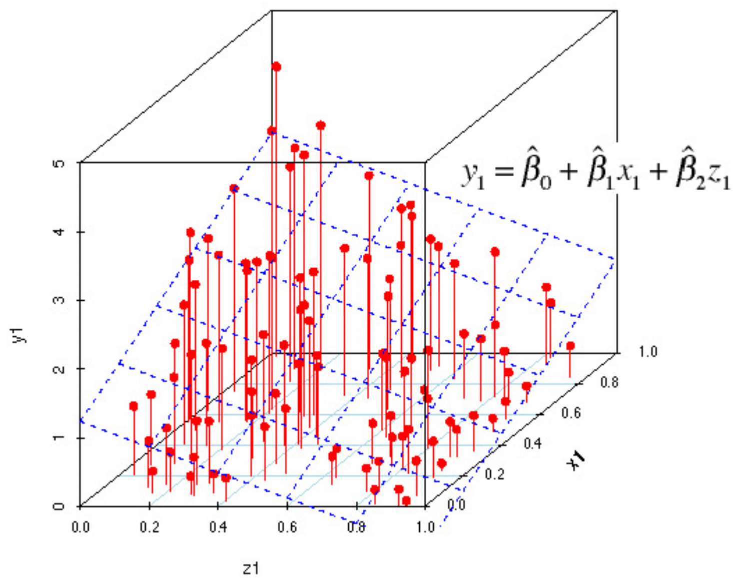
Linear Regression



Multiple Linear Regression

- Multiple linear regression models predict a continuous dependent (outcome) variable y from continuous and categorical independent variables x_i:

$$E[Y|X]=\beta_0+\beta_1 x_1 \ldots \beta_i x_i$$



- The regression line is the best-fit line through the points in the data
  - □ β_0, β_1, …, β_k are parameters
  - □ X_1, X_2, …,X_k are known constants
  - □ β_k = change in average outcome (difference in mean outcome) per unit change in Xi holding all other X's constant

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