Test of No Exposure-Disease Association

$$Z^2 = \frac{\left[X - \hat{E}(X \mid H_0)\right]^2}{\hat{Var}(X \mid H_0)} \sim \chi_1^2$$

	H_0	X	$\hat{E}(X \mid H_0)$	$\hat{Var}(X \mid H_0)$	
Methods for Count Data (closed cohort and cross-sectional studies)					
E $\bar{E}$ cases a b M_1 Non-cases N_1-a N_0-b M_0 N_1 N_0 T	$C_1=C_o$ $C_1/C_o=1$ $C_1-C_o=0$	a	$\frac{N_1M_1}{T}$	$\frac{M_1M_0N_1N_0}{T^3}$	Unstratified
		$\sum a_i$	$\sum \frac{N_{1i}M_{1i}}{T_i}$	$\sum \frac{M_{1i}M_{0i}N_{1i}N_{0i}}{T_i^3}$	Stratified
Methods for Person-Time Data (open cohort and closed cohort studies)					
E $\bar{E}$ cases a b M_1 PT N_1 N_0 T	$I_1=I_o$ $I_1/I_o=1$ $I_1-I_o=0$	a	$\frac{N_1M_1}{T}$	$\frac{N_1N_0M_1}{T^2}$	Unstratified
		$\sum a_i$	$\sum \frac{N_{1i}M_{1i}}{T_i}$	$\sum \frac{N_{1i}N_{0i}M_{1i}}{T_i^2}$	Stratified
Methods for Case-control Data					
E $\bar{E}$ cases a b M_1 controls c d M_0 N_1 N_0 T	OR=1 $I_1/I_o=1$	a	$\frac{N_1M_1}{T}$	$\frac{M_1M_0N_1N_0}{T^2(T-1)}$	Unstratified
		$\sum a_i$	$\sum \frac{N_{1i}M_{1i}}{T_i}$	$\sum \frac{M_{1i}M_{0i}N_{1i}N_{0i}}{T_i^2(T_i-1)}$	Stratified

Test of Homogeneity of Effect Measures

$$H = \sum_{i=1}^{I} \frac{[\hat{X}_i - \hat{X}_{summary}]^2}{\hat{Var}_i[\hat{X}_i]} \sim \chi^2_{I-1}$$

	H_0	$\hat{X}_i$	$\hat{X}_{summary}$	$\hat{Var}_i(\hat{X}_i)$
Methods for Count Data (closed cohort and cross-sectional studies)				
Difference measure	$CID_1=CID_2=\ldots=CID_i$ $CID_i=CID_j$ for all i,j	CID_i	$CID_{summary}$	$\frac{a_i c_i}{N_{1i}^3}+\frac{b_i d_i}{N_{0i}^3}$
Ratio measure	$CIR_1=CIR_2=\ldots=CIR_i$ $CIR_i=CIR_j$ for all i,j	$\ell n\,(CIR_i)$	$\ell n(CIR_{MH})$	$\frac{c_i}{a_i N_{1i}}+\frac{d_i}{b_i N_{0i}}$
Methods for Person-Times Data (open cohort and closed cohort studies)				
Difference measure	$IRD_1=IRD_2=\ldots=IRD_i$ $IRD_i=IRD_j$ for all i,j	IRD_i	$IRD_{summary}$	$\frac{a_i}{N_{1i}^2}+\frac{b_i}{N_{0i}^2}$
Ratio measure	$IRR_1=IRR_2=\ldots=IRR_i$ $IRR_i=IRR_j$ for all i,j	$\ell n(IRR_i)$	$\ell n(IRR_{MH})$	$\frac{1}{a_i}+\frac{1}{b_i}$
Methods for Case-control Data				
Ratio measure	$OR_1=OR_2=\ldots=OR_i$ $OR_i=OR_j$ for all i,j	$\ell n(OR_i)$	$\ell n(OR_{MH})$	$\frac{1}{a_i}+\frac{1}{b_i}+\frac{1}{c_i}+\frac{1}{d_i}$

Relative Excess Risk due to Interaction

$$RERI = CIR_{11}-CIR_{10}-CIR_{01}+1$$

E-value

$$\text{E-value} = RR + \sqrt{RR(RR-1)}$$

If RR < 1, take the reciprocal before computing the E-value

Confidence Intervals for Ratio and Difference Measures

$$X \pm Z_{1-\alpha/2}\sqrt{\hat{Var}(X)}$$

	X	w_i	$\hat{Var}(X)$
Methods for Count Data (closed cohort and cross-sectional studies)			
Cumulative incidence difference	$\frac{a}{N_1} - \frac{b}{N_0}$	--	$\frac{ac}{N_1^3} + \frac{bd}{N_0^3}$
Summary cumulative incidence difference	$\frac{\sum w_i\left[\frac{a_i}{N_{1i}} - \frac{b_i}{N_{0i}}\right]}{\sum w_i}$	$\frac{N_{1i}N_{0i}}{T_i}$	$\frac{\sum\left(\frac{a_i c_i N_{0i}^2}{T_i^2(N_{1i}-1)} + \frac{b_i d_i N_{1i}^2}{T_i^2(N_{0i}-1)}\right)}{\left(\sum\frac{N_{1i}N_{0i}}{T_i}\right)^2}$
Cumulative incidence ratio (ℓn)	$\ell n\left\{\frac{a}{N_1}\Big/\frac{b}{N_0}\right\}$	--	$\frac{c}{aN_1} + \frac{d}{bN_0}$
Summary cumulative incidence ratio (ℓn)	$\ell n\left\{\frac{\sum w_i\left[\frac{a_i}{N_{1i}}\Big/\frac{b_i}{N_{0i}}\right]}{\sum w_i}\right\}$	$\frac{b_i N_{1i}}{T_i}$	$\frac{\sum\left(M_{1i}N_{1i}N_{0i} - a_i b_i T_i\right)/T_i^2}{\left[\sum\frac{a_i N_{0i}}{T_i}\right]\left[\sum\frac{b_i N_{1i}}{T_i}\right]}$
Methods for Person-Time Data (open cohort and closed cohort studies)			
Rate difference	$\frac{a}{N_1} - \frac{b}{N_0}$	--	$\frac{a}{N_1^2} + \frac{b}{N_0^2}$
Summary rate difference	$\frac{\sum w_i\left[\frac{a_i}{N_{1i}} - \frac{b_i}{N_{0i}}\right]}{\sum w_i}$	$\frac{N_{1i}N_{0i}}{T_i}$	$\frac{\sum\left(\frac{a_i N_{0i}^2 + b_i N_{1i}^2}{T_i^2}\right)}{\left(\sum\frac{N_{1i}N_{0i}}{T_i}\right)^2}$
Rate ratio (ℓn)	$\ell n\left\{\frac{a}{N_1}\Big/\frac{b}{N_0}\right\}$	--	$\frac{1}{a} + \frac{1}{b}$
Summary rate ratio (ℓn)	$\ell n\left\{\frac{\sum w_i\left[\frac{a_i}{N_{1i}}\Big/\frac{b_i}{N_{0i}}\right]}{\sum w_i}\right\}$	$\frac{b_i N_{1i}}{T_i}$	$\frac{\sum\left(M_{1i}N_{1i}N_{0i}\right)/T_i^2}{\left[\sum\frac{a_i N_{0i}}{T_i}\right]\left[\sum\frac{b_i N_{1i}}{T_i}\right]}$
Methods for Case-control Data			
Odds ratio (ℓn)	$\ell n\left\{\frac{ad}{bc}\right\}$	--	$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$
Summary odds ratio (ℓn)	$\ell n\left\{\frac{\sum w_i\frac{a_i d_i}{b_i c_i}}{\sum w_i}\right\}$	$\frac{b_i c_i}{T_i}$	RGB variance

Computational Form of the Mantel Haenszel Estimators

Methods for Count Data (closed cohort and cross-sectional studies)

Summary cumulative incidence ratio

$$\hat{CIR}_{MH} = \frac{\sum_{i=1}^{I} \frac{a_i N_{0i}}{T_i}}{\sum_{i=1}^{I} \frac{b_i N_{1i}}{T_i}}$$

Summary cumulative incidence difference

$$\frac{\sum \left(\frac{a_i N_{0i} - b_i N_{1i}}{T_i} \right)}{\sum \frac{N_{1i} N_{0i}}{T_i}}$$

Methods for Person-Time Data (open cohort and closed cohort studies)

Summary rate ratio

$$\hat{IRR}_{MH} = \frac{\sum_{i=1}^{I} \frac{a_i N_{0i}}{T_i}}{\sum_{i=1}^{I} \frac{b_i N_{1i}}{T_i}}$$

Summary rate difference

$$\frac{\sum \left(\frac{a_i N_{0i} - b_i N_{1i}}{T_i} \right)}{\sum \frac{N_{1i} N_{0i}}{T_i}}$$

Methods for Case-control Data

Summary odds ratio

$$\hat{OR}_{MH} = \frac{\sum_{i=1}^{I} \frac{a_i d_i}{T_i}}{\sum_{i=1}^{I} \frac{b_i c_i}{T_i}}$$

Formulae for 1:1 matched case-control studies:

		Controls: E	Controls: $\bar{E}$
Cases: E		f_{11}	f_{10}
Cases: $\bar{E}$		f_{01}	f_{00}

$OR_{MH} = f_{10} / f_{01}$

Test statistic:
H_O: $OR_{MH} = 1$
H_A: $OR_{MH} \neq 1$

$$Z^2 = \frac{(f_{10} - f_{01})^2}{f_{10} + f_{01}} \sim \chi_1^2$$

Variance formula for confidence interval:

$$Var\left[ln(OR_{MH})\right] = \frac{1}{f_{10}} + \frac{1}{f_{01}}$$

Computational Form for Standardization

Indirectly standardized rate ratio: $$SMR = \frac{\sum a_i}{\sum (N_{1i}) I_{0i}}$$

Directly standardized rate ratio: $$SRR = \frac{\sum (N_{0i}) I_{1i}}{\sum b_i}$$

Indirectly standardized rate difference: $$SMD = \frac{\sum a_i}{\sum N_{1i}} - \frac{\sum (N_{1i})(I_{0i})}{\sum N_{1i}}$$

Directly standardized rate difference: $$SRD = \frac{\sum (N_{0i})(I_{1i})}{\sum N_{0i}} - \frac{\sum b_i}{\sum N_{0i}}$$

Misclassification Correction

Observed

	E	$\bar{E}$	
cases	**A**	**B**	m_1
controls	**C**	**D**	m_0
	N_1	N_0	**T**

Truth

	E	$\bar{E}$	
cases	**a**	**b**	m_1
controls	**c**	**d**	m_0
	N_1	N_0	**T**

Truth

	E	$\bar{E}$	
cases	$a = \frac{\text{specificity} * m_1 - B}{\text{sensitivity} + \text{specificity} - 1}$	$b = \frac{\text{sensitivity} * m_1 - A}{\text{sensitivity} + \text{specificity} - 1}$	m_1
controls	$c = \frac{\text{specificity} * m_0 - D}{\text{sensitivity} + \text{specificity} - 1}$	$d = \frac{\text{sensitivity} * m_0 - C}{\text{sensitivity} + \text{specificity} - 1}$	m_0
	N_1	N_0	**T**