Department of Economics - ESG-UQAM
Topics in Macroeconomics

Problem Set 3 - Local Linear Methods for Solving Macroeconomic Models

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Instructions:

Answer all questions. Show all steps and include any code or plots you use. You can use any programming language you want (Python, Julia, Matlab, R, etc.). Please submit your answers in a single PDF file. You can use Overleaf or any other LaTeX editor to write your answers.

Question 1 - New Keynesian Model

Consider the following log-linearised New Keynesian model:

$$\pi_t = \kappa x_t + \beta E_t[\pi_{t+1}], \qquad x_t = E_t[x_{t+1}] - \frac{1}{\gamma}(i_t - E_t[\pi_{t+1}]) + \varepsilon_t^x,$$

where π_t is inflation, x_t the output gap, i_t the nominal interest rate, and ε_t^x an IID shock.

a) Set the policy rule as $i_t = \delta\pi_t + \varepsilon_t$. Prove that $\delta > 1$ is necessary and sufficient for a unique stable equilibrium (Taylor principle). Explain the intuition.

b) Suppose the rule is $i_t = \delta_\pi \pi_t + \delta_x x_t + \varepsilon_t$. Derive the conditions for a unique stable equilibrium.

Question 2 - Log-linearisation and Solution with Dynare

Consider the following model. Households maximize expected discounted lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t (\log C_t + AL_t).$$

over consumption C_t, leisure L_t, capital K_t, and investment I_t, subject to the budget constraint:

$$C_t + K_t - K_{t-1} = (1-\tau)(R_t - \delta)K_{t-1} + (1-\xi)W_t N_t,$$

where $N_t \equiv 1 - L_t$ denotes hours worked and the evolution of capital is determined by:

$$K_t = (1-\delta)K_{t-1} + I_t,$$

where R_t, W_t denote respectively the price of capital rental and the wage rate. The parameters β and δ denote the discount factor and the depreciation rate of capital respectively. Furthermore, $\tau, \xi \in [0, 1)$ denote tax rates on capital and labor income respectively. Taxes on capital are after depreciation allowances.

The representative firm produces output Y_t, rents capital, hires efficiency units of labor at a rate W_t, and maximizes profits:

$$Y_t - W_t N_t - R_t K_{t-1},$$

in every period, subject to a Cobb–Douglas production function:

$$Y_t = Z_t K_{t-1}^{\alpha} N_t^{1-\alpha},$$

with $0 < \alpha < 1$, and to a technological shock, which evolves according to:

$$\log Z_t = \rho \log Z_{t-1} + \varepsilon_t,$$

with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $0 < \rho < 1$.

Finally, the government has expenditures G_t that are financed by levying taxes on capital and labor income, and maintains a balanced budget:

$$G_t = \tau(R_t - \delta)K_{t-1} + \xi W_t N_t.$$

This model is an extension of Hansen's (1985) model that includes government expenditures and taxation of capital and labor income.

a) Write down the **equations characterizing the competitive equilibrium**.

b) Derive the **steady-state equations**.

c) **Solve the log-linearized version** of the model **using Dynare**. Build a Dynare '.mod' file to:
   - Define parameters $\alpha = 0.36$, $\delta = 0.025$, $\beta = 1/1.01$, $\rho = 0.95$, $\sigma = 0.00712$, $A = 2.5846$;
   - Include capital and labor income taxes (τ and ξ);
   - Linearize and simulate the model around steady state using Dynare's stochastic simulation tools;
   - Report impulse responses and discuss the results.

d) Comment on how changing τ and ξ affects macroeconomic outcomes. What happens when $\tau = \xi = 0$ compared to positive tax rates? **Show this numerically using Dynare.**

Question 3 - A Basic New Keynesian Model (solve in Dynare)

Consider the following baseline New Keynesian (NK) model in (log) deviations from steady state. All variables are demeaned and interpreted as percentage deviations unless otherwise stated.

Equations.

$$\text{IS curve:} \quad x_t = E_t[x_{t+1}] - \frac{1}{\sigma}\left(i_t - E_t[\pi_{t+1}] - r_t^n\right), \tag{1}$$

$$\text{NK Phillips curve:} \quad \pi_t = \beta\, E_t[\pi_{t+1}] + \kappa\, x_t + u_t, \tag{2}$$

$$\text{Monetary policy (Taylor with inertia):} \quad i_t = \rho_i\, i_{t-1} + (1-\rho_i)\,(\phi_\pi\, \pi_t + \phi_x\, x_t) + \varepsilon_t^i, \tag{3}$$

where x_t is the output gap, π_t inflation, i_t the nominal interest rate. The natural-rate shock r_t^n (demand/IS shock) and the cost-push shock u_t follow AR(1) processes:

$$r_t^n = \rho_r\, r_{t-1}^n + \varepsilon_t^r, \qquad u_t = \rho_u\, u_{t-1} + \varepsilon_t^u. \tag{4}$$

Parameters. $\sigma > 0$ (inverse IES), $\beta \in (0,1)$, $\kappa > 0$, $\rho_i \in [0,1)$, $\phi_\pi > 0$, $\phi_x \geq 0$, $\rho_r, \rho_u \in [0,1)$. Shocks $\varepsilon_t^r, \varepsilon_t^u, \varepsilon_t^i$ are i.i.d. mean zero with standard deviations $\sigma_r, \sigma_u, \sigma_i$.

Calibration (baseline). Use:

$$\beta = 0.99, \quad \sigma = 1, \quad \kappa = 0.10, \quad \rho_i = 0.7, \quad \phi_\pi = 1.5, \quad \phi_x = 0, \quad \rho_r = \rho_u = 0.8,$$

and shock standard deviations:

$$\sigma_r = 0.20, \quad \sigma_u = 0.10, \quad \sigma_i = 0.10.$$

(a) **Model block (Dynare).** Create a Dynare `.mod` file that declares the variables x, π, i, r_n, u and shocks $\varepsilon^r, \varepsilon^u, \varepsilon^i$. Enter the IS, NKPC, Taylor rule, and AR(1) laws for r_t^n and u_t. Use the calibration above.

(b) **Steady state and checks.** Explain (briefly) why the deterministic steady state is $(x, \pi, i, r^n, u) = (0, 0, 0, 0, 0)$ in the linear model. In Dynare, run `steady;` and `check;` and report whether Blanchard–Kahn (BK) conditions hold.

(c) **Impulse responses.** Using `stoch_simul(order=1, irf=20)`, generate IRFs of $\{x_t, \pi_t, i_t\}$ to one-standard-deviation innovations in (i) the natural-rate shock ε_t^r, (ii) the cost-push shock ε_t^u, and (iii) the policy shock ε_t^i. Comment briefly on signs and persistence.

(d) **Determinacy vs. indeterminacy (Taylor principle).** Vary ϕ_π while keeping other parameters fixed:

$$\phi_\pi \in \{0.8,\ 1.1,\ 1.5\}.$$

For each case, re-run `check;` and record whether BK conditions are satisfied. Discuss how IRFs change when $\phi_\pi < 1$ vs. $\phi_\pi > 1$.

(e) **Role of interest-rate smoothing.** Set $\rho_i \in \{0,\ 0.7,\ 0.9\}$ with $\phi_\pi = 1.5$. Compare IRFs to ε_t^i. Explain how higher ρ_i affects the speed and amplitude of inflation/output-gap responses.

(f) **Cost-push vs. demand shocks.** With $\phi_\pi = 1.5$, $\rho_i = 0.7$, compare IRFs to ε_t^u vs. ε_t^r. Which shock produces a larger inflation-output trade-off? Relate your explanation to the NKPC and IS curve.

(g) **Sensitivity to slope of NKPC.** Let $\kappa \in \{0.05,\ 0.10,\ 0.20\}$ (keeping baseline elsewhere). How do inflation and output-gap IRFs change after a policy shock ε_t^i? Give a short intuition linking price stickiness (via κ) to inflation dynamics.

(h) **Welfare loss metric.** Define a period loss $L_t = \pi_t^2 + \lambda_x x_t^2$ with $\lambda_x = 0.25$. For each shock type, compute and compare the 20-period cumulative loss under $(\phi_\pi, \rho_i) = (1.5, 0.7)$ vs. $(0.8, 0.7)$. Briefly interpret.

Notes.

- This is a linear model, so the steady state is at zero for demeaned variables, and `order=1` suffices.
- If BK fails (e.g., $\phi_\pi < 1$), Dynare will warn about indeterminacy—document this and proceed to discuss.

Appendix: Minimal Dynare template (starter)

```
var x pi i r_n u;
varexo e_r e_u e_i;

parameters beta sigma kappa rhoi phi_pi phi_x rho_r rho_u
         sig_r sig_u sig_i;

beta=0.99; sigma=1; kappa=0.10;
rhoi=0.7; phi_pi=1.5; phi_x=0;
rho_r=0.8; rho_u=0.8;
sig_r=0.20; sig_u=0.10; sig_i=0.10;

model(linear);
  // IS
  x = x(+1) - (1/sigma)*( i - pi(+1) - r_n );
  // NKPC
```

```
  pi = beta*pi(+1) + kappa*x + u;
  // Taylor
  i = rhoi*i(-1) + (1-rhoi)*( phi_pi*pi + phi_x*x ) + e_i;
  // Shock processes
  r_n = rho_r*r_n(-1) + e_r;
  u   = rho_u*u(-1) + e_u;
end;

initval; x=0; pi=0; i=0; r_n=0; u=0; end;

shocks;
  var e_r; stderr sig_r;
  var e_u; stderr sig_u;
  var e_i; stderr sig_i;
end;

steady; check;
stoch_simul(order=1, irf=20);
```