Department of Economics - Sciences Po
Macroeconomics I

Problem Set 5

Professors: Nicolas Coeurdacier & Diego Rodrigues
TA: Aymeric Ortmans

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Question 1

Consider the following **Labor Search Model.** In this model the idea is that the agent is either employed or unemployed in each period of time. The unemployed worker will receive an unemployment insurance, which we will denote by $b > 0$ and receive job offers. There is a trade-off, since by accepting the job offer the worker receives the wage but lose the opportunity to receive new offers. First we will consider the model where the time is discrete. The parameters of the model will be the following ones:

- β: this is the discount factor, which will be given by $\frac{1}{1+r}$, where r is the discount rate. Notice that when $\beta = 1$ the agent is indifferent between *today* and *tomorrow* and when $\beta = 0$ the agent only cares about *today.*
- b: this will be the unemployment insurance.
- α: this is the probability of receiving an offer when unemployed.
- $F(w)$ represents the i.i.d. distribution of wages in this economy.

a) Write the agent's problem when employed and the agent's problem when unemployed.

b) Find the reservation wage, $\overline{w}$.

c) How does the reservation wage changes when we increase b or α or β?

d) Now assume that there is a possibility to the agent be fired. Denote this probability by λ, which will be exogenous. Rewrite the agent's problem when employed and when unemployed.

e) What is the reservation wage, $\overline{w}$, in this new scenario?

f) What is the effect of an increase in the probability to be fired in the reservation wage you found in the previous item?

g) Now let the probability to be fired goes to 1. What is the relationship between the reservation wage and the unemployment insurance? Do you have an intuition for this result?

h) Observe that in the steady state the number of people leaving unemployment must be the same number of people becoming unemployed. Denote by u the unemployment rate in the steady state. Find this value as a function of the parameters of the model, considering the case where there is a possibility to the agent be fired.

i) Now assume that $\beta = \dfrac{1}{1 + r\Delta}$, where Δ represents a time interval. Notice that in the cases we worked previously (discrete) we had $\Delta = 1$. From now on we are interested in cases where $\Delta \to 0$. Also observe that in the continuous case, there will be a change in the probability of receiving new offers, which now will be given by $\alpha\Delta$ (i.e., this can be interpreted as the process's average to generate new offers). Observe that in this new case the wage per time will be Δw and the unemployment insurance will be Δb. Given this, derive the agent's problem when employed and unemployed (assume the agent cannot be fired once employed.)

j) Take the case when $\Delta \to 0$, what are the equations of the employed and the unemployed agents in continuous time.

k) Now assume there is a possibility λ to be fired. Rewrite the problem of the employed and the unemployed worker in this new scenario.

l) Now assume the unemployment agent can exert an effort $g(\alpha)$, which will bring disutility but will increase the probability of receiving an offer. Let $g'(\alpha) > 0$ and $g''(\alpha) > 0$. By doing this we will have that α is endogenous in the model. Assume also that there is still a probability λ of losing the job. Find the equation that represents the optimal level of effort.

m) What is the reservation wage, $\overline{w}$, in this model? What happens with the reservation wage when we increase α?

n) Will there be a unique pair $(\overline{w}, \alpha)$ that solves this problem? You can argue graphically or throguh the equations you found in the previous two parts.

o) Now assume the employed agents can receive new job offers. Denote this probability for α_0. Assume also that there is a probability for the employed agent to be fired and this value continues to be λ. The unemployed agents on the other hand, receive new offers with a probability α_1. In this case the equations that represent the payoff of the individuals in continuous time are given by:

$$\begin{array}{c} rW(w) = w + \alpha_1 \int_w^\infty \left[W\left(w'\right) - W(w)\right] dF\left(w'\right) + \lambda[U - W(w)] \\ rU = b + \alpha_0 \int_{\overline{w}}^\infty \left[W\left(w'\right) - U\right] dF\left(w'\right) \\ rW(\overline{w}) = rU = \overline{w} \end{array} \tag{1}$$

Find the reservation wage and discuss how it depends on the relation between α_0 and α_1.

Question 2

Suppose the wage distribution is Pareto

$$F(w) = 1 - \left(\frac{\alpha}{w}\right)^\beta \qquad \alpha, \beta > 0$$

Note that $w \geq \alpha$ and assume $\beta > 1$, which ensures that there is a well-defined mean.

a) Derive the mean of the distribution and the conditional mean for $w \geq \xi$ and compare them.

b) Assume there there is unemployment benefit $b \geq 0$ and job offers arrive at the rate a. Show that the optimal stopping rule is a reservation wage. Use the reservation wage in the value of unemployment to derive analytically what the option value of unemployment is. Explain intuitively why there is an option value.

c) Derive the impact of a change in the arrival of job offers, a, on the hazard rate

$$H = a\left(1 - F(\xi)\right)$$

(the hazard rate is the probability of leaving unemployment). Explain your result.

d) Suppose that all workers have the same reservation wage and so the firms that offer wages below it exit the market. Derive the mean of the wage distribution and the hazard rate in terms of the parameters of the model and show that the mean depends both on the level of unemployment compensation and the offer arrival rate.

Question 3

Learning on the job. An unemployed worker receives UI benefit b and job offers arrive at rate a from a stationary distribution $F(w)$. Once the job is accepted wages are expected to grow at rate γ forever, where $r > \gamma > 0$, because of learning on the job. Derive the reservation wage and show that it decreases in γ. Explain why.

Question 4

Economic Growth. Suppose now the worker is searching in an economy that is growing at rate $\gamma > 0$ independently of what he does, i.e. b and all wage offers are growing at rate γ. Assume that at each time t the wage distribution is $F(w)$, which is independent of time, but the wage rate drawn is $we^{\gamma t}$. Derive again the reservation wage and show that it now increases in γ. Explain your reasons and compare with your results in question 2. (Hint: note that in steady state asset values also grow at the economy's growth rate. Treat the rate of growth of the asset value of unemployment as a capital gain accruing to the unemployed; i.e. assume $\dot{U} = \gamma U$.)