Department of Economics - Sciences Po
Macroeconomics I

Problem Set 4

Professors: Nicolas Coeurdacier & Diego Rodrigues
TA: Aymeric Ortmans

Fall 2022

Question 1 – Growth through externalities

Based on Romer (1986): Increasing Returns and Long-Run Growth.

The basic idea is that knowledge is accumulated as a by-product of capital accumulation, it is external to the firm. Assume that there is no population growth, $L = 1$. Assume further that there are a continuum of firms indexed by $i \in [0, 1]$, which operate in perfect competition. Each firm has the following production function:

$$Y_{i,t} = F(K_{i,t}, A_t L_{i,t}),$$

where F is a neoclassical production function, common to all firms; A_t is common to all firms, it is non-rival and non-excludable; $K_{i,t}$ and $L_{i,t}$ are capital and labor rented by firm i at time t.

a. What are the capital and labor market clearing conditions? (Denote aggregate capital by K_t and aggregate labor by L.)

b. Express the first order conditions of firm i using $\widehat{k}_{i,t} \equiv \frac{K_{i,t}}{A_t L_{i,t}}$ and the wage rate and the rental rate of capital! Given that there is a single rental rate for capital, can firms have different $\widehat{k}_{i,t}$? What does this imply about the wage rate and the rental rate?

Learning-by-doing assumption: technology improves as more and more capital is accumulated:

$$A_t = BK_t,$$

where $B > 0$ is an exogenous parameter. This implies that A_t can grow continuously at the economy level. We assume that firms take A_t as exogenous, even though it is endogenous for the economy as a whole.

c. Using what you found in point b. and the CRS property of the function F, show that output for the entire economy can be written as:

$$Y_t = F(K_t, BK_t L).$$

d. Again using that F is CRS, express output per capital, $\frac{Y_t}{K_t}$, as a function of B and L, and denote $\widetilde{f}(BL) \equiv F(1, BL)$.

e. Express output per person as a function of $\widetilde{f}$ and capital per person, k_t. What does this imply about returns to capital in the economy as a whole?

f. Express the market clearing wage and rental rate of capital as a function of $\widetilde{f}$, B, L and K_t. What can you say about the rental rate?

The consumer side is exactly the same as in the NGM, i.e. consumer optimality is guaranteed by the usual transversality condition and the usual Euler equation, and owners of the capital stock receive $r_t = R_t - \delta$ returns. The Euler equation is: $\frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r_t - \rho)$.

g. Using what you found in point f. what can you say about the growth rate of consumption (per person), g_C?

h. Now use the resource constraint of the aggregate economy to express the growth rate of capital, g_K!

i. Analyse the three possible cases of 1. $g_K = g_C$, 2. $g_K > g_C$, 3.$g_K < g_C$. Which is compatible with balanced growth, which violates the transversality condition, and which leads to zero capital?

j. Is there a scale effect in this economy? Explain why and what it implies.

k. Do you expect the decentralized equilibrium to be Pareto optimal? If not, what is the source of externality?

l. Set up the planner's problem. Use $\widetilde{f}$ in the constraint!

m. Solve the planner's problem by setting up the present value Hamiltonian, derive the FOCs and define the transversality condition. What is the growth rate of consumption and output in this economy? How does it compare to the growth rate in the decentralized economy?

Question 2 – Scale effects

Question 6.6 from the Barro and Sala-i-Martin book.

a. Why does the varieties model of technological change from section 6.1 exhibit a scale effect in the sense that the growth rate rises with the aggregate quantity of labor, L? Is it reasonable to identify L empirically with a country's population?

b. What happens in this model if population, L, grows at a constant positive rate?

c. What types of modifications to the model would eliminate the scale effects?