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Department of Economics - Sciences Po
Macroeconomics I

Problem Set 3

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Question 1 – Social security in the Diamond model
Question 2.16 from the Romer book.
Consider the canonical OLG model from the lectures, where production is Cobb-Douglas, there is no technological progress, population grows at rate n, and utility is logarithmic.

a. **Pay-as-you-go social security.** Suppose the government taxes each young individual an amount T and uses the proceeds to pay benefits to the old individuals; thus each old person receives $(1+n)T$.

   i. How, if at all, does this change affect the equation which gives k_{t+1} as a function of k_t?

   ii. How, if at all, does this change affect the balanced growth path value of k?

   iii. If the economy is initially on a balanced growth path that is dynamically efficient, how does a marginal increase in T affect the welfare of current and future generations? What happens if the initial balanced growth path is dynamically inefficient?

b. **Fully funded social security.** Suppose the government taxes each young person an amount T and uses the proceeds to purchase capital. Individuals born at t therefore receive $(1+r_{t+1})T$ when they are old.

   i. How, if at all, does this change affect the equation which gives k_{t+1} as a function of k_t?

   ii. How, if at all, does this change affect the balanced growth path value of k?

Question 2 – The basic OLG model
Question 2.17 from the Romer book.
(This follows Samuelson, 1958, and Allais, 1947.) Suppose, as in the Diamond model, that L_t two-period lived individuals are born in period t and that $L_t = (1+n)L_{t-1}$. For simplicity, let utility be logarithmic with no discounting: $U_t = \ln c_{1,t} + \ln c_{2,t+1}$.

The production side of the economy is simpler than in the Diamond model. Each individual born at time t is endowed with A units of the economy's single good. The good can be either consumed or stored. Each unit stored yields $\chi > 0$ units of good in the following period.

Finally, assume that in the initial period, period 0, in addition to the $L(0)$ young individuals each endowed with A units of the good, there are $L(0)/(1+n)$ individuals, who are alive only in period 0. Each of these 'old' individuals is endowed with some amount Z of the good; their utility is simply their consumption in the initial period, $C_{2,0}$.

a. Describe the decentralized equilibrium of the economy. (Hint: Given the overlapping generations structure, will the members of any generation engage in transactions with members of another generation?)

b. Consider paths where the fraction of agents' endowments that is stored is constant over time, and denote it by f. What is the total consumption (that is, consumption of all the young plus consumption of all the old) per person on such a path as a function of f? If $\chi < 1+n$, what value of f satisfying $0 \leq f \leq 1$ maximizes consumption per person? Is the decentralized equilibrium Pareto-efficient in this case? If not, how can a social planner raise welfare?