# Department of Economics - Sciences Po
# Macroeconomics I

## Problem Set 1

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**Question 1 – Taking the Solow Model to the Data**

We are now going to solve the canonical Solow in discrete time and try to see if there is hope to explain the income distribution across countries with the model. We know that without population growth, there will be no growth in the long-run. Hence, we will not be able to see anything about the time-series behavior (after all: most countries did grow in the last 50 years). Hence, we focus on the cross-section of countries in the year 2010.

Consider the standard Solow environment. Time is discreet, the saving rate is $s$, production in country $i$ is given by:

$$Y_{i,t} = F(K_{i,t}, Z_i L_i)$$

where $F$ is a neoclassical production function and $Z_i$ is a labor-augmenting technology term. Note that $L_i$ and $Z_i$ are assumed to be constant over time but allowed to be different for different countries. Capital depreciates at rate $\delta$. Both the saving rate and the rate of depreciation is assumed to be equal across countries.

a. State the accumulation equation for capital-per-capita $k_{i,t} = \frac{K_{i,t}}{L_i}$ (i.e. the equation for $k_{i,t+1}$) and express the level of output per worker $y_{i,t}$, wages $w_{i,t}$ and capital returns $R_{i,t}$ as a function of $k_{i,t}$ and $Z_i$. How do $(y_{i,t}, w_{i,t}, R_{i,t})$ depend on $Z_i$ and $k_{i,t}$?

b. Derive the condition for the steady-state capital-labor ratio $k_i^*$. How does $k_i^*$ depend on $Z_i$? Taking $k_i^*$ as a function of $Z_i$, how does $y^*, w^*, R^*$ depend on $Z_i$?

c. Now suppose that $F$ takes Cobb-Douglas form:

$$Y_{i,t} = F(K_{i,t}, Z_i L_i) = K_{i,t}^{\alpha}(Z_i L_i)^{1-\alpha}$$

Derive the expression for the steady state capital-labor ratio $k_i^*$. Derive an expression for $\ln y_{i,t}$ as of function of $\ln Z_i, \ln k_{i,t}$. Derive an expression of $\ln y_i^*$ as a function of parameters.

Now let us go to the data. The most important cross-country dataset are the Penn World Tables., find this on the internet, and use the newest release, PWT 9.1. In particular, download the data on real GDP (output-side and expenditure-side real GDP at chained PPPs (in mil. 2011US$)), on real capital stock (capital stock at constant 2011 national prices (in mil. 2011US$)), on population and on employment for the year 2010 for all countries available. From these compute output and capital per person and output and capital per worker. Use the measure which provides the most number of observations.

d. To get a rough idea about the magnitude of income differences across the world, calculate per-capita income relative to the US for the following countries: China, India, France, Vietnam, Nigeria. Report your results or plot them in a graph.

e. Now we are going to test how well the Solow model does to explain the differences in income across the world. We still assume that $Y_t = K_{i,t}^{\alpha}(Z_i L_i)^{1-\alpha}$. Let

$$\ln y_i = \phi(Z_i, \ln k_i)$$

be the relationship between $\ln y_i$ and $Z_i, \ln k_{i,t}$ derived in part c. Suppose that $\alpha = 1/3$.

(i) Assume that technologies are equal across the world, i.e. $Z_i = Z^{Solow}$ for all countries $i$. Then variation in income across countries is fully driven by the variation in the capital labor ratio. Let $Z^{Solow}$ satisfy:

$$(1-\alpha)\ln Z^{Solow} = \frac{1}{N}\sum_{i=1}^{N}\ln y_i - \alpha\frac{1}{N}\sum_{i=1}^{N}\ln k_i$$

where $N$ is the number of countries. The predicted income by the model is

$$\ln \hat{y}_i^{Solow} = \phi(Z^{Solow}, \ln k_i)$$

Plot $\ln \hat{y}_i^{Solow}$ against $\ln y_i$. Does the model do a good job in predicting income differences across the world? Does it over- or underestimate the inequality across countries? What does this tell you about the assumption that $Z_i = Z$?

(ii) Now suppose that we wanted to make the model consistent with data. Given the data on $(\ln y_i, \ln k_i)_i$ find the implied values of log productivity $\ln Z_i$ such that the model fits the data perfectly. Plot $\ln Z_i$ against $\ln y_i$. How does productivity in rich countries compare the one in poor countries? Did you expect this result from your answer in part (i)?

(iii) Up to now we have taken the data on $(k_i)$ as given, i.e. we have not used the model's formula for the steady-state. The steady-state capital-labor ratio you found above follows a relationship:

$$k_i^* = \psi\left(\left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}, Z_i\right)$$

i.e. the variation of capital across countries is informative about productivity differences. Let $\ln\left(\left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}\right)$ satisfy:

$$\ln\left(\left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}\right) = \frac{1}{N}\sum_{i=1}^{N}\ln k_i - \ln Z^{Solow},$$

Use part a. to find $\tilde{Z}_i$ such that $k_i^* = k^i$, i.e. that the observed capital-labor ratios are consistent with a steady state. Now predict per capita income by

$$\ln \hat{y}_i^{Fullmodel} = \phi\left(\tilde{Z}_i, \ln k_i\right).$$

How well does this model do? What does this suggest about the Solow model?

## Question 2 – Government in the Solow model

Question 2.7 in the Acemoglu book

Let us introduce government spending in the basic Solow model. Consider the basic model without technological change and suppose that $Y_t = C_t + I_t + G_t$, with $G_t$ denoting government spending at time $t$. Imagine that government spending is given by $G_t = \sigma Y_t$.

a. Discuss how the relationship between income and consumption should be changed. Is it reasonable to assume that $C_t = cY_t$, where $c = 1 - s$ from the lectures?

b. Suppose that government spending partly comes out of private consumption, so that $C_t = (c - \lambda\sigma)Y_t$, where $\lambda \in [0, 1]$. What is the effect of higher government spending (in the form of higher $\sigma$) on the equilibrium of the Solow model?

c. Now suppose that a fraction $\phi$ of $G_t$ is invested in the capital stock, so that total investment at time $t$ is given by

$$I_t = (1 - c - (1-\lambda)\sigma + \phi\sigma)Y_t.$$

Show that if $\phi$ is sufficiently high, the steady-state level of capital-labor ratio will increase as a result of higher government spending (corresponding to higher $\sigma$). Is this reasonable? How would you alternatively introduce public investments in this model?