Indebted Demand[*]

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Abstract

We propose a theory of *indebted demand*, capturing the idea that large debt burdens lower aggregate demand, and thus the natural rate of interest. At the core of the theory is the simple yet under-appreciated observation that borrowers and savers differ in their marginal propensities to save out of permanent income. Embedding this insight in a two-agent perpetual-youth model, we find that recent trends in income inequality and financial deregulation lead to indebted household demand, pushing down the natural rate of interest. Moreover, popular expansionary policies—such as accommodative monetary policy—generate a debt-financed short-run boom at the expense of indebted demand in the future. When demand is sufficiently indebted, the economy gets stuck in a debt-driven liquidity trap, or *debt trap*. Escaping a debt trap requires consideration of less conventional macroeconomic policies, such as those focused on redistribution or those reducing the structural sources of high inequality.

1 Introduction

Rising debt and falling rates of return have characterized advanced economies over the past 40 years. As shown in Figure 1, debt owed by households and the government in the United States has increased almost 100 percentage points of GDP since 1980, and real rates of return on financial assets have fallen by 3 to 5 percentage points for different securities. How did the twin phenomena of high debt levels and low rates of return come to be? What are the implications of high debt levels and low rates of return for the evolution of the economy and macroeconomic policymaking?

Figure 1: Debt and Returns.
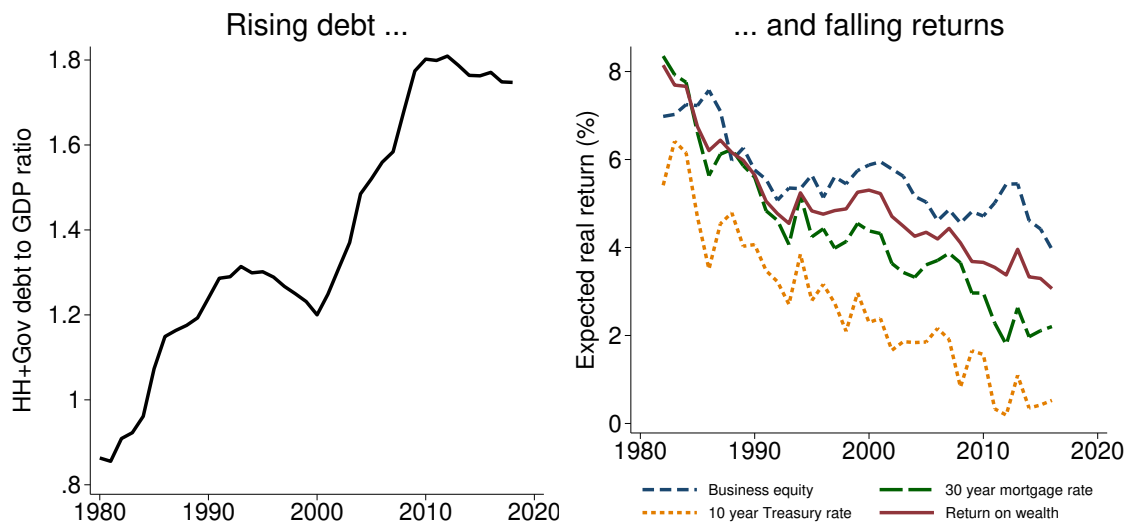
The left panel shows the household plus government debt to GDP ratio for the United States. The right panel shows expected real returns on a variety of assets for the United States. Please see appendix C for more information.

This study develops a new framework to tackle these difficult questions. The framework shows how rising income inequality and the deregulation of the financial sector can push economies into a low rate-high debt environment. Traditional macroeconomic policies such as monetary and fiscal policy turn out to be less effective over the long term in such an environment. On the other hand, less standard policies such as macro-prudential regulation, redistribution policy, and policies addressing the structural sources of high inequality are more powerful and long-lasting.

The model introduces non-homothetic consumption-saving behavior (e.g. Carroll 2000, De Nardi 2004, Straub 2019) into an otherwise conventional, deterministic two-agent endowment economy. The assumption of non-homotheticity implies that the saver in the model saves a larger fraction of lifetime income than the borrower. This is not a new idea in economics. In fact, it is pervasive in the work of luminaries such as John Atkinson Hobson, Eugen von Böhm-Bawerk, Irving Fisher, and John Maynard Keynes, and empirically supported by recent work (e.g., Dynan, Skinner and

Zeldes 2004, Straub 2019, and Fagereng, Holm, Moll and Natvik 2019). In the model, the wealthy lend to the rest of the population, which makes household debt an important financial asset in the portfolio of the wealthy. This implication of the model fits the data, as shown in Mian, Straub and Sufi (2020): a substantial fraction of household debt in the United States reflects the top 1% of the wealth distribution lending to the bottom 90%.[1]

The assumption of non-homotheticity in our model generates the crucial property that large debt levels weigh negatively on aggregate demand: as borrowers reduce their spending to make debt payments to savers, the latter, having greater saving rates, only imperfectly offset the shortfall in borrowers' spending. We refer to a situation in which demand is depressed due to elevated debt levels as *indebted demand*.

In general equilibrium, indebted demand thus implies that greater levels of debt go hand in hand with reduced natural interest rates. From the perspective of savers, reduced interest rates are necessary to balance the greater desire to save in response to greater debt service payments. In an interest rate - debt diagram, the savers' indifference condition is therefore represented by a downward-sloping saving supply schedule. We use the equivalence between indebted demand and the downward-sloping saving supply schedule extensively in our analysis.

The concept of indebted demand has broad implications for understanding what has led to the current high debt and low interest rate environment, and for evaluating what policies can potentially help advanced economies escape this equilibrium. An overarching theme of the model is that shifts or policies that boost demand today through debt accumulation necessarily reduce demand going forward by shifting resources from borrowers to savers; therefore, such shifts or policies actually contribute to persistently low interest rates.

The indebted demand framework predicts a number of patterns found in the data that models without non-homotheticity in the consumption-saving behavior of agents have a difficult time explaining. For example, since the 1980s, many advanced economies have experienced a large rise in top income shares (Katz and Murphy 1992, Piketty and Saez 2003, Piketty 2014, Piketty, Saez and Zucman 2017), in conjunction with a substantial decline in interest rates and increases in household and government debt. The model predicts exactly such an outcome: a rise in top income shares in the model shifts resources from borrowers to savers, pushing down interest rates due to savers' greater desire to save. Lower interest rates stimulate more debt, causing indebted demand—as debt is nothing other than an additional shift of resources in the form of debt service payments from borrowers to savers.

The framework also predicts that financial deregulation, which has been a prominent feature of advanced economies since the 1980s, leads to a decline in interest rates, a result that is difficult to generate in most macroeconomic models (e.g., Justiniano, Primiceri and Tambalotti 2017). In the indebted demand model, financial deregulation increases the amount of debt taken on by borrowers, which redistributes resources to savers. For the goods markets to clear, such a redistribution

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[1]For example, households in the top 1% of the wealth distribution financed 30% of the rise in the net household debt position of the bottom 90% of the wealth distribution from 1982 to 2007. See Figure 9 in Mian, Straub and Sufi (2020).

requires interest rates to fall given that savers have a lower marginal propensity to consume out of these larger debt payments.

The concept of indebted demand also provides insight into discussions of monetary and fiscal policy. For example, deficit-financed fiscal policy in the model is associated with a short run rise in natural interest rates, which reverses into a reduction in interest rates in the long-run, as the government needs to raise taxes or cut spending in order to finance the greater government debt burden.[2] As long as some of the taxes are ultimately imposed on borrowers, deficit-financed government spending is similar to any policy which attempts to boost demand through debt accumulation. Ultimately, such a policy shifts resources from borrowers to savers, depressing aggregate demand and therefore interest rates in the long run.

A similar argument applies to monetary policy, for which we extend our model to include nominal rigidities. Empirical evidence suggests that an important channel of accommodative monetary policy operates through an increase in debt accumulation (e.g., Bhutta and Keys 2016, Beraja, Fuster, Hurst and Vavra 2018, Di Maggio, Kermani and Palmer 2019, Cloyne, Ferreira and Surico 2019). This channel is also active in our model, boosting demand in the short-run. However, this boost reverses as monetary stimulus fades and debt needs to be serviced, beginning to drag on demand. Due to the presence of indebted demand, this drag can cause a persistent shift in natural interest rates after temporary monetary policy interventions. It is for this reason that monetary policy has *limited ammunition* in the model: successive monetary policy interventions build up debt levels, thereby lowering natural rates. This forces policy rates to keep falling with them to avoid a recession, thus approaching the effective lower bound.

When savers command sufficient resources in our economy, for instance due to high income inequality and large debt levels, the natural rate in our economy can be persistently below its effective lower bound. At that point, our economy is in a debt-driven liquidity trap, or *debt trap*, which is a well-defined stable steady state of our economy.

Once inside the debt trap, conventional policies that are based on debt accumulation only work in the short run. Eventually, the economy is "pulled back" into the debt trap. Certain unconventional policies, however, can facilitate an escape from the debt trap. For example, redistributive tax policies, such as wealth taxes, or structural policies that are geared towards reducing income inequality generate a sustainable increase in demand, persistently raising natural interest rates away from their effective lower bound. One-time debt forgiveness policies can also lift the economy out of the debt trap, but need to be combined with other policies, such as macroprudential ones, to prevent a return to the debt trap over time.

The idea of indebted demand helps explain the predicament faced by the world's leading central bankers, especially the absence of interest rate normalization. For example, a recent Wall Street Journal article cites monetary authorities worldwide in asserting that "borrowing helped pull countries out of recession but made it harder for policy makers to raise rates." Mark Carney,

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[2]As we discuss below, we find that a similar result holds up in the presence of spreads between government bond yields and the returns on other assets.

Governor of the Bank of England observed that "the sustainability of debt burdens depends on interest rates remaining low." Philip Lowe, Governor of the Reserve Bank of Australia has warned that "if interest rates were to rise . . . many consumers might have to severely curtail their spending to keep up their repayments."[3] This paper formalizes these intuitions.

Literature. Our paper is part of a burgeoning literature on the causes of the recent fall in natural interest rates, referred to as "secular stagnation" by Summers (2014). Among the existing explanations are population aging (Eggertsson, Mehrotra and Robbins 2019), income risk and income inequality (Auclert and Rognlie 2018, Straub 2019), the global saving glut (Bernanke 2005, Coeurdacier, Guibaud and Jin 2015) and a shortage of safe assets (Caballero and Farhi 2017, Caballero, Farhi and Gourinchas 2017).[4] Our theory suggests a new force for reduced natural interest rates, namely indebted demand. It can act both as an amplifier of existing explanations—as we demonstrate for rising income inequality—or give rise to new explanations, as we demonstrate for financial deregulation, which is commonly thought to be a force against low interest rates.[5]

The central element of our theory is the assumption of non-homothetic preferences, generating heterogeneous saving rates out of *permanent* income transfers.[6] As we mentioned above, such heterogeneity was an important aspect of many early studies of (non-optimizing) consumption behavior. Among the more recent papers in this tradition are Stiglitz (1969), Von Schlicht (1975), and Bourguignon (1981), who study the implications of such behavior on inequality. The earliest models of optimal consumption behavior that we know of and that allow for such preferences are Strotz (1956), Koopmans (1960) and Uzawa (1968). More recently, Carroll (2000), De Nardi (2004), and Benhabib, Bisin and Luo (2019) argue that non-homothetic preferences are important to understand wealth inequality, and Straub (2019) studies their implications for a rise in income inequality.

Our implications for monetary policy are related to the debate around "leaning vs. cleaning" (Bernanke and Gertler 2001, Stein 2013, Svensson 2018) and to the nascent academic literature surrounding the idea that monetary policy might have limited ammunition. McKay and Wieland (2019) explore this idea in a model of durables spending, Caballero and Simsek (2019) in a model with asset price crashes.

The closest antecedents to our paper are Kumhof, Rancière and Winant (2015), Cairó and Sim (2018) and Rannenberg (2019). Kumhof, Rancière and Winant (2015) study a two-agent endowment economy, where savers are more patient than borrowers and savers have non-homothetic preferences. They find that a rise in income inequality leads to greater debt levels and a greater likelihood of a financial crisis due to endogenous default, but no change in long-run interest rates.

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[3]See also Borio and White (2004), Koo (2008), Borio and Disyatat (2014), Lo and Rogoff (2015), Turner (2015), Dalio (2018) and Pettis (2019) for similar ideas.

[4]For an overview of multiple forces see Rachel and Summers (2019). For an alternative theory of secular stagnation based on preferences for wealth, see Michau (2018).

[5]For a notable exception, see Iachan, Nenov and Simsek (2015).

[6]This is not to be confused with heterogeneity in marginal propensities to consume out of *transitory* income transfers, which, as we explain below, are not sufficient to generate indebted demand.

The driving force behind this result is the specific structure and heterogeneity of preferences. It generates a higher saving rate of savers out of labor income, compared to borrowers, but a lower saving rate out of financial income. This is why the model does not feature indebted demand: in fact, an increase in debt raises aggregate demand in the model and thus dampens the effects of income inequality. The model in Cairó and Sim (2018) builds on Kumhof, Rancière and Winant (2015) and studies implications for a richer set of shocks and for the conduct of monetary policy. The recent paper by Rannenberg (2019) also builds on Kumhof, Rancière and Winant (2015) but shows that income inequality can also reduce natural interest rates in addition to generating greater debt.

Finally, as a paper about household and government debt, it relates to a vast empirical and theoretical literature on the origins and consequences of high debt levels. Among the empirical papers, Schularick and Taylor (2012) document the well known "financial hockey stick" behavior of private debt; Mian and Sufi (2015), Jordà, Schularick and Taylor (2016), Mian, Sufi and Verner (2017) document that expansions in household debt predict weak future economic growth; Reinhart and Rogoff (2010) assess the consequences of large government debt. Among the theoretical papers, Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017) study the effects of debt deleveraging on the economy. Our model emphasizes that even without deleveraging, debt reduces aggregate demand. This aspect is shared with Illing, Ono and Schlegl (2018), who show that debt can lead to persistent stagnation in the context of insatiable preferences for money (Ono 1994).

Layout. Section 2 introduces the model, and Section 3 studies equilibrium in the model, introducing the concept of indebted demand. Section 4 provides evidence to support the key feature of the model that long-run saving supply schedule for the rich are downward sloping. Section 5 examines how income inequality and financial deregulation affect debt levels and interest rates in the economy. Next, we study the implications of fiscal and monetary policy (Section 6), and what indebted demand means for an economy in a liquidity trap (Section 7). Section 8 provides two extensions, and Section 9 offers the perspective of a richer model. Section 10 concludes.

2   Model

The model is a deterministic, infinite-horizon endowment economy, populated by two separate dynasties of agents trading debt contracts. Endowments can be thought of as dividends of real assets, or "Lucas trees", owned by the two dynasties. Each such asset produces one unit of the consumption good each instant. There are Y real assets in total, where we normalize $Y = 1$ for now.

The agents in the two dynasties share the same preferences and only differ by their endowments of the real asset. For reasons that will become clear below, we refer to the poorer ("non-rich") dynasty as *borrowers* $i = b$ and wealthier ("rich") dynasty as the *savers* $i = s$. At any point in

time, there is a mass $\mu^b = 1 - \mu$ of borrowers and a mass $\mu^s = \mu$ of savers. We sometimes simply refer to all dynasties of type i as "agent" i.

The model is intentionally kept simple and tractable for now; several extensions can be found in Section 8 and Appendix B.

2.1 Preferences

We begin by setting up the agents' common preferences. An agent in dynasty $i \in \{b, s\}$ dies at rate $\delta > 0$ and discounts future utility at rate $\rho > 0$. At any date t, total consumption by dynasty i is c_t^i and total wealth by dynasty i is a_t^i. The average type-i agent therefore consumes c_t^i/μ^i and owns wealth a_t^i/μ^i, with a utility function given by[7]

$$\int_0^\infty e^{-(\rho+\delta)t} \left\{ \log\left(c_t^i/\mu^i\right) + \frac{\delta}{\rho} v(a_t^i/\mu^i) \right\} dt \tag{1}$$

Utility is derived from two components: each instant, utility over flow consumption per capita c_t^i/μ^i; and, arriving at rate δ, a warm-glow bequest motive captured by the function $v(a)/\rho$. We assume for now that upon death, the entire asset position of an agent is bequeathed to a single newborn offspring, ruling out any cross-dynasty mobility.[8] The consolidated budget constraint of all agents of type i is therefore simply given by

$$c_t^i + \dot{a}_t^i \leq r_t a_t^i \tag{2}$$

where r_t is the endogenous flow interest rate at date t.

The function $v(a)$ represents a crucial aspect of this model. It characterizes the relationship between wealth of a dynasty and its saving rate. To see this, consider the special case where $v(a) = \log a$. This choice of $v(a)$ makes the preferences in (1) homothetic: the borrower and saver dynasties would exhibit the exact same saving behavior, just scaled by their current wealth positions.[9]

This is no longer true as $v(a)$ deviates from $\log a$. To capture such deviations, we define $\eta^i(a)$ to be the marginal utility of v relative to the marginal utility of log, that is,

$$\eta^i(a) \equiv a/\mu^i \cdot v'(a/\mu^i). \tag{3}$$

$\eta^i(a)$ is defined in per-capita terms and therefore depends on i. $\eta^i(a)$ plays an important role in the analysis, especially $\eta^s(a)$ which henceforth we also denote by $\eta(a)$. When $\eta^i(a)$ is constant, for instance $\eta^i(a) = 1$ when $v(a) = \log a$, utility is homothetic as marginal utility of bequests and marginal utility of consumption are proportional. When $\eta^i(a)$ is decreasing, the marginal utility of

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[7]Our results also hold with utility functions over consumption different from log, see Appendix B.5.

[8]We relax this assumption in Section B.3.

[9]In fact, given the normalization with $1/\rho$, $v(a_t) = \log a_t$ exactly corresponds to an altruistic bequest motive in an equilibrium in which $r_t = \rho$.

bequeathing assets decreases relatively more quickly than the marginal utility of consumption; in this case, wealthier agents save relatively less. When $\eta^i(a)$ is increasing, marginal bequest utility decays more slowly than that of consumption, implying that wealthier agents have a stronger desire to save. As shown in Section 4 below, the empirical evidence supports the *non-homothetic* case in which $\eta^i(a)$ is increasing. As a result, the development of the model in this section and in Section 3 emphasizes this non-homothetic case.

2.2 Borrowing constraint

The two types of agents in the model maximize utility (1) subject to the budget constraint (2) and a borrowing constraint. To formulate the borrowing constraint, we separate type-i agents' wealth positions into two components: their *real assets* h_t^i and their *financial assets*, which if negative, we refer to as *debt* d_t^i, that is,

$$a_t^i = h_t^i - d_t^i \tag{4}$$

We assume for now that the agents' debt is adjustable-rate long-term debt which decays at some rate $\lambda > 0$.

Agents of type i own a fixed total endowment of $\omega^i \in (0,1)$ of real assets (trees), where $\omega^s + \omega^b = 1$. Within the endowment, we assume that $\ell^i < \omega^i$ are *pledgeable real assets* (e.g. land, houses, businesses, etc) and $\omega^i - \ell^i$ are *non-pledgeable real assets* (e.g. human capital). Denoting

$$p_t \equiv \int_t^\infty e^{-\int_t^s r_u du} Y ds \tag{5}$$

the price of a single real asset (tree), type-i agents' total wealth in real assets is

$$h_t^i = p_t \omega^i \tag{6}$$

and type-i agents' pledgeable wealth is $p_t \ell^i$. Henceforth we assume that pledgeable wealth (per capita) is equal across agents, $\ell^b/(1-\mu) = \ell^s/\mu$, and denote $\ell \equiv \ell^b$, so that the only source of heterogeneity between the two agents are the endowments ω^i, or equivalently, the agents' real-asset earning shares. We assume that savers' per capita earnings exceed those of borrowers, $\omega^s/\mu^s > \omega^b/\mu^b$.

We impose the borrowing constraint

$$\dot{d}_t^i + \lambda d_t^i \le \lambda p_t \ell \tag{7}$$

where, due to asset market clearing, $d_t^s + d_t^b = 0$.[10] We henceforth focus exclusively on the borrowers' total debt position $d_t \equiv d_t^b$, the key state variable for our analysis. d_t essentially captures how much borrowers have spent beyond earnings $\omega^b Y$ in the past, and how much of a debt burden

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[10] We multiply the right hand side by λ so that in a steady state, the constraint simplifies to $d^i \le p\ell$. This is immaterial to our results.

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borrowers need to service in the future.

According to borrowing constraint (7), new debt issuance $\dot{d}^i + \lambda d_t^i$ is bounded above by the value of pledgeable assets. As we emphasize below, most of our results do not rely on the specific constraint (7).[11]

2.3 Homothetic benchmark

Throughout the analysis, we compare the model to a *homothetic benchmark* model. This model is characterized by $\eta(a) = 1$, so that agents' preferences are indeed homothetic. Moreover, to avoid a continuum of steady state equilibria in the homothetic model, we allow the saver's discount factor to be different from, and smaller than, the borrower's discount factor, $\rho^s < \rho$. Heterogeneity of discount factors is not assumed in the non-homothetic model.

2.4 Equilibrium

We formally define equilibrium next.

Definition 1. Given initial debt $d_0 = d_0^b$ a *(competitive) equilibrium* of the model are sequences $\{c_t^i, a_t^i, d_t^i, h_t^i, p_t, r_t\}$ such that both agents choose $\{c_t^i, a_t^i\}$ to maximize utility (1) subject to the budget constraint (2) and the borrowing constraint (7); d_t^i is determined by (4); h_t^i is determined by (6); p_t is determined by (5); and financial markets clear at all times, that is, $d_t^s + d_t^b = 0$. The goods market clears by Walras' law.

A *steady state (equilibrium)* is an equilibrium in which $c_t^i, a_t^i, d_t^i, h_t^i$ and r_t are all constant.

A steady state with debt d is *stable* if there exists an $\epsilon > 0$ such that any equilibrium with initial debt $d_0 \in (d - \epsilon, d + \epsilon)$ has debt converge back to d, $d_t \to d$. All other steady states are *unstable*.

2.5 Discussion

What does $\eta(a)$ capture? The literature has pointed out numerous examples of why agents might care about their wealth beyond its value for financing their own consumption behavior. This includes bequests (De Nardi 2004), out-of-pocket medical expenses in old age (De Nardi, French, Jones and Gooptu 2011), utility over status (Cole, Mailath and Postlewaite 1992, Corneo and Jeanne 1997), inter-vivos transfers (Straub 2019), and numerous other reasons that are documented in other papers in the literature (e.g. Carroll 2000, Dynan, Skinner and Zeldes 2004, Saez and Stantcheva 2018). Many of these examples are more salient or applicable to wealthier agents and can be captured in reduced form by assuming a specific shape $\eta(a)$. In addition to these examples, $\eta(a)$ could also capture the idea that assets other than a given stock of liquid assets or human capital are illiquid and therefore being saved "by holding" (Fagereng, Holm, Moll and

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[11]In fact, we can allow for a more general constraint of the form $\dot{d}_t^i + \lambda d_t^i \leq \lambda \mathcal{L}(\{r_s\}_{s \geq t})$ where $\mathcal{L}$ is a general function of current and future interest rates. Denoting by $\mathcal{L}(r)$ the function $\mathcal{L}(\{r_s\}_{s \geq t})$ in the case where rates are constant $r_s = r$ for all $s \geq t$, our results require that $\mathcal{L}$ is decreasing in r. We show in Appendix B.6 that many alternative models of borrowing have this feature.

Natvik 2019). Observe that a standard altruistic saving motive would correspond to $\eta(a) \propto \log a$ and thus be equivalent to our homothetic benchmark model (cf footnote 9).[12]

Aggregate scale invariance. Our baseline non-homothetic model, with increasing $\eta(a)$, is not scale-invariant in aggregate. If aggregate output Y doubles, all agents are wealthier and thus, in line with a rising $\eta(a)$, would raise their savings by more than double. Taken at face value, this would generate rising saving rates in all growing economies, which seems counterfactual.

We believe that the key to understanding why a non-homothetic model, which breaks *individual scale invariance*, need not necessarily break *aggregate scale invariance* is that many of the motives for non-homothetic saving are relative to some economy-wide aggregates. For example, bequests are likely especially valued among the rich if they are large relative to the average wage or income in the economy, relative to the price of land, or relative to the average bequest. This suggests that $\eta(a)$ should really be thought of as a function of a relative to Y or aggregate wealth, i.e. $\eta(a/Y)$ or $\eta(a/(a^b + a^s))$. To incorporate this idea and reduce clutter in the formulas, we henceforth assume that η is of the form $\eta(a/Y)$ but output Y is normalized to 1, $Y = 1$. We demonstrate in Appendix B.5 that our results carry over to the case where η is of the form $\eta(a/(a^b + a^s))$, as in Corneo and Jeanne (1997).

Trading debt vs. trading assets. In the model, households trade debt contracts, rather than real assets. There are two simple reasons behind this assumption. First, in a deterministic model like ours, debt contracts and real assets are priced with the same rate of return, so trading one versus the other does not matter other than for one-time revaluation effects. Second, debt contracts have been and continue to be a very important vehicle for saving and dissaving across the U.S. wealth distribution. This fact is shown in Mian, Straub and Sufi (2020), where saving and borrowing across the income and wealth distribution are explored for the United States between 1963 and 2016. The analysis there shows that a substantial amount of borrowing by households in the bottom 90% of the wealth distribution was financed through the accumulation of financial assets by the top 1%. Even though much of this debt was collateralized by housing, the bottom 90% did not actually accumulate additional housing assets while borrowing.[13] In fact, Mian, Straub and Sufi (2020) show that the bottom 90% actually decumulated other assets, aggravating their dissaving.

Savers and borrowers. In the model, the rich agents are savers, and the non-rich agents are borrowers. While this is a simplifying assumption, it also fits with empirical evidence in the United States, as shown in Mian, Straub and Sufi (2020). In particular, from 1998 to 2016, individuals in the top 1% of the income or wealth distribution saved on average between 6 and 8 percentage points of national income annually depending on the methodology used. Individuals in the next

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[12] We discuss evidence in Section 4 that is incompatible with the altruistic model.

[13] These findings are also in line with Bartscher, Kuhn, Schularick and Steins (2018).

9% saved between 2 and 4 percentage points of national income. Estimates of savings by individuals in the bottom 90% range from -4% to 0% of national income per year. So while the model provides a simplified view of who in the economy saves and who borrows, this simplified view is not too far from reality.

Rate of return. The precise rate of return r_t in the model is the expected return on the loans extended by savers to borrowers, which can be thought of as the expected return on consumer or home mortgage debt. More broadly, the rate of return should include both the expected return on household debt and the expected return on other financial assets that savers have been accumulating relative to non-savers since the 1980s. Mian, Straub and Sufi (2020) show that the non-rich in the United States have indeed boosted their borrowing from the rich significantly since the early 1980s. However, the non-rich have also been decumulating financial asset holdings relative to the rich, who have boosted their holdings of both household debt and other financial assets. As a result, r_t in the model can also be thought of as the general return on wealth for households. As shown in Figure 1, real rates of return across asset classes have fallen substantially since the early 1980s.

3 Indebted Demand

We next characterize the equilibria in our model. We focus exclusively on equilibria in which debt is positive $d_t > 0$, that is, the borrower actually borrows and the saver actually saves.[14] Such equilibria always exist in our economy.

3.1 Saving supply schedules

The saver's Euler equation is given by

$$\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta \frac{c_t^s}{\rho a_t^s} \eta(a_t^s). \tag{8}$$

In a steady state, quantities and prices are constant, so that the budget constraint reads $c^s = ra^s$. Substituting this into the Euler equation (8), we find our first key steady state equilibrium condition

$$r = \rho \cdot \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(a^s)}. \tag{9}$$

This equation can be understood as a long-run saving supply schedule, describing the saving behavior of a possibly non-homothetic saver. Specifically, for each wealth position a^s, it describes the interest rate r that is necessary for a saver to find it optimal to keep his wealth constant at a^s.

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[14]If we assumed away heterogeneity in per-capita real earnings ω^i/μ^i, "borrowers" and "savers" become entirely symmetric, so that for each equilibrium in which borrowers borrow and savers save, strictly speaking there would also exist one in which savers borrow and borrowers save. With a realistic gap in ω^i/μ^i, this possibility vanishes.

Figure 2: Long-run saving supply schedules.
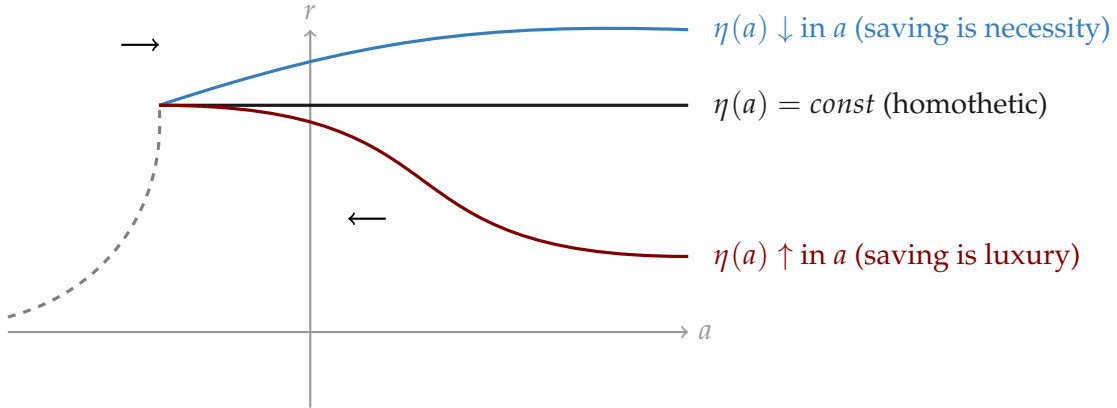
Equation (9) can thus be thought of as an indifference condition. It is defined as the unique interest rate at which borrowers are indifferent between saving and dissaving.[15]

The crucial object that determines the shape of the saving supply schedule is the function $\eta(a)$, as illustrated in Figure 2. In the homothetic benchmark economy, where $\eta(a)$ is equal to 1 (or another constant), we recover the standard infinitely elastic long-run supply schedule, $r = \rho$. When $\eta(a)$ falls in a, in which case saving is treated as a necessity by agents, the saving supply schedule slopes *up*. Finally, and most importantly, when $\eta(a)$ rises in a and thus saving is treated as a luxury, the saving supply schedule slopes *down*. This is the key property of our non-homothetic model. We summarize it in the following proposition.

Proposition 1. *The long-run saving supply schedule* (9) *is downward sloping if and only if wealthier agents have a greater marginal propensity to save, that is, when $\eta(a)$ is increasing in a.*

What is the intuition behind the negative slope? In a model in which wealthier agents save at higher rates, the higher an agent's wealth is, the *lower* must be the the return on wealth for the agent to be indifferent between saving and dis-saving.[16]

3.2 Steady state equilibria

Steady states are the intersections of saving supply schedules with *debt demand* curves, as we characterize in the following proposition.

Proposition 2. *Any steady state with positive debt $d > 0$ corresponds to an intersection of a long-run saving supply schedule*

$$r = \rho \cdot \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(\omega^s/r + d)} \tag{10}$$

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[15]It is not necessarily like a conventional "supply curve", which typically describes the level of wealth a^s savers tend towards for a fixed interest rate r.

[16]One may think that the individual saving dynamics displayed by the arrows in Figure 2 imply our economy is unstable. This is not the case, as we show below. Our model gives rise to a unique stable steady state. Moreover, note that the aggregate saving supply schedule can slope down in even as individual saving dynamics are stationary, as we illustrate in an extension in Appendix B.10.

Figure 3: Steady state equilibrium.
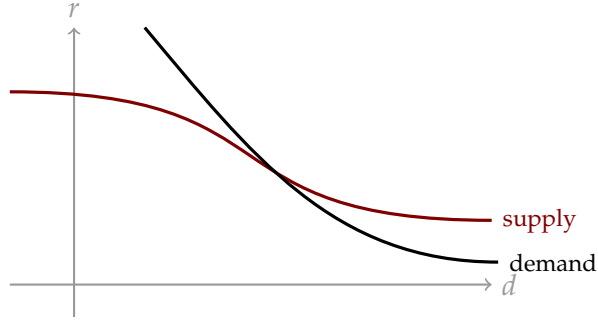
with a long-run debt demand curve

$$d = \frac{\ell}{r}. \tag{11}$$

There is at most a single such steady state.

Proposition 2 shows that the relevant saving supply schedule is that of the saver, and that the relevant debt demand curve is given by the borrowing constraint of the borrower. We write both conditions in terms of the interest rate (return on wealth) r and debt d. Similar to models with discount rate heterogeneity, the borrower is up against the borrowing constraint in the steady state. In this formulation, the debt demand curve slopes down in r. The slope of the saving supply schedule depends on the slope of $\eta(a)$. If $\eta(a)$ is strictly increasing in a, then the saving supply schedule slopes downward.

We illustrate the saving supply schedule of the saver and the debt demand curve and their intersection in Figure 3. We prove in Appendix A there can only be a single intersection, and hence a single steady state, in the model.[17]

Steady state in the homothetic economy. In the homothetic economy, the interest rate in the unique steady state is necessarily pinned down by the saver's discount rate, $r = \rho^s$. The associated debt level is then $d = \ell/\rho^s$.

Analytical example. The steady state conditions in Proposition 2 can be solved analytically in a simple special case, where $\eta(a)$ is a linear function in the relevant region of the state space. For example, assuming $\eta(a) = a$, there is a unique stable steady state in this region, with interest rate

$$r = \rho + \delta - \delta/\rho(\omega^s + \ell)$$

and associated debt level

$$d = \frac{\ell}{\rho + \delta - \delta/\rho(\omega^s + \ell)}.$$

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[17]Multiple steady states can occur under more general borrowing constraints (see footnote 11).

13

3.3 Indebted demand

At the core of many of the results in this paper is the idea that an increase in debt service costs by some dx, e.g. caused by a greater level of debt da so that $dx = r\, da$, may lower aggregate demand. We next explore this idea starting at the steady state and in partial equilibrium, holding the interest rate r fixed.

Proposition 3 (Indebted demand). *Assume the economy is in its steady state and hold r fixed. A permanent increase dx in debt service costs, or equivalently a permanent transfer from borrowers to savers, moves aggregate spending on impact by*

$$dC = dc^b + dc^s = -\frac{\rho + \delta}{r}\frac{1}{2}\left(1 - \sqrt{1 - 4\left(1 - \frac{r}{\rho + \delta}\right)\frac{r}{\rho + \delta}\epsilon_\eta}\right)dx \qquad (12)$$

Here, $\epsilon_\eta \equiv \frac{\eta'(a)a}{\eta(a)}$ is a measure of the degree of non-homotheticity in preferences. In particular, aggregate spending falls, $dC < 0$, if and only if $\epsilon_\eta > 0$.

Proposition 3 highlights that any increase in debt service costs weighs down on aggregate demand, $dC < 0$, precisely if and only if $\epsilon_\eta > 0$, a phenomenon we henceforth call *indebted demand*.

Why is demand indebted in this case? The increase in debt service costs dx passes through to the borrower's spending one-for-one, $dc^b = -dx$. But, since savers have a greater saving propensity, their spending initially rises by less than the transfer, $dc^s < dx$. Thus, aggregate spending falls, $dC < 0$. For the goods market to clear, the equilibrium interest rate must therefore fall. As this mechanism only relies on heterogeneity in saving propensities out of a small permanent transfer dx, *any* model that generates such heterogeneity along the wealth distribution exhibits the property of indebted demand. The model studied in this paper can be regarded as an example of such an economy.

The sign of dC in Proposition 3 is directly related to the slope of the saving supply schedule in Figure 2. The indebted demand property holds, i.e. dC is negative, precisely when savers are situated on a downward-sloping saving supply schedule. This is because, holding r fixed, a marginal increase in wealth da corresponds to a permanent transfer of $dx = rda$. When savers' consumption dc^s responds to this transfer less than one for one, $dc^s < dx$, their saving must become positive, $d\mathring{a}^s > 0$. But this implies that the shift in wealth da, without an offsetting shift in interest rates, must have moved savers above their saving supply schedule, into the region where wealth increases.

Therefore, a downward-sloping saving supply schedule is isomorphic to a marginal propensity to consume out of a permanent transfer of less than one. Indebted demand emerges if and only if the saving supply schedule slopes down. Given the critical role of the slope of the saving supply schedule in the model, Section 4 below provides both microeconomic and macroeconomic evidence to support the plausibility of the idea that the the saving supply schedule is in fact downward sloping.

The homothetic model, despite its discount rate heterogeneity, has $\epsilon_\eta = 0$ and thus does not generate indebted demand. The reason for this is that there is no heterogeneity in saving propensities out of a small *permanent* transfer dx: borrowers do not save out of a small transfer as they are hand-to-mouth; savers do not either as they smooth their consumption perfectly, with $r = \rho^s$.

As a side remark, observe that our non-homothetic model predicts a positive consumption response, $dC > 0$, to a reduction in debt service payments, $dx < 0$. Such a reduction could occur in reality when households refinance their mortgages to bring down the interest rate ("rate refi"). In homothetic models, as $\epsilon_\eta = 0$, there is no effect of "rate refis" on aggregate consumption (Greenwald 2018), which quantitatively limits their macroeconomic relevance (Berger, Milbradt, Tourre and Vavra 2018). In non-homothetic models, such as ours, "rate refis" could instead have sizable consequences for aggregate consumption.

3.4 Transitions

Having discussed the set of steady state equilibria in this economy, we now explore the entire set of equilibria, including the transitions along which the economy approaches the steady state.

The transitions follow along a system of ordinary differential equations (ODEs), with a single backward-looking state variable, debt d_t, and a single endogenous equilibrium price, the interest rate r_t. One can show that borrowers are always up against their borrowing constraint along the transition unless debt is below some threshold $\underline{d}$, which lies below the steady state debt position. Figure 4 illustrates the transitional dynamics in the interest rate - debt space. We next describe the equations characterizing these transitions, for simplicity for the case of a binding borrowing constraint (that is for $d_t \geq \underline{d}$).

Due to the binding borrowing constraint, debt evolves as in (7), that is

$$\dot{d} + \lambda d_t = \lambda p_t \ell. \tag{13a}$$

Here, the price of real assets p_t, defined in (5), is the first forward-looking state variable which follows the ODE

$$\dot{p}_t = r_t p_t - 1 \tag{13b}$$

The second forward-looking state variable is the consumption of savers, which is determined by the Euler equation (8)

$$\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta \frac{c_t^s}{\rho a_t^s} \eta(a_t^s) \tag{13c}$$

where wealth of savers can be expressed as $a_t^s = \omega^s p_t + d_t$. Finally, the interest rate is pinned down by the budget constraint of savers (2), which can be cast as

$$c_t^s + \dot{d}_t = r_t d_t + \omega^s. \tag{13d}$$

Together, the four equations (13a)–(13d) jointly determine the evolution of the three state variables

Figure 4: Equilibrium transitions in the baseline model.
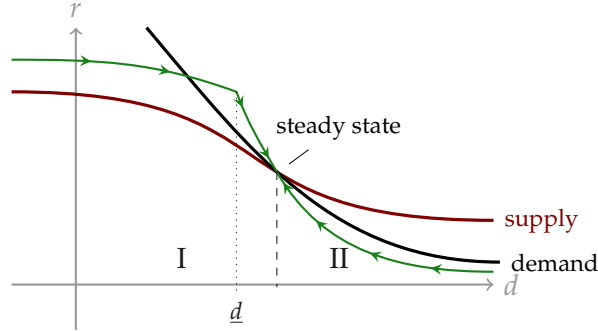
Note. Red: saving supply schedule. Black: debt demand curve. Green: transitional dynamics.

(d_t, p_t, c_t^s) as well as the interest rate r_t.[18] It turns out that this evolution is unique for any given initial level of debt $d_0 > 0$. We verified this using phase diagrams, confirmed it in our numerical simulations, and provide an analytical local uniqueness & existence result in Appendix A.

If d_0 is to the left of the steady state (region I), the borrower levers up, hitting the borrowing constraint as soon as d_t crosses $\underline{d}$, and ultimately converging to the steady state d. If d_0 is to the right of the steady state (region II), the borrower has a desire to deleverage, pushing interest rates down. The magnitude of the decline in interest rates depends on the degree of non-homotheticity, as when there is more non-homotheticity, the saver spends less of the additional debt payments.

Observe that the black line in Figure 4 only corresponds to the borrowing constraint in steady state, $d = \ell/r$. Along the transition from the left, the expectation of lower interest rates in the future implies an asset price p_t that lies above ℓ/r_t. Thus, there can be points (d_t, r_t) during the transition that lie to the right of the black line. The opposite happens during transitions from the right.

3.5 Illustrative calibration of the basic model

We next provide an illustrative calibration of our model. The calibration is meant to capture the US economy in the 1980s, before the recent increase in income inequality. We interpret the saver as comprising the top 1% earning households of the economy, i.e. with a population share $\mu = 0.01$, and the borrower as the bottom 99%. We choose the saver's real (non-bond) earnings share ω^s to match the post-tax income share (excluding returns to household debt) to be consistent with the calibration of our richer model in Section 9, giving $\omega^s = 0.06$. This ensures that the steady state distribution of income is the same as in our richer model.

We assume an initial interest rate of 5.5%, consistent with an expected real return on wealth of 7.5% (see Figure 1 and discussion in Section 2.5) net of 2% productivity growth. We calibrate ℓ to match the US household debt to GDP level in 1980 of 45%, giving $\ell = 0.0248$. We choose

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[18]The three boundary conditions are (a) an initial level of debt d_0; (b) the terminal level of the asset price $\lim_{t \to \infty} p_t = 1/r$; and (c) the terminal level of savers' consumption $\lim_{t \to \infty} c_t^s = c^s$. r and c^s are the steady state values.

$\delta = 0.025$ corresponding to an expected duration of a generation of 40 years.[19] The discount rate ρ, which approximately corresponds to the discount rate of borrowers as the bequest motive is less relevant to them, is chosen at 10%. Whenever we refer to the homothetic benchmark model, we use a discount rate of savers of $\rho^s = r = 0.055$.

We directly calibrate $\eta(a) = v'(a/\mu)a/\mu$, letting it take a flexible functional form,

$$\eta(a) = 1 + \frac{1}{\tilde{\eta}\tilde{a}} \log\left(1 + e^{\tilde{\eta}(a-\tilde{a})}\right) \tag{14}$$

where $\tilde{\eta}, \tilde{a} > 0$. This form is arguably the simplest "activation function", with the following desirable properties:[20] it is positive and strictly increasing everywhere; it is flat at 1 for low levels of assets a, implying near-homothetic behavior then; it rises linearly for large asset levels a, with slope $\tilde{a}^{-1}$; when $\tilde{a} \to \infty$, $\eta(a)$ remains flat for all a; the speed at which $\eta(a)$ moves from flat to linear is parametrized by $\tilde{\eta}$; its elasticity $\epsilon_\eta(a) = \frac{\eta'(a)a}{\eta(a)} = 1 - \frac{-v''(a)a}{v'(a)}$ always lies in $(0,1)$, consistent with $v(a)$ being a concave function. We jointly calibrate $\tilde{\eta}$ and $\tilde{a}$ to ensure that the steady state Euler equation (10) is satisfied and that savers have an MPC out of wealth of 0.01 in line with our discussion in the next section.[21]

The remaining parameter to be determined is λ, which is less important for our results as it only matters for the transitional dynamics. It governs the speed of the debt response. To calibrate it, we compare the impulse response of household debt over GDP to a monetary policy shock implied by our model to that commonly found to identified monetary policy shocks. In particular, we feed a 100 basis point interest rate cut with a half-life of 2 years (similar to (20) below) into the Section 6.2 variant of our model. We compare the household debt / GDP response at its peak (approximately 0.75 percentage points after 2 years) to the response of US household debt / GDP to a Romer and Romer (2004) shock. This procedure implies a λ approximately equal to 0.5.

4 Evidence for a Downward Sloping Saving Supply Schedule

The slope of the long-run saving supply schedule is a crucial aspect of the model. This section provides both microeconomic and macroeconomic evidence supporting the plausibility of a downward saving supply schedule among individuals at the top of the income or wealth distribution.

4.1 Saving rates out of lifetime income

As shown in Figure 2, the saving supply schedule slopes downward in our model if and only if $\eta(a)$, which is the marginal utility of wealth relative to the homothetic benchmark, is strictly increasing in a for savers in the economy. In the model, savers are those in the top of the permanent

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[19] This is conservative given the alternative reasons for non-homothetic saving, see e.g. the discussion in Section 2.5.

[20] The functional form in (14) is a transformation of a "SoftPlus" function commonly used in Machine Learning.

[21] The formula for the MPC is $r - \frac{\rho+\delta}{2}\left(1 - \sqrt{1 - 4\left(1 - \frac{r}{\rho+\delta}\right)\frac{r}{\rho+\delta}\epsilon_\eta}\right)$.

income distribution, which we interpret as the top 1%. As mentioned in Section 2.5, Mian, Straub and Sufi (2020) show that most of the savings in the U.S. economy come from those in the top 1% of the wealth or income distribution. The critical empirical question is whether $\eta(a)$ is strictly increasing in a for those at the top of the distribution.

Empirical research measuring saving rates out of lifetime income can inform us on the slope of $\eta(a)$ for the rich. In the homothetic benchmark, saving rates are constant across the lifetime income distribution. In contrast, saving rates are rising in lifetime income if $\eta(a)$ is increasing in a. There is a long line of influential work that supports the view that saving rates out of lifetime income are higher for wealthy individuals. For example, this idea features prominently in the writings of John Atkinson Hobson, Eugen Bohm von Bawerk, Irving Fisher, and John Maynard Keynes, among others. More recently, formal empirical work has validated the notion that saving rates are highest at the top end of the lifetime income distribution.

Dynan, Skinner and Zeldes (2004) use panel data from the Survey of Consumer Finances (SCF) to show that individuals in the top 20% of the income distribution have saving rates out of lifetime income that are substantially larger than the rest of the population. The saving rates for the top 1% and top 5% out of income are estimated to be particularly large, almost four times larger than at the median of the distribution (0.51 compared to 0.13 out of a dollar of permanent income, Table 4, column (2)).[22]

Straub (2019) uses the Panel Study of Income Dynamics (PSID) to estimate an elasticity of consumption to lifetime income. If preferences were homothetic, then the elasticity of consumption with respect to lifetime income should be one, implying that changes in permanent income inequality do not impact aggregate consumption. However, the paper estimates that the elasticity of consumption with respect to permanent income is around 0.7, which is evidence in favor of non-homothetic preferences and a concave relationship between consumption and lifetime income.

The advantage of these two studies is that they seek to estimate the saving rate out of lifetime income, which is the main object of interest in determining the shape of $\eta(a)$. In addition, there also exists recent evidence from studies estimating saving rates out of income more generally.

Fagereng, Holm, Moll and Natvik (2019) use administrative panel data from Norway to estimate saving rates out of income across the wealth distribution. The study finds substantially higher saving rates for wealthier households, with saving rates for the top 1% estimated to be almost double the saving rates for the median of wealth distribution.

The empirical strategy of Fisher, Johnson, Smeeding and Thompson (2018) estimates a consumption share and after-tax income share of the top 1% of the income distribution of $s_c = 0.066$ and $s_y = 0.171$, respectively, for the 2004 to 2016 period. Together with an estimate of the average propensity to consume out of income APC in the aggregate, one can estimate the saving rate of the top 1% as $1 - APC \cdot s_c/s_y = 0.649$. Here, the APC is measured as personal consumption expenditures divided by disposable personal income from the National Accounts. This same calculation

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[22]See also the influential article by Carroll (2000) which highlights some of the empirical work on the subject from the 1990s.

implies a saving rate of -0.025 for the bottom 99% as a whole. The top 1% have a much higher saving rate than the bottom 99%. In addition to showing a higher saving rate of the top 1%, the SCF evidence also provides further support to the idea that most of the saving in the economy is done by the top 1%.

4.2 MPCs and the return on wealth

The slope of the saving supply schedule can also be discerned through a comparison of the observed marginal propensity to consume out of a change in wealth versus the expected return on wealth for the rich. More specifically, let $\mathcal{C}(r, a)$ be the steady state consumption of rich households in an economy. The definition of the saving supply schedule $r(a)$ as a function of rich households' wealth requires that

$$\mathcal{C}(r(a), a) = r(a)a.$$

Total differentiation of this equation with respect to a allows us to isolate the local slope of the saving supply schedule

$$\frac{dr}{d \log a} = \frac{MPC^{\text{wealth}} - r}{1 - \epsilon_r} \tag{15}$$

where $\epsilon_r \equiv \frac{\partial \log \mathcal{C}}{\partial \log r}$ is the elasticity of consumption with respect to a permanent shift in interest rates, and MPC^{wealth} is the marginal propensity to consume out of wealth for the rich. Given the preponderance of illiquid wealth among the rich, this ought to be interpreted as the MPC out of illiquid wealth or capital gains. The denominator of the right hand side is necessarily positive, and so the sign of $MPC^{\text{wealth}} - r$ for the rich gives us the local slope of the saving supply schedule.[23]

Recent studies using data from a number of European countries suggest that the MPC^{wealth} of the rich is about 1.0%. More specifically, Arrondel, Lamarche and Savignac (2015) estimate MPC^{wealth} across the wealth distribution in France and find that the top 10% has an MPC^{wealth} of 0.6%. Garbinti, Lamarche, Lecanu and Savignac (2020) estimate MPC^{wealth} for the top 10% of the wealth distribution across five European countries, and they find estimates of 0.3% for Cyprus, 0.6% for Germany, 0.8% for Spain, 1.2% for Belgium, and 2.3% for Italy. Using administrative data from Sweden, Di Maggio, Kermani and Majlesi (2019) estimate an MPC^{wealth} of 2.8% for the top 5% of the wealth distribution. The median and mean of these estimates across countries suggests an MPC^{wealth} of about 1.0% for those in the top of the wealth distribution.[24]

How does this compare to r, or the expected return on wealth for the rich? As shown in Figure 1, the expected real return on wealth for the U.S. economy as a whole has averaged about 5% from

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[23]$\epsilon_r < 1$ holds for any function $\mathcal{C}(r, a)$ describing the response of initial consumption c_0 that is the solution to a standard utility maximization problem with monotone and concave preferences over paths $\{c_t\}$ with prices e^{-rt} relative to the present and initial wealth a.

[24]To the best of our knowledge, there are no estimates of how the MPC^{wealth} varies across the wealth distribution in the United States. Chodorow-Reich, Nenov and Simsek (2019) estimate 2.8% in the aggregate. The estimates by Garbinti, Lamarche, Lecanu and Savignac (2020) in other countries suggest an MPC^{wealth} for the bottom 90% that is on average five times larger than the top 10%. Applying this pattern to the United States implies that the MPC^{wealth} of the top of the wealth distribution in the United States is likely to be substantially lower than 2.8%.

1982 to 2016. It started out at around 7.5% and has fallen to about 3% in recent years. In order to apply it in (15), we need to subtract real GDP growth in the United States, which is about 2%, as (15) was derived in a model without growth. Given these facts on expected returns in conjunction with the estimates of the MPC^{wealth} for the top 10% above, the numerator $MPC^{\text{wealth}} - r$ is almost assuredly negative, thereby indicating a downward sloping saving supply schedule.

We can also use formula (15) to get a sense of magnitudes. To do so, we assume $\epsilon_r = 0$, which would be implied by log preferences.[25] Using an estimate of 1% for MPC^{wealth}, and 3% for r net of GDP growth, the average for the United States from 1982 to 2016, we obtain:

$$\frac{dr}{d \log a} \approx -2\%.$$

In words, this implies that if the richest households' wealth rises by 10%, the interest rate has to come down by 20 basis points. While this is not a precise calculation, it gives a rough sense of the magnitudes that are at play in the model.

4.3 Evidence using wealth to income ratios

Another implication of higher saving rates of the rich is a positive correlation between top income shares and wealth to income ratios.[26] This implication is robustly supported by time series data in the United States, as shown in the left panel of Figure 5. The share of income earned by the top 1% of the income distribution is strongly positively correlated with the aggregate wealth to income ratio across years from 1913 to 2019.

One may be concerned, however, that other time series factors could have influenced both inequality and wealth to income ratios in the aggregate. To try to identify more cleanly the effect of top income shares on wealth to income ratios, the right panel of Figure 5 uses cross-sectional variation across states in the rise in the top 1% share from 1982 to 2007.[27] As it shows, there is a strong positive correlation. States in which the top 1% earned a larger share of the state's total income over time also experienced larger wealth accumulation. While the figure only displays a correlation, Mian, Straub and Sufi (2020) show that this result is robust to variety of controls.[28]

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[25]Despite log utility over consumption, ϵ_r is slightly negative in our model due to the bequest motive in the utility function (1).

[26]Wealth to income is independent of the lifetime income distribution when saving rates are constant in lifetime income (Straub, 2019). Since we do not have data on top lifetime income shares, we use top (current) income shares. We believe this is appropriate given that the rise in (current) income inequality was largely driven by rising inequality in lifetime income (Kopczuk, Saez and Song, 2010, Guvenen, Kaplan, Song and Weidner, 2019).

[27]These years are chosen as they are the years for which state-level information is available to construct the wealth to income ratio in a state. See Mian, Straub and Sufi (2020) for more details.

[28]The slope of the relationship in the time series is larger than the slope of the relationship across states. One reason for this is that the time series relationship includes the endogenous response of interest rates to a rise in inequality. If interest rates fall due to a rise in top income shares (as we argue below), then wealth to income ratios will rise even further. The cross-sectional specification holds fixed the interest rate which is why this endogenous response is absent in the cross-sectional specification. In this sense, the cross-sectional relationship is the more direct test of non-homotheticity without general equilibrium effects.

Figure 5: Top Income Shares and Wealth to Income Ratios.
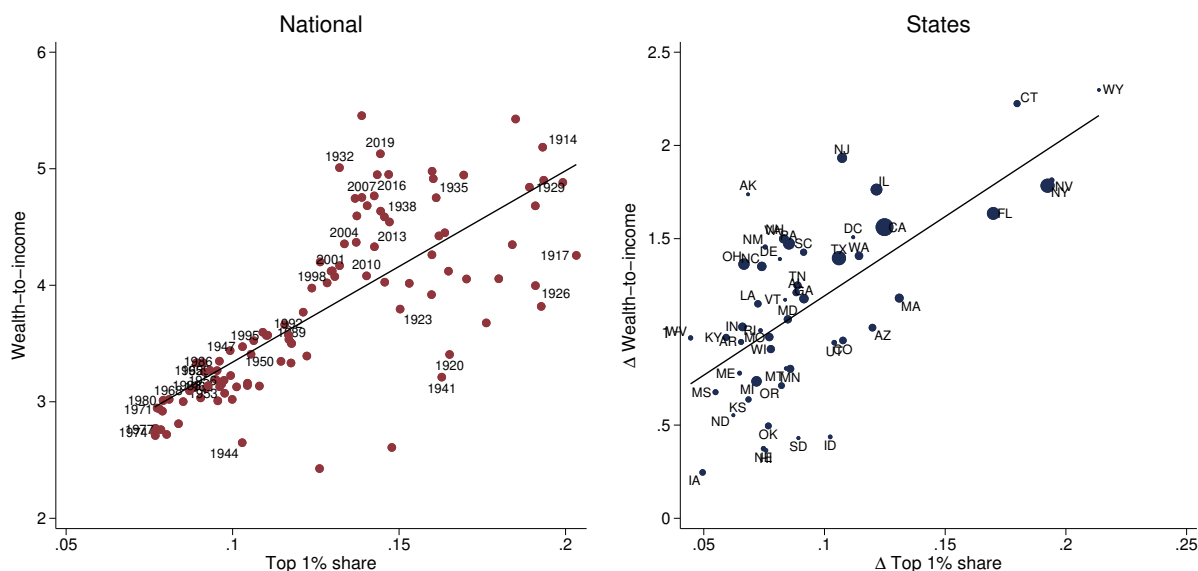
The left panel plots the aggregate wealth to income ratio against the share of income going to the top 1% for the United States from 1913 to 2019. Every third year is labeled. The data are from Saez and Zucman (2020). The left panel plots the change from 1982 to 2007 in the ratio of total household wealth to household income at the state level against the change in the top 1% share of income over the same period. For full details on data in the right panel, see Mian, Straub and Sufi (2020).

4.4 Comparing saving supply schedules across models

The evidence is supportive of the view that the saving supply schedule slopes downward for rich households. However, our model does not imply a downward sloping saving supply across the entire distribution. The borrowers in our model are on the upward part of their saving supply schedule as in other models used in the literature.

Perhaps the most prominent of models with an upward sloping saving supply schedule is Aiyagari (1994). In this model, there is a precautionary saving motive for households given the potential for hitting a borrowing constraint after negative idiosyncratic productivity shocks. In the Aiyagari (1994) model, a permanent transfer to households acts as additional insurance, cushioning the household in states of low realizations of idiosyncratic productivity. A household therefore responds to the transfer by raising consumption more than one for one, decumulating wealth. As a result, a permanent transfer leads to a lower saving rate.

While this logic is sound for households near a borrowing constraint, it is unlikely to be relevant for those near the top of the lifetime income distribution. Such households already have ample resources to buffer negative idiosyncratic shocks, and therefore it is unlikely that they will

have lower saving rates if they become richer.[29]

However, it is important to note that borrowers in our model, which we calibrate to correspond to the bottom 99% of the income distribution, are all on the upward-sloping part of the saving supply schedule, as in Aiyagari (1994). In fact, since debt is negative saving, their downward-sloping debt demand curve is the exact mirror image of their upward-sloping saving supply schedule. In our model, the upward slope stems from a borrowing constraint, but as we emphasize in Appendix B.6, many other formulations are possible, including a precautionary savings motive.

Borrowers are on the upward sloping part of the saving supply schedule while savers are on the downward sloping part. This is possible despite the fact that all agents share the same preferences. Each agent's saving supply schedule is first upward-sloping for low levels of wealth near the borrowing constraint, and then flattens out as wealth increases. Only if wealth is sufficiently high does it turn down again. Recall from Section 2.5 that most of the saving in the U.S. economy over the past 20 years has come from those in the top 1%, which supports the view that most saving in the United States is done by rich households that are likely to be on the downward sloping part of the saving supply schedule.

5 Inequality, Financial Deregulation, and Indebted Demand

The framework developed in the previous sections may help understand the underlying factors that contributed to the simultaneous increase in debt and decline in interest rates that many advanced economies have experienced in the past 40 years. We explore this next.
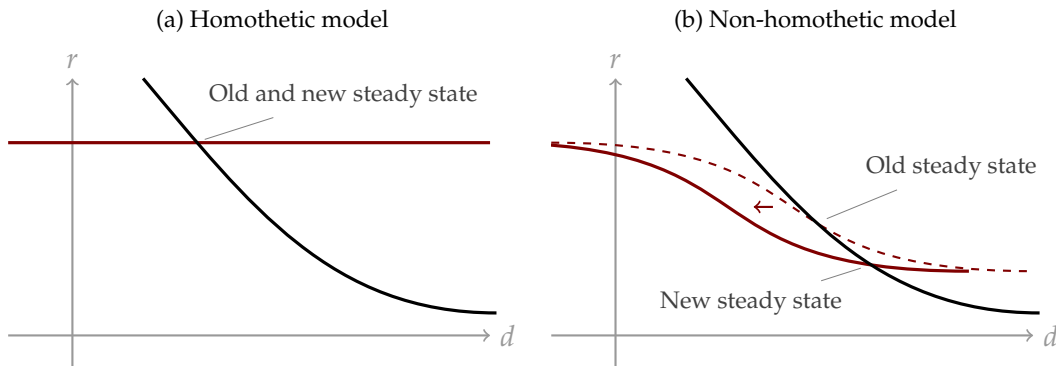
5.1 Inequality

As is well understood by now, many advanced economies have experienced a significant rise in income inequality (Atkinson, Piketty and Saez 2011). In the model, a rise in income inequality can be captured as an increasing share ω^s of real earnings going to savers, and a corresponding fall in $\omega^b = 1 - \omega^s$.[30] The following proposition characterizes the long-run implications of rising income inequality.

Proposition 4. *An increase in income inequality (greater ω^s) unambiguously reduces long-run equilibrium interest rates and raises household debt. In the homothetic model, long-run interest rates and household debt are unaffected by rising income inequality.*

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[29]Even in the Aiyagari (1994) model, saving supply schedules go from upward sloping to flat at the highest wealth levels. The reason is that the precautionary motive ceases to materially influence saving rates for the wealthy. There is no force such as non-homotheticity in the Aiyagari (1994) model to generate a downward sloping saving supply schedule at higher wealth levels.

[30]The rise in income inequality in the model is a rise in the permanent income of the high endowment agents relative to the low endowment agents. This experiment matches the data. Both Kopczuk, Saez and Song (2010) and Guvenen, Kaplan, Song and Weidner (2019) show that lifetime income inequality has increased substantially in the United States since the early 1980s.

Figure 6: The effects of rising income inequality for long-run saving supply and debt demand.

(a) Homothetic model   (b) Non-homothetic model

The long-run implications of rising inequality are best understood in the context of our model's saving supply schedule and debt demand curve. Figure 6 shows supply and demand diagrams for the homothetic economy in panel (a), and the non-homothetic economy in panel (b). In the homothetic case, the supply schedule is pinned down by the discount factor and thus independent of inequality. The demand curve is also independent of inequality, and therefore the old and new steady states coincide.

In the non-homothetic economy, savers have a greater propensity to save. Thus, if they earn a greater share of income, total saving increases. This manifests itself in a shift of the saving supply schedule (10) to the left: for a given level of debt d, savers earn more resources and are willing to save more. As Proposition 4 shows, and as is illustrated in Figure 6, the equilibrium interest rate falls and the amount of debt in the economy rises in response to the rise in inequality. The non-homothetic model thus helps rationalize the close empirical association between the rise in inequality and the simultaneous increase in debt and decline in interest rates across advanced economies.
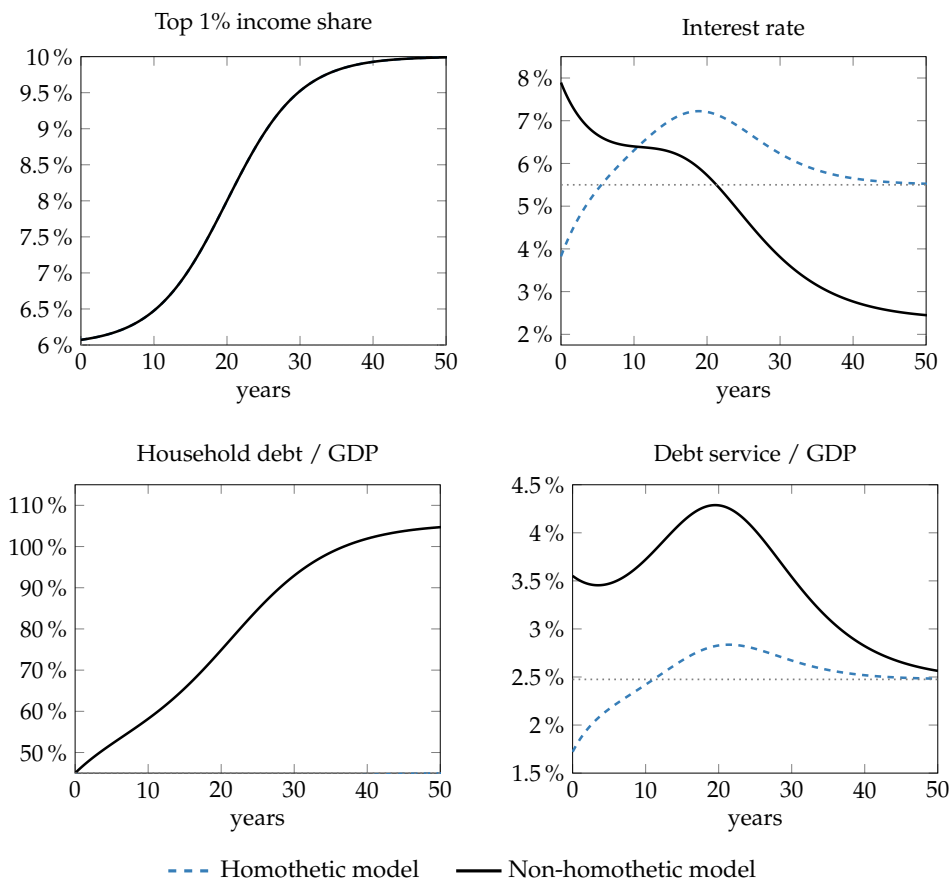
Transition. This is confirmed numerically in Figure 7, which simulates the responses of a homothetic and a non-homothetic economy to a permanent increase in income inequality. Since this is a perfect-foresight transition, borrowers begin raising their debt levels already early on, in anticipation of lower interest rates in the future, which raises demand and thus interest rates initially.[31]

Interestingly, the transition shows a hump-shaped profile in the debt service ratio, which ultimately falls back to its pre-transition value. This demonstrates that the debt service ratio is a highly endogenous object, which can be low either when there is little debt (early in the transition), or, when there is high debt but interest rates are low (late in the transition).

One reaction to the strong increase in debt in Figure 7 may be to point out that in the data, borrowers typically use debt to acquire assets (houses) and that their net worth actually remained more or less constant (Bartscher, Kuhn, Schularick and Steins 2018). Shouldn't this be reflected in

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[31]Similarly, the homothetic economy shows an on-impact drop in the interest rate, below its initial steady state value (dashed gray line) before converging back to it.

Figure 7: Rising income inequality and debt.

Top 1% income share

Interest rate

Household debt / GDP

Debt service / GDP

- - - Homothetic model ——— Non-homothetic model

Note. Plots show transitions from our calibrated steady state with saver income share $\omega^s = 0.06$ (dotted gray line) to a steady state with $\omega^s = 0.10$. The dashed blue line corresponds to the homothetic model with $\rho^s = 0.055$.

the model?

It turns out that it already is. Clearly, most of the run-up in debt over the last few decades is mortgage debt, and thus ultimately collateralized by housing. As we show in our companion paper, Mian, Straub and Sufi (2020), however, when taken together, the bottom 90% of the wealth distribution did not use the increase in debt to accumulate more housing. Instead, housing was bought and sold within the bottom 90%, likely from old homeowners to young homebuyers, and thus ultimately financed consumption expenditure by old homeowners (Bartscher, Kuhn, Schularick and Steins 2018). Net worth only remained stable because house prices were rising.

At a stylized level, this is precisely the mechanism in our model. A natural measure of borrowers' financial net worth is their pledgeable wealth net of debt, $p_t\ell - d_t$, where ℓ can be interpreted as land or housing owned by borrowers. Figure 7 shows how borrowers' net worth evolves, and splits it up into its components, $p_t\ell$ and d_t. Similar to the data, net worth remains stable in the transition. Underlying the stability, however, are two opposing trends. On the one hand, pledgeable wealth increased tremendously, as asset prices p_t rise; one the other, greater pledgeable wealth

Figure 8: Decomposing borrowers' net worth.
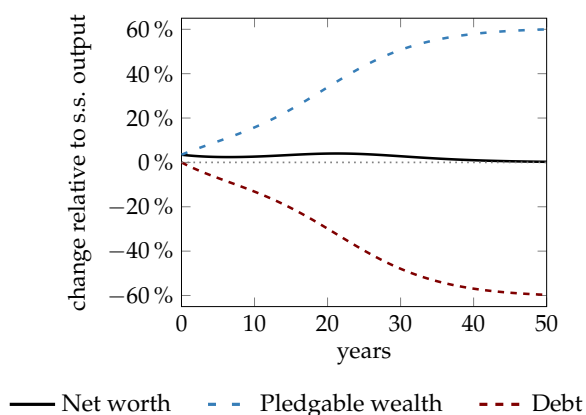
relaxes the borrowing constraint and thus leads to greater debt accumulation.[32]

If net worth of borrowers did not change, why then is there indebted demand? Couldn't borrowers sell their assets, annihilate their debt and finance the same level of consumption as before? The answer is no. What matters for borrowers' consumption stream—and hence their contribution to aggregate demand—is *not* their net worth; instead it is their income stream after making debt payments. Valuation effects from lower discount rates and greater asset prices do not alter the income stream. Thus, indebted demand occurs when rich households save and non-rich households dissave; this may or may not coincide with a reduction in borrowers' net worth.
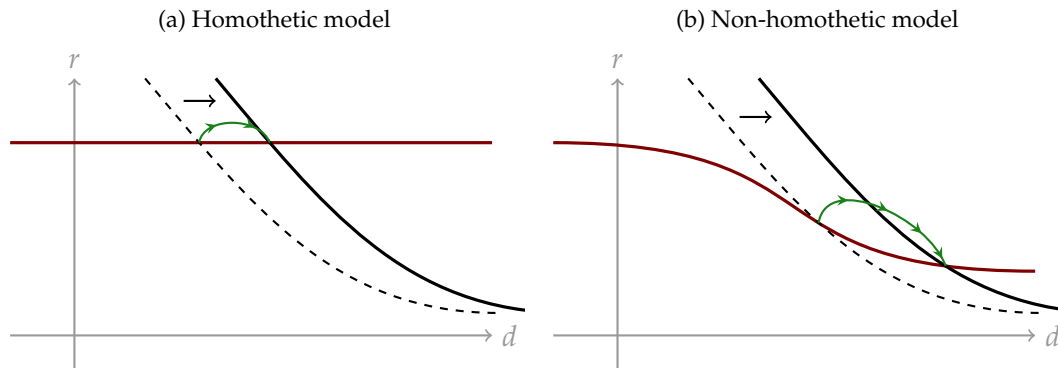
5.2 Financial deregulation

Another widespread recent trend in advanced economies has been financial liberalization and deregulation. Especially the "mortgage finance revolution" of the 1970s and 1980s allowed new institutions to enter mortgage markets, led to securitization of mortgages and to a general loosening of borrowing constraints (Ball, 1990). For example, Bokhari, Torous and Wheaton (2013) document large increases in the fractions of mortgages originated with an LTV ratio above 90% and a debt-to-income ratio above 40% from 1986 to 1995. One tension in the literature noted by Justiniano, Primiceri and Tambalotti (2017) and Favilukis, Ludvigson and Van Nieuwerburgh (2017) is that in most standard models, a loosening of such borrowing constraints should be associated with an increase in interest rates. We next explore the effects of financial deregulation on debt and interest rates in the model developed here.

To do so, financial deregulation is modeled as an increase in the pledgability ℓ of real assets.[33] We find the following result.

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[32] An important caveat here is that this is a perfect foresight transition with rational expectations. In practice, especially in the early 2000s, house prices and borrowing partly increased (and later reversed) due to optimism (see, e.g., Kaplan, Mitman and Violante 2017) and relaxed collateral constraints. Both can be captured to some extent by shifts in ℓ. We analyze such shifts in the next section.

[33] In our housing application in Section B.6.1 we show that a rising LTV ratio amounts to an increase in ℓ.

Figure 9: The effects of financial deregulation for long-run saving supply and debt demand.

(a) Homothetic model

(b) Non-homothetic model

Proposition 5. *Financial deregulation (greater ℓ) unambiguously reduces long-run equilibrium interest rates and increases household debt. By contrast, in the homothetic model, long-run interest rates are unaffected by financial deregulation and household debt rises by less.*

Figure 9 plots the implied shifts in the debt demand curve, as well as the qualitative transitional dynamics from the old steady state to the new one (green arrows). As can be seen, in both homothetic and non-homothetic models, the *short-run* saving supply schedule is upward-sloping: the loosening of borrowing constraints initially *increases* interest rates, as household demand grows in response. In the long run, the saving supply schedule is flat in the homothetic benchmark model, so that there is no long run effect of deregulation on interest rates.

In the non-homothetic model, by contrast, the increased debt burden ultimately leads to a fall in equilibrium interest rates, as the long-run saving supply schedule is downward-sloping.[34] This then again contributes to increasing debt further. Interestingly, this resolves the puzzle faced in the literature: the model shows that financial deregulation might only put upward pressure on interest rates in the short run, and it actually contributes to a declining interest rate in the long-run. This is in line with a common narrative for the early and mid 2000s in the US (Summers 2015): despite sharp credit growth, the economy did not experience a booming economy.

One common argument in favor of financial deregulation is that it enables households to better smooth consumption over the life cycle and insure themselves against financial shocks. Our results in this section imply that this benefit has to be weighed against the potential cost of pushing overall debt levels higher and interest rates towards the zero lower bound. The cost of hitting the zero lower bound is an aggregate demand externality (see Section 7) and thus not internalized by individual borrowers (e.g., Korinek and Simsek 2016). A deeper discussion of the implications of this trade-off for optimal financial regulation is left for future research.

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[34]We conjecture that the point at which interest rates fall below their prior steady state level is related to the point at which resources start flowing in net from borrowers to savers, similar to the evidence in Drehmann, Juselius and Korinek (2018).

6 Implications for Fiscal and Monetary Policy

6.1 Fiscal policy: Deficits and redistribution

The previous section showed how private deficits lead to the accumulation of household debt, and thus indebted demand. A considerable portion of the recent increase in debt, however, has been public debt. According to conventional wisdom, a rise in government debt exerts upward pressure on interest rates (e.g., Blanchard 1985, Aiyagari and McGrattan 1998).

What are the implications of a rise in government debt in our non-homothetic model? This section focuses on this question in the context of the equilibrium introduced in Section 2.4, in which output is fixed at $Y = 1$, and therefore interest rates endogenously adjust to clear the goods market. Section 7 revisits fiscal policy in the presence of nominal rigidities and a binding zero-lower bound.

We consider fiscal policy in this section, as well as other policies in subsequent sections, mainly from a positive perspective, documenting its effects in our model without any notion of welfare. The reason for this choice is that there are several real-world considerations that are first-order for welfare but outside our model. For example, high debt levels and low interest rates are often associated with instability and risk-taking in the financial sector, and thus raise the likelihood of a financial crisis (e.g. Reinhart and Rogoff 2009, Schularick and Taylor 2012, Stein 2012). Low interest rates may also reduce growth (Liu, Mian and Sufi 2019). Behavioral aspects, such as time-inconsistent preferences, would lead borrowers to accumulate too much debt. One important dimension of welfare an extension of our model can speak to is the potential for a liquidity trap when the (natural) interest rate is sufficiently depressed. We discuss the welfare implications of our model in this context in Section 7.

Government. We introduce a standard government sector into the economy. Specifically, the government is assumed to choose a debt position B_t, government spending G_t, and proportional income taxes τ_t^i on agent i such that its flow budget constraint

$$G_t + r_t B_t \leq \dot{B}_t + \tau_t^s \omega^s + \tau_t^b \omega^b \tag{16}$$

is satisfied at all times t. Ponzi schemes are ruled out by assuming that B_t is bounded above, uniformly in t. In the baseline model, government bonds pay the same interest rate as other assets. We discuss the implications for when this is not the case below.

For simplicity, government spending is treated here as purchases of goods that are either wasted, or—which is equivalent for the purposes of this current positive exercise—enter agents' utilities in an additively-separable form. Taxes are assumed to enter agents' real wealth in the natural way, $r_t h_t^i = (1 - \tau_t^i)\omega^i + \dot{h}^i$. Taking fiscal policy as given, the definition of a competitive equilibrium is unchanged from before, with the exception that the bond market clearing condition is now given by $d_t^b + d_t^s + B_t = 0$.

Figure 10: Long-run effect of an increase in public debt B.
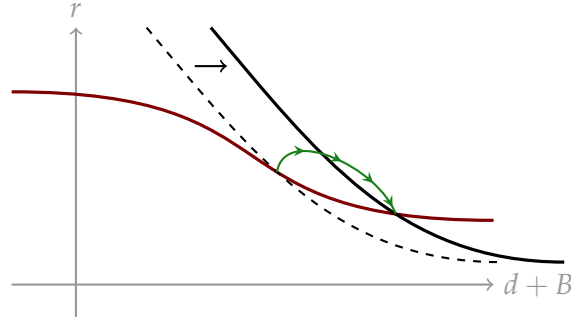
Long-run effects of fiscal policy. We begin by studying the long-run effects of fiscal policy, focusing on constant policies (G, B, τ^s, τ^b). In this case, the equilibrium conditions for steady state equilibria are given by

$$r = \rho \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(a)} \tag{17}$$

$$a = (1 - \tau^s)\frac{\omega^s}{r} + \frac{\ell}{r} + B \tag{18}$$

Equations (17) and (18) characterize the long-run implications of fiscal policy. We are specifically interested in increases in B, financed by raising taxes τ^i on both agents or cutting expenditure G; as well as tax-financed increases in G. This yields the following result.

Proposition 6 (Long-run effects of fiscal policy on interest rates and debt.). *In the long run,*

a) *larger government debt ($B \uparrow$) depresses the interest rate ($r \downarrow$) and crowds in household debt ($d \uparrow$).*

b) *tax-financed government spending ($G \uparrow$) increases the interest rate ($r \uparrow$) and crowds out household debt ($d \downarrow$).*

c) *fiscal redistribution ($\tau^s \uparrow$, $\tau^b \downarrow$) increases the interest rate ($r \uparrow$) and crowds out household debt ($d \downarrow$).*

With a homothetic saver, none of these policies have any effect on the long-run interest rate and on household debt.

An intuition for these results can be explained with the help of Figure 10. Consider the first policy in Proposition 6, and assume the greater debt level B is entirely paid for by a reduction in government expenditure G. As savers do not raise their consumption one-for-one with the increase in debt service payments by the government, aggregate demand would fall were it not for a reduction in interest rates. Graphically, the policy corresponds to an increase in the economy's total demand for debt, $d + B$, which shifts out to the right. Notably, the reduction in interest rates will crowd-*in* household debt.

When the greater level of debt is not paid for by government spending cuts, but instead by greater taxation, the result in Proposition 6 is qualitatively the same. However, the exact magnitude of the interest rate decline now depends on the distribution of taxation: in the corner case

where borrowers pay all additional taxes, the interest rate decline is as large as when government spending is cut; in the corner case where savers pay all additional taxes, interest rates do not respond.[35]

Tax-financed government spending and fiscal redistribution reallocate resources from the saver to a "spender", which is either the government—in the case of government spending—or the borrower—in the case of redistribution. Such resource reallocation would raise aggregate demand were it not for an increase in interest rates.

Proposition 6 and Figure 10 prescribe a very different role for fiscal policy in influencing interest rates than is typically assumed. What helps in the long-run is first and foremost redistribution between spenders and savers, not redistribution of taxes over time in the form of public deficits, which, paradoxically, *lowers* long-run interest rates even further as government demand becomes indebted.

One caveat to Proposition 6 is that the interest rate on government debt B, r^B, might differ form the interest rate r on other debt d. Here, the result depends on why r^B is different from r and, crucially, whether r^B is above or below the growth rate, which is zero in the baseline model. For example, it is conceivably that the first result in Proposition 6 breaks when the interest rate on government debt r^B lies sufficiently below zero. We leave a characterization of the interaction of convenience yields on government debt and indebted demand for future research.

Fiscal policy in the analytical example. We can illustrate the effects of fiscal policy in the analytical example in Section 3.2. It is straightforward to obtain the steady state given a set of tax policies (G, B, τ^s, τ^b)

$$r = \frac{\rho + \delta - \frac{\delta}{\rho}\left((1-\tau^s)\omega^s + \ell\right)}{1 + \frac{\delta}{\rho}B} \qquad \text{and} \qquad d = \frac{\left(1 + \frac{\delta}{\rho}B\right)\ell}{\rho + \delta - \frac{\delta}{\rho}\left((1-\tau^s)\omega^s + \ell\right)}.$$

G and τ^b do not enter the expressions, as they are implicitly used to balance the government budget for any values of B, τ^s; it does not matter which of the two is used. We see that greater redistribution and greater spending (both financed through greater τ^s) raises r and lowers d. Greater public debt B lowers r and crowds in d. Finally, greater B reduces the sensitivity of r to changes in τ^s, ω^s or ℓ.

Short run effects. Despite its novel long-run effects of government debt, the model predicts conventional short-run effects of debt-financed fiscal stimulus programs (whether through government spending or tax cuts). As before, in terms of saving supply schedule and debt demand curve, this is due to an upward-sloping short-run saving supply schedule. We illustrate this in

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[35]Interest rates are constant in government debt when savers are taxed because savers are Ricardian. In a non-Ricardian model with downward-sloping saving supply schedule, e.g. the one in Appendix B.9, interest rates would increase with greater government debt *if* it is financed by savers alone. Importantly, whether savers are Ricardian or not has no bearing on the cases in which borrowers are being taxed or the government cuts back its spending.

Figure 11: Deficit spending.

| Gov. debt / GDP | Interest rate | Household debt / GDP |

—— Non-homothetic model   - - - Homothetic model
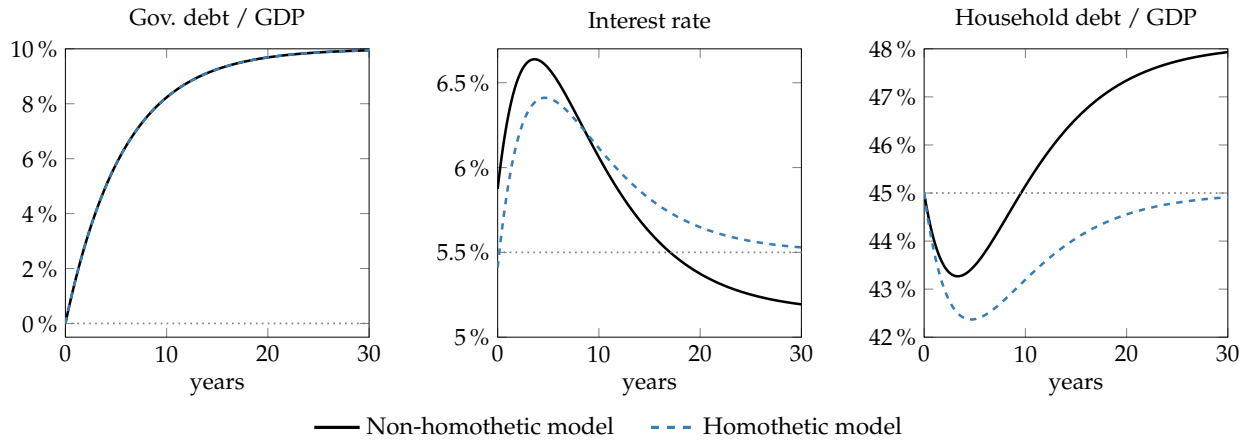
Note. Plot shows response of non-homothetic economy to temporary government spending shock (AR(1) spending path with $g_0 = 5.5\%$ of GDP and a half-life of 2 years). Fiscal rule: $\tau^i = r_\infty B$ where r_∞ is eventual steady state interest rate.

Figure 11, which plots the dynamic response of the economy to temporary deficit-financed government spending. There is a short-run rise in the natural interest rate, lasting about as long as the fiscal stimulus itself. During this time, household debt is crowded out by higher interest rates. Afterward, however, the interest rate declines, falling below its original level and allowing debt to increase. The opposite of the dynamics in Figure 11 would materialize in response to an austerity program, causing a short-term reduction in the natural rate but raising natural rates in the longer term.

In practice, this suggests a dilemma for economies that are currently stuck in a steady state with low interest rates and high public debt, but, for some reason outside our model, wish to raise rates going forward. If they expanded public debt even further, rates would rise in the short run, but subsequently fall again, even below their already undesirable previous levels. If they contracted public debt, rates will fall in the short-run—possibly below the effective lower bound, causing a recession—despite the prospect of greater rates in the long run.

6.2 Monetary policy: Limited ammunition

In the previous section, we saw that deficit-financed fiscal stimulus reduces natural interest rates in our model in the long run. We next argue that monetary stimulus also moves natural interest rates, possibly persistently so.

Extending the model. To do this, it is necessary to move away from an endowment economy, where output Y is fixed at 1 and interest rates endogenously adjust to clear goods markets. Instead, we now let actual output, henceforth denoted by $\hat{Y}_t$, adjust endogenously in response to monetary policy and differ from potential output Y_t. In the interest of space, we only explain here the main ingredients of the model, as they closely follow the existing literature, e.g. Werning

(2015) and Auclert, Rognlie and Straub (2018). We describe the full model extension in great detail in Appendix B.7.

To allow output $\hat{Y}_t$ to be endogenous, we assume it is produced using efficiency units of labor N_t, $\hat{Y}_t = N_t$, which are supplied by both types of agents. We assume that dynasty i has labor productivity ω^i and supplies hours n_t^i, such that total labor units are $N_t = \omega^b n_t^b + \omega^s n_t^s$. We modify dynasty i's utility function to include a standard additively separable disutility from labor supply. Dynasty i's total real asset wealth h_t^i is now given by its human capital, $h_t^i = \int_t^\infty e^{-\int_t^s r_u du} \omega^i n_s^i ds$, so that dynasty i's budget constraint simply becomes

$$c_t^i + r_t d_t^i \leq \dot{d}_t^i + \omega^i n_t^i.$$

We follow Werning (2015) and Auclert, Rognlie and Straub (2018) in assuming that prices are flexible, nominal wages are perfectly rigid, and that aggregate labor demand N_t is allocated across agents using a simple uniform allocation rule, namely $n_t^i = N_t$.[36] This implies that real earnings by type i are $\omega^i \hat{Y}_t$, and thus that the income distribution is unaffected by the level of aggregate output $\hat{Y}_t$. Moreover, as we show in Appendix B.7, the disutility can be specified such that the allocation with $\hat{Y}_t = N_t = n_t^i = 1$ is the *natural allocation*. We continue to denote potential output by $Y_t = 1$.

To ensure continued tractability of the model, we treat monetary policy as controlling the real rate directly, as in Werning (2015), McKay, Nakamura and Steinsson (2016) and Auclert, Rognlie and Straub (2020).[37] Transmission of monetary policy then works in the standard way: monetary policy changes the interest rate, which steers aggregate demand and thus output (and labor) in the economy. With exogenous interest rates $\{r_t\}$, the goods market now clears because $\{\hat{Y}_t\}$ is endogenous. We define as *natural interest rates* the sequence of real interest rates $\{r_t^n\}$ that achieves the natural allocation, that is, it implements a path of aggregate demand at potential, $\hat{Y}_t = Y = 1$. Given the assumptions above, this model is "backwards compatible", in that its natural allocation precisely corresponds to our baseline model from Section 2.

Monetary policy shocks. We consider two types of monetary policy shocks, which hit the economy at a stable steady state (c^b, c^s, r, d). The first type is a T-period long interest rate reduction, before a reversal back to the original interest rate

$$r_t = \begin{cases} \hat{r} & t \leq T \\ r & t > T \end{cases}. \tag{19}$$

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[36] The precise formulation of wage stickiness is actually irrelevant for this section as we make the simplifying assumption below that monetary policy sets the real interest rate directly. See Auclert, Rognlie and Straub (2018) for a related argument.

[37] This corresponds to a Taylor rule with a Taylor coefficient of 1. One can easily study other Taylor coefficients, after specifying the wage Phillips curve. The transmission from real rates to real economy outcomes remains unaffected. Note that, different from a textbook New-Keynesian model, our economy is locally determinate under a real-rate rule.

The second type also starts with a T-period long interest rate reduction, but then reverses back to the path of *natural* interest rates

$$r_t = \begin{cases} \hat{r} & t \leq T \\ r_t^n & t > T \end{cases}, \tag{20}$$

ensuring that for any $t > T$ after the intervention $\hat{Y}_t = Y = 1$ in this case.

Monetary policy and debt.   We begin by studying monetary policy shocks of the first kind. In our model, they stimulate the economy via two separate channels. First, they relax borrowing constraints and encourage borrowers to use additional household debt for spending (*debt channel*). Second, through income and substitution effects, they provide incentives for savers to spend more (*saver channel*). To study the role of these channels for monetary transmission, we define the following present values

$$PV^\tau(\{c_t^i\}) = \int_0^\tau e^{-\int_0^t r_s ds} c_t^i dt - \int_0^\tau e^{-rt} c^i dt$$

$$PV^\tau(\{\hat{Y}_t\}) = \int_0^\tau e^{-\int_0^t r_s ds} \hat{Y}_t dt - \int_0^\tau e^{-rt} Y dt$$

The first is the increase in the present value of agent i's spending until period τ; the second is the increase in the present value of output until period τ. The next proposition shows that the two channels have asymmetric implications for the path of aggregate demand.

Proposition 7. *The τ-period present value of the output response to the monetary policy shock* (19) *is given by*

$$PV^\tau(\{\hat{Y}_t\}) = \underbrace{\frac{1}{\omega^s} PV^\tau(\{c_t^s\})}_{\text{saver channel}} + \underbrace{\frac{1}{\omega^s} e^{-\int_0^\tau r_s ds} (d_\tau - d)}_{\text{debt channel}}. \tag{21}$$

In the long run ($\tau = \infty$), the present value of output is entirely determined by the saver channel,

$$PV^\infty(\{\hat{Y}_t\}) = \frac{1}{\omega^s} PV^\infty(\{c_t^s\}). \tag{22}$$

This implies that any output stimulus generated by debt accumulation necessarily weighs negatively on output going forward.

Proposition 7 shows that the two channels of monetary transmission have vastly different implications for the path of output. While the saver's consumption response to the interest rate change affects output permanently, the debt channel only has a temporary effect. In fact, as any additional debt taken out by borrowers eventually has to be serviced or even repaid, future demand is reduced by an active debt channel. Put differently, when monetary policy is used to stimulate the economy, any resulting increase in demand that is debt-financed does not sustainably raise demand and will contribute to reduced demand in the future.

One implication of the result in Proposition 7 is that if monetary policy is accommodative now,

Figure 12: Debt limits the ammunition of monetary policy.
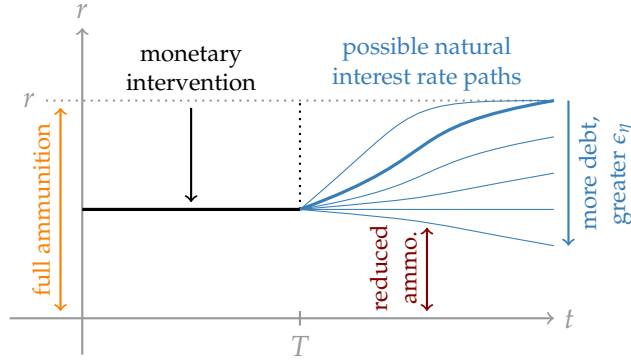
it endogenously limits its room to be accommodative in the future, as it also needs to ensure that the accumulated debt burden from past interventions does not cause a shortfall in demand. The accurate object summarizing the "room to be accommodative in the future" is the natural rate of interest r_t^n. Our next proposition studies the effect that monetary policy has on r_t^n.

Proposition 8. *To first order, a monetary policy shock as in* (19) *or* (20) *causes debt to rise and the natural rate to fall, $r_t^n < r$ for any t. For a given increase in debt, the natural rate falls by more (as measured by $\int_s^\infty e^{-r(t-s)} r_t^n dt$ for any s) if there is more non-homotheticity (as measured by the elasticity ϵ_η of η); and if interest rates are lower (lower $\hat{r}$) for longer (larger T).*

Accommodative monetary policy systematically reduces natural interest rates in our model, and thus endogenously limits the "ammunition" that is available to monetary policy in the future, before economy approaches the effective lower bound. This happens because in the presence of a greater debt burden, natural rates r_t^n cannot possibly be equal to r after $t = T$ as this would tighten borrowing constraints, and lead to the borrower severely contracting demand. Therefore, natural rates r_t^n are below r at least for some time after $t = T$ while the borrower deleverages (see Figure 12).

This logic operates even absent non-homotheticity. However, in a homothetic model, the convergence process $r_t^n \to r$ is sped up significantly by the fact that the saver's consumption rises significantly due to the increase in the saver's permanent income, pushing the natural rate up, closer to r. In a non-homothetic model such as ours, an additional reason for a decline in r_t^n emerges—indebted demand—which leads to lower natural rates and a significantly reduced convergence rate back to r. In other words, non-homotheticity and indebted demand significantly aggravate the "limited ammunition" property of monetary policy (see Figure 12). This can be sufficiently strong to permanently lower natural rates. Such behavior occurs when the economy exhibits multiple steady states (see our discussion in Section 3.2), and the monetary intervention is "too low for too long", that is, $\hat{r}$ is sufficiently low and T sufficiently long.

Relationship to literature. A number of economists have recently emphasized how the effectiveness of monetary policy interventions can be reduced by past interventions (e.g. Eichenbaum, Rebelo and Wong 2019, Berger, Milbradt, Tourre and Vavra 2018). In this paper, we do not consider consecutive interventions. Instead, we focus on how much "ammunition" in terms of the natural interest rate, a single intervention costs. This aspect of our paper is closest to McKay and Wieland (2019). To give an analogy with the IS curve, we focus on the effect of monetary policy on the future level of the IS curve as opposed to its slope.[38]

Practical implications for the conduct of monetary policy. Monetary policy can have long-lasting effects on natural rates through the accumulation of debt. This should be taken into account when contemplating the force with which to respond to different kinds macroeconomic shocks. Temporary shocks to borrowers' ability to borrow for instance—e.g. during a financial crisis—can be met with aggressive monetary easing as debt is unlikely to rise in this context. However, when reacting to shocks that do not directly affect borrowers' demand for debt—e.g. negative shocks to business investment as during the 2001 recession—aggressive monetary policy could lead to significant and persistent increases in household debt, and therefore reduce monetary policy ammunition going forward.

When used in conjunction with macroprudential policies that are designed to keep debt in check, thereby dampening the debt channel, monetary policy can be used more aggressively. That way, the economy does not merely "pull forward" demand through debt, demand which it then lacks in the future.

7 Debt Trap and Policies to Escape It

The most serious implications of indebted demand occur when it tips the economy into a liquidity trap. We next discuss ways in which an economy can slide into a liquidity trap, and evaluate policy options that may help the economy recover.

To do so, we focus on the case of our model where the interest rate associated with our model's steady state is possibly below its effective lower bound (ELB). Let $\underline{r} > 0$ denote that (real) effective lower bound. It needs to be positive as we take r to be the real return on wealth in our model. To get a number for $\underline{r}$, we propose to take an estimate of the real return on wealth during the ELB period—e.g. around 3.5% in Figure (1)—which then needs to be de-trended by productivity growth during the ZLB episode—e.g. 1.5%—to obtain $\underline{r}$. In this example, $\underline{r} = 2\%$.[39]

We next study a situation where the steady state natural interest rate r lies below $\underline{r}$, so that monetary policy cannot achieve full employment in steady state.

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[38] Aside from these implications, our model also finds that forward guidance is less powerful than in standard New-Keynesian models, for a reason similar to the one in Michaillat and Saez (2019).

[39] $\underline{r}$ can be microfounded in the context of our model in Appendix B.2, where it would be equal to the convenience yield. In Appendix B.8, we show how allowing for wage deflation amplifies our findings.

Figure 13: Falling into the liquidity trap in response to greater income inequality.

Household debt / GDP   Interest rate   Output gap

- - - Without ZLB   —— ZLB at $r = 2\%$
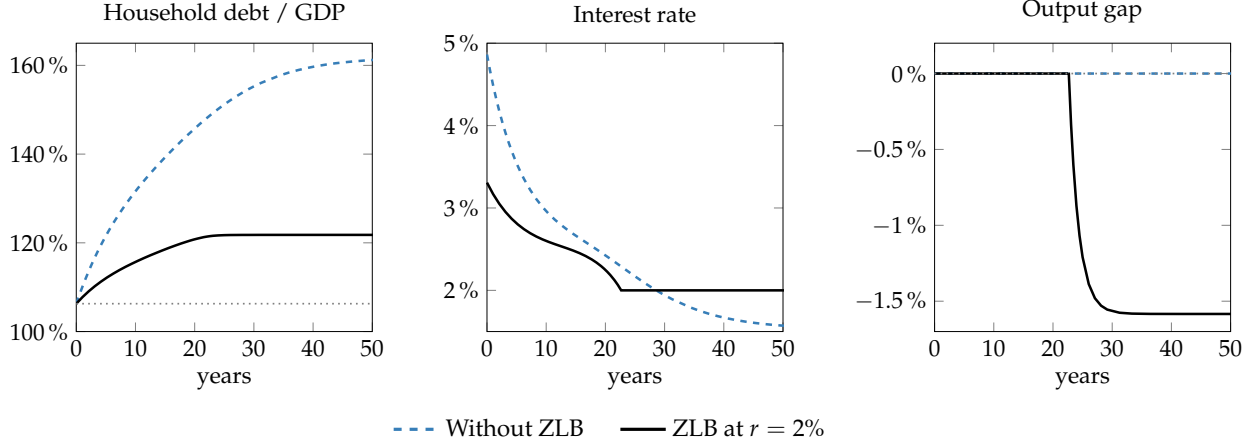
Note. These plots simulate an increase in income inequality from $\omega^s = 0.10$ to $\omega^s = 0.11$ in the non-homothetic economy. The black line assumes a lower bound of $\underline{r} = 2\%$, in line with the real return on wealth during the US ELB period (3.5% in Figure 1 net of 1.5% growth).

7.1 The debt trap steady state

We focus on a steady state (r, d) with a natural interest rate r below the effective lower bound, $r < \underline{r}$, that is,

$$\underline{r} < \rho \frac{1 + \delta/\rho}{1 + \delta/\rho\, \eta\, (\omega^s/\underline{r} + d)} \tag{23}$$

In this case, the economy gives rise to a stable liquidity trap steady state, which we henceforth also call *debt trap* due to its association with high levels of debt.

Proposition 9. *In the presence of an effective lower bound with* (23), *there exists a stable liquidity trap steady state ("debt trap"), in which output is reduced to*

$$\hat{Y} = Y \frac{\underline{r}}{\omega^s + \ell} \cdot \eta^{-1} \left(\frac{\rho}{\underline{r}} (1 + \rho/\delta) - \rho/\delta \right) < Y \tag{24}$$

In the debt trap, household debt is high and output is permanently reduced due to indebted demand. The reduction in output in (24) is larger the greater income inequality is (greater ω^s) and the higher the effective lower bound $\underline{r}$ is. Moreover, in our model, household debt is the key endogenous state variable that determines whether or not an economy is able to generate sufficient demand to avoid a liquidity trap. This implies that more household debt (greater ℓ) makes the trap more likely, and the output reduction greater. Any force or policy that boosts household debt in the debt trap will push output even lower in the long run.

It also means that the economy can slide into the trap over time, as debt levels increase (see Figure 13). Interestingly, the prospect of falling into debt trap itself accelerates its arrival. The reason for this is that both agents anticipate a recession in the debt trap, and thus, in an attempt to smooth consumption, cut back on their spending already in advance. This, however, only pushes down the natural rate further, closer to the effective lower bound. We illustrate this in Figure 13

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by plotting the transition path without imposing an ELB, which crosses the ELB later (blue dashed line).

The presence of debt as endogenous state variable sets our model apart from several prominent recent papers modeling secular stagnation, e.g. Benigno and Fornaro (2019), Caballero and Farhi (2017), Eggertsson, Mehrotra and Robbins (2019), and Ravn and Sterk (2018). Moreover, the liquidity trap here is indeed a *trap*, meaning associated with a stable steady state, rather than a relatively brief episode driven by household deleveraging, as in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017).

7.2 Potential policies to escape the trap

Fiscal policy in the debt trap. Once an economy finds itself in the debt trap, how does it get back out again? We can formally introduce the government exactly as in Section 6.1 and obtain the following result.[40]

Proposition 10. *With proportional income taxes τ^s, τ^b on the saver and borrower, output in the debt trap steady state is given by*

$$\hat{Y} = Y \frac{r}{(1-\tau^s)\omega^s + \ell} \cdot \eta^{-1} \left(\frac{\rho}{r} \left(1 + \rho/\delta \right) - \frac{\rho}{\delta} \right) \tag{25}$$

In particular: greater redistribution through taxes raises output and greater tax-financed government spending raises output.

This result is the mirror image of Proposition 6, except that at the effective lower bound, adjustments in the natural rate correspond to adjustments in aggregate demand and output. In particular, Proposition 10 suggests that redistributive tax policies—greater τ^s—can raise output $\hat{Y}$, reducing the severity of the liquidity trap. What would be the consequences of such a policy for the two agents? As long as the effective lower bound still binds, steady state consumption and wealth are given by

$$c^s = \underline{r}\eta^{-1} \left(\frac{\rho}{\underline{r}} \left(1 + \rho/\delta \right) - \frac{\rho}{\delta} \right) Y, \qquad a^s = \frac{c^s}{\underline{r}}, \qquad c^b = \hat{Y} - c^s. \tag{26}$$

The only object that is endogenous to the tax choice is $\hat{Y}$. Remarkably, greater redistribution therefore leaves the steady state consumption and wealth of savers entirely unaffected, while boosting the consumption of borrowers. The reason for this result is that the income loss of greater taxation of savers' incomes is exactly offset by rising overall incomes. This can happen in the liquidity trap due to aggregate demand externalities, as in Korinek and Simsek (2016) and Farhi and Werning (2016).

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[40]We leave out policies related to government debt here as those require a well-microfounded model of the interest rate spread between government and private debt that is beyond the scope of this paper.

What are the implications for welfare? While (26) only holds across steady states, observe that any policy change that raises $\hat{Y}$ also relaxes the borrowing constraint and thus generates additional consumption for borrowers during the transition period. Moreover, one can show that consumption and wealth of savers are constant at the levels in (26) throughout the transition. Thus, our model implies that, in the liquidity trap, greater redistribution is Pareto-improving.

Wealth taxes. Our framework provides an interesting and novel perspective to the current debate (as of January 2021) surrounding wealth taxes (e.g. Saez and Zucman 2019, Guvenen, Kambourov, Kuruscu, Ocampo-Diaz and Chen 2019, Sarin, Summers and Kupferberg 2020). A (progressive) wealth tax τ^a in our model taxes the saver's wealth $a^s = d + \frac{\omega^s}{r} + B$ each period, that is, the savers' consolidated budget constraint (2) becomes

$$c_t^s + \dot{a}_t^s = (r_t - \tau^a) \, a_t^s \tag{27}$$

The returns to this policy are rebated to borrowers. As can be seen in (27), the wealth tax effectively reduces the after-tax return on wealth realized by savers. This changes their steady state saving supply schedule (9) to

$$r - \tau^a = \rho \cdot \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(a^s)}$$

The relevant interest rate for savers' saving behavior is the after-tax return on wealth $r - \tau^a$. Thus, the presence of τ^a effectively relaxes the effective lower bound, raising output (25) during the liquidity trap according to

$$\hat{Y} = Y \frac{\underline{r} - \tau^a}{(1 - \tau^s)\omega^s + \ell(\underline{r})} \cdot \eta^{-1} \left(\frac{\rho}{\underline{r} - \tau^a} (1 + \rho/\delta) - \frac{\rho}{\delta} \right).$$

Thus, a progressive wealth tax is successful in mitigating the impact of secular stagnation in our framework.[41]

Macroprudential policies in the debt trap. A commonly prescribed remedy for economies with high debt burdens is macroprudential policy designed to bring down debt. Being the opposite of financial deregulation, we think of such a policy as a reduction in ℓ. Equation (24) immediately implies that such a policy raises demand and output $\hat{Y}$ in the long run, mitigating the recession. However, during the period of deleveraging, the economy goes through a significant short-run bust. This emphasizes that debt is best reduced by reducing saving supply rather than the demand for debt.

Debt jubilees. An alternative way to deal with a liquidity trap caused by high levels of debt is a debt jubilee. We define a *debt jubilee of size* $\Delta > 0$ as an immediate reduction in both private debt d

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[41]Interestingly, our model also suggests that a consumption (or VAT) tax would reduce natural interest rates and hence worsen the liquidity trap.

and saver's assets a^s by Δ. The following result lays out the long-run implications of a debt jubilee policy:

Proposition 11 (Bailouts and debt jubilee.)**.** *A debt jubilee raises output in the short run, but unless combined with structural changes to inequality, redistribution or debt limits, there is no change to the economy in the long-run.*

A debt jubilee amounts to a sudden reduction in the state variable d_t. With a unique stable steady state, this cannot have a long-run effect. In other words, borrowers would get themselves into debt again. However, a debt jubilee has positive short run effects, as it lifts the economy temporarily out of the debt trap. To prevent it from falling into it again, it can be paired with other policies, e.g. redistributive or macroprudential policies (see above). Such a combination can jointly address short and long-run issues.

8 Two Extensions

Our baseline model was intentionally kept simple. We now present two extensions to the baseline model in Section 2, beginning with investment.[42]

8.1 Role of Investment

To introduce investment, we allow output Y to be produced from three factors, capital K and both types of agents' labor supply n^i, and we write the net-of-depreciation production function as[43]

$$Y = F(K, n^b, n^s).$$

Dynasty i is assumed to have a labor endowment of $n^i = 1$ whose factor income is capitalized in its real asset wealth, or human capital, h_t^i.[44] We assume that without loss that Y has constant returns to scale; otherwise we include a fixed factor owned by savers and/or borrowers. Thus, total income Y can be split up into income going to savers and income going to borrowers. The income shares only depend on the level of capital K, which itself is pinned down by the interest rate,

$$F_K = r.$$

As only savers hold capital in our economy, the agents' income shares are then described by the following functions of the interest rate

$$\omega^s(r) = \frac{F_K K}{F} + \frac{F_{n^s} n^s}{F} \qquad \text{and} \qquad \omega^b(r) = \frac{F_{n^b} n^b}{F} = 1 - \omega^s(r).$$

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[42]Appendix B shows a number of other important extensions.

[43]We assume the typical: F is strictly concave, satisfying Inada conditions.

[44]We assume that the economy is not in the debt trap for this section. Investment would be negatively affected in the debt trap and thus amplify the output loss there, similar to the logic in Benigno and Fornaro (2018).

Figure 14: Capital and indebted demand.
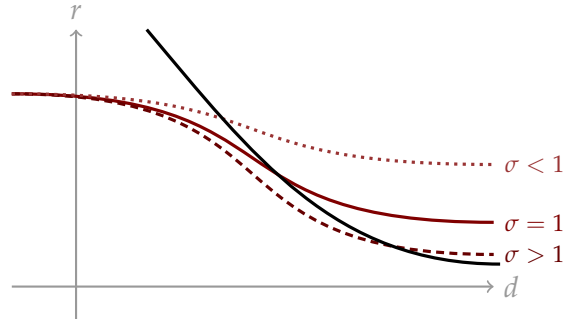
With these income shares, we can now characterize our economy's steady states as

$$r = \rho \cdot \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(\omega^s(r)/r + d)} \qquad \text{and} \qquad d = \frac{\ell}{r}. \tag{28}$$

Crucially, $\omega^s(r)$ is now possibly a function of r. As we demonstrate in the following proposition, the shape of $\omega^s(r)$ depends on the (Allen) elasticity of substitution $\sigma \equiv F_{n^b}F_K/(F_{Kn^b}F)$ between capital and borrowers' labor supply.

Proposition 12. *Let σ be the elasticity of substitution between capital and borrowers' labor supply in the economy with investment. Denote by $\omega^s(r)$ the savers' income share.*

a) *If $\sigma = 1$: $\omega^s(r)$ is independent of the interest rate r. The steady state is identical to the one in the economy without investment.*

b) *If $\sigma < 1$: $\omega^s(r)$ falls with lower r. This flattens the saving supply schedule.*

c) *If $\sigma > 1$: $\omega^s(r)$ rises with lower r. This steepens in the saving supply schedule.*

Proposition 12 precisely characterizes the role of investment for the long-run economy. When an increase in capital leaves the income distribution unchanged, investment will have no effect in the long-run (case (a)). So to what extent capital matters depends on whether it crowds the share of income going to borrowers in or out. If capital is complementary to borrowers' labor supply, it reduces the extent of indebted demand (even if it can never fully undo it) as lower interest rates go hand in hand with greater capital and a more equitable income distribution (case (b)). If capital is substitutable with borrowers' labor supply—one may think of capital-skill complementarity and automation as in Krusell, Ohanian, Rios-Rull and Violante (2000) and Autor, Levy and Murnane (2003),—the opposite is the case. Lower interest rates endogenously lead to a more unequal income distribution, effectively steepening the saving supply schedule and amplifying the problem of indebted demand (case (c)).[45] We illustrate the three cases in Figure 14.

This discussion focused on the long run. Investment contributes to demand in the short run as interest rates fall, irrespective of the structure of the production function F. For example, if F

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[45]See Straub (2019) for a related point.

is Cobb-Douglas, and the economy sees a shift in income inequality, investment picks up initially, temporarily slowing the decline in r. As the investment boom recedes, however, the fall in r accelerates again, eventually falling to the exact same steady state level as would have occurred without investment (as in Section 5).

Observe that the recent US experience does not align well with this description of investment. It did not seem that investment (as a fraction of GDP) rose as interest rates fell. This is subject of a recent literature (e.g. Gutiérrez and Philippon 2017, Liu, Mian and Sufi 2019, Farhi and Gourio 2018, Eggertsson, Robbins and Wold 2018).

Which kind of debt causes indebted demand? Productive vs. unproductive debt. Investment may be funded with (corporate) debt, raising the question, which kind of debt actually causes indebted demand? In Section 3, we argued that, starting in some steady state, a one-time exogenous increase in debt by some dD, holding r fixed, causes a response of aggregate spending of

$$dC = -\frac{\rho + \delta}{2} \left(1 - \sqrt{1 - 4 \left(1 - \frac{r}{\rho + \delta} \right) \frac{r}{\rho + \delta} \epsilon_\eta} \right) dD \tag{29}$$

(see Proposition 12). The debt in this experiment is *unproductive*, that is, it is being used for consumption.

We now repeat this exercise with *productive* debt, that is, debt that is being used to raise the capital stock of the economy, $dK = dD$. Holding r and household debt d fixed, what are the implications for aggregate spending?

Proposition 13 (Indebted demand when debt is productive). *Starting from a steady state and holding r and d, fixed, an exogenous increase in debt dD that raises the capital stock by the same amount affects aggregate spending by*

$$dC = -\frac{\rho + \delta}{2} \left(1 - \sqrt{1 - 4 \left(1 - \frac{r}{\rho + \delta} \right) \frac{r}{\rho + \delta} \epsilon_\eta \chi} \right) dD. \tag{30}$$

where $\chi \equiv \left(1 - \sigma^{-1} \right) \omega^b - r \frac{rd}{Y} < 1$.

A first observation about Proposition 13 is that productive debt, (30), always causes strictly less indebted demand than unproductive debt, (29). To see this, note that $\chi = \left(1 - \sigma^{-1} \right) \omega^b - r\frac{rd}{Y} < 1$. Thus, even if capital is perfectly substitutable with borrowers' labor supply, $\sigma = \infty$, productive debt does not cause as much indebted demand. The reason for this is that even if $\sigma = \infty$, capital raises aggregate output. A second observation is that the negative effect of dD on aggregate spending dC falls for lower values of σ, as one would expect given Proposition 12. In the case $\sigma = 1$, the effect is positive at first, $dC > 0$. Recall that (30) is the contemporary effect on spending—but once we allow household debt to increase with Y, the positive effect fades.

The distinction between productive and unproductive debt is not always obvious. Consider, for example, investment in infrastructure (e.g. airports, buildings, public transport) in remote

locations, as some argue can be found in China today. Debt that is financing such investments ought to be thought of as unproductive.

A special case where debt is used productively but χ is still very high is residential investment into borrowers' owner-occupied housing. For example, imagine borrowers remodel or extend their houses. Clearly this is productive, as a greater housing stock produces more housing services. However, these additional housing services are consumed by borrowers themselves and do not increase borrowers' marginal product of labor. Thus, in this case, $\sigma = \infty$ and $\chi = \omega^b - r\frac{d}{Y}$, the largest possible value for productive debt.

In sum, our results suggest that productive debt always causes weaker indebted demand than unproductive debt. The degree to which it is weaker depends on the elasticity of substitution σ between the capital that is accumulated using debt and the borrowers' labor supply. This suggests that debt-financed productive investments made by firms or governments need not necessarily contribute to indebted demand.

Does student debt contribute to indebted demand?   Since the end of the Great Recession, the US has witnessed a significant increase in student debt, begging the question whether student debt is a likely contributor to indebted demand. At the face of it, it seems that the answer is no. After all, student debt finances investment in human capital, and thus can be analyzed like investment in physical capital.

There is, however, an important difference. The rise in student debt partly reflects the rising cost of college tuition. To the extent that this is the case, student debt is indeed a source of indebted demand, as borrowers have to service larger piles of student debt without having accumulated greater human capital.

8.2   Longer duration debt

A recent literature highlighted that responses of economies to interest rate changes differ according to the type of debt contract agents hold (e.g. adjustable-rate vs. fixed-rate contracts) as well as the debt's maturity (e.g. Campbell 2013, Calza, Monacelli and Stracca 2013, Di Maggio et al. 2017). In this extension, we briefly investigate the conceptual role of debt duration for indebted demand.

Consider a version of our baseline model in Section 2. Assume debt has an entirely fixed rate, equal to the steady state debt payment ℓ (which is equal across steady states). How does this change the economy?

While fixed rate (FR) debt does not change the steady state, it does affect the transitional dynamics. To show this, consider first the experiment of rising income inequality in Figure 7. In that figure, due to the anticipated fall in interest rates, the present value of pledgeable wealth increases, while the present value of debt remains unchanged, as debt is adjustable rate. Thus, borrowers have more room to spend, pushing interest rates up in the short run.

With fixed rate debt, the present value of debt jumps on impact, in lockstep with pledgeable wealth. In fact, given our assumption of completely fixed debt, the value of debt exactly equals

pledgeable wealth. This implies that borrowers have no additional room to spend, and the economy adjusts *immediately* on impact.

Proposition 14 (Transition with completely fixed rate debt.)**.** *When debt carries a completely fixed rate ℓ, an unexpected permanent change in income inequality (greater ω^s) lets the economy jump immediately to its new steady state.*

Thus, the presence of fixed rate debt implies weaker aggregate demand during transitions with rising debt levels and falling interest rates, which speeds up the transition.

Now consider a shock that pushes real interest rates up, such as a reduction in inequality or an increase in progressive taxation. The present value of adjustable-rate (AR) debt is again unchanged, but the present value of FR debt *falls,* again speeding up the transition. Thus, in this sense, AR debt makes it harder to leave a steady state with high levels of debt since any increase in interest rates leads to an immediate sharp fall in demand without favorable revaluation effect.

These discussions highlight that AR debt contracts slow down transitions into states with low r and high debt, and FR debt contracts speed up transitions away from such states. Fixed-rate contracts with automatic refinancing achieve both of these arguably favorable outcomes.[46] Policies that raise the share of refinancing among US fixed-rate mortgage owners are therefore beneficial from this perspective.

9 Quantitative Exploration

Thus far, we illustrated several important properties in various extensions of the baseline model. In this section, we combine those extensions into a single richer model. Relative to the baseline model in Section 2, the model in this section includes additional forces: taxation and government bonds in the steady state; land; investment subject to Hayashi (1982) type capital adjustment cost; and the possibility of a transition without perfect foresight. We use the model to revisit the effect of inequality on interest rates and debt.

Extending the model.   *Households* — Dynasties continue to maximizes preferences (1), subject to their budget constraints (2). Their wealth a_t^i is still given by $h_t^i - d_t^i$. Real asset wealth is now given by the discounted stream of after-tax human wealth as well as the dynasty's share κ^i of capital holdings

$$h_t^i = \int_t^\infty e^{-\int_t^s r_u du} (1 - \tau_s^i) w_s \mu^i n_s^i ds + \kappa^i J_t$$

where $\kappa^s + \kappa^b = 1$; J_t is the value of land and capital combined. We continue to assume that agents can pledge part of their real asset wealth, in this case part of their land holdings, giving rise to the same borrowing constraint (7), in which p_t is the price of land.

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[46]Lengthening the maturity of debt when debt is high, as in Campbell, Clara and Cocco (2018), would further speed up transitions back to lower debt states as they would lengthen the duration of debt. Automatic refinancing is reminiscent of the idea to convert FR into AR contracts (Guren, Krishnamurthy and McQuade 2017), albeit such a conversion would slow down transitions back to states with lower debt levels.

Government — The government finances a fixed amount of government spending G and a fixed amount of government debt B using proportional taxes $\tau_t^i = \tau_t$ that are set to satisfy the government budget constraint (16).

Investment — There is a representative firm making investment decisions subject to quadratic capital adjustment costs. The firm uses a nested CES production function of the form

$$Y = F(\mathcal{L}, K, n^b, n^s) = \mathcal{L}^\psi \left(\frac{\alpha}{\alpha + \omega^b} \left(\frac{K}{K^*} \right)^{\frac{\sigma-1}{\sigma}} + \frac{\omega^b}{\alpha + \omega^b} \left(n^b \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\left(\alpha + \omega^b \right)} (n^s)^{\omega^s}$$

where ψ is the share of income going to owners of land $\mathcal{L}$ and, as before, σ is the Allen elasticity between K and n^b.[47] The normalization parameter K^* will be chosen to be equal to the initial steady state capital stock, in order for there not to be a direct effect of changes in σ on output. When $\sigma = 1$, the production function is Cobb-Douglas, with capital share α and labor income shares ω^i. We assume $\psi + \alpha + \omega^b + \omega^s = 1$. The firm maximizes its value

$$r_t J_t(K_t) = \max_{n_t^b, n_t^s, I_t} F(K_t, n_t^b, n_t^s) - w_t^b n_t^b - w_t^s n_t^s - I_t - \zeta \left(\frac{I_t - \delta_K K_t}{K_t} \right) K_t + \frac{d}{dt} J_t(K_t) \tag{31}$$

subject to the law of motion of capital $\dot{K}_t = I_t - \delta_K K_t$. The adjustment cost function is quadratic $\zeta(x) = \frac{1}{2\delta_K \epsilon_K} x^2$. The price of capital is given by $p_t \equiv \partial J_t(K_t)/\partial K_t$.

Equilibrium — The definition of equilibrium is like in Section 2.4. Households maximize their utility (1), firms maximize their value (31), and markets clear:

$$c_t^s + c_t^b + I_t + G = Y_t \qquad\qquad d_t^s + d_t^b + B = 0.$$

Calibration — We pursue a similar calibration strategy to the one in Section 3.5. We continue to use the same values for $\mu, \rho, \delta, \ell, r, \lambda$. The land share is set to $\psi = 0.05$, depreciation δ_K is set to 0.06, and the capital share α is set to 0.20, so that the value of capital to output is approximately 2.5 in the steady state, a standard value. Government spending G is 15% of GDP, government debt is 40% of GDP. Parameters $\tilde{\eta}, \tilde{a}$ are still chosen to match a steady state interest rate of 5.5% and an MPC out of wealth of savers of 0.01. Here, this gives $\tilde{\eta} = 1.14$, $\tilde{a} = 0.51$. ω^s and κ^s, which for simplicity we choose to be the same, are picked to match a top 1% income share of 10% (Smith, Yagan, Zidar and Zwick 2019) in 1980. This gives $\omega^s = \kappa^s = 4\%$.[48] The adjustment cost parameter ϵ_K is chosen to match a semi-elasticity of investment to Tobin's Q, p_t, of 1 as in Auclert and Rognlie (2018), so that $\epsilon_K = 1$. Our baseline assumes an Allen elasticity of $\sigma = 1$. We allow both ϵ_K and σ to vary below.

Effects of rising inequality. We revisit the experiment of rising income inequality from Section 5.1, studying the implications of a number of assumptions. We increase inequality from

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[47]Here, it is more convenient to define σ directly on the gross production function.

[48]The fact that we choose κ^s to be the same as ω^s has almost no bearing on our results.

Figure 15: Rising income inequality—Robustness in richer models.
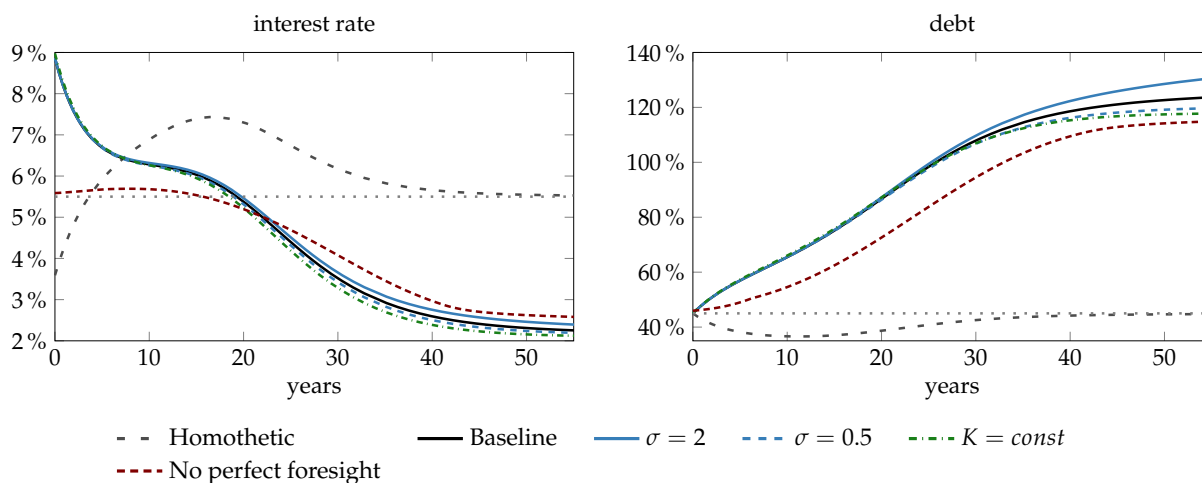
Note. Plots show transitions from our calibrated steady state with $\omega^s = 0.04$ to one with $\omega^s = 0.11$.

$\omega^s = 4\%$ to $\omega^s = 11\%$, in line with the findings of Smith, Yagan, Zidar and Zwick (2019).[49]

Figure 15 shows the resulting paths of interest rates and debt levels, for six different version of the quantitative model. The dashed gray line shows the homothetic economy, in which debt barely moves, and just like before, natural rates fall briefly but then converge back to their steady state level of 5.5%. The solid black line shows the transition in our baseline non-homothetic model. It shows a continuous upward trend in debt levels, along with a strong decline in the natural interest rate. The trends in debt levels are amplified somewhat when the elasticity of substitution σ is assumed to be 2, and mitigated somewhat when $\sigma = 0.5$.[50] Interestingly, both trends are similar when the capital stock is assumed to be constant at its initial level, with infinite adjustment cost, $\epsilon_K = 0$.

The red densely dashed line shows the baseline transition, but without the assumption of perfect foresight. In particular, we assume that each instant, when presented with a greater ω^s level, agents simply expect ω^s to remain constant at the current level forever after. While this has no bearing on the eventual natural rate and debt levels, it no longer exhibits an increase of the natural interest rate on impact, in line with our discussion of Figure 7.

Quantitatively, the simulations predict an eventual interest rate decline of around 3%, from one steady state to another, which is in the range of estimates for the decline of the natural interest rate over the past four decades (Laubach and Williams 2016). Debt levels are predicted to eventually increase by about 80 percentage points relative to GDP (steady state to steady state), which is greater than the actual increase that happened in the US so far (55*pp* until the financial crisis of 2008/09, 40*pp* until Q2 2020). This is likely due to the fact that our analysis in this section worked with a simplified borrowing constraint (7) and did not include an effective lower bound, which—

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[49]See Figure IX in Smith, Yagan, Zidar and Zwick (2019). The share of labor income plus 75% of the share of business income going to the top 1% rose by approximately 7 pp.

[50]These forces are harder to see for the interest rate as it tends to converge more slowly to its long-run level.

as shown in Figure 13—mitigates the increase in debt levels.

10   Conclusion

In this paper, we proposed a new theory connecting several recent secular trends: the increase in income inequality, financial deregulation, the decline in natural interest rates, and the rise in debt by households and governments.

The central element in our theory are non-homothetic preferences, which lead richer households to have greater saving rates out of a permanent income transfer. This gives rise to the idea of indebted demand: greater debt levels mean a greater transfer of income in the form of debt service payments from borrowers to savers, and thus depress demand.

We identified three main implications of indebted demand. First, secular economic shifts that raise debt levels (e.g. income inequality or financial deregulation) also lower natural interest rates, which then itself has an amplified effect on debt. Second, monetary and fiscal policy, to the extent that they involve household or government debt creation, can persistently reduce future natural interest rates. This means that there is only a limited number of such policy interventions that can be used before economies approach the effective lower bound. Finally, when the lower bound is binding, the economy is in a liquidity trap with depressed output. In this "debt trap", debt-financed stimulus deepens the recession in the future, whereas redistributive policies and policies addressing the structural sources of inequality mitigate it.

Our results suggest that economies face a sort of "budget constraint for aggregate demand". They can stimulate aggregate demand through debt creation, but that reduces future demand (and thus natural interest rates). This logic suggests a new trade-off for debt-based stimulus policies. We view an exploration of this trade-off in an optimal policy setting as a promising avenue for future research.

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Online Appendix to

Indebted Demand

Atif Mian   **Ludwig Straub**   **Amir Sufi**

A   Proofs

Proof of Proposition 1.   This proof is an immediate consequence of (9).

Proof of Proposition 2.   Given that debt $d > 0$, savers cannot be up against their borrowing constraint. Thus, the Euler equation has to hold with equality, implying (10). Moreover, the Euler equation cannot hold with equality for borrowers, as their per capita wealth is much smaller than that of savers. Thus, their borrowing constraint is binding, implying (11).

    Proof of unique steady state.

    To show that there is a unique steady state if it exists, use $p = 1/r$ and rewrite the steady state conditions (10) and (11) as

$$\rho p \left(1 + \delta/\rho\right) = 1 + \delta/\rho \cdot \eta \left(\left(\omega^s + \ell\right) p\right).$$

Assume p is a steady state. We next show that at any such p, the slope of the left hand side is greater than the slope of the right hand side. This then shows no two intersections can exist, or else at least one of them must have the reverse order of slopes.

    The left hand side is a linear function (in p) with constant slope $\rho \left(1 + \delta/\rho\right)$. At steady state p, the slope of the right hand side is given by

$$\delta/\rho \cdot \eta' \left(\left(\omega^s + \ell\right) p\right) \left(\omega^s + \ell\right) < \frac{1}{p} \delta/\rho \eta \left(\left(\omega^s + \ell\right) p\right) = \rho \left(1 + \delta/\rho\right) - \frac{1}{p} \tag{32}$$

which is clearly below $\rho \left(1 + \delta/\rho\right)$. The inequality in (32) follows from concavity of $v(a)$ and the fact that it implies $\eta'(a) < \eta(a)/a$.

Proof of Proposition 3.   As borrowers are borrowing constrained, their consumption responds one for one, $dc^b = -dx$. To analyze the consumption response by savers define their consumption-wealth ratio $\chi_t \equiv c_t^s/a_t^s$. Expressing budget constraint and Euler equation in terms of (a_t^s, χ_t), we find

$$\frac{\dot{a}_t^s}{a_t^s} = r - \chi_t \qquad \text{and} \qquad \frac{\dot{\chi}_t}{\chi_t} = \chi_t - \rho - \delta + \frac{\delta}{\rho} \chi_t \eta(a_t^s).$$

To get the on-impact consumption response of savers, we need to solve the ODEs for a small change in wealth at date 0 away from the steady state, in which $\chi^* = r$ and $r = \rho + \delta - \delta/\rho r \eta(a^{s*})$. Let $\hat{a}$, $\hat{\chi}$ be the deviations (in levels, not logs) from the steady state. Defining $\epsilon_\eta \equiv \eta'(a^{s*})a^{s*}/\eta(a^{s*})$, we have

$$\dot{\hat{a}}_t = -a^{s*}\hat{\chi}_t \qquad \text{and} \qquad \dot{\hat{\chi}}_t = (\rho + \delta)\hat{\chi}_t + (\rho + \delta - r) r\epsilon_\eta \frac{\hat{a}_t}{a^{s*}}. \tag{33}$$

We guess and verify that $\hat{\chi} = -k\hat{a}/a^{s*}$ for $k > 0$. Using the equations in (33) we find a quadratic for k,

$$k^2 - (\rho + \delta) k + (\rho + \delta - r)r\epsilon_\eta = 0.$$

The only solution that leads to a positive consumption response to transfers is

$$k = \frac{\rho + \delta}{2}\left(1 - \sqrt{1 - 4\left(1 - \frac{r}{\rho + \delta}\right)\frac{r}{\rho + \delta}\epsilon_\eta}\right).$$

Since $\hat{c}_t^s = a^{s*}\hat{\chi}_t + r\hat{a}_t = (r - k)\hat{a}_t$, and $\hat{a}_0 = dx/r$, we have

$$dc_0^s = dx - k/rdx$$

which together with dc_0^b implies (12).

Local uniqueness and existence result. Here we prove that locally around a stable steady state, i.e. for d_0 in a neighborhood of steady-state debt d^*, there is a unique equilibrium. To do this, we first collect the equations describing equilibrium when the borrower is up against a binding borrowing constraint. The pricing equation for real assets is

$$r_t p_t = 1 + \dot{p}_t. \tag{34}$$

The debt level evolves as

$$\dot{d}_t = \lambda \ell p_t - \lambda d_t \tag{35}$$

which we can plug into the savers' collective budget constraint, we get the interest rate

$$r_t = \frac{1}{d_t}\left\{c_t^s + \lambda \ell p_t - \lambda d_t - \omega_t^s\right\}. \tag{36}$$

Finally, savers' Euler equation is given by

$$\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta\frac{c_t^s}{\rho\left(\omega^s p_t + d_t\right)}\eta(\omega^s p_t + d_t). \tag{37}$$

Equations (34)–(37) describe a system of 3 ODEs in variables (p_t, d_t, c_t^s), after using (36) to substitute out the interest rate r_t. Denote by (p^*, d^*, c^{s*}) the respective steady state values of the three variables. We define two auxiliary variables

$$w \equiv \frac{\omega^s}{\ell} \quad \text{and} \quad \Delta \equiv \rho + \delta - r^*.$$

In the homothetic model, $\Delta = 0$. Using them, we find for the steady state variables

$$p^* = \frac{1}{r^*}, \quad d^* = \frac{\ell}{r^*}, \quad \frac{c^*}{d^*} = r^*(1 + w), \quad \frac{p^*}{d^*} = \frac{1}{\ell}.$$

Denote by $\tilde{p}, \tilde{d}, \tilde{c}^s$ the log-linearized versions of p, d, c^s. Denote by $\tilde{r}$ the linearized version of r. Linearizing (34)–(37) we find

$$\tilde{r} = -r^*\tilde{d} + \tilde{c}r^*(1 + w) + \lambda\tilde{p} - \lambda\tilde{d}$$

$$\dot{\tilde{p}} = \tilde{r} + r^*\tilde{p} \tag{38}$$

$$\dot{\tilde{d}} = \lambda\tilde{p} - \lambda\tilde{d}$$

$$\dot{\tilde{c}} = \tilde{r} + \Delta \left[\tilde{c} - (1 - \epsilon_\eta) \frac{w\tilde{p} + \tilde{d}}{w + 1} \right]$$

We guess and verify that the solution takes the form

$$\tilde{r} = -R\tilde{d}, \quad \tilde{p} = P\tilde{d}, \quad \tilde{c} = C\tilde{d}.$$

We also verify that $\dot{\tilde{d}}/\tilde{d} < 0$, which is necessary for an equilibrium in the neighborhood of the steady state. After some algebra, we arrive at a condition for P,

$$F(P) = 0$$

where

$$F(P) = [\Delta + \lambda (2 - P) + r^*(1 + w)] \left(\lambda(1 - P) + r^* \right) P + \Delta P \lambda$$
$$- \lambda(\Delta + \lambda(1 - P)) - r^* \lambda + r^* \Delta \left[(1 - \epsilon_\eta) (1 + Pw) - 1 \right].$$

As one can see easily, $F(0) < 0$, $F(1) > 0$, $F''(1) < 0$, $F'''(0) > 0$ (the coefficient on P^3 is positive). Together, they imply that F admits a unique root P below 1. This root is therefore unique in implying stability, $\dot{\tilde{d}}/\tilde{d} = \lambda(P - 1) < 0$. It also implies that

$$R = P\lambda(1 - P) + r^* P > 0, \tag{39}$$

so that the (natural) interest rate is lower if the economy has a higher debt level than steady state debt. Moreover, observe that F strictly declines in ϵ_η. Thus,

$$\frac{\partial P}{\partial \epsilon_\eta} > 0. \tag{40}$$

Proof of Proposition 4. The proof is an immediate consequence of Proposition 2.

Proof of Proposition 5. The proof is an immediate consequence of Proposition 2.

Proof of Proposition 6. This follows directly from (17) and (18).

Proof of Proposition 7. Savers' intertemporal budget constraint is given by

$$\int_0^\tau e^{-\int_0^t r_s ds} c_t^i dt = \omega^s \int_0^\tau e^{-\int_0^t r_s ds} \hat{Y}_t dt + d_0 - e^{-\int_0^\tau r_s ds} d_\tau \tag{41}$$

which implies (21) by subtracting from (41) its steady state analogue. (22) is a direct consequence of (21) and the fact that the steady state is stable, and so $d_\tau \to d$ as $\tau \to \infty$.[51]

Proof of Proposition 8. To start, we derive the system of equations characterizing the evolution of the economy in response to an arbitrary path of interest rates r_t. From the savers' budget

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[51]Stability of a steady state for a fixed interest rate $\hat{r}$ follows from (42) below. For fixed $\hat{r}$, $\hat{a}_t^s$ is immediately at its steady state value (see discussion below and Figure 16).

Figure 16: Phase diagram for (c_t^s, a_t^s) under an exogenous interest rate.
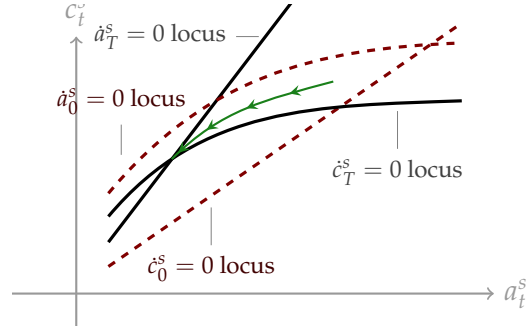
constraint and Euler equation we have

$$\dot{a}_t^s = r_t a_t^s - c_t^s$$

$$\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta \frac{c_t^s}{\rho a_t^s} \eta \left(a_t^s \right).$$

As one can easily see in a phase diagram (Figure 16), both equations are forward looking. Thus, under a monetary policy shock as in (19), c_t^s and a_t^s jump up on impact, converging back towards their steady state values, $c_t^s = c^s$ and $a_t^s = a^s$, which they reach at $t = T$. Noting that $a_t^s = \omega^s p_t + d_t$, we find that debt evolves as

$$\dot{d}_t = -\lambda(1 + w^{-1})d_t + w^{-1}a_t^s \tag{42}$$

implying that $d_t > d$ for all $t > 0$ ($w \equiv \omega^s/\ell$). Thus, under a monetary policy shock as in (19), debt strictly rises. An (first-order) increase in debt, as explained in (39), leads to a reduction in natural interest rates.

The increase in debt is greater for all t under monetary policy shock (20). The reason for this is that under (19), natural rates are still below steady state r after $t = T$. Thus, if $r_t = r_t^n$ is set after $t = T$, r_t is below what it would be under (19). But if r_t is lower, both a_t^s and c_t^s are greater everywhere (Figure 16), which by (42) must translate into a greater level of debt. Similarly, if we increase T or reduce $\hat{r}$, this further raises c_t^s and a_t^s in Figure 16, thus also increasing the path of debt d_t at all times.

Why do natural rates fall more with greater non-homotheticity? To prove this, we focus on a convenient measure of the path of natural rates, namely the present value, $R_s^n \equiv r \int_s^\infty e^{-r(t-s)} r_t^n dt$. We ask: how does $\tilde{R}_s^n \equiv R_s^n - r$ vary with the level of debt $\tilde{d}$? And how does that mapping depend on ϵ_η? By (38), we find that

$$\tilde{R}_t^n = -r P \tilde{d}_t.$$

Thus, natural interest rates always fall when $\tilde{d}_t > 0$, and more so the larger P is, e.g. when ϵ_η is greater (see (40)).

Finally, consider a situation with multiple steady states. Call $\underline{\zeta}$ the interest rate of the alternative high-debt, low-interest-rate steady state, and $\underline{c}^s, \underline{a}^s, \underline{d}$ the associated values of c^s, a^s and $\underline{d}$. Based on Figure 16, setting $\hat{r} = \underline{\zeta}$ forever ($T = \infty$) means $c_t^s = \underline{c}^s$ and $a_t^s = \underline{a}^s$ for any $t > 0$. Both variables jump immediately to their new values. Debt evolves as in (42), and converges to $\underline{d}$. Because the new steady state is stable, the economy does not converge back to its original values. Also, by continuity, there exists a threshold $\underline{T}$, such that any $T > \underline{T}$ will bring the economy sufficiently close to $\underline{d}$ that it will converge by itself (without any further stimulus) to the alternative

steady state.

Proof of Proposition 9. The expression for $\hat{Y}$ follows directly from inverting saving supply schedule at $\underline{r}$,

$$\underline{r} = \rho \frac{1 + \delta/\rho}{1 + \delta/\rho\eta \left(\frac{\omega^s + \ell}{r} \hat{Y} \right)}$$

to solve for $\hat{Y}$. The debt trap is stable since the economy, for a fixed interest rate $\underline{r}$, is described by constant c^s, a^s, and debt evolves as in (42), converging always back to the debt trap debt level.

Proof of Proposition 10. (25) again follows directly from inverting the saving supply schedule with fiscal policy at $\underline{r}$,

$$\underline{r} = \rho \frac{1 + \delta/\rho}{1 + \delta/\rho\eta \left(\frac{(1-\tau^s)\omega^s + \ell}{r} \hat{Y} + B \right)}.$$

Proof of Proposition 12. The result follows directly from (28) and the behavior of $\omega^s(r)$. Observe that

$$\frac{\partial \omega^s}{\partial K} = -\frac{\partial \omega^b}{\partial K} = \left(1 - \sigma^{-1} \right) \omega^b \frac{F_K}{Y} \tag{43}$$

implying that ω^s increases with K (and falls with r) precisely iff $\sigma > 1$, and is constant if $\sigma = 1$. The slope of the saving supply schedule is given by

$$\frac{\partial r}{\partial d} = -\frac{\delta \eta' r}{\rho + \delta \eta - \delta \eta' \frac{\omega^s(r)}{r} + \delta \eta' \omega^{s\prime}(r)}$$

where η, η' are evaluated at $\omega^s(r)/r + d$. Evidently, the larger (more positive) $\omega^{s\prime}(r)$ is, the flatter is the saving supply schedule. Vice versa for negative $\omega^{s\prime}(r)$.

Proof of Proposition 13. To derive (30), observe that

$$\frac{\partial (a^s/Y)}{\partial D} = \frac{\partial}{\partial D} \left(\frac{\omega^s}{r} + \frac{rd}{Y} \right) = \left(1 - \sigma^{-1} \right) \omega^b - \frac{rd}{Y} \frac{F_K}{Y} = \left(1 - \sigma^{-1} \right) \omega^b - r\frac{rd}{Y} \equiv \chi$$

where we used (43). Following steps as in the proof of Proposition 3, we then find that the savers' consumption response to the increase in their assets dD is given by

$$dc_0^s = (r - k)dD$$

where now

$$k = \frac{\rho + \delta}{2} \left(1 - \sqrt{1 - 4 \left(1 - \frac{r}{\rho + \delta} \right) \frac{r}{\rho + \delta} \epsilon_\eta \chi} \right).$$

Thus, $dC = -kdD$.

B Model extensions

B.1 Growth

To introduce growth into our model, we assume that each real asset (tree) now bears $Y_t = e^{gt}$ fruits, instead of $Y = 1$. For any quantity x (e.g. consumption, assets, asset prices) denote by $\hat{x}$ its de-trended version, that is, $\hat{x}_t \equiv e^{-gt} x_t$. As usual, the de-trended budget constraint of type i agents is then

$$\hat{c}_t^i + \dot{\hat{a}}_t^i \leq (r_t - g)\hat{a}_t^i.$$

Crucially, as we continue to assume that preferences are over wealth relative to income, $v(a_t^i/Y_t)$, as discussed in Section 2.5, we can express preferences (1) as function of de-trended variables,

$$\int_0^\infty e^{-(\rho+\delta)t} \left\{ gt + \log\left(\hat{c}_t^i/\mu^i\right) + \frac{\delta}{\rho} v(\hat{a}_t^i/\mu^i) \right\} dt$$

where clearly the term gt is irrelevant for consumption choices. Finally, the de-trended price of a tree is

$$\hat{p}_t \equiv \int_t^\infty e^{g(s-t) - \int_t^s r_u du} ds.$$

Following exactly the same steps as before, we can derive two conditions that characterize the steady state(s) with growth. As it turns out, both equations are simply shifted versions of the conditions without growth. The saving supply schedule and debt demand curve now read

$$r - g = \rho \cdot \frac{1 + \delta/\rho}{1 + \delta/\rho \cdot \eta(\omega^s/(r-g) + d)} \qquad \text{and} \qquad d = \frac{\ell}{r-g}. \tag{44}$$

Equation (44) shows that growth is orthogonal to the steady states in our model. It merely shifts both curves vertically in parallel. Similarly, one can show that growth does not change the dynamics of the economy either, after de-trending.

B.2 Convenience yields at the zero lower bound

We work with a lower bound $\underline{r} > 0$ on the private return on wealth r in Section 7. Here, we show that this can be microfounded by introducing a convenience yield on government bonds. We allow for such a convenience yield by assuming that savers, who are the equilibrium holders of government bonds, derive an additional utility from holding government bonds, as in Krishnamurthy and Vissing-Jorgensen (2012). In particular, we assume the per-period utility function of a representative saver is

$$\log\left((c_t^s + \xi B_t)/\mu^s\right) + \delta/\rho v\left(a_t^s/\mu^s\right)$$

for some $\xi \geq 0$ that captures the additional convenience yield. The saver's Euler equations now differ for bonds and other assets. In the steady state, we obtain

$$0 = r^B + \xi - \rho - \delta + \frac{\delta}{\rho} r\eta(a) \tag{45}$$

for government bonds and

$$0 = r - \rho - \delta + \frac{\delta}{\rho} r\eta(a) \tag{46}$$

for other assets. Subtracting (46) from (45) yields a steady-state government bond return of

$$r^B = r - \xi. \tag{47}$$

Due to the spread ξ in (47), r^B is always strictly lower than r. Thus, r^B might hit zero even as r is still positive. In that case, $r = \xi$ is the appropriate lower bound for the private return on wealth. Setting B to zero recovers all other equations in Section 2.

B.3 Intergenerational mobility

We have so far ignored mobility across saver and borrower dynasties. Yet, since current mobility levels in the US are low while income inequality is rising (Lee and Solon 2009, Chetty, Hendren, Kline, Saez and Turner 2014), several policies have been proposed to promote intergenerational mobility. To study the effects of such policies on household debt and interest rates, we next extend our framework to include mobility.

To do so without jeopardizing the tractability of our model, we assume that with some probability $q > 0$, savers' offsprings turn into borrowers; and with probability $q\frac{\mu}{1-\mu}$, borrowers' offsprings turn into savers. A saver-turned-borrower immediately consumes his wealth down to the level of an average borrower $a^b / (1 - \mu)$; and vice versa, a borrower-turned-saver immediately receives a transfer from all savers to achieve their average asset position a^s / μ.[52] Agents' preferences are unchanged.

This structure changes the saving supply schedule to

$$r = \rho \frac{1 + \delta/\rho}{1 + \delta/\rho\eta(a)} + \underbrace{q\gamma\delta \frac{\delta/\rho\eta(a)}{1 + \delta/\rho\eta(a)}}_{\text{contribution of mobility}} \tag{48}$$

where γ is a measure of steady-state income inequality, equal to 1 minus the ratio of per capita incomes

$$\gamma = 1 - \frac{\left(\omega^b - \ell\right) / (1 - \mu)}{\left(\omega^s + \ell\right) / \mu}.$$

We see from (48) that the intergenerational mobility term increases in a. This makes the saving supply schedule less downward-sloping, thus mitigating indebted demand. The effect of greater mobility (raising q) is particularly relevant when income inequality γ is high, i.e. close to 1, since in that case, it redistributes wealth within each generation in a similar fashion as redistributive taxation.

B.4 Open economy model

Many countries with large debt burdens are open economies. This begs the question whether our model can be generalized to an open economy setting, and if so, whether this generates new insights. This is what we briefly discuss next.

We assume that our two types of agents live in a small open economy (the "home" country), which trades a single good with the rest of the world. It has imperfect access to world financial markets: as in Gabaix and Maggiori (2015), a continuum of foreign-based financial intermediaries invests in the home country, earning a return spread equal to the domestic interest rate r minus

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[52]This may be interpreted as borrowers marrying into saver families.

the world interest rate r^*. Thus, home's net foreign asset position is given by

$$\text{nfa}_t = \frac{1}{\Gamma}\left[r^* - r_t\right]. \tag{49}$$

What do steady states look like in this economy? To understand this, it helps to view the rest of the world as another "borrower" in the economy. When the home interest rate r_t is lower than the world interest rate r^*, the rest of the world borrows an amount equal to nfa_t from the home economy. Thus, nfa_t expands total steady-state saving demand to $d_t + B_t + \text{nfa}_t$. Given (49), opening up the economy's financial account amounts to a counterclockwise twist around the point on the saving demand curve where $r = r^*$.

International credit boom-bust cycle. A similar open economy extension can be developed with nominal rigidities, in the spirit of Gali and Monacelli (2005), but with imperfect international capital markets as in (49). In that extension, a permanent expansion in ℓ would also induce a boom-bust cycle—much like the one in the closed economy in Section 7. Assuming that monetary policy follows a standard Taylor rule, the boom-bust cycle will be more pronounced in an open economy, due to amplification from capital flows. In particular, as the economy booms, imports and interest rates r_t rise, pulling in additional capital, which is ultimately lent to borrowers. Then, as borrowers start to cut back on their spending, and r_t falls, capital flows reverse and leave the country again. This international credit cycle bears similarities with the evidence in Atif Mian, Amir Sufi and Emil Verner (2017).

B.5 More general preferences

B.5.1 EIS different from one

Our model naturally generalizes to utility functions over consumption with an elasticity of intertemporal substitution (EIS) σ^{-1} different from one. Assume the preferences of a type-i agent are given by

$$\int_0^\infty e^{-(\rho+\delta)t}\left\{\frac{1}{1-\sigma}\left(c_t^i/\mu^i\right)^{1-\sigma} + \frac{\delta}{\rho^\sigma}v(a_t^i/\mu^i)\right\}dt.$$

Define $\eta(a) \equiv (a/\mu^s)^\sigma v'(a/\mu^s)$ so that the homothetic benchmark with $v'(a) \propto a^{-\sigma}$ continues to correspond to $\eta(a) = 1$. The Euler equation of savers is then given by

$$\sigma\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta\left(\frac{c_t^s}{\rho a_t^s}\right)^\sigma \eta(a_t^s)$$

which, at a steady state, reduces to

$$r = \rho + \delta - \delta\left(\frac{r}{\rho}\right)^\sigma \eta(a^s).$$

While not necessarily solvable in closed form for r, this equation still has a unique solution for r for any value of a^s, by the intermediate value theorem. Moreover, by the implicit function theorem, the slope of the implied saving supply schedule is still negative.

B.5.2 Recursive preferences over consumption

Our preferences (1) involve a warm-glow utility over bequests. Does the negative slope of the saving supply schedule hinge on bequests (or wealth more broadly) entering the utility function? We now argue that the answer is no. To do so, we give both agents a recursive utility function solely defined over consumption, as in Uzawa (1968) and Lucas and Stokey (1983). We thus assume preferences are given by

$$U^i = \int_0^\infty u(c_t^i/\mu^i)e^{-\Delta_t}dt$$

where $\dot{\Delta}_t = \rho\left(c_t^i/\mu^i\right)$, u is strictly increasing, continuously differentiable and concave, and ρ is continuously differentiable. If $\rho = const$, this corresponds to standard homothetic preferences, but in general, these preferences allow the discount factor to move with consumption.

After some math, we find that savers' steady state Euler equation is

$$r = \rho(ra^s/\mu^s).$$

This defines an implicit equation in r for any a^i. If ρ decreases, such that greater consumption levels are associated with less impatience, the saving supply schedule is downward-sloping, just as in the model of Section 2.

B.5.3 Preferences over relative wealth

As we mentioned in Section 2.5, an alternative way to set up the warm-glow bequest utility is to define it not relative to output Y, but relative to total wealth, following Corneo and Jeanne (1997). In this formulation, preferences are given by

$$\int_0^\infty e^{-(\rho+\delta)t}\left\{\log\left(c_t^i/\mu^i\right) + \frac{\delta}{\rho}v\left(\frac{a_t^i/\mu^i}{A_t}\right)\right\}dt$$

where we let $A_t \equiv a_t^s + a_t^b$. Observe that $A = Y/r = 1/r$ in a steady state (Y is normalized to 1). Defining η as before, we then obtain the savers' Euler equation and saving supply schedule

$$\frac{\dot{c}_t^s}{c_t^s} = r_t - \rho - \delta + \delta\frac{c_t^s}{\rho a_t^s}\eta\left(\frac{a_t^i}{A_t}\right) \qquad \Rightarrow \qquad r = \rho\frac{1+\delta/\rho}{1+\delta/\rho\eta\,(ra^s)}. \tag{50}$$

The only change in (50) relative to (9) is that there is now an additional "r" on the right hand side. Conceptually, this plays no role, however. When $\eta(a)$ is increasing, the right hand side falls in r, implying that there is a unique interest rate r for any a^s. Moreover, as the right hand side also falls in a^s, that interest rate must decline as a^s increases. Thus, we still have a negatively sloped saving supply schedule.

B.6 Borrowing constraints nested

We provide three alternative microfoundations for a borrowing constraint that fits the general description in footnote 11.

B.6.1 Housing as collateral asset

In Section 2, we derived a simple borrowing constraint based on the idea that agents can pledge an income stream ℓY (e.g. coming from land they own). We now show that one would obtain a

similar borrowing constraint when borrowers purchase houses instead, and use them as collateral. To do so, assume there is a fixed mass μ^b of housing units, freely traded at price p_t^h, over which borrowers have additive preferences $\alpha \log h_t$ each period, $\alpha > 0$. We assume that the collateral constraint is $\dot{d} + \lambda d \leq \theta p_t^h h_t$, where θ is the loan-to-value (LTV) ratio. In steady state, one can show that this implies a market clearing house price

$$p^h(r) = \frac{\alpha \omega^b}{(1 - \theta/\lambda)\rho + (1 + \alpha)\theta r/\lambda}$$

This fits into our earlier framework by assuming that $\ell(r) = rp^h(r)$.

In this version of the model, agents in the borrower dynasty bequeath both the house and their debt position to their offsprings. In a more realistic model with a life-cycle, older agents would sell their house, pay down their debts and consume the proceeds before death. Younger agents would purchase houses, partly debt-financed. A larger house price (due to lower interest rates), would stimulate consumption of existing homeowners and sellers of houses, raising aggregate consumption of the borrower dynasty, even if younger agents now pay more for the same-sized house.

B.6.2 Bewley-Aiyagari model

For this extension, we model borrowers as in Achdou, Han, Lasry, Lions and Moll (2017), that is, we deviate from homogeneous preferences across types of agents. We focus on a steady state with a constant interest rate r. Each borrower i maximizes utility

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \log c_t^i dt$$

subject to budget and borrowing constraint

$$\dot{d}_t^i = r d_t^i + c_t^i - y^b e_t^i \qquad \text{and} \qquad d_t^i \leq \bar{d} y^b$$

where $\bar{d} > 0$ and $e_t^i > 0$ is a random Markov process for productivity in continuous time; $y^b \equiv \omega^b Y/\mu^b$ is borrowers' average income. This specification implies that borrowers can borrow up to a fixed fraction (or multiple) of their average income. Following the logic in Achdou, Han, Lasry, Lions and Moll (2017), we see that total debt taken out by borrowers, $d(r, y^b) \equiv \int d_t^i di$ in a steady state with rate r is continuous and approaches $-\infty$ as $r \uparrow \rho$ and approaches $\underline{d} y^b$ as $r \downarrow 0$. Moreover, observe that $d(r, y^b)$ scales in y^b due to homothetic preferences, see Straub (2019). Therefore, we can write total debt as

$$d(r, y^b) = d(r, 1)y^b = \underbrace{rd(r, 1)\omega^b/\mu^b}_{\equiv \ell(r)} \cdot \frac{Y}{r}$$

in line with our general borrowing constraint in footnote 11. Thus, this model generates a steady-state debt demand curve that is "mostly" downward-sloping, in the sense that $d(r, y^b)$ is $-\infty$ for $r \uparrow \rho$, rising to $\bar{d} y^b$ for $r \downarrow 0$. There could be non-monotonicity in between, although that is unlikely given the results in Achdou, Han, Lasry, Lions and Moll (2017).

B.6.3 Simplified buffer-stock model

We have found that a simplified and tractable version of the full Bewley-Aiyagari model is a useful way to understand its implications. Specifically, we assume that there is a small Poisson

probability $v > 0$ that a borrower receives a one-time negative income shock of size $\varphi Y > 0$ (after which there are no more other shocks). We model the negative shock to be sufficiently large that it shows up in a borrower's asset position: when assets are a_t^b before the shock, they fall to $a_t^b - \varphi Y$ after the shock. As we make the shock probability very small, $v \to 0$, this setup converges to a model where the borrowing constraint is given by

$$\dot{d}_t^i + \lambda d_t^i \leq p_t \ell - \varphi Y$$

where the borrowing constraint is tightened by the potential income loss φY relative to (7). The right hand side can be rewritten in the form $\ell(\{r_s\}_{s \geq t})$.

B.7 Nominal rigidities

Households. We describe the model with nominal rigidities here in more detail. First, agents preferences are given by

$$\int_0^\infty e^{-(\rho+\delta)t} \left\{ \log\left(c_t^i/\mu^i\right) + \frac{\delta}{\rho} v(a_t^i/\mu^i) - \frac{\phi}{1+\eta^{-1}} \left(n_t^i\right)^{1+\eta^{-1}} \right\} dt \tag{51}$$

where η is the Frisch elasticity of labor supply, and ϕ is a disutility shifter. Observe that in the limit $\eta \to 0$, this model converges to an endowment economy, with labor endowments $n_t^i = 1$ per agent. The budget constraint is still given (2) with a total asset position equal to

$$a_t^i = h_t^i - d_t^i.$$

Real asset wealth, which is equal to human wealth here, is given by

$$rh_t^i = w_t \omega^i n_t^i + \dot{h}_t^i \tag{52}$$

where w_t is the real wage per efficiency unit and ω^i/μ^i denotes the efficiency units per hour worked of dynasty i. Total efficiency units supplied by dynasty i are then $\omega^i n_t^i$. Combining (2) and (52), we can also write the budget constraint in the more traditional form,

$$c_t^i + r_t d_t^i \leq \dot{d}_t^i + w_t \omega^i n_t^i \tag{53}$$

To be consistent with this description, the price of pledgeable wealth is then assumed to be equal to

$$p_t \equiv \int_t^\infty e^{-\int_t^s r_u du} w_s n_s^i ds \tag{54}$$

with the interpretation that a total amount $\ell < \omega^i$ of efficiency units can be pledged by each dynasty. This implies the same borrowing constrained as before, (7).

Production. Production in this economy follows a simple linear aggregate production function

$$\hat{Y}_t = N_t$$

where N_t equals total efficiency units supplied by the dynasties, $N_t = \omega^b n_t^b + \omega^s n_t^s$. We assume production is perfectly competitive and prices are flexible, so that the real wage is equal to 1 at all times, $w_t = 1$.

Nominal rigidities and monetary policy rule. Instead, we assume that nominal wages are perfectly rigid, $W_t = 1$. While this might seem extreme, our results will be amplified if a Phillips curve is assumed instead.[53] With a wage rigidity in place, workers may be off their labor supply schedules, so it is important to determine which worker works how much. Here, we keep with the existing literature, e.g. Werning (2015) and Adrien Auclert, Matthew Rognlie and Ludwig Straub (2018), and assume that both dynasties' hours move identically, $n_t^b = n_t^s = N_t = \hat{Y}_t$. The central bank in this model sets a path for the real interest rate $\{r_t\}$ directly (as in Adrien Auclert, Matthew Rognlie and Ludwig Straub 2018).

Observe that with these modifications, the above modeling equations can be simplified. For instance, the budget constraint (53) becomes

$$c_t^i + r_t d_t^i \leq \dot{d}_t^i + \omega^i \hat{Y}_t$$

and the price of pledgeable wealth (54) becomes

$$p_t \equiv \int_t^\infty e^{-\int_t^s r_u du} \hat{Y}_s ds$$

Natural allocation. The natural allocation is defined as the allocation that would materialize if wages were perfectly flexible. In that case, r_t is no longer determined by monetary policy. With labor supply as in (51), wealth effects on labor supply imply that the natural allocation need not necessarily feature $n_t^i = 1$ and thus not necessarily $\hat{Y}_t = Y = 1$. To establish continuity with our analysis before and focus on the case where the natural allocation indeed exhibits $\hat{Y}_t = Y = 1$, we focus on the case where $\eta \to 0$. This assumption essentially makes labor supply arbitrarily curved around 1. While not having any effects on our analysis of monetary policy, it does imply that the natural allocation is one in which $n_t^i = 1$ for both dynasties, implying that dynasty i's income is equal to ω^i just as in the endowment economy introduced in Section 2.

B.8 Wage deflation in the debt trap

In this section, we illustrate how the debt trap worsens in the presence of wage deflation (or disinflation). This is also known as the "paradox of flexibility" and has been widely studied in the literature.[54]

To show how wage inflation matters, we introduce downward nominal wage rigidity of the form (Schmitt-Grohé and Uribe 2016)

$$\pi = \dot{W}/W \geq \underline{\pi} \tag{55}$$

where $\underline{\pi}$ is lower bound on wage inflation (which could be zero). We assume that (55) is binding whenever the demand for labor falls below 1, where 1 corresponds to the amount of labor supplied in the flexible wage equilibrium.

As we continue to assume that prices are flexible, price inflation is also given by π, so that at the effective lower bound, the real return on wealth is now given by $\xi - \underline{\pi}$. Debt trap output is

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[53]This can be for instance done by integrating the heterogeneous-agent version of the Christopher J Erceg, Dale W Henderson and Andrew T Levin (2000) wage rigidity model developed in Adrien Auclert, Matthew Rognlie and Ludwig Straub (2018). We show in appendix B.8 how the liquidity trap is amplified by wage deflation.

[54]For an especially clear illustration, see Werning (2012).

then given by

$$\hat{Y} = Y \frac{\xi - \pi}{\omega^s + \ell} \cdot \eta^{-1} \left(\frac{\rho}{\xi - \pi}(1 + \rho/\delta) - \rho/\delta \right) < Y$$

which is lower, the more wage deflation $-\underline{\pi}$ there is in the economy.

B.9 Model with bonds in the utility

In our baseline model, agents derive utility from total wealth a, including human wealth. An alternative is to specify preferences as

$$\int_0^\infty e^{-(\rho+\delta)t} \left(\log c_t^i + \frac{\delta}{\rho} v(b_t^i/\mu^i) \right) dt \tag{56}$$

where b_t is an agents' net saving position. For simplicity, The budget constraint would then be given by

$$c_t^i + \dot{b}_t^i \leq r_t b_t^i + \omega^i \tag{57}$$

There are two complications with this approach. First, typical choices for v, such as $v = \log$, give the poorest agents the strongest saving motive, as their marginal utility from holding just a few bonds is nearly infinite. Aside from being implausible, this prevents those agents from ever borrowing. Second, (56) is somewhat less "clean" in that it implies savers' preferences are non-Ricardian, and is thus one step further away from the standard homothetic infinite-horizon consumption saving model compared to our baseline utility function (1).

Leaving these complications aside, we can still analyze the saving supply schedule of savers in this model and investigate whether that schedule can ever slope down. Together with a suitable choice of v which allows borrowers to borrow, the results in the main body of our paper will then go through unchanged.

The first order condition of savers is then given by

$$\frac{\dot{c}_t^s}{c_t^s} = r - \rho - \delta + \frac{\delta}{\rho} v'(b_t^s/\mu^s) c_t^s/\mu^s$$

We can again define

$$\eta(b^s) \equiv v'(b^s/\mu^s) b^s/\mu^s$$

with elasticity $\epsilon_\eta(b^s) \equiv \eta'(b^s) b^s/\eta(b^s)$. In a steady state with constant b^s and c^s, we then have

$$r = \rho + \delta - \frac{\delta}{\rho} \eta(b^s) \frac{rb^s + \omega^s}{b^s}$$

which can be reformulated to

$$r = \rho \frac{1 + \frac{\delta}{\rho} - \frac{\delta}{\rho}\eta(b^s)\frac{\omega^s}{\rho b^s}}{1 + \frac{\delta}{\rho}\eta(b^s)} \tag{58}$$

This is identical to (9) except for an additional term in the numerator. (58) slopes down precisely when

$$\epsilon_\eta > (1 + rb^s/\omega^s)^{-1} \tag{59}$$

In our baseline model, the condition was simply that $\eta(b)$ is increasing in b, that is, $\epsilon_\eta > 0$. In the bonds-in-the-utility model, the condition is slightly stricter, namely that ϵ_η lies above $(1 + rb^s/\omega^s)^{-1}$.

Both expressions are equivalent for large wealth positions, $b^s \to \infty$.

B.10 Model with financial intermediation and more than two savers

We now sketch an extension of the model of the previous appendix subsection B.9 to allow for financial intermediation. In particular, we now assume that all saving b^s of savers is intermediated by a banking system, issuing deposits and lending to borrowers, subject to some intermediation spread ζ that is earned by savers. This implies that the deposit rate r^b is exactly ζ below the lending rate r. Savers then maximize (56) subject to budget constraint

$$c_t^s + \dot{b}_t^s \leq r_t^b b_t^s + \omega^s + \pi_t$$

where π_t are profits from the banking system capturing the intermediation spread, $\pi_t = \zeta d$, paid as lump-sum transfers to savers. In the steady state, $d = b^s$.

Following the same steps as before, we find the same condition (59) for the *aggregate* saving supply schedule to slope down. However, what is interesting in this extension, is that the saving supply schedule of an *individual* saver can slope up as the aggregate slopes down. The condition for an individual saver's saving supply schedule to slope up is given by

$$\epsilon_\eta < \left(1 + r^b b^s / \left(\omega^s + \zeta b^s\right)\right)^{-1} \tag{60}$$

where the upper bound on ϵ_η is strictly greater than the lower bound on ϵ_η in (59). This framework can be used to extend our model of indebted demand to models with more than one saver, including savers subject to idiosyncratic income risk with well-defined stationary wealth distribution.

C Expected Return on Wealth Calculation

This section explains details on the construction of the expected returns in the right panel of Figure 1. The expected return on business equity is calculated using the methodology in Farhi and Gourio (2018). It is based on the Gordon growth model, and it estimates dividend growth using a smoothed estimate of real GDP growth around the year in question. The results are not materially affected if one uses an estimate of expected real GDP growth from the Survey of Professional Forecasters, as shown in the discussion of this study by Mark Gertler (Gertler and Papanikolaou 2018). The expected rates of return on fixed income assets are the yields from FRED (MORTGAGE30US and GS10) minus the 10-year expected inflation series from the Cleveland Fed. The Cleveland Fed expected inflation data are only available from 1982 onward, which is the main limitation in terms of the time series shown.

To obtain an expected return on wealth, data from B.101 of the Financial Accounts are used to calculate a portfolio share between corporate business equity and fixed income assets. The corporate business equity portfolio share includes direct holdings of corporate equity, and indirect holdings through mutual funds and pension entitlements. For the fixed income portfolio share, deposits, money market funds, debt securities, loans, and debt claims owned through mutual funds and pensions are included. The unfunded claims on pensions by households do not represent claims on any actual financial assets, and they therefore are excluded. This process gives us portfolio shares of the U.S. household sector in corporate business equity and fixed income assets.

These portfolio shares are then multiplied by the underlying expected returns to obtain an expected real return on financial assets. The baseline uses the Farhi and Gourio (2018) estimate

for the corporate business equity expected return, and the 30 year mortgage real interest rate for the fixed income expected return. The choice of which fixed income asset return to use affects the level of the expected return on financial wealth, but it does not affect the trend given the similar downward trend in all yields for fixed income assets.

The calculation of the expected return on wealth does not subtract expected debt service payments by households. The reason is that we want to calculate a gross financial asset expected return that proxies for the expected rate of return of a saver in the model who is considering making a loan to a borrower. We also exclude equity in non-corporate businesses from our measure of the expected return on financial wealth. Non-corporate business equity is not likely to be an "investable" asset by a passive investor, and it likely includes substantial value that is tied directly to the human capital of the owners and managers of these businesses (e.g., Smith, Yagan, Zidar and Zwick 2019).

Other researchers exploring the return on wealth or business capital have typically used realized returns as opposed to expected returns (e.g., Gomme, Ravikumar and Rupert 2011, Eggertsson, Robbins and Wold 2018, Auclert, Martenet, Malmberg et al. 2019). This strategy typically uses National Accounting data for the numerator of the return, and a measure of either business capital or household wealth in the denominator. Auclert, Martenet, Malmberg et al. (2019) show that the realized return on wealth has fallen during the 1982 to 2016 period. This contrasts with the relatively flat realized return on business capital calculated in Gomme, Ravikumar and Rupert (2011) and Eggertsson, Robbins and Wold (2018) because these studies do not use market values of business capital in the denominator of their return formula. See Auclert, Martenet, Malmberg et al. (2019) for a further discussion of this issue.