Short Speeches

Short Speech 1

In the Solow-Swan model, what is the long-run growth rate of output and output per capita? Why can't the economy grow by accumulating more capital in the long run? Which production function delivers a counter-example?

Short Speech 2

Explain the disagreement between Young and Hsieh over the East Asian miracle. Who do Fernald and Nieman side with? Explain the difference between the primal and dual approaches of measuring TFP growth.

Problem

Consider a household of consumers with constant elasticity of intertemporal substitution given by:

$$U_0 = \int_0^\infty e^{-(\rho-n)t}\frac{c^{1-\theta}-1}{1-\theta}, dt$$

The household gets income from assets and wages.

1. Write down the per capita budget constraint for this household if r is the real interest rate, w is the wage rate, n the constant rate of population growth and b is the amount of assets per capita.

2. Imagine now that the government introduces taxes on consumption (at rate τ_c), wages (at rate τ_w) and asset income (at rate τ_a). For now, these tax rates are not necessarily constant. Write down the new budget constraint in per capita terms with after-tax financial income ($(1-\tau_a)r$), labor income ($(1-\tau_w)w$) and consumption ($(1+\tau_c)c$). We assume that the government throws away the proceeds of the tax.

3. Households maximize utility subject to the constraint described in question 2. Set up the Hamiltonian and derive the set of first order conditions. Write down an equation for consumption growth as a function of the parameters of the model, the interest rate and the various tax rates. Hint: once again, τ_c is not necessarily constant over time; $\dot{\tau}_c$ should appear in your equation.

   (a) How does consumption growth depend on τ_c? Explain intuitively.

L = TOTAL POPULATION

1 B = TOTAL ASSETS

$$C_t L_t + B_{t+1} = W_t L_t + (1 + r_t) \cdot B_t$$

$$\underbrace{B_{t+1} - B_t} = W_t \cdot L_t + r_t B_t - C_t \cdot L_t$$

$$\boxed{\dot{B} = W \cdot L + r \cdot B - C \cdot L}$$

IN PER CAPITA TERMS

$$\frac{\dot{B}}{L} = W + r \cdot \frac{B}{L} - C$$

$$\frac{B}{L} = b$$

$$\boxed{\frac{\dot{B}}{L} = r \cdot b + W - C} \quad (1)$$

$$\dot{b} = \frac{\partial \left(\frac{B}{L}\right)}{\partial t} = \frac{\dot{B} \cdot L - B \cdot \dot{L}}{L^2} =$$

$$\frac{\dot{B}}{L} - \underbrace{\frac{\dot{L}}{L}}_{n} \cdot \underbrace{\frac{B}{L}}_{b} = \frac{\dot{B}}{L} - n \cdot b \Rightarrow$$

$$\boxed{\frac{\dot{B}}{L} = \dot{b} + n \cdot b} \quad (2)$$

$$\dot{b} + n \cdot b = r \cdot b + w - c$$

$$\boxed{\dot{b} = (r - n) \cdot b + w - c}$$

(2) $$\dot{B} = (1-\tau_w) \cdot w \cdot L + (1-\tau_a) \cdot r \cdot B - (1+\tau_c) \cdot c \cdot L$$

$$\boxed{\frac{\dot{B}}{L} = (1-\tau_a)\, r \cdot b + (1-\tau_w) \cdot w - (1+\tau_c) \cdot c} \quad (3)$$

REPLACE (2) INTO (3):

$$\dot{b} + n \cdot b = (1-\tau_a) \cdot r \cdot b + (1-\tau_w) \cdot w - (1+\tau_c) \cdot c$$

$$\boxed{\dot{b} = [(1-\tau_a) r - n] \cdot b + (1-\tau_w) \cdot w - (1+\tau_c) \cdot c}$$

3 CONTROL VARIABLE: c

STATE VARIABLE: b

$$\mathcal{H} = e^{-(\rho-n)t} \frac{c^{1-\theta}-1}{1-\theta} + \mu.\{[(1-\tau_a).r - n].b + (1-\tau_w).w - (1+\tau_c).c\}$$

F.O.C.s.

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \iff e^{-(\rho-n)t}.c^{-\theta} - \mu.(1+\tau_c) = 0 \iff$$

$$\boxed{e^{-(\rho-n)t} c^{-\theta} = \mu.(1+\tau_c)} \quad (1)$$

$$\frac{\partial \mathcal{H}}{\partial b} = -\dot{\mu} \iff$$

$$\boxed{\mu.[(1-\tau_a).r - n] = -\dot{\mu}} \quad (2)$$

$$\boxed{\lim_{t\to\infty} \mu.b = 0} \quad (3)$$

$\frac{\dot{c}}{c} = ?$

TAKE DERIVATIVE OF (1) WITH RESPECT TO t:

$$-(\rho - n)\cdot e^{-(\rho-n)t}\cdot c^{-\theta} + e^{-(\rho-n)t}\cdot \underbrace{-\theta\cdot c^{-\theta-1}\cdot \dot{c}}_{c^{-\theta}\cdot -\theta\cdot \frac{\dot{c}}{c}}$$

$$\left(-\rho + n - \theta\cdot\frac{\dot{c}}{c}\right)\cdot e^{-(\rho-n)t}\cdot c^{-\theta} =$$

$$\dot{\mu}\cdot(1+\tau_c) + \mu\cdot\dot{\tau}_c \qquad \mu$$

$$\dot{\mu} = \left(-\theta\cdot\frac{\dot{c}}{c} - \rho + n\right)\cdot \frac{e^{-(\rho-n)t}\cdot c^{-\theta}}{(1+\tau_c)}$$

$$-\mu\cdot\frac{\dot{\tau}_c}{(1+\tau_c)} = \dot{\mu}$$

$$\mu \cdot \left(-\theta \cdot \frac{\dot{c}}{c} - \rho + n \right) - \mu \cdot \frac{\dot{\tau}_c}{(1+\tau_c)} =$$

$$-\mu \cdot \left(\theta \cdot \frac{\dot{c}}{c} + \rho - n + \frac{\dot{\tau}_c}{1+\tau_c} \right) =$$

$$\dot{\mu} = -\mu \cdot \left[(1-\tau_a) \cdot x - n \right]$$

$$\theta \cdot \frac{\dot{c}}{c} + \rho - \cancel{n} + \frac{\dot{\tau}_c}{(1+\tau_c)} =$$

$$(1-\tau_a) \cdot x - \cancel{n} \Rightarrow$$

$$\theta \cdot \frac{\dot{c}}{c} = (1-\tau_a) \cdot x - \frac{\dot{\tau}_c}{(1+\tau_c)} - \rho \Rightarrow$$

$$\boxed{\frac{\dot{c}}{c} = \frac{1}{\theta} \cdot \left[(1-\tau_a) \cdot x - \frac{\dot{\tau}_c}{(1+\tau_c)} - \rho \right]}$$

(a) $\uparrow \tau_c \Rightarrow \downarrow \frac{\dot{c}}{c}$

CONSUMPTION BECOMES MORE EXPENSIVE

(b) $\hat{i} \uparrow w \Rightarrow$ NO IMPACT

AFFECT YOUR INCOME
BUT DOES NOT AFFECT
INTERTEMPORAL TRADE-OFF

(c) $\hat{i} \uparrow a \Rightarrow \downarrow \frac{\dot{c}}{c}$

REDUCE INCENTIVES
TO SAVE, PREFER CONSUME
TODAY THAN TOMORROW,
THEN $\frac{\dot{c}}{c}$ GOES DOWN

(d) $\hat{i} \uparrow \theta \Rightarrow \frac{\dot{c}}{c}$ GOES DOWN

(b) How does consumption growth depend on τ_w? Explain intuitively.

(c) How does consumption growth depend on τ_a? Explain intuitively.

Note: the intuition is important for the grade.

4. Imagine that the government also taxes firm's income at rate τ_f. The government defines "taxable earnings" equal to output less wage payments and depreciation so that after tax profits are:

$$(1-\tau_f)\left(F(K,L)-wL-\delta K\right)-rK$$

Firms choose K and L so as to maximize profits. $F(.)$ satisfies the neoclassical axioms. (There is no technological progress: $E=1$ and $x=0$.) Write down the first-order conditions for the firm.

5. Using asset market clearing, rewrite the budget constraint in terms of capital per capita. Don't make any assumption about tax rates for now. You should obtain:

$$\dot{k}=(1-\tau_w)f(k)-(1+\tau_c)c-\left((1-\tau_a)(1-\tau_f)-(1-\tau_w)\right)kf'(k)-(1-\tau_a)(1-\tau_f)\delta k-nk$$

6. Assume that all tax rates are 0. Write down the four equations that describe the solution in terms of two variables, k and c, assuming that $k(0)=k_0>0$ is given. Solve the model with a phase diagram. Justify with words—equations are not necessary—why we can rule out paths that start above or below the stable arm of the saddle path.

7. Assume that the economy is in steady state. At time 0, the government introduces a constant consumption tax: $\tau_c(t)>0$ and $\dot{\tau}_c(t)=0$ for all $t\geq 0$. Use the phase diagram to explain what happens. Explain the economics.

8. Assume that the economy is in steady state. At time 0, the government introduces a set of constant taxes such that $\tau_a(t)=\tau_w(t)=\bar{\tau}>0$ and $\tau_c(t)=\tau_f(t)=0$ for all $t\geq 0$. Use the phase diagram to explain what happens. Explain the economics.

(4) $(1-\tau Q)\cdot[F(K,L) - w\cdot L - \delta K]$
$- r\cdot K = \pi$

$$(1-\tau Q)\cdot F_K - (1-\tau Q)\cdot \delta - r = 0$$

$$\boxed{(1-\tau Q)\cdot F_K = r + (1-\tau Q)\cdot \delta}$$

$$(1-\tau Q)[F_L - w] = 0 \Rightarrow \quad (1)$$

$$\boxed{F_L = w} \quad (2)$$

$$\frac{F(K,L)}{L} = Q(k) \quad , \quad k \equiv \frac{K}{L}$$

$$\boxed{F(K,L) = L\cdot Q(k)}$$

$$F_K = L\cdot Q'(k)\cdot \frac{1}{L}$$

$$\boxed{F_K = Q'(k)}$$

$$-k$$

$$F_L = Q(k) + L\cdot Q'(k)\cdot\left(-\frac{K}{L^2}\cdot\frac{1}{L}\right)$$

$$\boxed{F_L = Q(h) - h \cdot Q'(h)}$$

5 $b = h + d$

$$\boxed{\dot{b} = [(1-\tau_a)r - n] \cdot b + (1-\tau_w) \cdot w - (1+\tau_c) \cdot c}$$

$$\cancel{\boxed{\dot{h} = [(1-\tau_a) \cdot r - n] \cdot h + (1-\tau_w) \cdot w - (1+\tau_c) \cdot c]}} \quad (3)$$

BY (1) WE HAVE:

$$r = (1-\tau_Q) \cdot [F_K - \delta]$$

$$\boxed{r = (1-\tau_Q) \cdot [Q'(h) - \delta]} \quad (4)$$

BY (2) WE HAVE:

$$w = F_L$$

$$\boxed{w = Q(h) - h \cdot Q'(h)} \quad (5)$$

REPLACING (4) & (5) INTO (3):

$$\dot{k} = [(1-\tau_a)(1-\tau_Q)(Q'(k)-\delta) - n] \cdot k + (1-\tau_w)(Q(k) - k \cdot Q'(k)) - (1+\tau_c) \cdot c$$

$$\boxed{\begin{aligned}\dot{k} = {} & (1-\tau_w) \cdot Q(k) - (1+\tau_c) \cdot c \\ & + [(1-\tau_a)(1-\tau_Q) - (1-\tau_w)] \cdot Q'(k) \cdot k - (1-\tau_a)(1-\tau_Q) \cdot \delta \cdot k \\ & - n \cdot k\end{aligned}}$$

D_t = PUBLIC DEBT

$$\underbrace{(1+r) \cdot D_{t-1}}$$

$$\boxed{G + \tilde{r} \cdot D = \tau_a \cdot r \cdot K + \tau_w \cdot w \cdot L + \tau_c \cdot c \cdot L + \dot{D}}$$

$\underline{D_t}$

b

Euler Equation:

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \cdot (r - \rho) =$$

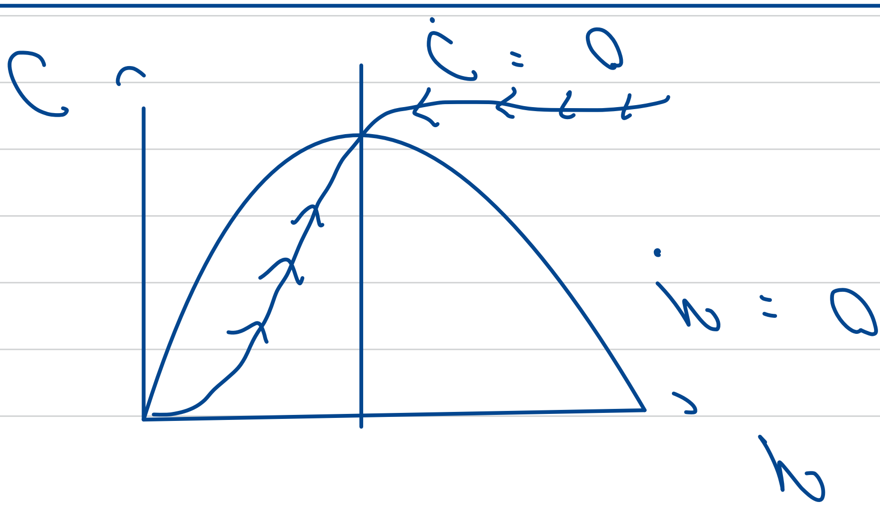
$$\boxed{\frac{\dot{c}}{c} = \frac{1}{\theta} \cdot (f'(k) - \delta - \rho)}$$

Initial Condition:

$$\boxed{k(0) = k_0 > 0}$$

Transversality Condition:

$$\boxed{\lim_{t \to \infty} e^{-(\rho - n)t} \cdot c^{-\theta} \cdot k = 0}$$



$$\dot{k} = Q(k) - c - \delta \cdot k - n \cdot k$$

$$\boxed{\dot{k} = Q(k) - c - (\delta + n) \cdot k}$$