Macroeconomics 1

Lecture - Structural Change

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Sciences Po

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Motivation

- Before Industrial Revolution, majority of workers in the agricultural sector.
- Today, only few percents of the workforce in agriculture in developed countries.
- With economic development, reallocation of resources across sectors, away from agriculture towards manufacturing and services. Richer country mostly produce services.
- Need multisector growth model to understand these patterns of long term development.

Structural Change

Roadmap

1. Facts of Structural Change
2. Theory of Structural Change
3. Application. Structural Change and Urbanization
   Coeurdacier, Oswald, Teignier (2021)

Facts of Structural Change

Structural Change

- One-sector growth model abstracts from several features of the economic growth process.
- A very important one is unbalanced sectoral growth. **Structural Change** (or **Structural Transformation**).
- Structural change defined as the reallocation of economic activity across the three broad sectors

1. The size of the agricultural sector falls with economic development.
2. The size of the manufacturing sector initially increases but then decreases with economic development.
3. The size of the service sector rises with economic development.

Measuring Structural Change

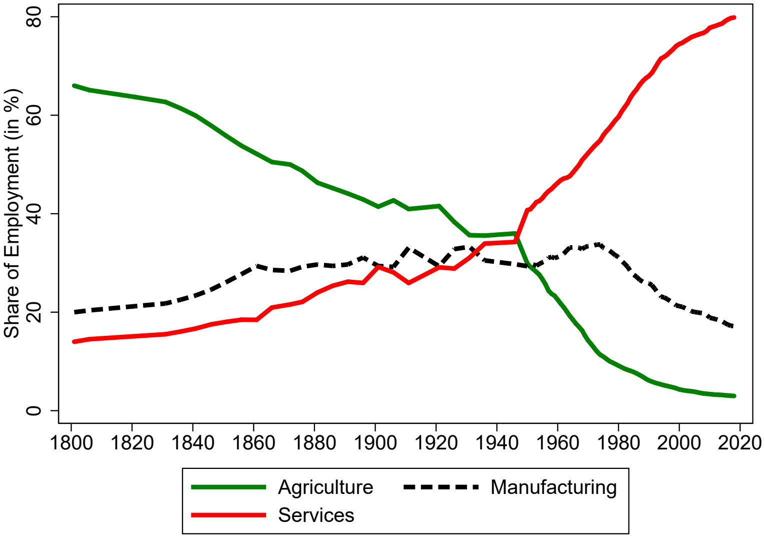
- Common measures of economic activity at the sectoral level

1. Employment shares (number of workers or hours worked).
2. Value added shares.
3. Final consumption expenditure shares.

- Data

  Historical National Accounts Database of the University of Groningen, EU KLEMS, Penn World Table, UN Statistics Division National Accounts, OECD Consumption Expenditure Data, WB World Development Indicators...

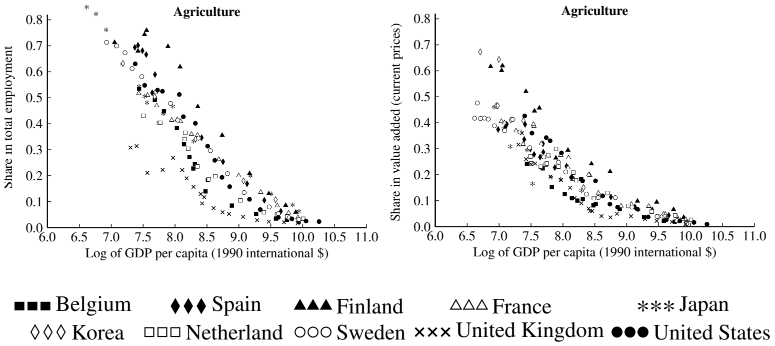
Evidence from France, 1800-2020



Source: Herrendorf, Rogerson and Valentinyi (2014, Handbook Economic Growth) and OECD

Evidence from long time series, 1800-2000

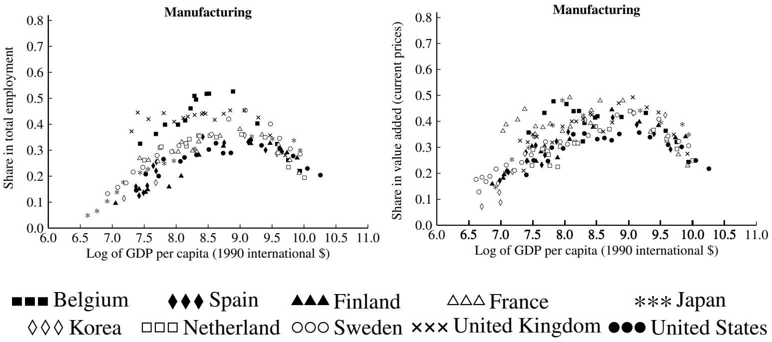
Agriculture



Source: Herrendorf, Rogerson and Valentinyi (2014, Handbook Economic Growth)

Evidence from long time series, 1800-2000

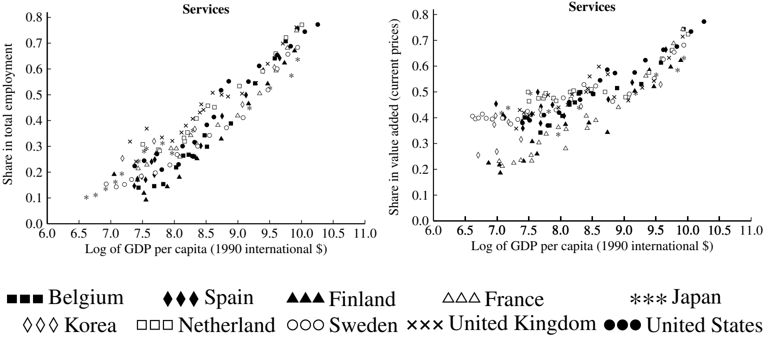
Manufacturing



Source: Herrendorf, Rogerson and Valentinyi (2014, Handbook Economic Growth)

Evidence from long time series, 1800-2000

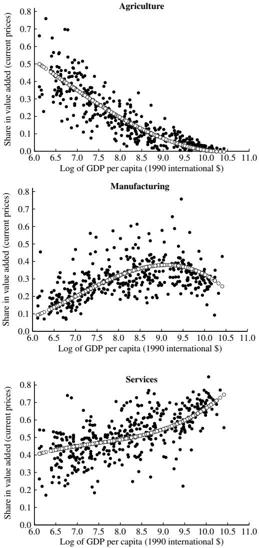
Services



Source: Herrendorf, Rogerson and Valentinyi (2014, Handbook Economic Growth)

Evidence from cross-country value added data

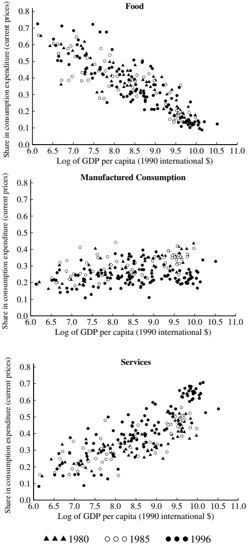
Cross-sections from UN National Accounts: 1975-2005 (103 countries)



Source: Herrendorf, Rogerson and Valentinyi (2014, Handbook Economic Growth)

Evidence from cross-country consumption data

Cross-sections from PWT Benchmark Studies



Source: Herrendorf, Rogerson and Valentinyi (2014, Handbook Economic Growth)

Theory of Structural Change

Barebone theory of structural change

- Multi-sector economy (agriculture, manufacturing, services).
- Perfectly competitive markets and perfect mobility of workers across sectors (long term perspective).
- Static model for simplicity to focus on intuitions.
  Changes over time = replication over time of static equilibria.
- **Technological view of structural change.**
  Drivers of structural change = exogenous productivity (TFP) changes, possibly sector-specific.

Production Function

- Three sectors, agriculture a, manufacturing m and services s.
- Only labor to produce for simplicity.
- $L(t)$ workers mobile across sectors.
- Technology to produce output at date t in each sector

$$Y_i(t) = Z_i(t)L_i(t)$$

  $i \in \{a, m, s\}$

  $Z_i(t)$ sector-specific TFP.

  $L_i(t)$ labor employed in i.
- **Labor market clearing.** $L_a(t) + L_m(t) + L_s(t) = L(t)$.

Firm Optimization

- Firms maximize profits in each sector $i \in \{a, m, s\}$,

$$\max_{\{L_i(t)\}} p_i(t) Z_i(t) L_i(t) - w(t) L_i(t)$$

  $p_i(t)$ price of good i.

  wage $w(t)$ equalized across sectors (**perfect mobility**)

- Workers paid their marginal productivity,

$$p_i(t) Z_i(t) = w(t)$$

- **Remark.** $\frac{p_i(t)}{p_j(t)} = \frac{Z_j(t)}{Z_i(t)}$ for $i \neq j$. Relative price reflects relative productivity. Intuition?

Preferences

- Consumption composite,

$$C(t) = \mathcal{C}\left(c_a(t), c_m(t), c_s(t)\right)$$

with $\mathcal{C}$ consumption function aggregator increasing in all its arguments and $c_i(t)$ consumption of good$-i$.

- Example. Cobb-Douglas aggregator,

$$\mathcal{C}\left(c_a(t), c_m(t), c_s(t)\right) = c_a(t)^{\omega_a} c_m(t)^{\omega_m} c_s(t)^{\omega_s}$$

with $\omega_i \geq 0$, the weight of each good, $\omega_a + \omega_m + \omega_s = 1$.

Consumer optimization

- **Budget constraint.**

  Total expenditures = labor income $w(t)L(t)$

  $$p_a(t)c_a(t) + p_m(t)c_m(t) + p_s(t)c_s(t) = w(t)L(t)$$

- **Optimization.**

  $$\begin{aligned} & \max_{\{c_a(t), c_m(t), c_s(t)\}} \mathcal{C}\left(c_a(t), c_m(t), c_s(t)\right) \\ \text{s.t} \quad & p_a(t)c_a(t) + p_m(t)c_m(t) + p_s(t)c_s(t) = w(t)L(t) \qquad (\lambda(t)) \end{aligned}$$

- **First-Order Conditions.** For $i \in \{a, m, s\}$

  $$\frac{\partial \mathcal{C}}{\partial c_i(t)} = \lambda(t)p_i(t)$$

Cobb-Douglas Preferences

- Consumption composite, with $\omega_i \geq 0$ and $\omega_a + \omega_m + \omega_s = 1$,

$$\mathcal{C}\left(c_a(t), c_m(t), c_s(t)\right) = c_a(t)^{\omega_a} c_m(t)^{\omega_m} c_s(t)^{\omega_s}$$

- **First-Order Conditions.** For $i \in \{a, m, s\}$

$$\omega_i C(t) = \lambda(t) p_i(t) c_i(t)$$

Summing across sectors and using budget constraint leads to constant expenditure share across goods,

$$C(t) = \lambda(t) w(t) L(t) \Rightarrow p_i(t) c_i(t) = \omega_i w(t) L(t)$$

- Consumption expenditures shares constant across sector. Does not look like a good model of structural change.

Cobb-Douglas Preferences

- **Market clearing for goods.** For $i \in \{a, m, s\}$,

$$c_i(t) = Z_i(t)L_i(t)$$

$$\Rightarrow \omega_i w(t)L(t) = p_i(t)Z_i(t)L_i(t) = w(t)L_i(t)$$

$$\frac{L_i(t)}{L(t)} = \omega_i$$

- Consumption shares, employment shares and value added shares in each sector are constant equal to ω_i.

- No structural change with Cobb-Douglas preferences. Result independent of sectoral productivities.

Relaxing Cobb-Douglas Preferences

- No structural change with Cobb-Douglas preferences.

1. **Unitary income elasticity of consumption.** Consumption of each good increases one for one with income (holding relative prices fixed). Counterfactual. Share of consumption towards agricultural (resp. service) goods falls (resp. increases) with income. Subsistence vs. luxury good.
2. **Unitary elasticity of substitution across goods.** Relative price movements exactly offset relative increase in output. Expenditure and value added shares constant across sectors.

- Need to relax at least one of these assumptions.

Non-Homothetic Preferences

- **Subsistence good.** Need to devote a large fraction of expenditures towards agriculture when poor. 'Food problem'.
- **Luxury good.** Share of expenditures towards luxury good increases with income. Case of many services.
- **Stone-Geary Preferences.** Consumption composite,

$$\mathcal{C}\left(c_a(t), c_m(t), c_s(t)\right) = \left(c_a(t) - \overline{c_a}\right)^{\omega_a} c_m(t)^{\omega_m} \left(c_s(t) + \overline{c_s}\right)^{\omega_s}$$

with $\omega_i \geq 0$, the weight of each good, $\omega_a + \omega_m + \omega_s = 1$.

$\overline{c_a}$ is the agricultural consumption subsistence level (generates an income elasticity smaller than one).

$\overline{c_s}$ is the service consumption endowment (generates an income elasticity larger than one).

Consumer optimization

- **Optimization.**

$$\begin{aligned} &\max_{\{c_a(t),c_m(t),c_s(t)\}} \left(c_a(t)-\overline{c_a}\right)^{\omega_a} c_m(t)^{\omega_m} \left(c_s(t)+\overline{c_s}\right)^{\omega_s} \\ &\text{s.t} \quad p_a(t)c_a(t)+p_m(t)c_m(t)+p_s(t)c_s(t)=w(t)L(t) \end{aligned}$$

- **Expenditures.** With $X(t)=(w(t)L(t)-p_a(t)\overline{c_a}+p_s(t)\overline{c_s})$,

$$\begin{aligned} p_a(t)\left(c_a(t)-\overline{c_a}\right) &= \omega_a X(t) \\ p_m(t)c_m(t) &= \omega_m X(t) \\ p_s(t)\left(c_s(t)+\overline{c_s}\right) &= \omega_s X(t) \end{aligned}$$

Trick towards solution. With $\tilde{c}_a(t)=(c_a(t)-\overline{c_a})$ and $\tilde{c}_s(t)=(c_s(t)+\overline{c_s})$, optimization same as before with Cobb-Douglas with budget constraint,

$$p_a(t)\tilde{c}_a(t)+p_m(t)c_m(t)+p_s(t)\tilde{c}_s(t)=w(t)L(t)-p_a(t)\overline{c_a}+p_s(t)\overline{c_s}=X(t)$$

Consumption expenditures and value added shares

- **Expenditures shares** = value added shares, $s_a(t) = \frac{p_i(t)c_i(t)}{w(t)L(t)}$

$$
\begin{aligned}
s_a(t) &= \omega_a \frac{X(t)}{w(t)L(t)} + \frac{p_a(t)\overline{c_a}}{w(t)L(t)} \\
s_m(t) &= \omega_m \frac{X(t)}{w(t)L(t)} \\
s_s(t) &= \omega_s \frac{X(t)}{w(t)L(t)} - \frac{p_s(t)\overline{c_s}}{w(t)L(t)}
\end{aligned}
$$

- Notice $\frac{p_i(t)\overline{c_i}}{w(t)} = \frac{\overline{c_i}}{Z_i(t)}$ negatively related to TFP. Intuition?
- With low agricultural TFP (= high subsistence needs), large share in agriculture, low shares in manufacturing/services.
- As productivity and income grow, $s_a(t)\downarrow$, $s_m(t)\uparrow$ and $s_s(t)\uparrow$ even more. Very much like the data on structural change.

  Remark. For large enough productivities, $s_i(t) \to \omega_i$.

Employment shares

- **Employment shares** equal to value added shares. For $i \in \{a, m, s\}$,
$$\frac{L_i(t)}{L(t)} = s_i(t)$$
- Use market clearing for goods to show this, $c_i(t) = Z_i(t)L_i(t)$
$$\Rightarrow \omega_a X(t) + p_a(t)\overline{c_a} = p_a(t)Z_a(t)L_a(t) = w(t)L_a(t)$$
$$\frac{L_a(t)}{L(t)} = \omega_a \frac{X(t)}{w(t)L(t)} + \frac{p_a(t)\overline{c_a}}{w(t)L(t)} = s_a(t)$$
- Employment share in agriculture drops with income. Agricultural TFP growth crucial for results ('food problem').
- Proper calibration of parameters $\overline{c_a}$ and $\overline{c_s}$ can generate patterns of structural change.

Constant Elasticity of Substitution (CES) Preferences

- **Elasticity of substitution.** Price elasticity of consumption across goods. Measures how demand for a good react to a (relative) price change. If goods close (poor) substitutes, strong (small) reaction of demand.
- **CES Preferences.** Consumption composite,

$$C(t) = \left[\omega_a^{1/\varepsilon} \left(c_a(t)\right)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_m^{1/\varepsilon} \left(c_m(t)\right)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^{1/\varepsilon} \left(c_s(t)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

  where

  - $\omega_i \geq 0$, $i = a, m, s$, weight of each good.
  - $\varepsilon > 0$ is the elasticity of substitution between the goods (i.e. the response of consumption to relative prices)

$$\text{if } \varepsilon \begin{cases} > 1 & \text{goods are substitutes} \\ < 1 & \text{goods are complements} \\ = 1 & \text{Cobb-Douglas} \end{cases}.$$

Consumer optimization

- **Optimization.**

$$\max_{\{c_a(t),c_m(t),c_s(t)\}} \left[\omega_a^{1/\varepsilon}\left(c_a(t)\right)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_m^{1/\varepsilon}\left(c_m(t)\right)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^{1/\varepsilon}\left(c_s(t)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\text{s.t} \quad p_a(t)c_a(t) + p_m(t)c_m(t) + p_s(t)c_s(t) = w(t)L(t) \qquad (\lambda(t))$$

- **First-Order Conditions.** Check that for $i \in \{a, m, s\}$,

$$\omega_i\left(c_i(t)\right)^{-\frac{1}{\varepsilon}} C(t)^{\frac{1}{\varepsilon}} = \lambda(t)p_i(t)$$

$$\Rightarrow \left(\frac{c_i(t)}{c_j(t)}\right) = \left(\frac{\omega_i}{\omega_j}\right)\left(\frac{p_i(t)}{p_j(t)}\right)^{-\varepsilon} \quad \text{for } i \neq j$$

$\varepsilon =$ (relative) price elasticity of (relative) consumption

Consumption expenditures and value added shares

- **Expenditures.** $p_i(t)c_i(t) = \omega_i \lambda(t)^{-\varepsilon} \left(p_i(t)\right)^{1-\varepsilon} C(t)$
- Using budget constraint summing across goods gives

$$p_i(t)c_i(t) = \omega_i \left(\frac{p_i(t)}{P(t)}\right)^{1-\varepsilon} \cdot w(t)L(t)$$

  with price index of consumption composite,

  $P(t) = \left[\omega_a \left(p_a(t)\right)^{1-\varepsilon} + \omega_m \left(p_m(t)\right)^{1-\varepsilon} + \omega_s \left(p_s(t)\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$

- **Expenditures shares** (= value added shares) depend on relative prices for $\varepsilon \neq 1$,

$$s_i(t) = \omega_i \left(\frac{p_i(t)}{P(t)}\right)^{1-\varepsilon}$$

Employment shares

- **Employment shares** still equal to value added shares due to market clearing for goods. For $i \in \{a, m, s\}$,

$$\frac{L_i(t)}{L(t)} = s_i(t) = \omega_i \left(\frac{p_i(t)}{P(t)} \right)^{1-\varepsilon}$$

- **Relative sectoral TFP**. Employment shares also equal to

$$\frac{L_i(t)}{L(t)} = s_i(t) = \omega_i \left(\frac{Z_i(t)}{Z(t)} \right)^{\varepsilon-1}$$

  with $Z(t)$ weighted average of sectoral TFPs (aggregate TFP),
  $Z(t) = \left[\omega_a \left(Z_a(t) \right)^{\varepsilon-1} + \omega_m \left(Z_m(t) \right)^{\varepsilon-1} + \omega_s \left(Z_s(t) \right)^{\varepsilon-1} \right]^{\frac{1}{\varepsilon-1}}$
- Changes in **relative** sectoral TFP generate changes in relative prices and thus sectoral reallocation for $\varepsilon \neq 1$.

Intuitions

- Employment shares equal to

$$\frac{L_a(t)}{L(t)} = s_i(t) = \omega_i \left(\frac{Z_i(t)}{Z(t)} \right)^{\varepsilon - 1}$$

- If $\varepsilon > 1$, the sector with a faster TFP growth (relative to aggregate) grow faster. Wages increase more in this sector, which attract more workers.
- If $\varepsilon < 1$, the sector with a faster TFP growth (relative to aggregate) shrinks. Alike 'Baumol Disease'. Wages increase faster in slow TFP growing sector due to price increases.

Insights from CES preferences

- Can **sector-specific productivity growth** account for structural change?
- Data do not systematically show faster or slower TFP growth in agriculture relative to manufacturing over long period. Perhaps faster in agriculture post-WW2 but more the opposite before. Not a great theory to account for the drop of employment in agriculture.
- Data suggest slower productivity growth in services. Combined with a low elasticity of substitution, $\varepsilon < 1$, theory can account for the rise in the service economy. Consistent with the relative price increase in services.
- Empirical measures of ε point towards a low value.

More General Preferences

- **CES Stone-Geary** combine both mechanisms.
- Consumption composite

$$C(t) = \left[\omega_a^{1/\varepsilon}\left(c_a(t) - \overline{c_a}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_m^{1/\varepsilon}\left(c_m(t)\right)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_s^{1/\varepsilon}\left(c_s(t) + \overline{c_s}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where,

  - $\omega_i \geq 0$, $i = a, m, s$, is the weight of each good.
  - $\varepsilon > 0$ is the elasticity of substitution between the goods
  - $\overline{c_a}$ is the agricultural consumption subsistence level
  - $\overline{c_s}$ is the service consumption endowment

- Can match the data on structural change pretty well (see Herrendorf, Rogerson and Valentinyi (AER, 2013)).

Wrap up of theory

- Two main views on technology driven structural change.

1. **Income effects.** As income grows, spending moves away from subsistence goods (e.g. food) towards more luxury goods (e.g. services). The agricultural sector shrinks, the others grow, and particularly the service sector. Likely to be the main driver of the drop in employment in agriculture.
2. **Substitution effects.** If goods of different sectors are poor substitutes ($\varepsilon < 1$), the sector with a lower TFP growth (e.g. services) should expand.
   Demand does not react much to the price increase and value added, expenditure and employment in this sector all increase. Can partly account for the rise of the service sector.

Application
Structural Change and Urbanization

Structural Change, Land Use and Urban Expansion

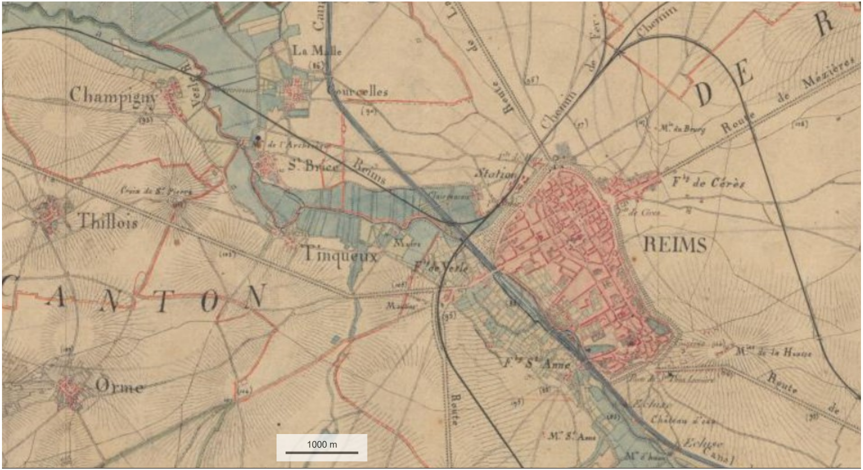
Coeurdacier, Oswald and Teignier (2021)

- **Spatial dimension of structural change.** Reallocation of workers away from agriculture related to urbanization. Migration from rural areas to cities.
- **Research question.**

  How do cities grow during the process of structural change?
- Investigates the question theoretically and empirically using historical data for France since 1840.

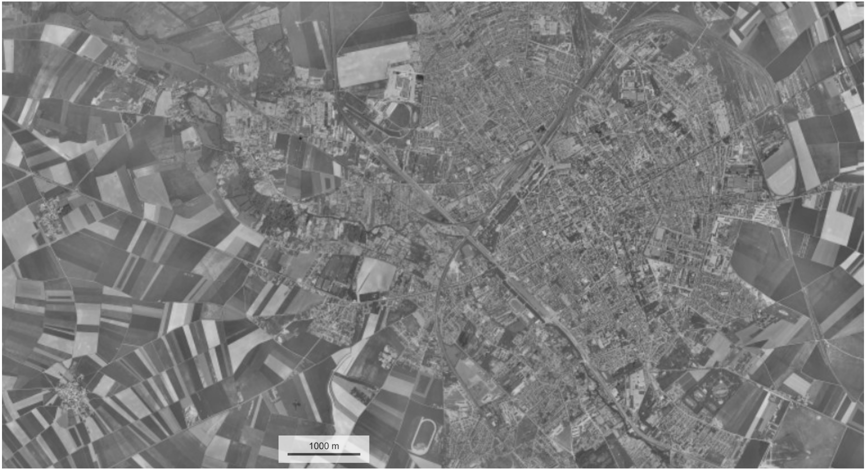
Motivation

Reims from above in 1866



Motivation

Reims from above in 1950



Motivation

Reims from above in 2015

Motivation

- In 1866, about half of the workers in the region of Reims are working in agriculture (about 1% today)
- Since 1866, the population of the urban area of Reims has been multiplied by almost 3.
- The urban surface was multiplied by about 25—largely expanding out of agricultural land.
- Average urban density fell by a factor close to 8.

$\rightarrow$ Need to understand the spatial reallocation of land use and workers with economic development.

Urban Expansion and Sprawling

Different views

1. Urban Economics
   - Development of commuting technologies push city fringe outwards.
   - Decline in commuting costs allows residing further away from dense city centre. Suburbanisation.
2. (Macro) Structural Change
   - Food subsistence constraint initially binding. High land values. Little income left for housing. Small and dense cities.
   - Rising agricultural productivity solves the 'food problem'. Downward pressure on land values. City expands easily to accomodate greater housing demand. Urban density falls.

This paper: try to reconcile these views in an unified framework.

This paper

A spatial general equilibrium model of structural change and land use

- Three sectors/goods: rural, urban and housing
  - Different intensity in the use of land as input
  - Rival Land Use = Agricultural or Housing
  - Fixed Supply of Land
- Urban versus Rural Land
  - Commuting costs for urban workers
- Drivers of structural change
  - Non-homothetic preferences
  - Transitory dynamics with rising productivity

The Story

Transitory dynamics with rising productivity

- **Old times.** Land is scarce. High values of farmland with respect to income due to low productivity ('food problem'). Small, dense and walkable cities.

- **Transition.** Agricultural productivity growth frees up labor and land for cities to expand. Urban workers use faster commuting modes. Cities getting large (in area) and much less dense without a large increase in land values.

- **Recent times.** Reallocation of factors/land use slowdowns. Cities expand less and land prices increase more with rising productivity.

Outline of the paper

1. Historical Evidence from France
    - Land use and urban expansion in France since 1840.
2. Baseline Theory [brief overview if time permits]
    - A baseline spatial general equilibrium model of structural change and land use
3. Quantitative Evaluation [not today]
    - Richer model calibrated to French data since 1840.

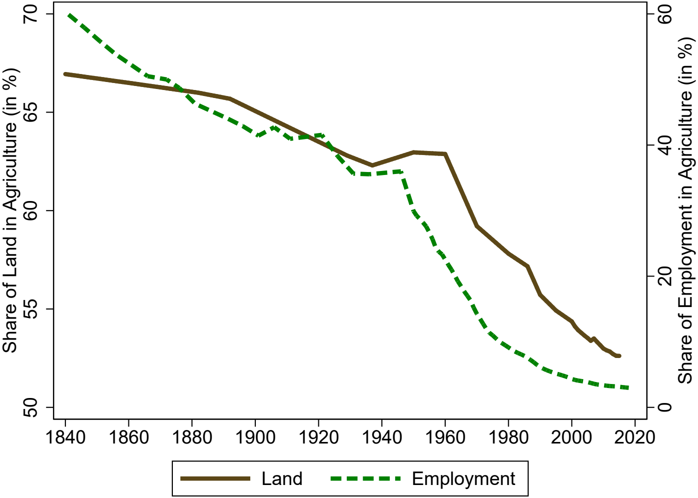
Historical Evidence from France

Data

- Land use in agriculture
  - Historical: mostly based on Agricultural Census. Post-1950, Ministry of Agriculture.
- Employment across sectors
  - Insee, Toutain (1993), Herrendorf et al. (2014).
- The expansion of cities
  - Carte Etat-Major (1866), IGN (1950), Satellite Data post-1975 (GHSL data). Census for Population. Toulouse
- Housing and Land Prices
  - Aggregate Historical: Piketty et al. (2014), Knoll et al. (2017).

Land and labor reallocation

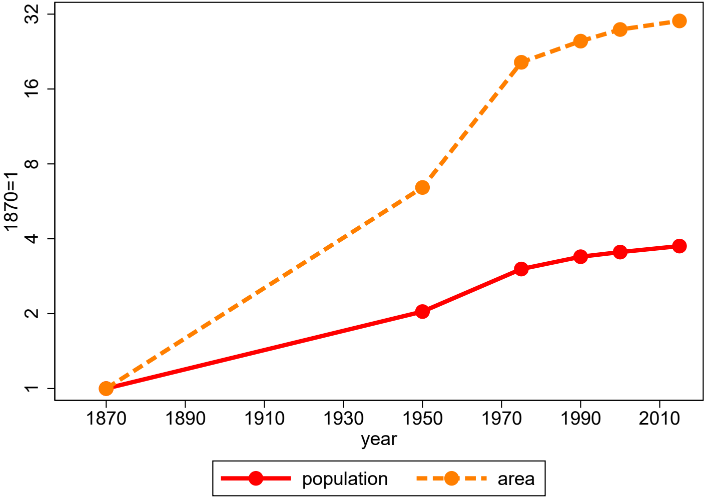
Aggregate for France



Source: Agricultural Census and Toutain (1993). Ministry of Agriculture.

Urban Expansion

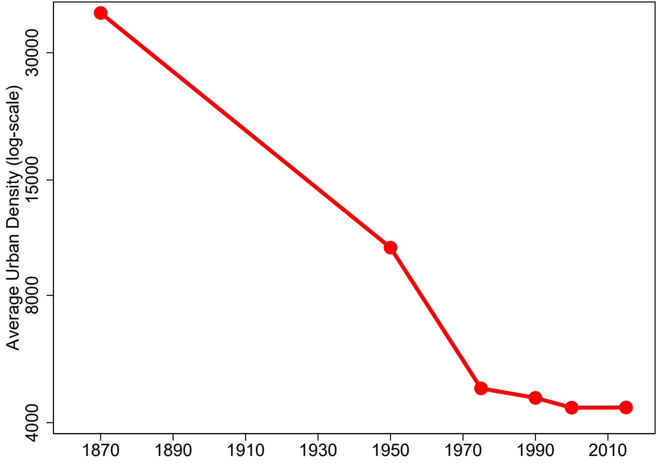
Average across 100 urban areas



Source: Carte Etat-Major, IGN and GHSL. Census

The historical fall in urban density

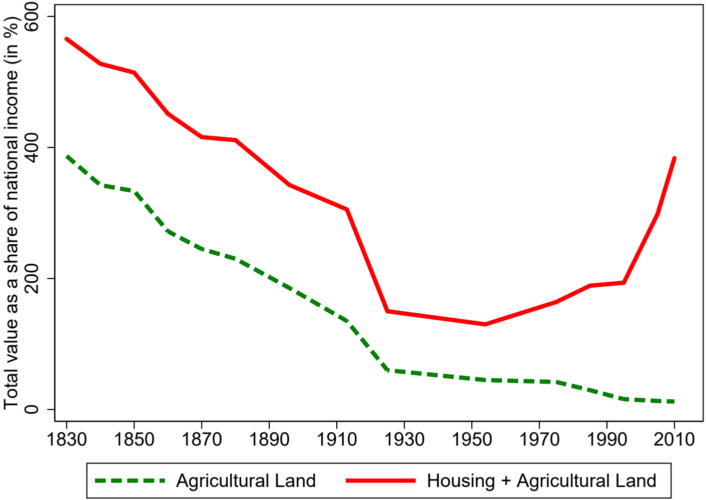
Average across 100 urban areas



▸ Paris Source: Carte Etat-Major, IGN and GHSL. Census

Housing and Agricultural Land Wealth

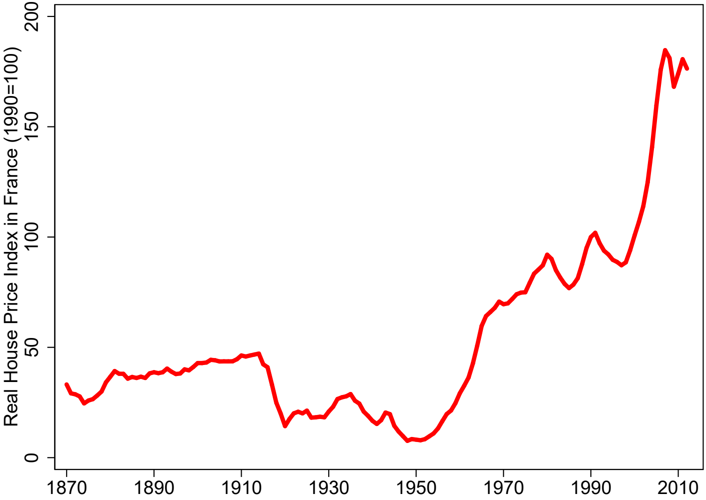
Share of National Income



Source: Piketty and Zucman (2014).

The hockey-stick in housing prices

Real Housing Price Index in France (1870-2010)



Source: Knoll et al. (2017).

Historical Evidence from France

Wrap up

- **Structural Change.** Reallocation of workers and land away from agriculture.
- **Urbanization.** Urban Expansion and Sprawling.
  Urban area increasing faster than population. Drop in average urban density.
- **Land value.** Reallocation of land value away from agricultural land towards urban land. Faster increase in urban land values in recent decades.

A baseline spatial general equilibrium model of land use

Set-up Description

- Economy endowed with a fixed amount of land and L ex-ante identical workers supplying one unit of labour.
- Three sectors/goods: rural (r), urban (u), housing (h).
  - Rural sector produces using labor and land.
  - Urban sector produces using labor.
  - Housing uses the urban good and land.
- Goods and factor markets perfectly competitive.
  Labor perfectly mobile.
- Goods perfectly tradable.

Technology

Urban and Rural good

For the urban good, only labor for simplicity,

$$Y_u = \theta_u L_u.$$

For the rural good,

$$Y_r = \theta_r (L_r)^{\alpha} (S_r)^{1-\alpha},$$

$\theta_i =$ TFP in sector i, $L_i =$ labor used in i, $S_r =$ land used in r.

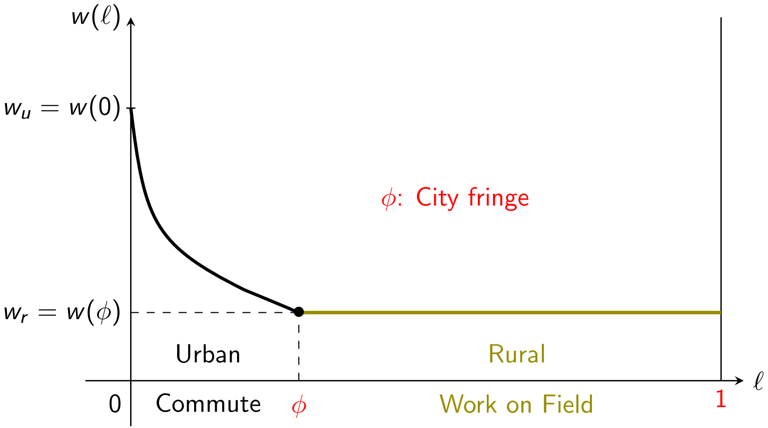
Rural good more intensive in land.

Spatial Structure

- Only one city [multiple-city extension].
- Continuous locations ℓ between $[0,1]$. Urban good produced in the city at location $\ell = 0$.
- Urban commuting costs
  - Depend on opportunity cost of time and distance of trips, $\tau(\ell) = a \cdot (w_u \cdot \ell)^{\xi}$. ▸ Microfoundation
  - A urban worker in ℓ pays $\tau(\ell)$ to work in u.
  - Earns $w(\ell) = w_u - \tau(\ell)$, with $\tau(0) = 0$, and $\partial\tau(\ell)/\partial\ell \geq 0$.
- Urban workers locate as close as possible to $\ell = 0$.
- Urban fringe: $\ell = \phi < 1$, the farthest away (endogenous) location of a urban worker.

Spatial Structure

Wages Net Of Commuting Costs, $w(\ell) = w_u - \tau(\ell)$



Preferences and budget constraint

- Non-homothetic preferences for an individual in location ℓ

$$C(\ell) = (c_r(\ell) - \underline{c})^{\nu(1-\gamma)} (c_u(\ell) + \underline{s})^{(1-\nu)(1-\gamma)} h(\ell)^{\gamma},$$

$c_i(\ell) =$ consumption of $i = \{r, u\}$, housing consumption $h(\ell)$.

$\underline{c} =$ subsistence consumption for rural good.

$\underline{s} =$ endowment of urban good.

- Budget constraint,

$$pc_r(\ell) + c_u(\ell) + q(\ell)h(\ell) = w(\ell) + r,$$

p denotes the relative price of the rural good.

$q(\ell)$ the rental price per unit of housing in location ℓ.

r land rental income per capita, equally distributed.

Equilibrium Sorting

Mobility conditions

- Indifference condition across locations:

$$\overline{C} = C(\ell) = \kappa \frac{w(\ell) + r - p\underline{c} + \underline{s}}{q(\ell)^\gamma}$$

  - Within the city, $q(\ell)$ is falling with ℓ to compensate workers who live in worse locations.
  - In the rural area, housing price is constant across locations:

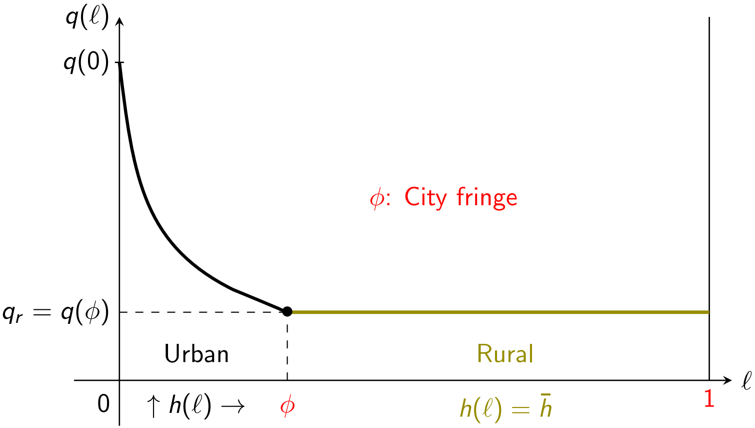
$$q(\ell \geq \phi) = q$$

- Indifference at the fringe,

$$w(\phi) = w_u(1 - \tau(\phi)) = w_r$$

Equilibrium Sorting

Housing Rental Price Gradient $q(\ell)$



Housing Market

Housing supply and land prices

- Developers supply housing using land and urban goods. Housing supply from profit maximization, ▸ Details

$$H(\ell) = q(\ell)^{\epsilon},$$

with housing supply elasticity $\epsilon \geq 0$.

- Zero-profit condition ties land prices to housing prices in ℓ,

$$\rho(\ell) = \frac{q(\ell)^{1+\epsilon}}{1+\epsilon}$$

- Indifference conditions across usage at the fringe ϕ,

$$\rho_r = \rho(\phi) = (1-\alpha)p\theta_r \left(\frac{L_r}{S_r}\right)^{\alpha}.$$

Characterization of the Equilibrium

- Equilibrium determines the labor and land use allocation together with land (or housing) and goods prices. Full Definition

- Housing market clears such that demand for housing in each urban location ℓ is equal to supply.

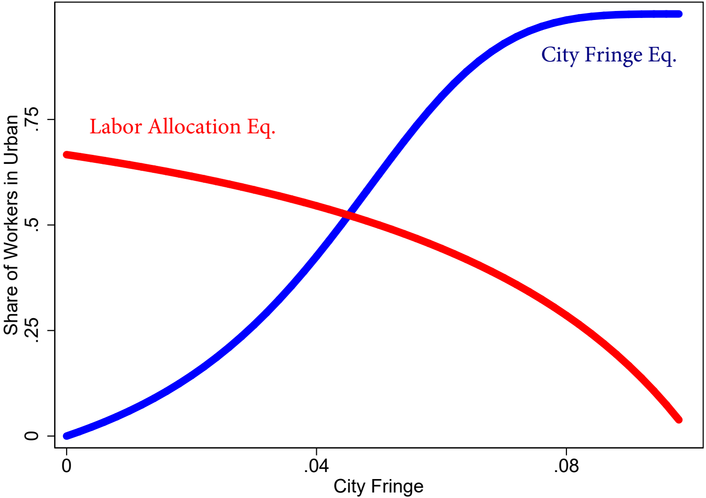
  Pins down the fringe ϕ for a given urban population. Details

- Labor market clears, $L_u + L_r = L$.
- Land market clears, $\phi + S_r + S_{hr} = 1$, with S_{hr} = the land demand for housing in the rural area.

- Goods markets clear for both goods, urban and rural. Details

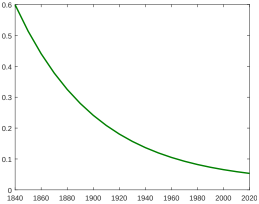
Characterization of the Equilibrium

The labor allocation and the city fringe equations

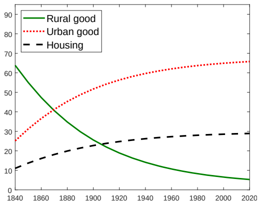


Numerical Illustrations

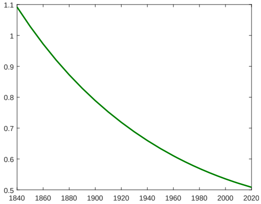
Baseline — common productivity growth across sectors



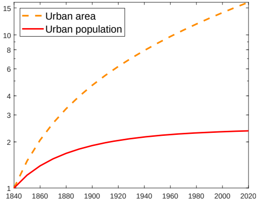
(a) Rural Labor



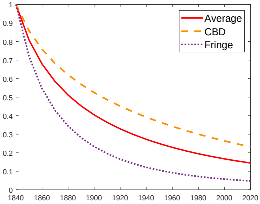
(b) Expenditure shares



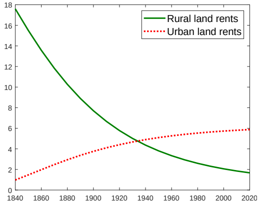
(c) Relative price



(d) City size



(e) Urban densities



(f) Land rents/income

▸ Parameters values ▸ Rural Productivity Growth ▸ Urban Productivity Growth ▸ $s >> c$

Take aways from theory

- **Structural Change.** With low initial productivity, rural land and goods expensive ('food problem'). Small and dense cities. Rural productivity growth reallocates workers and land away from agriculture.
- **Urbanization.** As workers move towards cities, cities expand in area and population. Faster in area as fringe prices getting cheap relative to income and workers can devote more resources to housing. Urban density drops. Amplified by the use of faster commuting modes by urban workers.
- **Land value.** Reallocation of land value away from agricultural land towards urban land along the transition.

Quantitative evaluation from France (1840-2015)

Summary

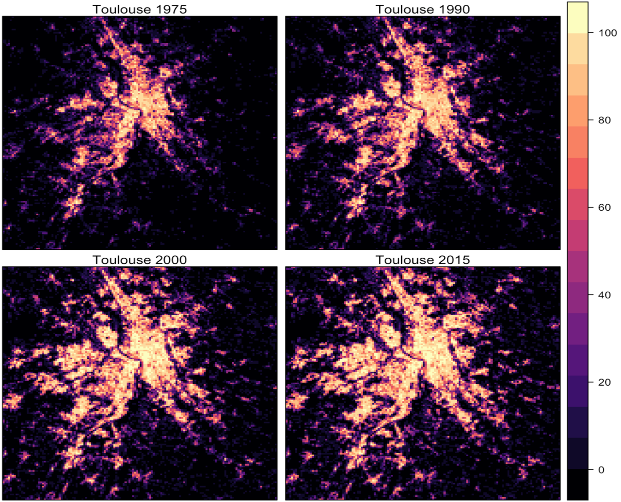
- Extended set-up calibrated using macro and micro data. (multiple circular cities, more general commuting costs, housing supply elasticity location-specific, dynamic model).
- Matches the evolution of allocation of economic activity and density of settlements across time and space.
- Explains a significant fraction of the reallocation of land value towards urban locations and of the increase in housing prices in recent decades.
- **Rural productivity growth** together with evolution of commuting costs quantitatively crucial. Quantitative evaluation matches the rise in average urban commuting speed since 1840.

Conclusion and Future Research

- **What makes a country rich?**
  Poor countries have a very low agricultural productivity.
  Sectoral TFP gaps important to understand income gaps?
- **Agricultural productivity gap.** Marginal productivity of labour in agricultural low relative to other sectors. Why? Misallocation? Mobility frictions, urban congestion, spatial differences in living costs, workers selection, ...
- Spatial general equilibrium of land use. Useful tool to evaluate the effects on welfare and aggregate productivity of land use/environmental regulations.
  - Is sprawling 'excessive'? Benefits of compact cities?
  - Implications of changing commuting costs for productivity?

Urban Expansion

Urban Area and Population Measurement



- 1866: Manual + Census
- 1950: Manual + Census
- 1975, 1990, 2000, 2015: GHSL

▸ Back

Endogenous commuting costs

Microfoundation

- Endogenous commuting mode/speed m
  - Commuting cost: operating cost $f(m)$ and time cost $t(\ell) = \frac{2\ell}{m}$,

$$\tau(\ell) = f(m) + 2\zeta w_u \cdot \frac{\ell}{m}$$

    Faster modes are more expensive, $f(m) = \frac{c_\tau}{\eta_m} m^{\eta_m}$.

  - Optimal commuting mode, $m = \left(\frac{2\zeta w_u}{c_\tau}\right)^{\frac{1}{1+\eta_m}} \ell^{\frac{1}{1+\eta_m}}$.
- Endogenous commuting cost

$$\tau(\ell) = a \cdot (w_u \cdot \ell)^{\xi},$$

where $\xi \equiv \frac{\eta_m}{1+\eta_m} \in (0, 1]$ and $a \equiv \left(\frac{1+\eta_m}{\eta_m}\right) c_\tau^{\frac{1}{1+\eta_m}} (2\zeta)^{\frac{\eta_m}{1+\eta_m}} > 0$

back

Housing Market Equilibrium

Land developers

- Housing supply provided by land developers.

- Technology: in each location, developers supply housing space H per unit of land with a convex cost,

$$\frac{H^{1+1/\epsilon}}{1+1/\epsilon},$$

where $\epsilon \geq 0$ is the cost parameter.

- Profits per unit of land at ℓ,

$$\pi(\ell) = q(\ell)H(\ell) - \frac{H(\ell)^{1+1/\epsilon}}{1+1/\epsilon} - \rho(\ell),$$

$\rho(\ell)$ the (rental) price of a unit of land in ℓ.

Housing Market Equilibrium

Housing supply and land prices

- Housing supply from profit maximization,

$$H(\ell) = q(\ell)^{\epsilon},$$

with housing supply elasticity $\epsilon \geq 0$.

- Free entry of developers pins down land prices in ℓ,

$$\rho(\ell) = \frac{q(\ell)^{1+\epsilon}}{1+\epsilon},$$

- Indifference conditions across usage at the fringe,

$$\rho_r = \frac{q_r^{1+\epsilon}}{1+\epsilon} = (1-\alpha)p\theta_r \left(\frac{L_r}{S_r}\right)^{\alpha}.$$

back

Equilibrium Allocation

Definition of the Equilibrium

For technological parameters $(\theta_u, \theta_r, \alpha)$, commuting cost parameters (η_m, c_τ, ζ) and resulting spatial frictions $\tau(\ell)_{\ell \in \mathcal{L}}$, housing supply conditions ϵ, and preference parameters, $(\nu, \gamma, \underline{c}, \underline{s})$, an equilibrium is a sectoral labor allocation (L_u, L_r), a city fringe (ϕ) and rural land for production (S_r), factors and goods prices (w_u, w_r, q_r, p) and land rents (r), such that:

- *Factors are paid their marginal productivity.*
- *Workers are indifferent across locations.*
- *The demand for urban land (or city fringe ϕ) satisfies the city size Eq. 1 below.*
- *Land and labor markets clear.*
- *Rural and urban goods markets clear.*
- *Land rents satisfy Eq. 2 below.*

Back

Housing Market Clearing in the City

- Within the city, $\ell \leq \phi$, density $D(\ell)$ in location ℓ.
- Housing market clearing, $H(\ell) = D(\ell)h(\ell)$, leads to

$$D(\ell) = \frac{H(\ell)}{h(\ell)},$$

where $h(\ell)$ denotes the demand for housing space.

- Total urban population

$$L_u = \int_0^{\phi} D(\ell) d\ell. \tag{1}$$

Back

Land and Labor Market Clearing

- Land market clearing:

$$\phi + S_r + S_{hr} = 1,$$

with S_{hr} = the land demand for housing in the rural area.

- Labor market clearing:

$$L_u + L_r = L.$$

- Land rental income:

$$rL = \int_0^{\phi} \rho(\ell)d\ell + \rho_r \times (1 - \phi) \tag{2}$$

Back

Goods Market Equilibrium

Goods Market Clearing

- Per capita consumption of the goods, $c_i = \frac{1}{L}\int_0^1 c_i(\ell)d\ell$.
- Per capita market clearing condition for the rural good,

$$c_r = y_r,$$

where $y_r = \frac{Y_r}{L}$ = production per worker of (r).

- For the urban good,

$$c_u + \frac{1}{L}\int_0^{\phi} \tau(\ell)D(\ell)d\ell + \frac{1}{L}\frac{\epsilon}{1+\epsilon}\int_0^1 q(\ell)H(\ell)d\ell = y_u,$$

where $y_u = \frac{Y_u}{L}$ = production per worker of (u).

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Numerical Illustrations

Parameter values

Technology. $\alpha = 0.75$. $L = 1$.
$\theta_{u,t} = \theta_{r,t}$ growing at 1% per year.

Preferences. Housing weight, $\gamma = 30\%$. Rural weight, $\nu = 2.5\%$. $\underline{s} = 0$ as baseline. $\underline{c}$ such that given initial θ about 60% of workers in (r). Sensitivity: $\underline{s} >> \underline{c}$.

Commuting costs. Commuting technology such that urban land small relative to rural land. $\xi = 2/3$ matching the increase in commuting speed.

Housing supply elasticity. $\epsilon = 4$.

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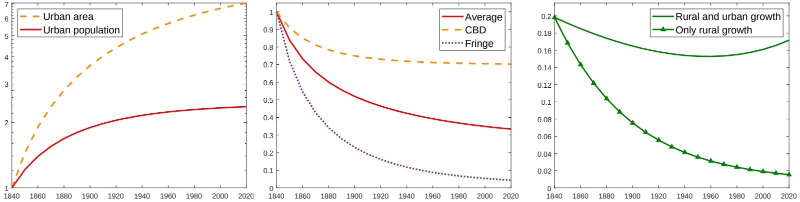
Numerical Illustrations

Rural Productivity Growth Only



(g) City size (h) Urban densities (i) Farmland price

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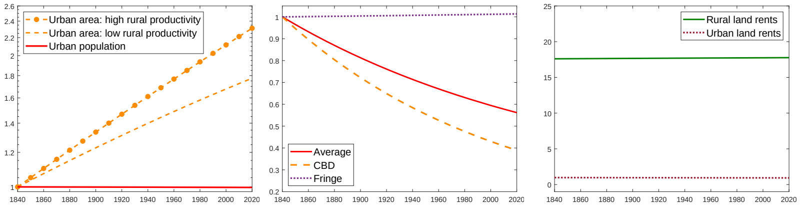
Numerical Illustrations

Urban Productivity Growth Only



(j) City size (k) Urban densities (l) Farmland price

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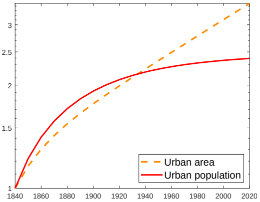
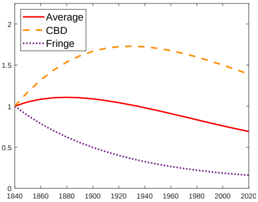
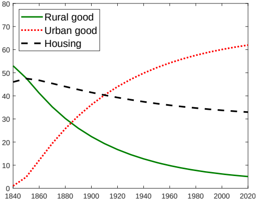
Numerical Illustrations: $\underline{s} > \underline{c} > 0$



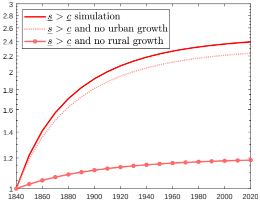
(m) City size



(n) Urban densities



(o) Expenditure shares



(p) Urban population

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