# Macroeconomics 1

## Lecture - Labor Markets

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# Overview of Labor Markets

# The neoclassical model of the labor market

A central question for macroeconomics and labor: **what determines the level of employment and unemployment in the economy?**

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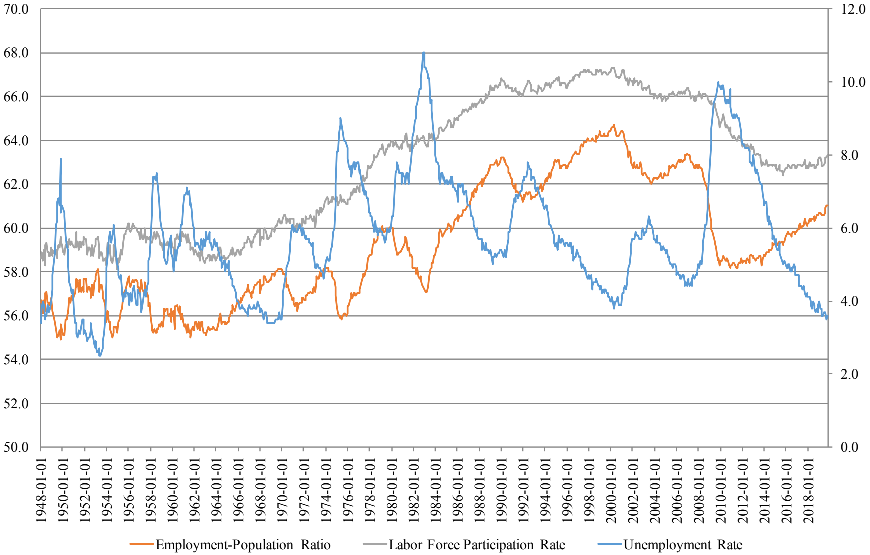
A central question for macroeconomics and labor: **what determines the level of employment and unemployment in the economy?**

**The neoclassical model:**

1. **Theoretically** cannot deal with involuntary unemployment:
   - ▶ There is supply and demand only:
   - $\sim$ The demand is determined by technology or by demand for output.
   - $\sim$ The supply is driven by inter-temporal substitution, where the idea is that given prices, some agents *optimally choose* to work zero hours.
   - $\Rightarrow$ **Unemployment is seen as leisure** ($\neq$ statistical definition).
   - $\Rightarrow$ If there is wage stickiness, there can be *under-employment*.
   - $\Rightarrow$ There is **no involuntary unemployment** in equilibrium.
2. **Empirically** cannot explain fluctuations in employment:
   - ▶ Shifting the demand curve according to the business cycle, employment and wage movements do not fit the data.

# Key labor market statistics US




Source: FRED, Bureau of Labor statistics, monthly, seasonally adjusted data

# Some facts about the labor market

- **Unemployment is a persistent phenomenon.**
  $\rightarrow$ Can wage/price stickiness be the reason? Not really.

- Large flows of workers between **employment, unemployment, and non-participation states**.

  $$\Delta u = \text{inflow} - \text{outflow}$$

  **inflow**: due to job loss or new entry from non-participation.

  **outflow**: due to job finding or exit into non-participation (retirement, school, inactivity).

- **Employed workers often change jobs** - with a wage gain or wage reduction.

# Search theory

- → Can we learn more about the macro equilibrium of the labor market by introducing **frictions**, by studying the **flows**?
- → Is getting information about the stocks through the flows more useful than studying the stocks directly?
- → By studying the question in this way we have a strong theoretical background for quantitative questions, which is useful for policy analysis.

# How should labor market frictions be modeled?

- **Incentive problems, efficiency wages**.
- **Wage rigidities, bargaining, non-market clearing prices**.
- **Search frictions**.

**Search and matching:** costly process for workers to find the right jobs & for firms to find the right workers.

- This is very similar to:
  - Searching for a flat.
  - Searching for a spouse.
  - Searching for the best loans on offer.
- Many applications of the search model.

# Shimer's exercise – role of separation & job finding rate

The change in the **unemployment rate** is:

$$u_{t+1} - u_t = s_t(1 - u_t) - f_t u_t.$$

- $u_t$ – **unemployment rate**.
- $s_t$ – **separation rate**.
- $f_t$ – **job finding rate**.
- Ignore exit from the labor force, and entry from out of labor force.

Denote average rates (over a period) by:

$$\overline{s} = \sum_{t=1}^{T} \frac{s_t}{T} \quad \text{and} \quad \overline{f} = \sum_{t=1}^{T} \frac{f_t}{T}.$$

# Shimer's exercise

Construct two **hypothetical unemployment rates**:

1. Using the **average separation rate**:

$$u_{t+1} - u_t = \overline{s}(1 - u_t) - f_t u_t.$$

$\rightarrow$ Remove fluctuations in the separation rate.

$\rightarrow$ **Changes are due to the job finding rate**.

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→ Remove fluctuations in the separation rate.
→ **Changes are due to the job finding rate**.

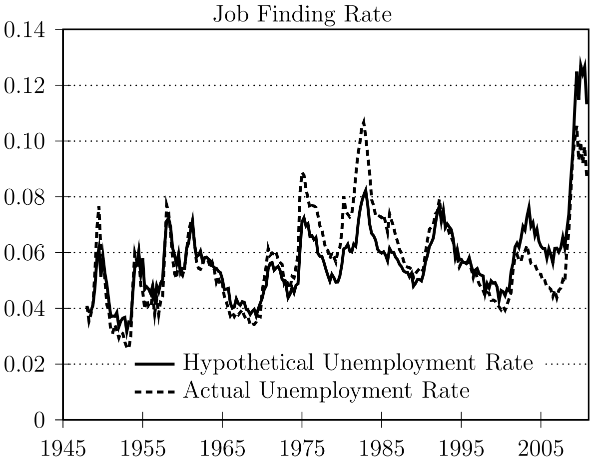
2. Using the **average job finding rate**:

$$u_{t+1} - u_t = s_t(1 - u_t) - \bar{f} u_t.$$

→ Remove fluctuations in the job finding rate
→ **Changes are due to the separation rate**

→ Compare to the **actual unemployment rate**.

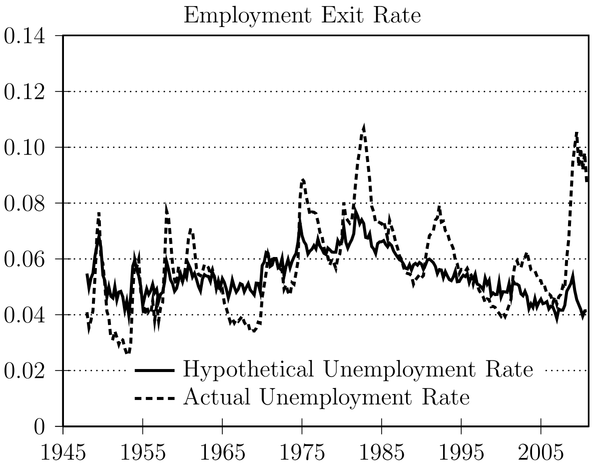
# The role of the job finding rate, by using $\bar{s}$




Source: Shimer (2012)

$\Rightarrow$ Varying only **the job finding rate** hypothetical $u$ is close to actual.

# The role of the separation rate, by using $\overline{f}$

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⇒ Varying only **the separation rate** hypothetical $u$ further from actual.

# Lessons from Shimer's exercise

- **Separation rate not so important** in the evolution of US unemployment (explains 25%).
- **Job finding rate is a more important** determinant of unemployment (explains 75%).

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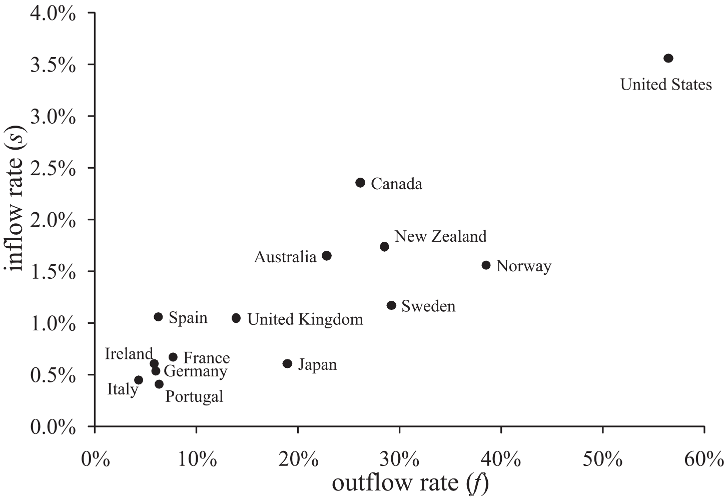
- **Separation rate not so important** in the evolution of US unemployment (explains 25%).
- **Job finding rate is a more important** determinant of unemployment (explains 75%).

- Why?
- **Separation rate increases during recessions**.
- But the average **job finding rate is high** in the US.
- Even if more workers get laid off, they find a job quickly.

# But countries are different - OECD 1968-2007

FIGURE 1.—AVERAGE INFLOW AND OUTFLOW RATES ACROSS COUNTRIES




Source: Elsby, Hobijn, Sahin (2013).

# Shimer's exercise for other countries

Changes in unemployment are due to:

- UK: **71% inflow rate**, **29% outflow rate** (Elsby, Smith, Wadsworth (2010))
- Spain: **57% inflow rate**, **43% outflow rate** (Petrongolo and Pissarides (2009))

$\Rightarrow$ Study the determinants of both the **outflow** and the **inflow**

# An overview of search models

1. **First generation: one-sided search**:
   - Focuses on the workers.
   - There is an exogenous job arrival.
   - Worker's optimal decision.

2. **Second generation: two-sided search**:
   - Endogenous job arrival where the idea is that somebody has to create the job – active job creation by firms.
   - Matching function $m = m(u, v)$
     $u$ – the stock of unemployed, state variable.
     $v$ – the number of vacancies, the control variable.

3. **Third generation**:
   - Endogenous job destruction, where jobs are destroyed if their productivity is not high enough.

4. **Fourth generation**:
   - Endogenous wage distribution.

# First generation models

# Search and Unemployment Models

- A useful subset of dynamic programming is those models that deal with **search and unemployment** for some agent workers.
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- A useful subset of dynamic programming is those models that deal with **search and unemployment** for some agent workers.
- Those models attempt to characterize the process that takes place between a **worker and hiring firms**.

- We will study the McCall model to introduce some of the concepts.
- While studying those models we can be aware that there will be a variety of **small adjustments** that can be introduced into the modeling environment to capture the real-life dynamics in the job matching market.

# Mathematical preliminaries

# Probability theory

- The **cumulative distribution** is given by $F(p) = \text{Prob}\{P \leq p\}$, where we assume that $F(0) = 0$, which implies that $p$ takes only non-negative values.
- We also assume that the distributions have some upper bound $B$ such that we observe no $p$ larger and where $F(B) = 1$.

# Probability theory

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- We also assume that the distributions have some upper bound $B$ such that we observe no $p$ larger and where $F(B) = 1$.
- This means that there exists **zero probability of observing an observation $\tilde{p}$ outside of the interval $[0, B]$**.

# Probability theory

- Given some CDF, the **expected value** of some random variable $p$, denoted by $\mathbb{E}[p]$ is defined by:

$$E[p] = \int_0^B p dF(p) = \int_0^B p f(p) dp,$$

where $f(p)$ denotes the **probability density function** for the random variable.

- Using integration by parts $\int_a^b u dv = uv - \int_a^b v du$ notice:

$$\begin{aligned}
\int_0^B p dF(p) &= pF(p)|_0^B - \int_0^B F(p) dp \\
&= [B(1) - 0F(0)] - \int_0^B F(p) dp \\
&= B - \int_0^B F(p) dp.
\end{aligned}$$

# Probability theory

- This means we also have the **equivalent expression for the mean**:

$$E[p] = B - \int_0^B F(p)dp.$$

- Note for $n$ **independent and identical draws** of $p_i$ from the CDF $F(p)$, we have:

$$\text{Prob}\{\max(P_1, P_2, \ldots, P_n) < p\} = F(p)^n.$$

# Mean-preserving spread

- We introduce now the concept of **mean-preserving spread**. This refers to analyzing/comparing multiple distributions which are characterized by the same mean.
- Consider a class of distributions indexed by some parameter $r$ in the set $R$. Assume $F(0, r) = 0$ and $F(B, r) = 1$ for all possible distributions in the set/class.
- These distributions carry the same expected value for the random variable $p$, thus:

$$\int_0^B \left[F\left(p, r_1\right) - F\left(p, r_2\right)\right] dp = 0. \tag{1}$$

# Single-crossing property

- Two distributions, indexed by $r_1$ and $r_2$ satisfy the **single-crossing property** if $\exists \hat{p} \in (0, B)$ such that:

$$F(p, r_2) - F(p, r_1) \leq 0 \quad \text{when} \quad p \geq \hat{p}, \tag{2}$$

  and vice-versa whereby both inequality signs are flipped.
- This means as $p$ increases in values from 0 to $B$, the difference will switch from negative to positive value at only one point $\hat{p}$.

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- This means as $p$ increases in values from 0 to $B$, the difference will switch from negative to positive value at only one point $\hat{p}$.
- This means that at $\hat{p}$, there is an equal probability of observing values below (or above) $\hat{p}$ across the two distributions

# Mean-preserving spread

- Equation (2) means that there is more probability mass to the left of $F(p, r_1)$ for all $p$ larger than $\hat{p}$.
- When (1) and (2) are satisfied, we say that the distribution indexed by $r_2$ has been obtained by $r_1$ by a **mean-preserving spread**. These imply:

$$\int_0^y \left[F\left(p, r_2\right) - F\left(p, r_1\right)\right] dp \geq 0 \quad \forall y \in [0, B].$$

# The McCall Model

# Introduction

- We begin analyzing the model in **discrete time**.
  1. At each period the worker receives a **job offer** (e.g., wage offers drawn from an i.i.d. distribution).
  2. The worker may either **reject** this offer and receive **unemployment insurance** $b > 0$ that period, or may **accept** the offer, receiving the **wage** $w$ for the period and every period forward.
  3. This game takes place over an **infinite horizon**.
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- What is the **trade-off** here regarding the worker decision?

- When the unemployed worker accepts the job he receives the wage, but loses the opportunity of receiving new offers.

# Some parameters

- $\beta$ is the **discount rate** given by $\beta = \dfrac{1}{1+r}$. When $\beta = 1$ the agent is indifferent between today and tomorrow and $\beta = 0$ the agent only cares about today.
- $b$ is the value of **unemployment insurance**.
- $\alpha$ is the **probability of receiving an offer when unemployed**.
- $F(w)$ is the **i.i.d. distribution of wages** in this economy.

# Agent's utility

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$$W(w) = w + \beta W(w) \implies W(w) = \frac{w}{1-\beta}.$$

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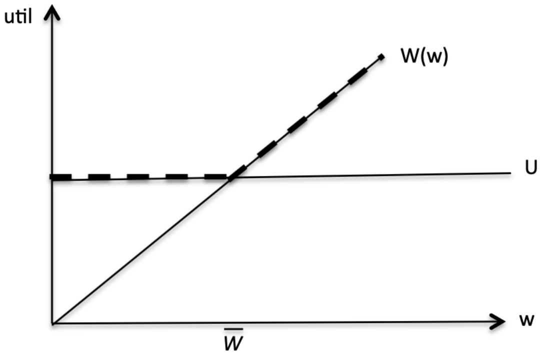
$$U = b + (1-\alpha)\beta U + \alpha \int_0^\infty \max\left\{\beta W\left(w'\right), \beta U\right\} dF\left(w'\right).$$

Notice we are assuming the utility is linear such that $u(w) = w$ and $u(b) = b$ and notice $F(w)$ does not depend on time.

# Agent's utility



# Agent's utility

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- In the end we have:

$$U = b + (1-\alpha)\beta U + \alpha\beta \left[ \int_{\bar{w}}^{\infty} W\left(w'\right) dF\left(w'\right) \right] + \alpha\beta F(\bar{w}) U.$$

# First approach to the problem

- By definition

$$W(\bar{w}) = U = \frac{\bar{w}}{1-\beta} \text{ and } W\left(w'\right) = \frac{w'}{1-\beta}$$

- Now use those definitions in the equations for $U$:

$$\frac{\bar{w}}{1-\beta} = b + (1-\alpha)\beta\frac{\bar{w}}{1-\beta} + \alpha\frac{\beta}{1-\beta}\int_{\bar{w}}^{\infty} w' dF\left(w'\right) + \alpha\beta F(\bar{w})\frac{\bar{w}}{1-\beta}.$$

1. Notice:
$\int_{\bar{w}}^{B} w' dF\left(w'\right) = w' F(w')|_{\bar{w}}^{B} - \int_{\bar{w}}^{B} F\left(w'\right) dw' = \int_{\bar{w}}^{B} \left[1 - F\left(w'\right)\right] dw' + \bar{w}[1 - F(\bar{w})]$

...

# First approach to the problem

- The **reservation wage** is:

$$\bar{w} = b + \frac{\alpha\beta}{1-\beta}\int_{\bar{w}}^{B}\left[1 - F\left(w'\right)\right] dw'. \tag{3}$$

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- **Static analysis**:
  1. $b \uparrow \Longrightarrow \bar{w} \uparrow$: ...
  2. $\alpha \uparrow \Longrightarrow \bar{w} \uparrow$: ...
  3. $\beta \uparrow \Longrightarrow \bar{w} \uparrow$: ...

# Second approach to the problem

- Observe:

$$\int_{\hat{w}}^{\infty}[1 - F(w')]dw' = \underbrace{\int_{\bar{w}}^{\infty}(w' - \bar{w})dF\left(w'\right)}_{\text{Expected surplus}}.$$

- Using this result in equation (3) we obtain as **reservation wage**:

$$\bar{w} = b + \frac{\alpha\beta}{1-\beta}\int_{\bar{w}}^{\infty}(w' - \bar{w})dF\left(w'\right). \tag{4}$$

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- Using this notation we can write:

$$\underbrace{\bar{w}-b}_{\textbf{Opportunity cost of searching one more time}} = \underbrace{\frac{\alpha\beta}{1-\beta}\int_{\bar{w}}^{\infty}(w'-\bar{w})dF\left(w'\right)}_{\textbf{Expected benefit of searching one more time}}.$$

# Is there a unique solution to this problem?

- Notice $h(\bar{w}) = \frac{\alpha\beta}{1-\beta}\int_{\bar{w}}^{\infty}(w' - \bar{w})dF(w')$.
    1. We know $h(0) = \frac{\alpha\beta}{1-\beta}\mathbb{E}(w)$ and $h(B) = 0$.
    2. Using Leibniz's rule we have $h'(w) = -\frac{\alpha\beta}{1-\beta}[1 - F(\bar{w})] < 0$.
    3. The second derivative is positive.
    4. Then the function will have intercept $\frac{\alpha\beta}{1-\beta}\mathbb{E}(w)$ and with a diminishing negative slope that tends to zero.

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  3. The second derivative is positive.
  4. Then the function will have intercept $\frac{\alpha\beta}{1-\beta}\mathbb{E}(w)$ and with a diminishing negative slope that tends to zero.
- Now plotting the line $\bar{w} - b$ and $h(w)$ we can obtain a **unique solution in the positive quadrant**.

# Third approach to the problem

- Notice we have:

$$\bar{w} - b = \frac{\alpha\beta}{1-\beta}\int_{\bar{w}}^{\infty}(w' - \bar{w})dF\left(w'\right).$$

- Now add the following to the expression above:

$$\frac{\alpha\beta}{1-\beta}\int_{0}^{\bar{w}}(w' - \bar{w})dF\left(w'\right) - \frac{\alpha\beta}{1-\beta}\int_{0}^{\bar{w}}(w' - \bar{w})dF\left(w'\right).$$

- This results in :

$$\bar{w}(1-\beta) - b(1-\beta) = \alpha\beta\mathbb{E}(w) - \alpha\beta\bar{w} + \alpha\beta\int_{0}^{\bar{w}} F(w')dw'. \tag{5}$$

# Third approach to the problem

- Notice equation (5) can be rewritten in the following way:

$$\bar{w}(1-\beta(1-\alpha)) - b = \beta(\alpha\mathbb{E}(w) - b) + \alpha\beta\int_0^{\bar{w}} F(w')dw'$$

1. Increasing $b$ will shift down both sides of the equation, but both are characterized by positive slopes where the RHS is increasing in $\bar{w}$. Thus the **new equilibrium wage** will be larger (i.e., $\bar{w}(b+\Delta b) > \bar{w}(b)$).

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1. Increasing $b$ will shift down both sides of the equation, but both are characterized by positive slopes where the RHS is increasing in $\bar{w}$. Thus the **new equilibrium wage** will be larger (i.e., $\bar{w}(b+\Delta b) > \bar{w}(b)$).
2. Holding ***b* constant**, **a mean-preserving increase in risk causes $\bar{w}$ to increase** as well. Take $r_2$ that is obtained through a mean-preserving spread over $r_1$:

$$\beta(\alpha\mathbb{E}(w) - b) + \alpha\beta\int_0^{\bar{w}} F(w', r_2)dw' - [\beta(\alpha\mathbb{E}(w) - b) + \alpha\beta\int_0^{\bar{w}} F(w', r_1)dw']$$

$$\implies \alpha\beta\Big[\int_0^{\bar{w}} F(w', r_2)dw' - \int_0^{\bar{w}} F(w', r_1)dw'\Big] \geq 0.$$

# Third approach to the problem

- An **increase in risk leads to a positive shift in the curve**, representing the expected benefit of search, which means an increase in the reservation wage. Observe that wages are bounded below 0, then the increase in risk generally means **more volatility in wage offers in the positive direction**, given the bound. Given this, the worker is more leaned to wait another period to see if an exceptionally high job offer is made. **What if the worker is allowed to quit the job?**...

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Now there will be three scenarios: **1) accept the wage and keep the job forever, 2) accept the wage but quit after $t$ periods, or 3) reject the wage and continue the job search**. The worker would never prefer the second scenario. Thus, the inclusion would add unnecessary complexity.

# Third approach to the problem

1. Given a reservation wage, a worker rejects an offer with probability $\lambda = \int_0^{\bar{w}} dF(w')$ and accepts with probability $(1-\lambda)$. **What is the probability of waiting $N$ periods until acceptance?**

   ...

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$$\lambda^{N-1}(1 - \lambda).$$

2. And what about the **average time until accepting an offer?**

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$$(1-\lambda)^{-1}.$$

# The McCall Model with probability to be fired

# Probability to be fired

- Introduce a **probability to be fired denoted by $\lambda$**, which will be exogenous.
- **Employed**:

$$W(w) = w + \lambda\beta U + (1-\lambda)\beta W(w) \Rightarrow W(w) = \frac{w + \lambda\beta U}{1 - \beta(1-\lambda)}.$$

- **Unemployed**:

$$U = b + (1-\alpha)\beta U + \alpha \int_0^\infty \max\left\{\beta W\left(w'\right), \beta U\right\} dF\left(w'\right).$$

# Probabilitty to be fired

- As before notice the only change right now is the **inclination of the $W(w)$ curve**.
- There will be the same rule such that $W(\bar{w}) = U$.
- This results $W(\bar{w}) = U = \dfrac{\bar{w}}{1-\beta}$.

# Probability to be fired

- Using the previous definitions we can calculate the reservation wage.

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- The **reservation wage** in this case:

$$\bar{w} = b + \frac{\alpha\beta}{1-\beta(1-\lambda)} \int_{\bar{w}}^{\infty} (w' - \bar{w}) dF\left(w'\right). \tag{6}$$

- Static analysis:
  1. $\lambda \uparrow \Longrightarrow \bar{w} \downarrow$: ...
  2. $\lambda \rightarrow 1 \Longrightarrow \bar{w} > b$ : ...

# Unemployment rate in the steady state

- The **number of people leaving unemployment must be equal to the number of people entering the job market**. Let $u$ denote the unemployment rate.
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$$u\alpha[1 - F(\bar{w})] = (1 - u)\lambda$$

.

- $$u = \frac{\lambda}{\lambda + \alpha(1 - F(\bar{w}))}.$$
- **Static analysis**:
  1. $\lambda \uparrow \Longrightarrow u \uparrow$

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  1. $\lambda \uparrow \Longrightarrow u \uparrow || \lambda \uparrow \Longrightarrow \bar{w} \downarrow \Longrightarrow (1 - F(\bar{w})) \uparrow \Longrightarrow u \downarrow$: ...
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  2. $\alpha \uparrow \Longrightarrow u \downarrow || \alpha \uparrow \Longrightarrow \bar{w} \uparrow \Longrightarrow (1 - F(\bar{w})) \downarrow \Longrightarrow u \uparrow$: ...

# Model in continuous time

# Introduction

- Let $\beta = \dfrac{1}{1 + r\Delta}$, where $\Delta$ represents the time interval.
- Notice that in the discrete case we have $\Delta = 1$.
- We are interested in cases where $\Delta \to 0$.

- The **probability of receiving new offers** now will be $\alpha\Delta$, which can be interpreted as the **average of the process to generate new offers**.
- Let $\Delta$ be the time and $w$ be the wage in this time interval.

# The Model in continuous time

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- Therefore the equations in **continuous time** will be:

$$rW(w) = w. \tag{7}$$

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- Notice, before we were measuring the payoff in terms of stock, now we are measuring in terms of **flow payoff**.
- The advantage is that right now we are interested in **events that lead to a change in each instant of time**. Events that do not change the flow are out of the equation.

# Model in continuous time with the possibility to be fired

# Possibility to be fired in continuous time

- Considering what we have just derived and commented about the flow payoff, **what would be the equation that characterize the flow payoff of an employed worker when there is a probability $\lambda$ to be fired?**...

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- 

$$rW(w) = w + \lambda(U - W(w)) \tag{9}$$

$$rU = b + \alpha \int_{\bar{w}}^{\infty} [W(w') - U] dF(w') \tag{10}$$

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- In the end we would have:

$$\bar{w} = b + \frac{\alpha}{r+\lambda}\int_{\bar{w}}^{\infty}(w' - \bar{w})dF(w'). \tag{11}$$

$$\bar{w} = b + \frac{\alpha}{r+\lambda}\int_{\bar{w}}^{\infty}(1 - F(w'))dF(w'). \tag{12}$$

# Model in continuous time with wage variation inside the job

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$$rW(w) = w + \lambda \int_{\bar{w}}^{\infty} (W(w') - W(w))dF(w') + \lambda F(\bar{w})(U - W(w)) \tag{13}$$

$$rU = b + \alpha \int_{\bar{w}}^{\infty} [W(w') - U]dF(w') \tag{14}$$

# Model with wage variation inside the job

- Using the same cutoff rule $rW(\bar{w}) = rU$ we have:
-

$$\begin{aligned}
\bar{w} + \lambda \int_{\bar{w}}^{\infty} (W(w') - U) dF(w') &= b + \alpha \int_{\bar{w}}^{\infty} [W(w') - U] dF(w') \\
\implies \bar{w} &= b + (\alpha - \lambda) \int_{\bar{w}}^{\infty} [W(w') - U] dF(w') \\
\therefore \bar{w} &= b + \frac{\alpha - \lambda}{r + \lambda} \int_{\bar{w}}^{\infty} (w' - \bar{w}) dF(w'.)
\end{aligned}$$

# Model in continuous time with ”job to job” transition

# Model "job to job" transition

- Now let $\lambda$ be the **exogenous probability of losing the job**. Let $\alpha_0$ **be the probability of an unemployed worker receiving an offer and** $\alpha_1$ **the probability of an employed worker receiving an offer**.
- $$rW(w) = w + \alpha_1 \int_w^\infty \left[W\left(w'\right) - W(w)\right] dF\left(w'\right) + \lambda[U - W(w)]$$
$$rU = b + \alpha_0 \int_{\bar{w}}^\infty \left[W\left(w'\right) - U\right] dF\left(w'\right)$$
$$rW(\bar{w}) = rU = \bar{w}$$
- What do you think would happen to the **reservation wage when** $\alpha_1 > \alpha_0$**?**...

# Model in continuous time with endogenous effort

# Model with endogenous effort

- Suppose the **unemployment agent can do an effort $g(\alpha)$** which brings **disutility** but increase the **probability of receiving an offer**.
- Let $g'(\alpha) > 0$ and $g''(\alpha) > 0$.
- In this way $\alpha$ is endogenous in the model.
- Assume we still have a probability $\lambda$ of losing the job (exogenous).

# Model with endogenous effort

- $$rW(w) = w + \lambda(U - W(w)). \tag{15}$$

  $$rU = b + \alpha \int_{\bar{w}}^{\infty} [W(w') - U] dF(w') - g(\alpha). \tag{16}$$

- The cutoff rule is $rW(\bar{w}) = rU = \bar{w}$.

# Model with endogenous effort

- The optimal level will be such that the marginal cost of the effort will be equal to the marginal benefit of increasing $\alpha$ in one unit.
- $\frac{\partial rU}{\partial \alpha} = 0$.
- $g'(\alpha) = \int_{\bar{w}}^{\infty} [W(w') - U] dF(w')$.

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# Model with endogenous effort

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  ...

- In the end we have:
$$\bar{w} = b + \alpha g'(\alpha) - g(\alpha). \tag{17}$$
- How do we guarantee there will be a unique $\bar{w}$ and $\alpha$?...

# Conclusion