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Department of Economics - Sciences Po
Macroeconomics III

Problem Set 6 - Incomplete Markets with Endogenous Labor

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Question 1

This problem set asks to compute an Aiyagari-style incomplete markets model as the one we have seen in class but now we have endogenous labor and income taxes. Here, small letters denote individuals choice and capital letters denote the aggregate ones. The setup of the model is as follows:

- There is a continuum of ex-ante identical agents who have preferences over consumption and leisure given by the following utility function:

$$\mathbb{E}_0 \sum \beta^t \left\{ \frac{\left(c_{i,t}^\eta l_{i,t}^{1-\eta}\right)^{1-\mu}}{1-\mu} \right\}$$

Households supply labor, save in a risk-free bonds subject to a debt limit. They also pay an income tax τ_y and receive transfers T_t. Let w_t and r_t denote the pre-tax wage rate and return on savings. The budget constraint of a typical households will be:

$$c_{i,t} + a_{i,t+1} \leq (1-\tau_y) e_{i,t} w_t (1-l_{i,t}) + T_t + (1 + (1-\tau_y) r_t) a_{i,t}$$

where $a_{i,t+1} \geq -\underline{a}, l_{i,t} \leq 1$ and $c_{i,t} \geq 0$, the labor supply is $N_t = \int e_{i,t}(1-l_{i,t})$.

- The government budget constraint is:

$$G_t + T_t + (1-\tau_y) r_t B_t = B_{t+1} + \tau_y (w_t N_t + r_t A_t)$$

where G_t is government spending and B_t is debt.

- Technology: There is a representative firm that uses capital, K_t, and labor, N_t, to operate a CRS technology that produces output according to:

$$Y_t = F(K_t, N_t) = AK_t^\theta N_t^{1-\theta}$$

The above will pin down the rental rate $r_t = F_K - \delta$ and wage rate $w_t = F_N$.

- Asset markets: Agents can trade claims to one-period risk-free bonds, capital and government bonds.

$$A_t = K_t + B_t$$

For your baseline calculations set $\beta = 0.98$, $\mu = 1.5$, and $\eta = 0.3$, $\tau_y = 0.4$, $\rho = 0.6$, $\sigma = 0.3$, $\delta = 0.075$, $\theta = 0.3$. It would be useful to set A such that the steady state $Y = 1$. You want to set transfers T_t suhc that aggregate transfers to output is roughly 10 % and G_t such that government expenses are 20 % of output. You can discretize the AR(1) process for skills using 5 states. Now answer the following questions:

a) Compute the stationary equilibrium for this economy.

b) Plot the consumption and savings functions as a function of individual assets for low, medium, and high skill agent.

c) Plot the supply and demand curves as a function of the interest rate and explain how to pin down the market clearing prices.

d) Plot the distribution of agents across states. Compute the mean, st, Gini coefficients and the Lorenz curve for wealth. In what respects do you think the model misses the wealth distribution in the data? Why?

e) Now lower the tax rate to 20 %. Make a table that describes how the aggregates and distributional moments change when you do this tax reform.