Macroeconomics III

Complete Markets Economy and Introduction to Incomplete Markets Economy

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The Baseline Model

The neoclassical growth model consists of the following objects:

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$$y_t = F\left(k_t, l_t\right).$$

2. **Consumers:**

$$c_t + i_t = y_t,$$

where investment, i_t, obeys the law of motion for the capital stock

$$k_{t+1} = (1-\delta)k_t + i_t.$$

Arrow-Debreu Competitive Equilibrium

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1. The Representative agent (AD):

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$\text{s.t.} \quad \sum_{t=0}^{\infty} p_t \left[c_t + k_{t+1} - (1-\delta)k_t\right] \leq \sum_{t=0}^{\infty} p_t \left[r_t k_t + w_t l_t\right]$$
$$0 \leq l_t \leq 1, c_t \geq 0, k_{t+1} \geq 0, k_0 \text{ known.}$$

Arrow-Debreu Competitive Equilibrium

2. Firm:

$$\begin{array}{cc} \max \sum_{t=0}^{\infty} p_t \left[y_t - r_t k_t - w_t l_t \right] \\ \text{s.t.} \quad y_t = F\left(k_t, l_t\right) \forall t \end{array}$$

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3. Resource feasibility constraint:

$$y_t = c_t + k_{t+1} - (1-\delta)k_t \forall t$$

Arrow-Debreu Competitive Equilibrium

Definition 1

In this setting, an Arrow-Debreu Competitive Equilibrium is defined as a **household allocation** decision $Z^H = \{(c_t, k_{t+1}, l_t)\}_{t=0}^{\infty}$, a **firm allocation** decision $Z^F = \left\{\left(k_t^f, l_t^f\right)\right\}_{t=0}^{\infty}$, and a **price system** $\{(p_t, r_t, w_t)\}_{t=0}^{\infty}$, such that, given prices,

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1. the representative agent maximizes utility subject to the budget constraint, resource constraints and the transversality condition
2. the firm maximizes profits subject to its resource feasibility constraint, and,
3. the aggregate resource feasiblity constraint is met (i.e. markets clear)

$$\begin{aligned} F(k_t, l_t) = c_t + k_{t+1} - (1-\delta)k_t & \qquad \textbf{(Goods)} \\ k_t^f = k_t & \qquad \textbf{(Capital)} \\ l_t^f = l_t. & \qquad \textbf{(Labor/Leisure)} \end{aligned}$$

Conditions for the Competitive Equilibrium

The representative agent chooses **consumption** and **capital** to maximize utility and the firm chooses **capital** and **employment** to maximize profits. Given a Lagrangean construction:

- $[c_t]:$
- $[k_{t+1}]:$
- $\left[k_t^f\right]:$
- $\left[l_t^f\right]:$

Conditions for the Competitive Equilibrium

Euler Equation:

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta\left[r_{t+1} + 1 - \delta\right] = \beta\left[F_k(k_{t+1}, 1) + 1 - \delta\right]$$

Feasibility Condition:

$$F(k_t, l_t) = c_t + k_{t+1} - (1-\delta)k_t$$

How do the primitives affect behavior?

1. **Smooth consumption:** if the utility function is strictly concave the individual prefers a smooth consumption stream.
   Example:

2. **Impatience:** a low β will be associated with low c_{t+1} and high c_t.

3. **The return on savings:** since k_{t+1} is endogenous we have that $F_k(k_{t+1}, l_t)$ non-trivially depends on it, so the effect cannot be signed directly.

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- From a mechanical standpoint, **all the activity in the previous economy occurred at date 0**: the consumer faces a maximization problem with a single lifetime budget constraint.
- Essentially, households take stock of future states of the world and make decisions about future consumption, labor and investment in the initial time period.
- **This representation does not capture the normal way in which we imagine interaction in an economy.**

Sequential Market Equilibrium

1. The Representative agent (SME):

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$$
$$\text{s.t. } c_t + k_{t+1} - (1-\delta)k_t \leq r_t k_t + w_t l_t \quad \forall t$$
$$0 \leq l_t \leq 1, c_t \geq 0, 0 \leq k_{t+1} \leq \overline{K}, k_0 \text{ known.}$$

Sequential Market Equilibrium

Definition 2

In this setting, a Sequential Markets equilibrium is an **allocation** $\{(c_t, k_{t+1}, l_t)\}_{t=0}^{\infty}$ and a **price system** $\{(r_t, w_t)\}_{t=0}^{\infty}$, such that

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1. the households maximize the utility subject to the budget constraint at each period t and the constraints,
2. the firm maximizes, that is, $F_k(k_t^f, l_t^f) = r_t$ and $F_l(k_t^f, l_t^f) = w_t$,
3. the aggregate resource feasiblity constraint is met (i.e. markets clear)

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  - Let the **economy be deterministic** and have an **infinite horizon with a finite number of types of agents**. Let there be I types and assume that there is an **equal mass of each type**:

$$\sum_i c_t^i = \sum_i e_t^i$$

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  - Thus, the allocation consists in choosing a sequence $\left\{\left\{c_t^i\right\}_t\right\}_i$ that solves

$$\max \sum_{t=0}^{\infty} u_i\left(c_t^i\right)$$
$$\text{s.t.} \quad \sum_t p_t c_t^i \leq \sum_t p_t e_t^i$$

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  We also need to impose a **Non-Ponzi** scheme such that

$$a_{t+1} \geq -\overline{A}, \quad \text{with} \quad \overline{A} \in \mathbb{R}_+$$

Solving the Simplified Example

...

Exercise

An economy consists of two infinitely lived consumers names $i = 1, 2$. There is one non-storable consumption good. Consumer i consumes c_t^i at time t. Consumer i ranks consumption streams by:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right),$$

where $\beta \in (0, 1)$ and $u(c)$ is increasing, strictly concave, and twice continuously differentiable. Consumer 1 is endowed with a stream of the consumption good $e_t^i = 1, 0, 0, 1, 0, 0, 1, \ldots$. Consumer 2 is endowed with a stream of the consumption good $e_t^i = 0, 1, 1, 0, 1, 1, 0, \ldots$ Assume that there are **complete markets with time 0 trading**.

- Define a competitive equilibrium.
- Compute a competitive equilibrium.

Exercise

Consider the following economy

$$u\left(c_0^i, c_1^i, \cdots\right) = \sum_{t=0}^{\infty} \beta^t \log c_t^i,$$

where $\left(e_0^1, e_1^1, e_2^1, e_3^1, \cdots\right) = (6, 4, 6, 4, \cdots)$ and

$$\left(e_0^2, e_1^2, e_2^2, e_3^2, \cdots\right) = (4, 6, 4, 6, \cdots)$$

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$$\left(e_0^2, e_1^2, e_2^2, e_3^2, \cdots\right) = (4, 6, 4, 6, \cdots)$$

- Define a **Sequential Market Equilibrium** and find it.
- Define an **Arrow-Debreu Equilibrium** and find it.

Introduction to Incomplete Markets Economy

1. In a **complete markets setting**, whether in an **Arrow-Debreu** or **Sequential Markets**, **agents are able to trade and hedge against various idiosyncratic shocks**. For example, an **agent can buy contingent claims on consumption next period, which depend upon whether or not a high or low endowment shock occurs.**

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2. To do so the agents must pay some price today (with a premium) to insure herself against the shock.
3. However there are situations in which **borrowing is limited**, which generates **Incomplete Markets**.
4. Around 20% of US households seem to be affected by credit constraints. Around (40%) firms as well (Japelli 1990; Hubbard 1998).

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2. For some of those risks the agents cannot fully insure against it.
3. For instance the unemployment risk study by Carrol (1992).

Heterogeneity matters

- Household heterogeneity in net worth is relevant for determining the behavioral adjustments triggered by a massive aggregate economic downturn.
- Therefore the shape of the wealth distribution is potentially an important determinant of aggregate outcomes.

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Next steps

- We will consider economies in which there exists a **continuum of agents which are ex ante identical**, but due to **stochastic shocks and limitations in the asset/insurance markets, have ex post heterogeneous asset holdings**.
- When borrowing is limited agents must self-insure against the shocks.
- As one risk-averse person may expect, when times are **good (shocks are positive), agents choose to accumulate a larger rainy day fund of assets**, and when times are **bad (shocks are negative), they are forced to consume their stockpile of assets in an attempt to smooth consumption over time.**