Reiter Method

Diego de Sousa Rodrigues

Sciences Po

November 30, 2022

1 Aiyagari model with aggregate productivity shocks

In this section we are going to solve the **Aiyagari model with aggregate productivity shocks**. First of all we tried to keep the model similar to the one we solved using the Krusell and Smith, but there was a problem to incorporate the joint distribution of the aggregate and the idiosyncratic shocks using the Reiter's method. Moreover, by keeping the Idiosyncratic shocks with just two states was concentrating the steady state assets distribution in a single point. Given that we modified the problem as follows:

- The **Aggregate productivity shock** z follows the following process:

$$z_{t+1} = \rho_z z_t + \sigma_z w_{t+1} \quad , w_{t+1} \sim N(0,1)$$

  We defined the variables to be: $\rho_z = 0.9$ and $\sigma_z = 0.0001$.

- The **Idiosyncratic productivity shock** e follows the following process:

$$e_{t+1} = 1 - \rho_e + \rho_e e_t + \nu_t \quad , \nu_t \sim N(0, \sigma_e)$$

  We defined the variables to be: $\rho_e = 0.6$, $\sigma_e = 0.3$.

- In the **Production side**, there is a representative firm which produces output Y_t according to the production side:

$$Y_t = e^{z_t} K_t^{\alpha} H_t^{1-\alpha},$$

where z_t is an aggregate productivity shock, K_t is the aggregate capital, and H_t is the aggregate labor.The firm's optimization problem will give us:

$$w(K_t, H_t, z_t) = (1-\alpha)e^{z_t} K_t^{\alpha} H_t^{-\alpha}$$

$$R(K_t, H_t, z_t) = 1 + \alpha e^{z_t} K_t^{\alpha-1} H_t^{1-\alpha} - \delta$$

,

Observe the wage and return in capital depends on the aggregate state variables, K_t and H_t.

- There is also also a continuum of ex-ante identical **Households** with unit mass. In each period the household faces an idiosyncratic shock e. An employed earns w per unit of labor. The markets will be incomplete and the agents will save through capital accumulation denoted by a and R is going to be the rate of return net of depreciation.

- The agent recursive problem is given by

$$V(a, e, K, H, z) = max_{c,a',n} \frac{\left(c^{\eta}(1-n)^{1-\eta}\right)^{1-\mu}}{1-\mu} + \beta E\left[V(a', e', K', H', z')|e, K, H, z\right]$$

subject to

$$c \leq R(K, H, z)a - a' + w(K, H, z)en$$

$$a' \geq 0$$

$$e_{t+1} = 1 - \rho_e + \rho_e e_t + \nu_t \quad , \nu_t \sim N(0, \sigma_e)$$

The parameters of the model were set such that $\beta = 0.99$, $\delta = 0.0025$, and $\mu = 1$.

- The markets clearing condition is such that:

$$L = \int ned\lambda(a, e)$$

$$K' = \int a' d\lambda(a, e)$$

2 Reiter's algorithm

1. Use a method to approximate the infinite dimensional equilibrium objects, which involves the policy functions of the households and cross-sectional distribution of individual states, by some finite dimensional object. By doing this we will find a finite parametrization of the model.

2. We create this local approximation to the economy in the neighborhood of the stationary equilibrium that has idiosyncratic shocks but no aggregate shocks.

3. Linearize the discrete model equations with respect to the finite parametrization around the steady state and compute the rational expectation solution of the linearized system.

By assuming the economy is in the steady state, the Euler equation is given by:

$$u_{ct} = \beta E[R_{ss} u_{ct+1} | e_t]$$

Let n_a^d be the number of gridpoints for the distribution approximation and n_ϵ the number of idiosyncratic states. The law of motion for the pdf, $n_\varepsilon \times (n_a^d - 1) \times 2$ equations:

$$\lambda_{\varepsilon', t+1}(a_{k+1}) = \sum_{\varepsilon \in E} \pi(\varepsilon, \varepsilon') \frac{a_{k+1} - g_a(a_j; \varepsilon)}{a_{k+1} - a_k} \lambda_{\varepsilon, t}(a_j)$$

If $a_k \leq g_a(a_j;\varepsilon) \leq a_{k+1}$

$$\lambda_{\varepsilon',t+1}(a_k) = \sum_{\varepsilon \in E} \pi(\varepsilon,\varepsilon') \frac{g_a(a_j;\varepsilon) - a_k}{a_{k+1} - a_k} \lambda_{\varepsilon,t}(a_j)$$

If $a_{k-1} \leq g_a(a_j;\varepsilon) \leq a_k$ and 0 otherwise.

The algorithm to find the steaty state is:

1. Guess R and w.

2. Get the aggregate variables implied by wages setting z to the steady state.

3. Solve the agent problem through the endogenous grid as in Krusell and Smith method we solved in the first part of this final problem set.

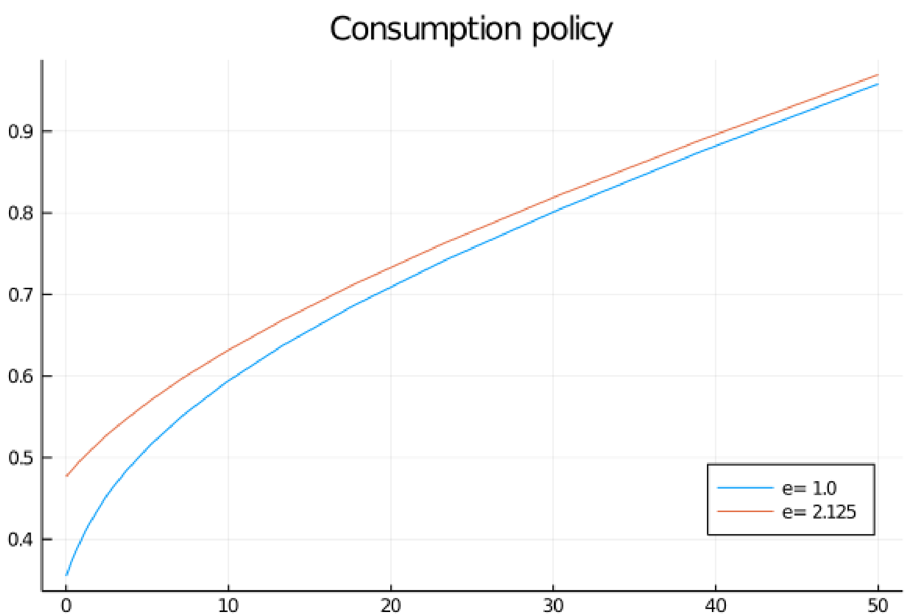
4. Find the invariant assets distribution implied by policy rules and aggregate the individual assets.

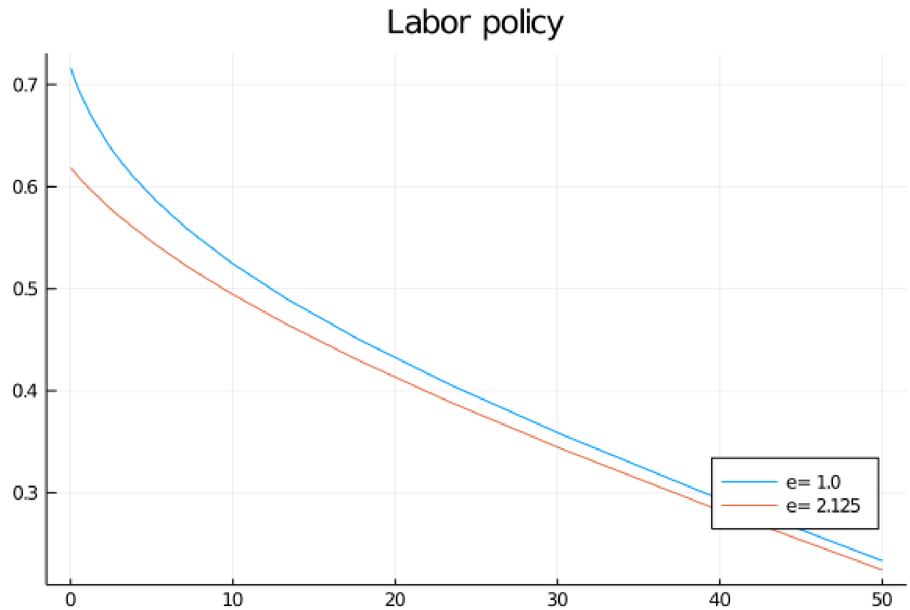
5. Using the formulas above calculate the new aggregate assets and returns (wages and return to capital) and check convergence.

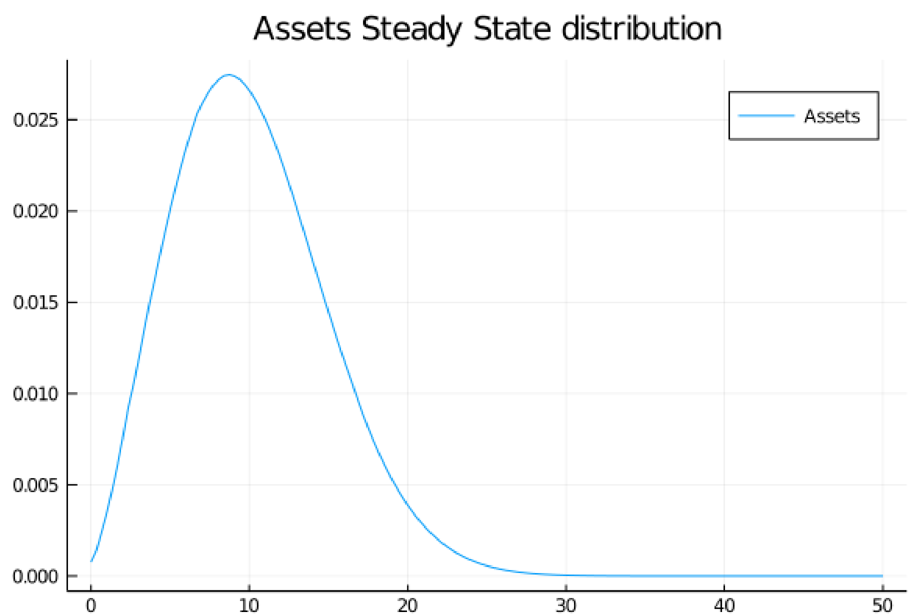
3 Results

Now we show the results of the model, by plotting the respective policy functions of the problem defined above as well as the asset distribution in the steady state.


Assets policy
e= 1.0
e= 2.125
45
0
10
20
30
40
50


Consumption policy
e= 1.0
e= 2.125
0.4
0.5
0.6
0.7
0.8
0.9
0
10
20
30
40
50


Labor policy
0.7
0.6
0.5
0.4
0.3
0
10
20
30
40
50
e= 1.0
e= 2.125


Assets Steady State distribution
0.025
0.020
0.015
0.010
0.005
0.000
0
10
20
30
40
50
Assets