

Macroeconomics III

Incomplete Markets and Heterogeneous Agent Model

Diego Rodrigues

SciencesPo
diego.desousarodrigues@sciencespo.fr

Fall 2023

Main ideas

- A representative agent is **not one agent**.
- **It does not exclude trade** - it just mean it occurs under the hoods.
- Another word for representative agent model - **complete markets economy**.

Lucas (1976) *Econometric policy evaluation - a critique*

Those points are necessary in any model that aims to **evaluate the effect and desirability of economic policy**.

- ① **Rational decisions** made by **consumers** and **firms** with assumptions in **preferences** and **technology**.

Lucas (1976) *Econometric policy evaluation - a critique*

Those points are necessary in any model that aims to **evaluate the effect and desirability of economic policy**.

- ① **Rational decisions** made by **consumers** and **firms** with assumptions in **preferences** and **technology**.
- ② **Savings and investment** decisions are **inter-temporal**.

Lucas (1976) *Econometric policy evaluation - a critique*

Those points are necessary in any model that aims to **evaluate the effect and desirability of economic policy**.

- ① **Rational decisions** made by **consumers** and **firms** with assumptions in **preferences** and **technology**.
- ② **Savings and investment** decisions are **inter-temporal**.
- ③ **Stochastic aggregate shocks** lead to downswings and upswings in the economy.

Lucas (1976) *Econometric policy evaluation - a critique*

Those points are necessary in any model that aims to **evaluate the effect and desirability of economic policy**.

- ① **Rational decisions** made by **consumers** and **firms** with assumptions in **preferences** and **technology**.
- ② **Savings and investment** decisions are **inter-temporal**.
- ③ **Stochastic aggregate shocks** lead to downswings and upswings in the economy.
- ④ **General equilibrium model**, where prices are endogenous.

- ① **Rational decisions** made by **consumers** and **firms** with assumptions in **preferences** and **technology**.
- ② **Savings and investment** decisions are **inter-temporal**.
- ③ **Stochastic aggregate shocks** lead to downswings and upswings in the economy.
- ④ **General equilibrium model**, where prices are endogenous.

- ① **Rational decisions** made by **consumers** and **firms** with assumptions in **preferences** and **technology**.
- ② **Savings and investment** decisions are **inter-temporal**.
- ③ **Stochastic aggregate shocks** lead to downswings and upswings in the economy.
- ④ **General equilibrium model**, where prices are endogenous.
- ⑤ **Heterogeneous population** with different wealth and idiosyncratic income shocks against which they cannot fully insure.

So, sometimes we wish to **depart from the representative agent model**:

- ① Distributions and aggregations matter.
- ② Precautionary savings from incomplete markets also matter.
- ③ This could lead to interesting dynamics.
- ④ More importantly, there is no full insurance in the world.

- Economies with a **continuum of agents** which are **ex ante identical**, but due to **stochastic shocks** and **limitations in the asset/insurance markets**, have **ex post heterogeneous asset holdings**.

- Economies with a **continuum of agents** which are **ex ante identical**, but due to **stochastic shocks** and **limitations in the asset/insurance markets**, have **ex post heterogeneous asset holdings**.
- Agents are hit by only **partially insurable idiosyncratic shocks**.

- One-period obligation contracts is the only source of insurance (**bonds**).

- One-period obligation contracts is the only source of insurance (**bonds**).
- When **borrowing is limited**, as is the case with incomplete markets, agents must **self-insure**: they are left with **stock-piling** quantities of some asset.

In this model, the environment is characterized by an infinitely-lived measure of households with preferences given by:

$$E_0 \sum_{t=0} \beta^t u(c_t)$$

- 1 Agents face an idiosyncratic productivity shock $y_t \in \mathcal{Y}$.

In this model, the environment is characterized by an infinitely-lived measure of households with preferences given by:

$$E_0 \sum_{t=0} \beta^t u(c_t)$$

- ① Agents face an idiosyncratic productivity shock $y_t \in \mathcal{Y}$.
- ② There is a probability to change the productivity shock in each period given by $\pi(y'|y)$.

The Model

In this model, the environment is characterized by an infinitely-lived measure of households with preferences given by:

$$E_0 \sum_{t=0} \beta^t u(c_t)$$

- ① Agents face an idiosyncratic productivity shock $y_t \in \mathcal{Y}$.
- ② There is a probability to change the productivity shock in each period given by $\pi(y'|y)$.
- ③ We will assume **there is no aggregate uncertainty**.

- ① Let $\Pi(y)$ be the unconditional stationary distribution of y :
- $\pi(y)$ provides the unconditional probability of receiving endowment y .
 - The fraction of the households on the unit interval that receive y is $\Pi(y)$.

The Model

- ① Let $\Pi(y)$ be the unconditional stationary distribution of y :
 - $\pi(y)$ provides the unconditional probability of receiving endowment y .
 - The fraction of the households on the unit interval that receive y is $\Pi(y)$.
- ② As mentioned before the aggregate productivity in this economy is constant and it is always given by \bar{y} .

The Model

- ① Let $\Pi(y)$ be the unconditional stationary distribution of y :
 - $\pi(y)$ provides the unconditional probability of receiving endowment y .
 - The fraction of the households on the unit interval that receive y is $\Pi(y)$.
- ② As mentioned before the aggregate productivity in this economy is constant and it is always given by \bar{y} .
- ③ Households are faced with the constraints:

$$\begin{aligned}a' + c &= wy + (1 + r)a, \\ a' &\geq -b\end{aligned}$$

$$\mathcal{L} = u(wy + (1+r)a - a') + \beta \sum_{y'} \pi(y' | y) V(a', y') + \mu(a' + b)$$

The model yields the following first-order conditions:

In order to define the equilibrium we must characterize the joint distribution of assets and idiosyncratic shocks in this economy:

$$\lambda(a_0, y_0) = \text{Prob}\{a \leq a_0, y \leq y_0\}$$

The Model

For each $i = 1, \dots, n$ in this economy:

$$V(a, y_i) = \max_{-b \leq a' \leq \bar{a}} \left\{ u((1+r)a + wy_i - a') + \beta \sum_{j=1}^n P_{ij} V(a', y_j) \right\}.$$

The Model

For each $i = 1, \dots, n$ in this economy:

$$V(a, y_i) = \max_{-b \leq a' \leq \bar{a}} \left\{ u((1+r)a + wy_i - a') + \beta \sum_{j=1}^n P_{ij} V(a', y_j) \right\}.$$

- 1 You can easily show that this is a **Contraction Mapping**.
- 2 Standard Dynamic Programming algorithm gives the optimal policy function $a' = g(a, y)$ and $c = g_c(a, y)$.

The Model

- OK, so suppose that we have found $g(a, y)$ conditional on some r , now what?

The Model

- OK, so suppose that we have found $g(a, y)$ conditional on some r , now what?
- As there are the idiosyncratic shocks, each individual in this economy will be exposed to different shocks at each period.

The Model

- OK, so suppose that we have found $g(a, y)$ conditional on some r , now what?
- As there are the idiosyncratic shocks, each individual in this economy will be exposed to different shocks at each period.
- We can identify an individual through his history of shocks.

- OK, so suppose that we have found $g(a, y)$ conditional on some r , now what?
- As there are the idiosyncratic shocks, each individual in this economy will be exposed to different shocks at each period.
- We can identify an individual through his history of shocks.
- Clearly, as the history of shocks affect the individual's wealth and individual will have experienced different types of histories, there will be a **cross-sectional distribution of wealth holdings**.

The Model

- OK, so suppose that we have found $g(a, y)$ conditional on some r , now what?
- As there are the idiosyncratic shocks, each individual in this economy will be exposed to different shocks at each period.
- We can identify an individual through his history of shocks.
- Clearly, as the history of shocks affect the individual's wealth and individual will have experienced different types of histories, there will be a **cross-sectional distribution of wealth holdings**.
- For simplicity, assume the state space for a is discrete.
- We will denote the distribution of (a, y) in t as $\lambda_t(a, y)$.

Law of motion for the wealth-shock distribution.

- **Unconditional distribution** of (a_t, y_t) is given by $\lambda_t(a_t, y_t)$.

$$\lambda_t(a_t, y_t) = Pr(a_t, y_t)$$

- Example: Suppose the following: $a_t \in [a_1 < a_2]$ and $y_t \in [y_1 < y_2]$
What is the following object: $Pr(a_{t+1} = a_1, y_{t+1} = y_1) =$

Observe in the end we have the following:

$$\begin{aligned} \Pr(a_{t+1} = a_1, y_{t+1} = y_1) = & \\ \Pr(a_{t+1} = a_1/a_t = a_1, y_t = y_1) \Pr(y_{t+1} = y_1/y_t = y_1) \Pr(a_t = a_1, y_t = y_1) + & \\ \Pr(a_{t+1} = a_1/a_t = a_1, y_t = y_2) \Pr(y_{t+1} = y_1/y_t = y_2) \Pr(a_t = a_1, y_t = y_2) + & \\ \Pr(a_{t+1} = a_1/a_t = a_2, y_t = y_1) \Pr(y_{t+1} = y_1/y_t = y_1) \Pr(a_t = a_2, y_t = y_1) + & \\ \Pr(a_{t+1} = a_1/a_t = a_2, y_t = y_2) \Pr(y_{t+1} = y_1/y_t = y_2) \Pr(a_t = a_2, y_t = y_2). & \end{aligned}$$

The Model

Observe in the end we have the following:

$$\begin{aligned} \Pr(a_{t+1} = a_1, y_{t+1} = y_1) = & \\ \Pr(a_{t+1} = a_1/a_t = a_1, y_t = y_1) \Pr(y_{t+1} = y_1/y_t = y_1) \Pr(a_t = a_1, y_t = y_1) + & \\ \Pr(a_{t+1} = a_1/a_t = a_1, y_t = y_2) \Pr(y_{t+1} = y_1/y_t = y_2) \Pr(a_t = a_1, y_t = y_2) + & \\ \Pr(a_{t+1} = a_1/a_t = a_2, y_t = y_1) \Pr(y_{t+1} = y_1/y_t = y_1) \Pr(a_t = a_2, y_t = y_1) + & \\ \Pr(a_{t+1} = a_1/a_t = a_2, y_t = y_2) \Pr(y_{t+1} = y_1/y_t = y_2) \Pr(a_t = a_2, y_t = y_2). & \end{aligned}$$

Therefore, in the end we have the following:

Aggregate Distribution of assets and shocks

$$\Pr(a_{t+1} = a_1, y_{t+1} = y_1) = \sum_i \sum_j \Pr(a_{t+1} = a_1/a_t = a_i, y_t = y_j) \Pr(y_{t+1} = y_1/y_t = y_j) \Pr(a_t = a_i, y_t = y_j)$$

Now observe:

$$\Pr(a_{t+1} = a_1, y_{t+1} = y_1) = \sum_{a_t} \sum_{y_t} \Pr(a_{t+1} = a_1 / a_t = a_i, y_t = y_j) \Pr(y_{t+1} = y_1 / y_t = y_j) \Pr(a_t = a_i, y_t = y_j)$$

The Model

Now observe:

$$\Pr(a_{t+1} = a_1, y_{t+1} = y_1) = \sum_{a_t} \sum_{y_t} \Pr(a_{t+1} = a_1 / a_t = a_i, y_t = y_j) \Pr(y_{t+1} = y_1 / y_t = y_j) \Pr(a_t = a_i, y_t = y_j)$$

Aggregate Distribution of assets and shocks

$$\lambda_{t+1}(a', y') = \sum_a \sum_y \lambda_t(a, y) \Pr(y', y) I(a', a, y)$$

Aggregate Distribution of assets and shocks

$$\lambda_{t+1}(a', y') = \sum_a \sum_y \lambda_t(a, y) \Pr(y', y) I(a', a, y),$$

where $I(a', a, y) = 1$ if $a' = g(a, y)$ and 0 otherwise.

Aggregate Distribution of assets and shocks

$$\lambda_{t+1}(a', y') = \sum_a \sum_y \lambda_t(a, y) \Pr(y', y) I(a', a, y),$$

where $I(a', a, y) = 1$ if $a' = g(a, y)$ and 0 otherwise.

$$\lambda_{t+1}(a', y') = \sum_y \sum_{\{a: a'=g(a,y)\}} \lambda_t(a, y) \Pr(y', y)$$

Aggregate Distribution of assets and shocks

$$\lambda_{t+1}(a', y') = \sum_a \sum_y \lambda_t(a, y) \Pr(y', y) I(a', a, y),$$

where $I(a', a, y) = 1$ if $a' = g(a, y)$ and 0 otherwise.

$$\lambda_{t+1}(a', y') = \sum_y \sum_{\{a: a'=g(a,y)\}} \lambda_t(a, y) \Pr(y', y)$$

A time-invariant distribution is such that $\lambda_{t+1} = \lambda_t = \lambda$.

The Model

- ① One can show that under quite weak assumptions, λ_t (and for any λ_0) converges to a unique stationary distribution λ such that,

$$\lambda(a', y') = \sum_y \sum_{\{a: a' = g(a, y)\}} \lambda(a, y) \Pr(y', y)$$

The Model

- 1 One can show that under quite weak assumptions, λ_t (and for any λ_0) converges to a unique stationary distribution λ such that,

$$\lambda(a', y') = \sum_y \sum_{\{a: a' = g(a, y)\}} \lambda(a, y) \Pr(y', y)$$

- 2 This stationary distribution is very important to us.
- 3 Given a **constant interest-rate**, the **optimal household decision yields a stationary distribution with a constant excess-demand for bonds**.
- 4 Moreover, even if aggregates are constant (aggregate wealth, consumption, endowments etc.) individual specific variables are not: **agents jump frequently around in the distribution, but aggregates never change**.

The problem induces an **Endogenous Markov Chain**:

$$\Pr(a_{t+1} = a', y_{t+1} = y' / a_t = a, y_t = y) = \Pr(a_{t+1} = a' / a_t = a, y_t = y) \Pr(y_{t+1} = y' / y_t = y) = I(a', a, y) P(y', y) = Q$$

The problem induces an **Endogenous Markov Chain**:

$$\Pr(a_{t+1} = a', y_{t+1} = y' / a_t = a, y_t = y) = \Pr(a_{t+1} = a' / a_t = a, y_t = y) \Pr(y_{t+1} = y' / y_t = y) = I(a', a, y) P(y', y) = Q$$

The formula defines an $N \times N$ matrix where N is the number of states for y and N is the number of grid points for a

Definition

A stationary equilibrium is an interest rate r , a policy function, $g(a, y)$, and a stationary distribution $\lambda(a, y)$, such that:

- 1 The policy function $g(a, y)$ solves $V(a, y)$;
- 2 The loan markets clears:

$$\sum_{y,a} \lambda(a, y) g(a, y) = 0, \left(\sum_{y,a} \lambda(a, y) g_c(a, y) = \bar{y} \right)$$

- 3 The stationary distribution $\lambda(a, y)$ is induced by (P, y) and $g(a, y)$:

$$\lambda(B) = \sum_{X=[-b, \bar{a}] \times \mathcal{Y} \in B} Q(X, B)$$

Solution algorithm:

- 1 Guess $r = r_j$.
- 2 Solve household's problem using dynamic programming to find $g_j(a, y)$ and find $\lambda_j(a, y)$.
- 3 Compute:

$$e = \sum_{(y,a)} \lambda_j(a, y) g_j(a, y).$$

- 4 If $e > \varepsilon$, update $r_{j+1} < r_j$ (if $e < \varepsilon$, update $r_{j+1} > r_j$) and go back to step (1). If $|e| < \varepsilon$ stop.

- 1 Sounds easy, right?

- ① Sounds easy, right?
- ② Not necessarily! **Calculating the cross-sectional distribution can be a pain.**
- ③ We will discuss two ways to solve this issue:
 - **Discretization:** approximate the distribution function on a discrete number of grid points over the assets;
 - **Monte-Carlo simulation:** we take a sample of households and we track them over time.

- When solving heterogeneous models, we will encounter stochastic variables with law of motion described by a probability density function:

$$\lambda(\theta_{t+1}, \theta_t)$$

Digression on Computing Distributions II

- What is the density of θ_{t+2} given θ_t ?

$$\lambda(\theta_{t+2}, \theta_t) = \int_{\theta_{t+1}} \Psi(\theta_{t+2}, \theta_{t+1}) \lambda(\theta_{t+1}, \theta_t)$$

- What is the density of θ_{t+3} given θ_t ?

$$\lambda(\theta_{t+3}, \theta_t) = \int_{\theta_{t+2}} \Psi(\theta_{t+3}, \theta_{t+2}) \Psi(\theta_{t+2}, \theta_t)$$

- In general

$$\lambda(\theta_{t+n}, \theta_t) = \int_{\theta_{t+n-1}} \Psi(\theta_{t+n}, \theta_{t+n-1}) \Psi(\theta_{t+n-1}, \theta_t)$$

- Many times we are interested in the unconditional, or long-run density:

$$\lambda(\theta) = \lim_{n \rightarrow \infty} \lambda(\theta_{t+n}, \theta_t)$$

- This density must satisfy the following equation:

$$\lambda(\theta') = \int_{\theta'} \psi(\theta', \theta) \lambda(\theta)$$

- Transition matrix:

$$T := \begin{bmatrix} \psi(\theta_1, \theta_1) & \cdots & \psi(\theta_N, \theta_1) \\ \vdots & \ddots & \vdots \\ \psi(\theta_1, \theta_N) & \cdots & \psi(\theta_N, \theta_N) \end{bmatrix}$$

- **Each row must sum to one!**
- What is the distribution of θ_{t+1} given $\theta_t = \theta_j$?

- Let v_0 be a $1 \times N$ vector, with zeros everywhere apart from the element j , where it is one. Then given $\theta_t = \theta_j$:

$$\psi(\theta_{t+1}) = v_0 T = v_1$$

- Let v_0 be a $1 \times N$ vector, with zeros everywhere apart from the element j , where it is one. Then given $\theta_t = \theta_j$:

$$\psi(\theta_{t+1}) = v_0 T = v_1$$

- Things get simple since right now we have:

$$\psi(\theta_{t+n}) = v_{n-1} T = v_n$$

- Let v_0 be a $1 \times N$ vector, with zeros everywhere apart from the element j , where it is one. Then given $\theta_t = \theta_j$:

$$\psi(\theta_{t+1}) = v_0 T = v_1$$

- Things get simple since right now we have:

$$\psi(\theta_{t+n}) = v_{n-1} T = v_n$$

- And the long-run unconditional distribution must solve:

$$vT = v \Rightarrow (T - I)v = 0$$

Transition matrix: example

- The job finding probability, f , in the United States is around 0.4 per month.
- How do I know that?
- In order to see this we can use the data and see that the unemployment duration is around 2.5 months.
- **Calculation:**

Transition matrix: example

- The separation rate in the United States is 3.4 % (data).
- Therefore, the **transition matrix between employed (1) and unemployed (2)** is given by:

$$T := \begin{bmatrix} 0.966 & 0.034 \\ 0.4 & 0.6 \end{bmatrix}$$

- The **Long-run distribution** is:

$$v = [0.9217, 0.0783]$$

Back to the Model

- Suppose we have 5 states for a and a policy function defined as:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \\ 4 \end{bmatrix}$$

Back to the Model

- Suppose we have 5 states for a and a policy function defined as:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \\ 4 \end{bmatrix}$$

- This can be written as a **transition matrix**:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Back to the Model

- But in the model we normally have one policy function for each states. So suppose we have:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \text{ if good state} \quad \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \\ 4 \end{bmatrix}, \text{ and} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \text{ if bad state} \quad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \\ 4 \end{bmatrix}$$

Back to the Model

- Now imagine we have the following transition matrix between good and bad states:

$$T = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

- With the two transition matrices:

$$M_g = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \text{ and } M_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Back to the Model

- The full transition matrix is given by:

Back to the Model

- The full transition matrix is given by:

$$\begin{bmatrix} T(1,1)M_g & T(1,2)M_g \\ T(2,1)M_b & T(2,2)M_b \end{bmatrix}$$

Back to the Model

- The full transition matrix is given by:

$$\begin{bmatrix} T(1,1)M_g & T(1,2)M_g \\ T(2,1)M_b & T(2,2)M_b \end{bmatrix}$$

- Endogenous transition matrix:

$$M = \begin{bmatrix} 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 \end{bmatrix}$$

Montecarlo Simulation

- Choose a sample size of Q individuals ($Q \approx 1000$).
- Initialize each individual i with an initial asset holding a_i^0 and as productivity shock y_i .
- Compute $a'_i = g(a_i, y_i) \quad \forall i = 1, \dots, Q$.
- Generate the next period productivity shock $y'_i \quad \forall i = 1, \dots, Q$.
- Calculate a set of statistics on the distribution of y and a (average and standard deviation).
- Iterate until convergence on the statistics.

Recall the Solution Algorithm

- 1 Guess $r = r_j$.
- 2 Solve household's problem using dynamic programming to find $g_j(a, y)$ and find $\lambda_j(a, y)$.
- 3 Compute:

$$e = \sum_{(y,a)} \lambda_j(a, y) g_j(a, y).$$

- 4 If $e > \varepsilon$, update $r_{j+1} < r_j$ (if $e < \varepsilon$, update $r_{j+1} > r_j$) and go back to step (1). If $|e| < \varepsilon$ stop.

How to adjust the Interest rate?

- The asset demand is given by:

$$e = \sum_{(y,a)} \lambda_j(a,y) g_j(a,y).$$

How to adjust the Interest rate?

- The asset demand is given by:

$$e = \sum_{(y,a)} \lambda_j(a,y) g_j(a,y).$$

We also know that: $e\left(\frac{1}{\beta} - 1\right) \geq 0$ and hopefully $e(0) < 0$.

How to adjust the Interest rate?

- The asset demand is given by:

$$e = \sum_{(y,a)} \lambda_j(a,y) g_j(a,y).$$

We also know that: $e\left(\frac{1}{\beta} - 1\right) \geq 0$ and hopefully $e(0) < 0$.

- Luckily, $e(r)$ is continuous.

How to adjust the Interest rate?

- The asset demand is given by:

$$e = \sum_{(y,a)} \lambda_j(a,y) g_j(a,y).$$

We also know that: $e\left(\frac{1}{\beta} - 1\right) \geq 0$ and hopefully $e(0) < 0$.

- Luckily, $e(r)$ is continuous.
- This means that in order to find the interest rate we can use **bisection method**.