Macroeconomics III

Incomplete Markets and Heterogeneous Agent Model

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Main ideas

- A representative agent is **not one agent**.
- **It does not exclude trade** - it just mean it occurs under the hoods.
- Another word for representative agent model - **complete markets economy**.

Main ideas

Lucas (1976) *Econometric policy evaluation - a critique*

Those points are necessary in any model that aims to **evaluate the effect and desirability of economic policy**.

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3. **Stochastic aggregate shocks** lead to downswings and upswings in the economy.
4. **General equilibrium model**, where prices are endogenous.
5. **Heterogeneous population** with different wealth and idiosyncratic income shocks against which they cannot fully insure.

Main ideas

So, sometimes we wish to **depart from the representative agent model**:

1. Distributions and aggregations matter.
2. Precautionary savings from incomplete markets also matter.
3. This could lead to interesting dynamics.
4. More importantly, there is no full insurance in the world.

The Model

- Economies with a **continuum of agents** which are **ex ante identical**, but due to **stochastic shocks** and **limitations in the asset/insurance markets**, have **ex post heterogeneous asset holdings**.

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- Agents are hit by only **partially insurable idiosyncratic shocks**.

The Model

- One-period obligation contracts is the only source of insurance (**bonds**).

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- When **borrowing is limited**, as is the case with incomplete markets, agents must **self-insure**: they are left with **stock-piling** quantities of some asset.

The Model

In this model, the environment is characterized by an infinitely-lived measure of households with preferences given by:

$$\mathrm{E}_0 \sum_{t=0} \beta^t u\left(c_t\right)$$

1. Agents face an idiosyncratic productivity shock $y_t \in \mathcal{Y}$.

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1. Agents face an idiosyncratic productivity shock $y_t \in \mathcal{Y}$.
2. There is a probability to change the productivity shock in each period given by $\pi(y'|y)$.
3. We will assume **there is no aggregate uncertainty**.

The Model

1. Let $\Pi(y)$ be the unconditional stationary distribution of y:
    - $\pi(y)$ provides the unconditional probability of receiving endowment y.
    - The fraction of the households on the unit interval that receive y is $\Pi(y)$.

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2. As mentioned before the aggregate productivity in this economy is constant and it is always given by $\overline{y}$.
3. Households are faced with the constraints:

$$a' + c = wy + (1+r)a,$$
$$a' \geq -b$$

The Model

$$\mathcal{L} = u\left(wy + (1+r)a - a'\right) + \beta \sum_{y'} \pi\left(y' \mid y\right) V\left(a', y'\right) + \mu\left(a' + b\right)$$

The model yields the following first-order conditions:

The Model

In order to define the equilibrium we must characterize the joint distribution of assets and idiosyncratic shocks in this economy:

$$\lambda\left(a_0, y_0\right) = \mathrm{Prob}\left\{a \leq a_0, y \leq y_0\right\}$$

The Model

For each $i = 1, ..., n$ in this economy:

$$V\left(a, y_i\right) = \max_{-b \leq a' \leq \bar{a}} \left\{ u\left((1+r)a + wy_i - a'\right) + \beta \sum_{j=1}^{n} P_{ij} V\left(a', y_j\right) \right\}.$$

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1. You can easily show that this is a **Contraction Mapping**.
2. Standard Dynamic Programming algorithm gives the optimal policy function $a' = g(a, y)$ and $c = g_c(a, y)$.

The Model

- OK,so suppose that we have found $g(a, y)$ conditional on some r, now what?

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- Clearly, as the history of shocks affect the individual's wealth and individual will have experienced different types of histories, there will be a **cross-sectional distribution of wealth holdings.**

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- Clearly, as the history of shocks affect the individual's wealth and individual will have experienced different types of histories, there will be a **cross-sectional distribution of wealth holdings.**
- For simplicity, assume the state space for a is discrete.
- We will denote the distribution of (a, y) in t as $\lambda_t(a, y)$.

The Model

Law of motion for the wealth-shock distribution.

- **Unconditional distribution** of (a_t, y_t) is given by $\lambda_t(a_t, y_t)$.

$$\lambda_t(a_t, y_t) = Pr(a_t, y_t)$$

- Example: Suppose the following: $a_t \in [a_1 < a_2]$ and $y_t \in [y_1 < y_2]$
  What is the following object: $\Pr\left(a_{t+1} = a_1, y_{t+1} = y_1\right) =$

The Model

Observe in the end we have the following:

$$
\begin{aligned}
&\Pr\left(a_{t+1}=a_1, y_{t+1}=y_1\right)= \\
&\Pr\left(a_{t+1}=a_1/a_t=a_1, y_t=y_1\right)\Pr\left(y_{t+1}=y_1/y_t=y_1\right)\Pr\left(a_t=a_1, y_t=y_1\right)+ \\
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&\Pr\left(a_{t+1}=a_1/a_t=a_2, y_t=y_1\right)\Pr\left(y_{t+1}=y_1/y_t=y_1\right)\Pr\left(a_t=a_2, y_t=y_1\right)+ \\
&\Pr\left(a_{t+1}=a_1/a_t=a_2, y_t=y_2\right)\Pr\left(y_{t+1}=y_1/y_t=y_2\right)\Pr\left(a_t=a_2, y_t=y_2\right).
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\end{aligned}$$

Therefore, in the end we have the following:

Aggregate Distribution of assets and shocks

$$\Pr\left(a_{t+1}=a_1, y_{t+1}=y_1\right)=\sum_i\sum_j\Pr\left(a_{t+1}=a_1/a_t=a_i, y_t=y_j\right)\Pr\left(y_{t+1}=y_1/y_t=y_j\right)\Pr\left(a_t=a_i, y_t=y_j\right)$$

The Model

Now observe:

$$\Pr\left(a_{t+1}=a_1, y_{t+1}=y_1\right)=\sum_{a_t}\sum_{y_t}\Pr\left(a_{t+1}=a_1/a_t=a_i, y_t=y_j\right)\Pr\left(y_{t+1}=y_1/y_t=y_j\right)\Pr\left(a_t=a_i, y_t=y_j\right)$$

Now observe:

$$\Pr(a_{t+1} = a_1, y_{t+1} = y_1) = \sum_{a_t} \sum_{y_t} \Pr(a_{t+1} = a_1 / a_t = a_i, y_t = y_j) \Pr(y_{t+1} = y_1 / y_t = y_j) \Pr(a_t = a_i, y_t = y_j)$$

Aggregate Distribution of assets and shocks

$$\lambda_{t+1}(a', y') = \sum_a \sum_y \lambda_t(a, y) \Pr(y', y)\, I(a', a, y)$$

The Model

Aggregate Distribution of assets and shocks

$$\lambda_{t+1}\left(a',y'\right)=\sum_{a}\sum_{y}\lambda_{t}\left(a,y\right)\Pr\left(y',y\right)I\left(a',a,y\right),$$

where $I(a',a,y)=1$ if $a'=g(a,y)$ and 0 otherwise.

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$$\lambda_{t+1}\left(a',y'\right)=\sum_{y}\sum_{\{a:a'=g(a,y)\}}\lambda_{t}\left(a,y\right)\Pr\left(y',y\right)$$

A time-invariant distribution is such that $\lambda_{t+1}=\lambda_{t}=\lambda$.

1. One can show that under quite weak assumptions, λ_t (and for any λ_0) converges to a unique stationary distribution λ such that,

$$\lambda\left(a', y'\right) = \sum_{y} \sum_{\{a: a' = g(a,y)\}} \lambda\left(a, y\right) \Pr\left(y', y\right)$$

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2. This stationary distribution is very important to us.
3. Given a **constant interest-rate**, the **optimal household decision yields a stationary distribution with a constant excess-demand for bonds.**
4. Moreover, even if aggregates are constant (aggregate wealth, consumption, endowments etc.) individual specific variables are not: **agents jump frequently around in the distribution, but aggregates never change.**

The Model

The problem induces an **Endogenous Markov Chain**:

$$\Pr\left(a_{t+1} = a', y_{t+1} = y' / a_t = a, y_t = y\right) = \Pr\left(a_{t+1} = a' / a_t = a, y_t = y\right)\Pr\left(y_{t+1} = y' / y_t = y\right) = I\left(a', a, y\right) P\left(y', y\right) = Q$$

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The formula defines an NXN matrix where N is the number of states for y and N is the number of grid points for a

The Model

Definition

A stationary equilibrium is an interest rate r, a policy function, $g(a, y)$, and a stationary distribution $\lambda(a, y)$, such that:

1. The policy function $g(a, y)$ solves $V(a, y)$;
2. The loan markets clears:

$$\sum_{y,a} \lambda(a, y) g(a, y) = 0, \left(\sum_{y,a} \lambda(a, y) g_c(a, y) = \overline{y} \right)$$

3. The stationary distribution $\lambda(a, y)$ is induced by (P, y) and $g(a, y)$:

$$\lambda(B) = \sum_{X=[-b,\bar{a}]\times\mathcal{Y}\in B} Q(X, B)$$

Solution algorithm:

1. Guess $r = r_j$.
2. Solve household's problem using dynamic programming to find $g_j(a, y)$ and find $\lambda_j(a, y)$.
3. Compute:

$$e = \sum_{(y,a)} \lambda_j(a, y) g_j(a, y).$$

4. If $e > \varepsilon$, update $r_{j+1} < r_j$ (if $e < \varepsilon$, update $r_{j+1} > r_j$) and go back to step (1). If $|e| < \varepsilon$ stop.

Computing issues

1. Sounds easy, right?

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2. Not necessarily! **Calculating the cross -sectional distribution can be a pain.**
3. We will discuss two ways to solve this issue:
   - **Discretization:** approximate the distribution function on a discrete number of grid points over the assets;
   - **Monte-Carlo simulation**: we take a sample of households and we track them over time.

The Model

- When solving heterogeneous models, we will encounter stochastic variables with law of motion described by a probability density function:

$$\lambda(\theta_{t+1}, \theta_t)$$

Digression on Computing Distributions II

- What is the density of θ_{t+2} given θ_t?

$$\lambda(\theta_{t+2}, \theta_t) = \int_{\theta_{t+1}} \Psi(\theta_{t+2}, \theta_{t+1}) \lambda(\theta_{t+1}, \theta_t)$$

- What is the density of θ_{t+3} given θ_t?

$$\lambda(\theta_{t+3}, \theta_t) = \int_{\theta_{t+2}} \Psi(\theta_{t+3}, \theta_{t+2}) \Psi(\theta_{t+2}, \theta_t)$$

- In general

$$\lambda(\theta_{t+n}, \theta_t) = \int_{\theta_{t+n-1}} \Psi(\theta_{t+n}, \theta_{t+n-1}) \Psi(\theta_{t+n-1}, \theta_t)$$

Digressing on Computing Distributions

- Many times we are interested in the unconditional, or long-run density:

$$\lambda(\theta) = \lim_{n\to\infty} \lambda\left(\theta_{t+n}, \theta_t\right)$$

- This density must satisfy the following equation:

$$\lambda\left(\theta'\right) = \int_{\theta'} \psi\left(\theta', \theta\right) \lambda(\theta)$$

Transition Matrices

- Transition matrix:

$$T := \begin{bmatrix} \psi\left(\theta_1, \theta_1\right) & \cdots & \psi\left(\theta_N, \theta_1\right) \\ \vdots & \ddots & \vdots \\ \psi\left(\theta_1, \theta_N\right) & \cdots & \psi\left(\theta_N, \theta_N\right) \end{bmatrix}$$

- **Each row must sum to one!**
- What is the distribution of θ_{t+1} given $\theta_t = \theta_j$?

Distributions

- Let v_0 be a $1XN$ vector, with zeros everywhere apart from the element j, where it is one. Then given $\theta_t = \theta_j$:

$$\psi\left(\theta_{t+1}\right) = v_0 T = v_1$$

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- And the long-run unconditional distribution must solve:

$$vT = v \Rightarrow (T - I)v = 0$$

Transition matrix: example

- The job finding probability, f, in the United States is around 0.4 per month.
- How do I know that?
- In order to see this we can use the data and see that the unemployment duration is around 2.5 months.
- **Calculation:**

Transition matrix: example

- The separation rate in the United States is 3.4 % (data).
- Therefore, the **transition matrix between employed (1) and unemployed (2)** is given by:

$$T := \begin{bmatrix} 0.966 & 0.034 \\ 0.4 & 0.6 \end{bmatrix}$$

- The **Long-run distribution** is:

$$v = [0.9217, 0.0783]$$

Back to the Model

- Suppose we have 5 states for a and a policy function defined as:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \\ 4 \end{bmatrix}$$

Back to the Model

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$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \\ 4 \end{bmatrix}$$

- This can be written as a **transition matrix**:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Back to the Model

- But in the model we normally have one policy function for each states. So suppose we have:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \text{ if good state } \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \\ 4 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \text{ if bad state } \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \\ 4 \end{bmatrix}$$

Back to the Model

- Now imagine we have the following transition matrix between good and bad states:

$$T = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

- With the two transition matrices:

$$M_g = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \text{ and } M_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

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- Endogenous transition matrix:

$$M = \begin{bmatrix} 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0.7 & 0 \end{bmatrix}$$

Montecarlo Simulation

- Choose a sample size of Q individuals ($Q \approx 1000$).
- Initialize each individual i with an initial asset holding a_i^0 and as productivity shock y_i.
- Compute $a_i' = g(a_i, y_i) \quad \forall i = 1, ..., Q$.
- Generate the next period productivity shock $y_i' \quad \forall i = 1, ..., Q$.
- Calculate a set of statistics on the distribution of y and a (average and standard deviation).
- Iterate until convergence on the statistics.

Recall the Solution Algorithm

1. Guess $r = r_j$.
2. Solve household's problem using dynamic programming to find $g_j(a, y)$ and find $\lambda_j(a, y)$.
3. Compute:

$$e = \sum_{(y,a)} \lambda_j(a, y) g_j(a, y).$$

4. If $e > \varepsilon$, update $r_{j+1} < r_j$ (if $e < \varepsilon$, update $r_{j+1} > r_j$) and go back to step (1). If $|e| < \varepsilon$ stop.

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- The asset demand is given by:

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- Luckily, $e(r)$ is continuous.
- This means that in order to find the interest rate we can use **bisection method.**