Food crisis and debt distress

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PRELIMINARY VERSION, August 2023

Abstract

This paper studies how food crises and sovereign debt defaults are connected in developing economies reliant on food imports. Fluctuations in food prices can worsen these economies' debt situations, leading to defaults. To examine this, we use unexpected harvest shocks as an instrument to isolate the impact of food price changes. Focusing on Ghana, which defaulted in 2022, we find that positive price shocks raise import prices, inflation, trade imbalances, currency depreciation, and external debt. We use a small open economy model to clarify this interplay. The results indicate that even countries reliant on food imports with low debt can default due to high food import prices. On the other hand, highly indebted nations can default even with minor price increases. These findings emphasize the need for urgent international aid to mitigate the combined effects of food and debt crises, especially for countries heavily reliant on commodity imports.

Keywords: Food crisis, Sovereign debt defaults, Commodity Price Shocks, Debt crisis

JEL Classification: C32, E21, E23, E31, E32, E43, F34, O11, O19, O55, Q18

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1 Introduction

The COVID-19 crisis and the war in Ukraine, coupled with subsequent increases in global commodity food prices, have delivered a tragic blow to many of the world's poorest countries. Currently, a considerable number of nations facing a serious threat of a food crisis are also contending with debt distress. Following the events of 2019, Zambia became the first African country to default on its sovereign debt. However, the count of countries seeking aid to restructure debt is steadily rising, with the majority of low-income countries on the verge of a debt crisis. For instance, in 2022, both Ghana and Sri Lanka defaulted on their external debt, while Pakistan and Egypt are teetering on the brink of default.[1] These debt crises were followed by an unprecedented surge in food-import bills, particularly affecting impoverished countries for which the import of wheat, rice, and maize plays a crucial role in their trade balance. In contrast to previous debt crises, particularly those in the 1980s that arose in the aftermath of declining commodity prices, the ongoing crisis impacting these economies is characterized by an escalation in commodity prices.

This disparity arises primarily due to certain countries on the brink of debt distress being net importers of commodities. Some of these nations, particularly in Africa, rely on food imports from Russia and Ukraine.[2] This essentially signifies that a notable increase in commodity prices, particularly in sectors featuring goods with low elasticity of substitution, such as food, can impact economic activity. This, in turn, affects public debt and can trigger a subsequent debt crisis. A food crisis alone is already highly detrimental, but when it is compounded by debt distress, the consequences can be devastating, particularly for impoverished nations.

The goal of this paper is, therefore, to explore the connection between food crises and a country's ability to repay its debt, which might lead to a sovereign default. Our claim is that a supply shock, which means a sudden increase in food prices, could have negative effects on a country's debt repayment ability. To understand the impact of this supply shock as an independent factor affecting economic indicators linked to a higher chance of default, it's important to examine shifts in food prices as a distinct driver. This step is crucial because prices are also influenced by global business cycles, and if we do not distinguish those shocks from macroeconomic activities, it's hard to clearly show a direct link between food crises and debt problems.

Food prices can be affected by business cycles and disruptions in supply. That's why it's really important to create a tool (instrumental variable) if we want to understand the impact of food crises on the economy. In our paper, we use and expand on the methods from De Winne & Peersman (2016) and Peersman (2022) to create an instrumental variable that helps us analyze the shocks in food prices. To do this, we first make a quarterly list of unexpected harvest shocks for the four most important food commodities in the world: corn, wheat, maize, and soybeans. Then we use these harvest shocks as tools to identify external shocks in food prices using a VAR model. We extend and update this methodology to the most extensive data set available for all relevant series, besides building a shock that takes into account events in whole world.

Given these price shocks, we offer real-world evidence of how food crises worsen Ghana's economic conditions. This evidence is based on estimating the impulse response function (IRFs)

[1] So far, Pakistan has successfully secured a preliminary \$3 billion funding deal with the International Monetary Fund, aimed at alleviating short-term pressures on its debt situation.

[2] According to the World Bank as many as 25 African economies import at least one third of their wheat from those countries and for 15 of those the proportion is greater than 50 percent.

of selected variables to price shocks. The IRFs are estimated using Jordà (2005)'s local projection approach. Those estimates provides three main findings. First, when commodity prices suddenly rise, we observe an increase in the cost of imported goods and a rise in overall inflation. This highlights the importance of those shocks as an independent driver of inflation. Second, these price shocks push up the cost of imports, but they do not affect how much Ghana earns from exports. This leads to a drop in the country's terms of trade and causes the local currency to lose value. Lastly, our investigation into how this affects government debt and the difference in interest rates (spreads) demonstrates that the price shock does not impact domestic debt , but it does lead to higher external debt. It also causes interest rate spreads to widen. This last piece of evidence shows that Ghana heavily relies on international markets, and the higher interest rates suggest that the likelihood of the country having trouble repaying its debts is increasing.

While we contend that we use an external instrument to identify the price shock, the discussed transmission mechanism lacks a neat identification. To tackle this concern, we build upon the foundational work of Arellano (2008) and incorporate the elements explored in the empirical section. Our model centers around a small open economy where the government chooses between consuming two types of goods—one produced locally and the other imported (e.g., food) — while also deciding the extent of debt issuance. In our model, the government is a benevolent one, and, consistent with the observations in the empirical evidence, we assume that the imported goods, specifically food items, possess a low elasticity of substitution. This method of identification is crucial to emphasize the connection between food crises and debt troubles. For goods that the government cannot readily replace, such as these, external sources become necessary to navigate challenging economic circumstances, as a sudden increase in food prices. Our findings reveal that when import prices surge to a certain level, even countries with modest debt levels could default.

However, it is worth noting that the issue of debt has been on the rise for low-income countries long before the COVID-19 crisis struck.[3] In our model, we demonstrate that even minor increases in import prices can push heavily indebted nations into a crisis that leads to default. If this scenario holds true, these countries are left with minimal capacity to address this challenge. We perceive these findings as a strength of our modeling approach, as they allow us to examine the cause-and-effect relationship between food crises and debt distress and is able to address also this scenario where the debt is already high enough before the sudden shock.

Lastly, we assess the importance of a country's degree of openness in relation to the likelihood of default. Our analysis demonstrates that as the degree of openness rises, a nation enhances its capacity to engage in financial markets and secure international aid. Consequently, a higher degree of openness translates to a lower susceptibility to default for the country.

These facts highlight that the primary challenge revolves around small, heavily indebted economies with limited openness—precisely the focus of our analysis. The outcomes presented in this paper do not imply that the ongoing debt crisis is a universal occurrence. Rather, as we contend, it might be predominantly restricted to developing economies, particularly those dependent on food imports. As demonstrated by our quantitative findings, in a scenario where countries are already burdened by substantial debt, a food crisis leaves them with minimal options for effective action.

[3]Based on estimates from the World Bank, by the end of 2020, the total public and publicly guaranteed debt of low-income countries had reached approximately $124 billion, marking a substantial increase of about 75 percent compared to the levels seen in 2010.

In terms of policy making, our paper offers a straightforward framework that underscores the connections between these two issues. As per our results, for countries already entrenched in debt, the most viable path to address debt distress in the face of a food crisis lies in seeking international assistance. The specifics of crafting such assistance extend beyond the scope of this paper and remain a subject for future research.

Literature review This paper primarily relates to the literature concerning quantitative models of sovereign default, tracing back to the seminal framework developed by Eaton & Gersovitz (1981), which was subsequently expanded upon by Aguiar & Gopinath (2006) and Arellano (2008). A body of the literature has focused on the link between terms of trade and sovereign default (for example, Min (1998), Cuadra & Sapriza (2006)). However, the literature has to some extent concentrated on the significant influence of fluctuations in commodity prices on a country's likelihood of default, it has predominantly examined this phenomenon from an export-oriented standpoint (Hilscher & Nosbusch (2010) and Roch (2019)). In contrast, our paper shifts its focus towards the impact of price fluctuations from an import-oriented perspective, specifically centering on food commodities.

This paper also relates to the literature on the impact of world price shocks. Kose (2002) shows that world prices shocks are key drivers of business cycles in small, open, emerging economies, accounting for approximately 88 percent of the variations in aggregate output, underscoring their significant explanatory power. In addition, Caballero & Panageas (2008) along with Calvo et al. (2008), have observed that a decline in commodity prices raises the probability of a sudden stop in capital flows, which increases the probability of sovereign default (Reinhart et al. (2016)). In the case of Ecuador, Hatchondo et al. (2007) underscore the importance of commodity prices within the realm of sovereign default. They elucidate that the decline in commodity prices played a pivotal role in exacerbating the country's macroeconomic conditions, ultimately culminating in a sovereign default in 1999. Finally, Farah-Yacoub et al. (2022) estimates that after a sovereign default, the country faces a consistent shortfall in calorie availability. This gap, when compared to their synthetic control groups, increases to 4 percentage points a decade after the default occurs, and highlights how sovereign debt distress can lead to food crisis.

To conclude, this paper also relates to the literature on commodity prices shocks identification, with the papers of De Winne & Peersman (2016) and Peersman (2022).

The remainder of the paper is organized as follows. Section 2 presents the motivating facts for choosing Ghana as a laboratory exercise. Section 3 describes the construction of the harvest shocks, the estimation methodology, and presents the arguments for using these shocks as an external instrument for price shocks. Section 4 provides empirical evidence of the mechanisms at hand, using the shocks built before to show that indeed those shocks affect the variables of interest. Section 5 builds a toy model where we make certain assumptions to better illustrate the intuitions; more precisely, the analysis in this section depends on import prices. Section 6 presents a more complex model where we allow the analysis to depend on the terms of trade. Finally, Section 7 concludes the paper.

2 Motivation facts

As said previously, Ghana defaulted on its external debt in December 2022, following a series of events, as depreciation of its currency, which lost more than 50 % of its value from January to October 2022. This subsequent event led to an increase in the debt burden of around \$6bn.[4] Some general observations we can do about Ghana's economy that make it a good laboratory for the present paper are as follows:

Observation 1: Increase in the commodity prices. Amidst the economic vulnerabilities inflicted by the COVID-19 crisis, particularly impacting low-income and emerging nations, the beginning of the Ukraine-Russia war in February 2022 introduced new challenges for these countries. The war was associated by an overall and unprecedented rise of commodity prices, as shown in Figure 1. When we narrow our focus to three specific commodities - rice, maize, and wheat - we can see that the prices reached in 2022 are among the highest observed since 1960 - Figure 2.

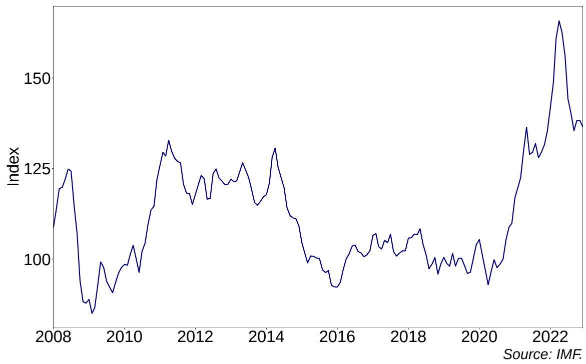
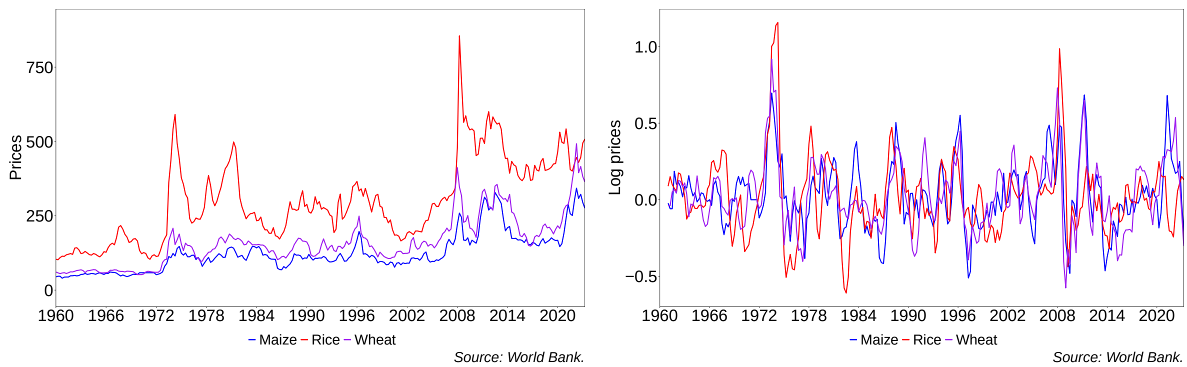


Figure 1: Global price of Food Index.



(a) Prices.

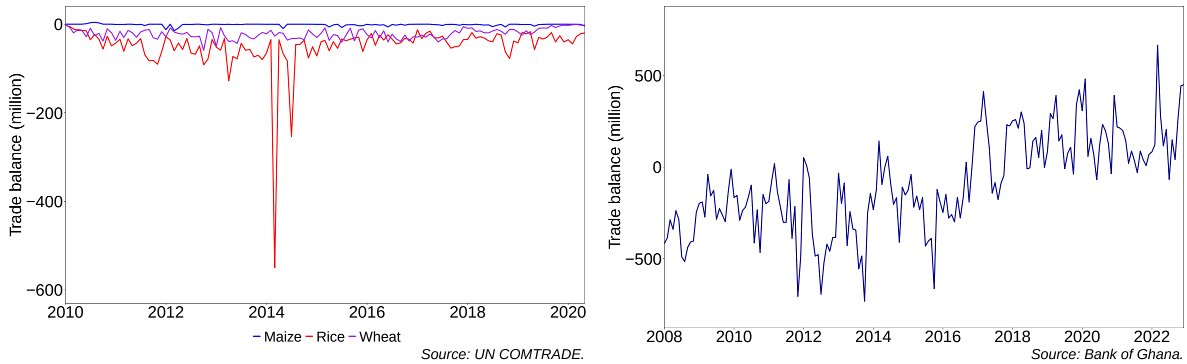
(b) Year-on-year log of prices.

Figure 2: Comparison of prices and log prices for selected commodities (USD).

Observation 2: Ghana is a net-importer of commodities. For countries that are net-importers of those commodities, and given the low elasticity of substitution regarding those

[4] After the default the Ghana goal is to reduce its external debt repayment of \$20bn by half to secure a loan deal from the IMF to be able to restructure its debt.

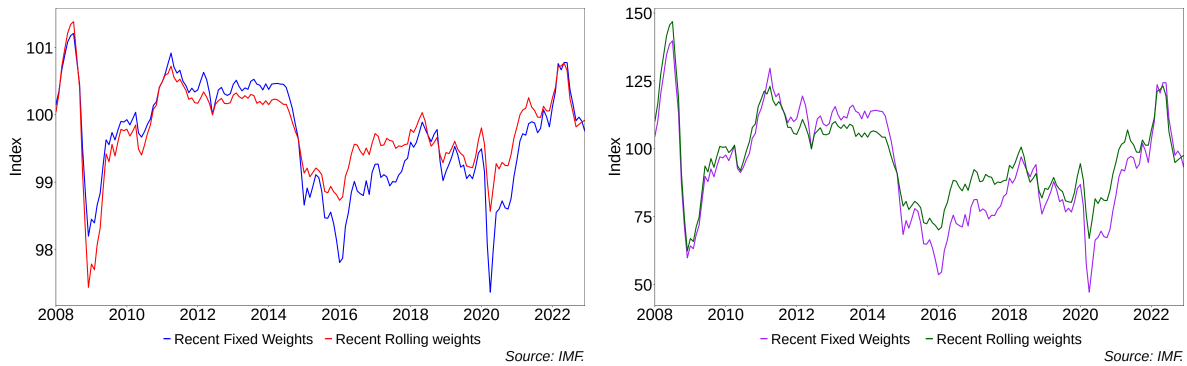
goods, such an increase can have massive economic consequences. In 2022, Ghana found itself in this situation. Over the past few decades, Ghana has consistently been a net importer of the three specified commodities. This is the case even though there has been a positive change in its trade balance in recent years, as illustrated in the accompanying Figure 3. Given its trade



(a) For selected commodities. (b) Overall.

Figure 3: Ghana's trade balance (USD).

situation and the increase in commodity prices, Ghana's import terms of trade increased in 2022, as captured by the commodity import price index of individual commodities shown in Figure 4.



(a) Individual commodities weighted by ratio of imports to GDP. (b) Individual commodities weighted by ratio of imports to total commodity.

Figure 4: Comparison of commodity import price index.

Observation 3: Surge in inflation. In a historic turn of events, Ghana experienced its first instance of debt default in December 2022. Our analysis suggests that the substantial surge in commodity prices, along with its repercussions on Ghana's economy, could provide partial insight into the factors that led the country to a state of sovereign default by year's end. Indeed, an increase in import prices for a net-importer first translates into an increase of inflation, especially to food items when it comes to commodity prices. Certainly, in 2022, Ghana experienced a remarkable and unprecedented rise in its overall inflation rate, as shown in Figure 5. Notably, this trend was most pronounced in the food sector, where prices surged by over 60%. As import price increases, the country theoretically try to decrease its import. However, when it comes to food and commodities, the elasticity of substitution is usually low, as the market is concentrated among a small number of exporters and prices are determined on international markets.

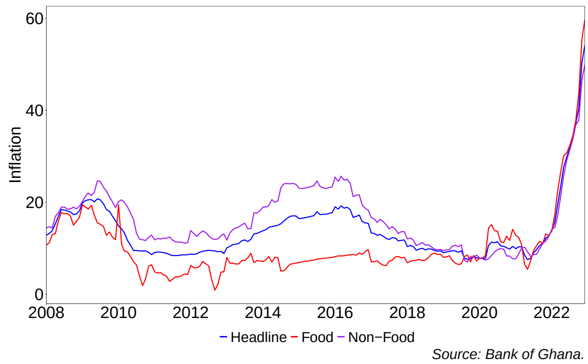


Figure 5: Ghana's year-on-year inflation.

Observation 4 : Depreciation of the currency. As import price increases, the trade balance in value of the country deteriorates. It also implies that the country's currency depreciates. This effect appears to be even stronger in a context of gradual increase in Fed's interest rate since February 2022 due the worldwide increase in inflation. This makes the dollar more attractive, especially in periods of high uncertainty, which accentuates the safe-haven property of the dollar. Consequently, Ghana's currency, the Ghanaian cedi, has experienced depreciation against the dollar throughout 2022, as depicted in Figure 6.

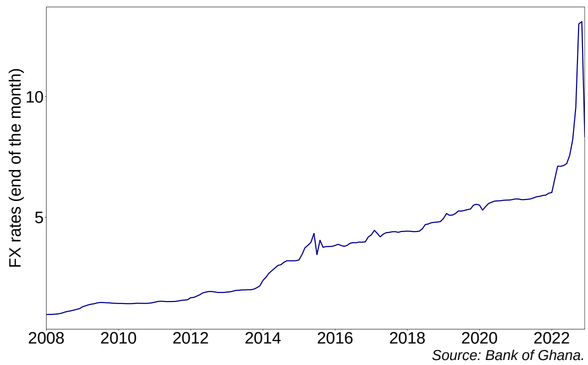


Figure 6: Historical interbank FX rates in Ghana (end of the month).

Observation 5 : Increase in public debt. For Ghana, a depreciation of its currency also implies an increase of the service of its debt, as almost half of the debt is denominated in dollar, as shown in Figure 7.

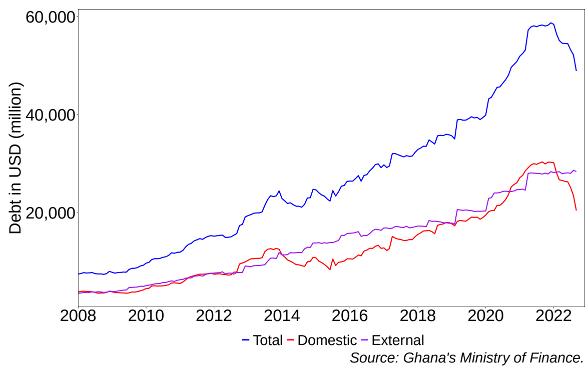


Figure 7: Ghana's public debt (USD millions).

During the year 2022, a self-fulfilling mechanism was feared by international markets regarding Ghana. First, it is due to the fact that a significant proportion of the country's debt is denominated in dollars. In the case of Ghana, this represents more than half of its debt in the early 2022, as shown in Figure 7 (ending in September 2022). Its debt has massively increased in the recent years, going from almost 20% of GDP in 2008 to above 80% of GDP in 2022, as shown in Figure 8. Therefore, a depreciation raises the burden of the government's debt that is held in foreign currencies. As the cedi's value diminishes, the equivalent amount in local currency required to service the debt increases, intensifying the debt burden. However, while the debt burden continues to grow, market starts to loss confidence in the ability of the government to reimburse its debt, which increases the interest rates on government debt's. This increase the external debt, leading to another depreciation of the currency. Overall, these two elements reinforce each other and can lead to a sovereign default, precisely what Ghana faced by the end of 2022.

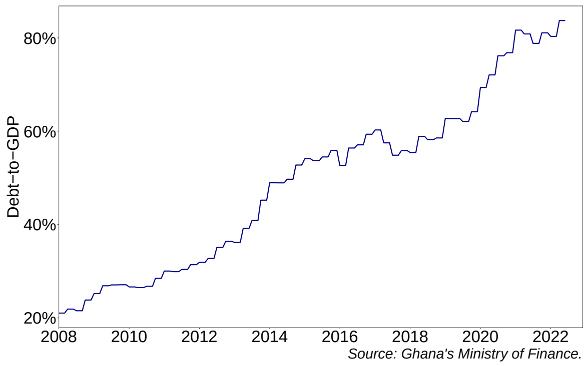


Figure 8: Ghana debt (in percent of GDP).

3 Supply shocks

We propose that the sequence of events leading Ghana to default in 2022 was further exacerbated by a food crisis. Here, by *food crisis*, we are referring to an unprecedented surge in food commodity prices. This escalation, in turn, has the potential to trigger adverse reactions within the economic environment. To truly understand how much this problem with food prices affected the other issues we discussed, especially regarding a surge in external debt and its spread, we need to isolate changes in food prices. This is important because food prices are influenced not only by external factors, such as insufficient food availability, but also by business cycle properties, which could, in the end, also affect debt distress. Therefore, we need to find a way to measure the impact of the food crisis separately from business cycles if we wish to study this connection.

Our plan is to employ a specialized method to measure this impact in a way that provides us with clearer answers. This method involves examining specific factors that affect food prices uniquely. These factors help us understand the relationship between the food crisis and debt distress. In the following sections, we will first explain how we construct a measure for exogenous variations in international food commodity prices, and then we will explain the methodology used to obtain these estimates.

3.1 Construction

To identify exogenous variations in international food commodity prices, we elaborate on De Winne & Peersman (2016) and Peersman (2022) to construct an instrumental variable. Since the instrument's validity relies on satisfying the exogeneity condition, we adopt the approach of De Winne & Peersman (2016).

This involves first constructing a quarterly global food production index, aligning with the annual production cycle and planting and harvesting schedules for specific crop-country pairs. Despite the possibility of commodity prices swiftly reacting to macroeconomic shocks, there remains a delay of at least one quarter between the decision to produce (through planting) and the actual production following the harvest. Consequently, macroeconomic shocks might not immediately impact harvest volumes; instead, such volumes are influenced only by exogenous shocks to the economy, like weather conditions or crop diseases.

More precisely, the procedure starts by using the annual production data for each of four commodities - corn, maize, wheat and soybeans - for 192 countries over the period 1960-2021 that are published by the FAO. For each commodity, the production is converted into edible calories according to the conversion parameters estimated by Roberts & Schlenker (2013). They estimate for each commodity the conversion parameters to go from bushel to pound and from pound to calorie, then rescale the caloric conversion ratios so the average price in 1961-2010 of all four crops equals that of maize. In a later stage, we plan to estimate on more recent data this rescaling parameters. By multiplying the two conversion coefficients and dividing by the price rescaler, we derive a conversion factor for each individual commodity, facilitating the transition from bushels to calories.

Given that our data is denominated in tons, an additional converter is employed to facilitate the shift from tons to calories. We therefore use the crop calendar for each individual country developed by De Winne & Peersman (2016) where they are able to determine a specific quarter to which allocate two-thirds of the country's annual production for this specific crop. More details on the determination of the quarter for each pair of crop and country can be found in their paper. We therefore assign one-ninth of the annual production to the remaining quarters. This enables to have for each pair of crop and country, a quarterly production quantified in edible calories. The quarterly production data are then aggregated across crops and countries to give a global quarterly production series. Our motivation is rooted in the events of 2022, yet the currently available FAO data only extend up to 2021. In order to still be able to compute the quarterly production for 2022, we temporarily rely on estimates provided by the United States Department of Agriculture regarding global production for each crop. We follow the previously described conversion method to transition from ton to edible calories. Given that we possess data solely for global production and lack country-specific values, we are unable to employ the assignment approach by the crop calendar.

Consequently, we distribute for each crop the production across quarters based on the average quarterly share for each year within our dataset. We then convert our quarterly production into an index which takes as reference 2010. Following De Winne & Peersman (2016), we then seasonally adjust it using the U.S. Census Bureau's X-13ARIMA-SEATS seasonal adjustment program. In Peersman (2022), the author focused on the effect of fluctuations in international food prices on euro-area inflation dynamics and therefore, do not include the harvests of European countries in the index. In our case, as Ghana can be considered as a small open economy among the 192 countries, we do not consider necessarily to exclude Ghana from our index computation, as it is unlikely that specific shocks applying only to Ghana would impact the world commodity production and prices. In the Appendix, we compute an alternative quarterly production index without Ghana and use it in our analysis. Overall, our quarterly global food production index (including 2022) is represented in Figure 9.

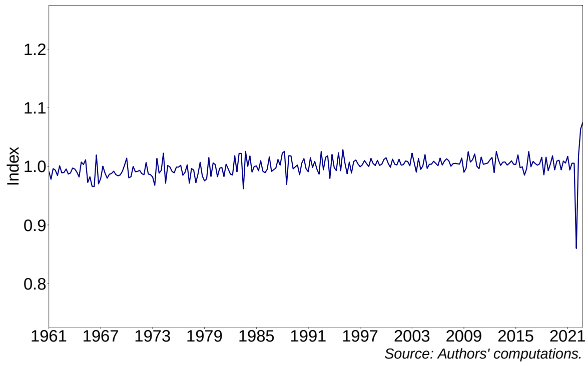


Figure 9: Quarterly global food production index.

In addition, we also follow De Winne & Peersman (2016) and Peersman (2022), and construct a cereal price index based on prices of our four selected commodities: wheat, corn, rice and soybeans. Once again, we use the FAO annual production data for each of the four commodities. This data is aggregated among countries for each specific commodity and year. Incorporating the 2022 estimations by the United States Department of Agriculture, pertaining to the global production for each crop, we achieve a comprehensive annual production record spanning from 1960 to 2022. For each commodity, we compute the trend production volumes by applying a Hodrick-Prescott filter to annual production data, with smoothing parameter of 100. This approach diverges from both De Winne & Peersman (2016) and Peersman (2022), as we compute the trend using annual global production data. Extending this methodology to each crop, we duplicate the annual global production data for every quarter within a year. This enables us to compute the crop's proportion of the annual global production, thereby obtaining the appropriate weight for price calculations. This dataset is then harmonized with quarterly price data for each crop, spanning from 1960 to 2022, sourced from the World Bank Commodity Price Data (The Pink Sheet). For every quarter of each year and for each crop, the share is multiplied by the corresponding quarterly price. Subsequently, the weighted prices are averaged for each quarter of each year. We finally apply seasonal adjustment to the averaged data using the U.S. Census Bureau's X-13ARIMA-SEATS program. Overall, our quarterly cereal price index (including 2022) is represented in Figure 10.

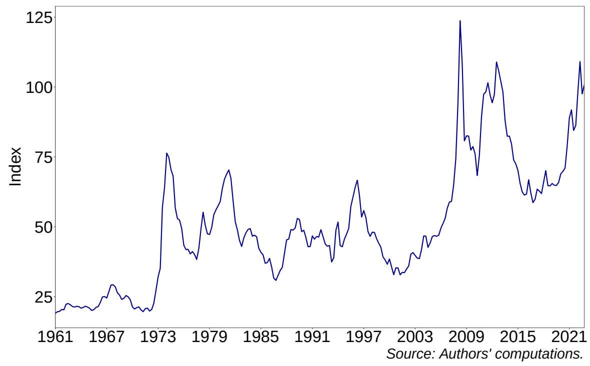


Figure 10: Quarterly cereal price index.

3.2 Estimation

The series of instrumental variables is obtained through the estimation of the subsequent supply shocks equation:

$$y_t = \beta_0 + \beta_1 t + \beta_2 \Psi_t + \sum_{i=1}^{p} \lambda_{i,x} X_{t-i} + \sum_{i=1}^{p} \gamma_i y_{t-i} + \xi_t, \tag{1}$$

where Ψ_t is a vector of the Multivariate El Niño/Southern Oscillation index (MEI), the Oceanic Niño Index (ONI) collected from the National Oceanic and Atmospheric Administration (NOAA) and a dummy variable based on the US National Oceanic and Atmospheric Administration (NOAA) definition of El Niño, and should control for the global weather phenomena.

X_t is a vector of control variables that could have a lagged influence on global food production. It includes the Industrial Production Total Index of the Board of Governors of the Federal Reserve System (US) in order to account for current economic activity and the MSCI World Equity Price Index collected from Refinitiv Eikon and the G-7 value of the OECD Composite Leading Indicator to account for expected economic activity. These data are available quarterly from 1960 to 2022. In addition, it includes the nominal crude oil prices from 1974 to 2022, and global oil production from 1973 to 2022, both collected from the U.S. Energy Information Administration. The crude oil prices are seasonally adjusted using the U.S. Census Bureau's X-13ARIMA-SEATS seasonal adjustment program. As some variables are expressed in nominal USD, we also include the US personal consumption expenditures implicit price deflator index collected from the U.S. Bureau of Economic Analysis from 1960 to 2022. Finally, it includes the cereal price index previously explained and the IMF global price of Food index, where those value represents the benchmark prices (period averages in nominal U.S. dollars) which are representative of the global market and are determined by the largest exporter of a given commodity. This includes prices regarding cereals, vegetable oils, meat, seafood, sugar, bananas, and oranges. The IMF global price of Food index is also seasonally adjusted using the U.S. Census Bureau's X-13ARIMA-SEATS seasonal adjustment program. Finally the sum represents the lag operator, where we use $p = 6$ in the estimation.

The estimation of this equation is conducted on a quarterly basis from 1961 to 2022, which represents the most extensive dataset available for all the relevant series. Presuming that the information available to local farmers is not more comprehensive than what's encompassed by equation 1, the residuals ξ_t can be interpreted as a sequence of unexpected harvest shocks, that we will refer as supply shocks. These shocks can then serve as an external instrument to identify exogenous shocks in international food commodity prices. The corresponding data is depicted in Figure 11.

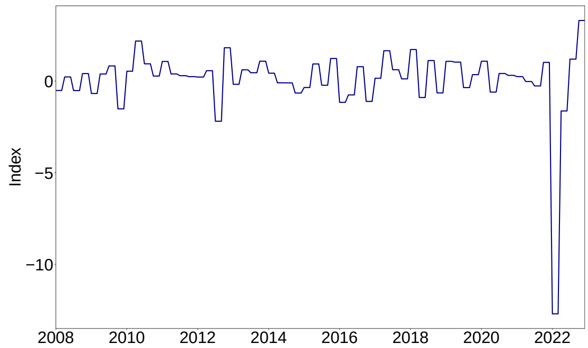


Figure 11: Supply shock.

4 Empirical Evidence

This section presents empirical evidence that serves three main purposes: (i) supporting the upcoming model's predictions, (ii) validating certain aspects of the model's mechanisms, and (iii) emphasizing the pivotal role of our instrumental variable, ξ_t. This variable function as an external instrument to identify shocks in international food commodity prices.

By employing this variable as a shock, we can effectively determine whether there exists empirical support for the notion that food crises can trigger adverse economic conditions leading to debt distress. To accomplish this, we exploit the fact that the information available to producers is no greater than that in equation (1). Consequently, the residuals ξ_t can be treated as a sequence of unforeseen harvest shocks. These shocks then serve as an external instrument, aiding in the identification of shocks in the international food commodity prices.

The use of this instrument is essential. Without it, one could say that countries can anticipate shifts in commodity prices and take measures to hedge against the associated risks. In such a scenario, the link between food crises and debt distress would not be as straightforward. Our approach, therefore, centers on scrutinizing the impact of unanticipated supply shocks on a country's economic and financial conditions, particularly in terms of defaults.[5]

4.1 Commodity prices transmission mechanisms

First, we begin by estimating harvest shocks in the data following the procedure described in equation (1). We then utilize these harvest shocks as an external instrument to price shocks. Once this data is obtained, we analyze its impact on the variables of interest, specifically the variables discussed in Section 2. To quantitatively assess this impact, we estimate the following

[5]For example, in light of recent events like the Ukraine-War, many analysts were unable to predict their occurrence. Within our model, these events are considered unanticipated shocks. This underlines our focus on understanding the effects of such shocks on a country's economic dynamics, particularly in the context of defaults. For a recent study using the war as an unanticipated event see Afunts et al. (2023).

model:

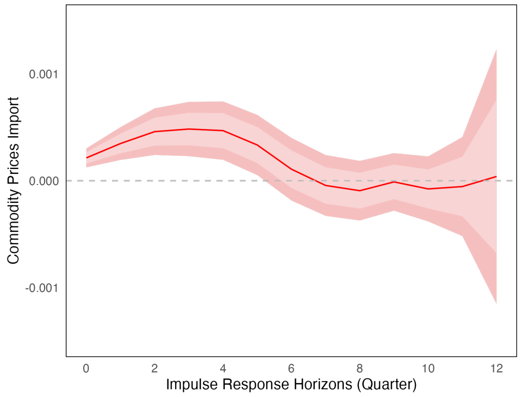
$$y_{t+h} = \beta_0^h + \beta_1^h \varepsilon_t^p + controls_t + \varepsilon_{t+h}, \tag{2}$$

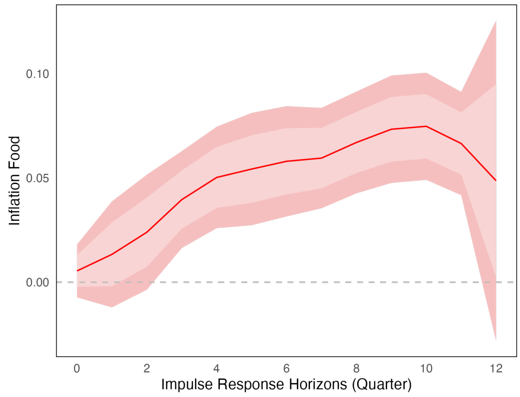
where y_t represents the variable under investigation for which we wish to study the effect of the price shock, ε_t^p denotes the commodity price shock, and ε_t is the residual. In this expression, $controls_t \equiv \sum_{i=1}^{p} \beta_{i,x}^h X_{t-i} + \gamma_1^h trend_t$, where X_t represents a vector of control variables, and $trend_t$ denotes the time trend. The null hypothesis is $\beta_1^h = 0$, and $\beta_1^h \neq 0$ indicates that a negative harvest shock, understood here as a negative supply shock that will lead to a positive price shock, renders supply shocks as having an effect in the variable under consideration. The specific vector of control variables will be discussed for each case we present subsequently. We include four lags ($p = 4$) of control variables since we are using quarterly data.

In order to accurately estimate equation (2) and investigate whether price shocks affect the variables of interest, it is imperative to have an exogenous measure of price shocks to causally analyze their effects. This justifies our reliance on the points outlined in Section 3.1, ensuring that these shocks are plausibly uncorrelated with other macroeconomic shocks, which fulfills the required exogeneity condition.

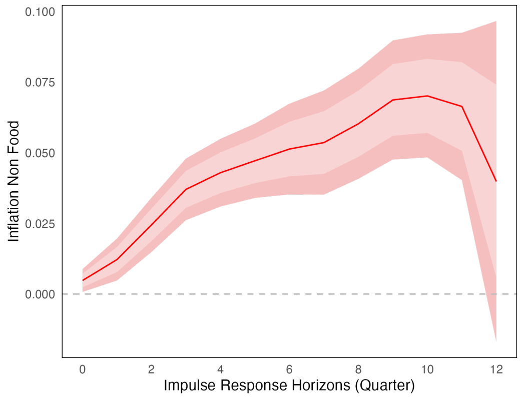
To estimate the Impulse Response Functions (IRFs) presented next, we utilize the local projection methodology proposed by Jordà (2005). To account for potential cointegration between the variables, the model in equation (2) is estimated in levels, adhering to the recommendations of Sims et al. (1990). An advantage of this approach is its ability to delve into the transmission mechanism in greater detail, particularly the pass-through effect onto the variables of interest.

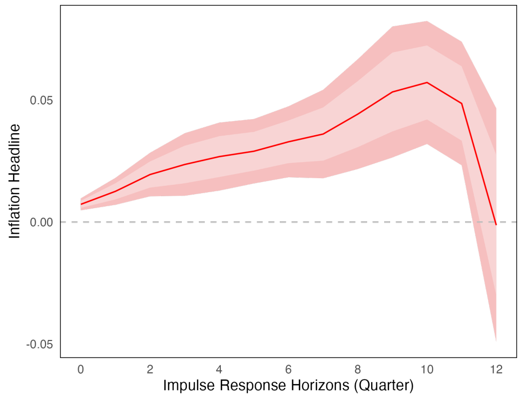
Figure 12 illustrates the outcomes derived from estimating model (2) for each value of $h = 0, 1, ..., 12$, considering a one-unit escalation in price shocks. The figure also includes the 68% and 90% confidence intervals, which are constructed using Newey-West standard errors.





(a) Impulse Response Commodity Prices. (b) Impulse Response Inflation Food.





(c) Impulse Response Inflation Non Food. (d) Impulse Response Inflation Headline.

Figure 12: Impact of a Positive Shock in International Food Commodity Prices.

Note: This figure illustrates the Impulse Response Functions (IRFs) of selected prices and inflation following an unforeseen positive supply shock, characterized as an unanticipated increase in commodity prices on the international market. In particular, Panel (a) displays $\hat{\beta}_1^h$, estimated from the model: $y_{t+h} = \beta_0^h + \beta_1^h \varepsilon_t^p + controls_t + \varepsilon_{t+h}$. Here, y_t represents the logarithm of Commodity Price Imports, and the control vector X_t encompasses the logarithm of Commodity Price Imports, the logarithm of Consumer Price Index (CPI) for Food, and the logarithm of FX Debt Value. Panel (b) exhibits the estimated $\hat{\beta}_1^h$ from the model specified in equation (2), where y_t is the logarithm of Consumer Price Index (CPI) for Food, and the control vector X_t includes the logarithm of CPI for Food and the logarithm of External Debt in National currency. Panel (c) showcases the estimated $\hat{\beta}_1^h$ resulting from estimating the model in equation (2), with y_t representing the logarithm of Consumer Price Index (CPI) for Non-Food items. The control vector X_t encompasses the logarithm of CPI for Non-Food items and the logarithm of External Debt in National currency. Lastly, panel (d) presents the estimated $\hat{\beta}_1^h$ derived from estimating the model in equation (2), where y_t denotes the logarithm of Headline Consumer Price Index (CPI). The control vector X_t includes the logarithms of Headline CPI and External Debt in National currency. The lightly shaded region corresponds to a 68% confidence interval, while the more prominently shaded area indicates the 90% confidence interval. Both intervals are constructed employing Newey-West Standard Errors.

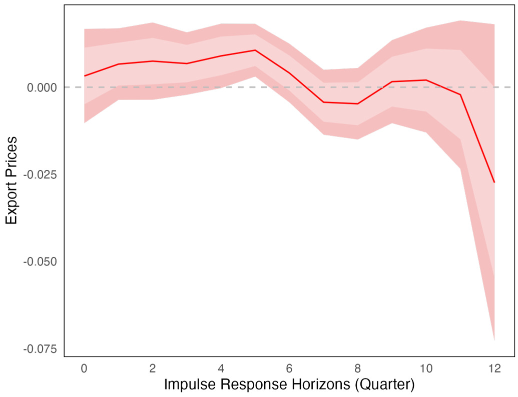
Figure 12 conveys a crucial message: the null hypothesis that price shocks do not influence internal prices in Ghana can be dismissed across multiple time horizons, particularly in the initial quarters. Notably, the food commodity price shock triggers a transient and statistically significant surge in commodity price imports. Examining panel (a), we discern that a unit increase in the price shock results in an approximately 0.08% increase in the commodity price of imports. This impact endures for about five quarters before losing its statistical significance, underscoring the temporal and diminishing nature of the shock's effect. Panels (b) and (c) illuminate the contrasting impacts on Inflation for Food items and Inflation for Non-Food items. As anticipated due to the nature of the shock, the impact on inflation for Food items is higher than for the Non Food items.

These findings demonstrate the discernible influence of international food commodity prices on retail prices. Specifically, regarding Inflation in Food, a noteworthy escalation of nearly 10% emerges after ten quarters — a magnitude twice that observed in core inflation, as depicted in panel (d). This phenomenon may be attributed to two channels. Firstly, as international prices rise due to the supply shock, imported food commodities become costlier for domestic residents. This circumstance is evident in the increased commodity import prices depicted in panel (a). Moreover, this increase in prices can lead to a weakened Ghanaian currency against the dollar, which in turn translates to currency depreciation, leading to a subsequent elevation in CPI for Food. If that is the case, the plausible depreciation of the currency has wider implications, impacting non-food import prices as well — an aspect elucidated in panel (c). This seems to be the mechanism here.

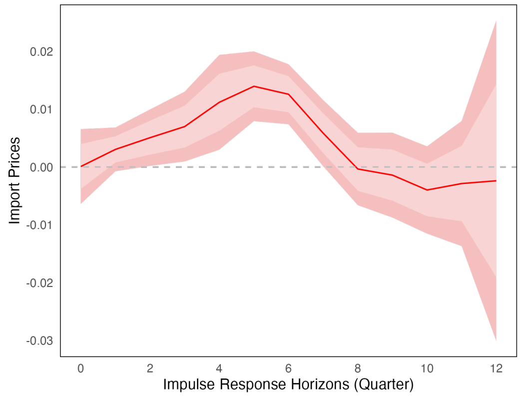
Secondly, given integrated and competitive markets for food commodities, a rise in international food prices could potentially trigger domestic price hikes, which could benefit the country. However, the ensuing negative impact on terms of trade, especially considering Ghana's status as a net importer of commodities, accentuates that the price surge predominantly affects imports rather than translating into substantial gains through higher export prices. This impacts even more the inflation in food.

It's worth noting that across all scenarios, a one-unit increase in the price shock continues to exhibit effects even after several quarters, peaking around the quarter 10. This sustained influence underscores the persistence of the impact on prices, indicating the likely existence of indirect consequences stemming from fluctuations in international food commodity prices on the Consumer Price Index (CPI). Observe that despite their persistence, all of these effects ultimately diminish and vanish over time.

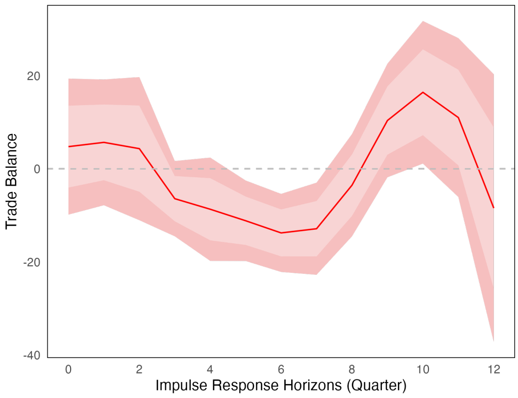
Moving forward, our analysis delves into the ramifications of the supply shock on Ghana's Terms of Trade.



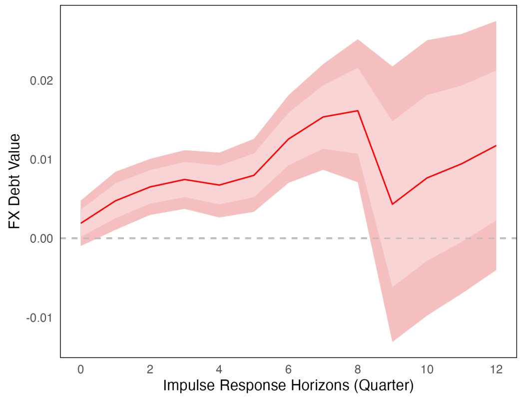
(a) Impulse Response Export Prices.



(b) Impulse Response Import Prices.



(c) Impulse Response Trade Balance.



(d) Impulse Response FX Debt Value.

Figure 13: Impact of a Positive Shock in International Food Commodity Prices.

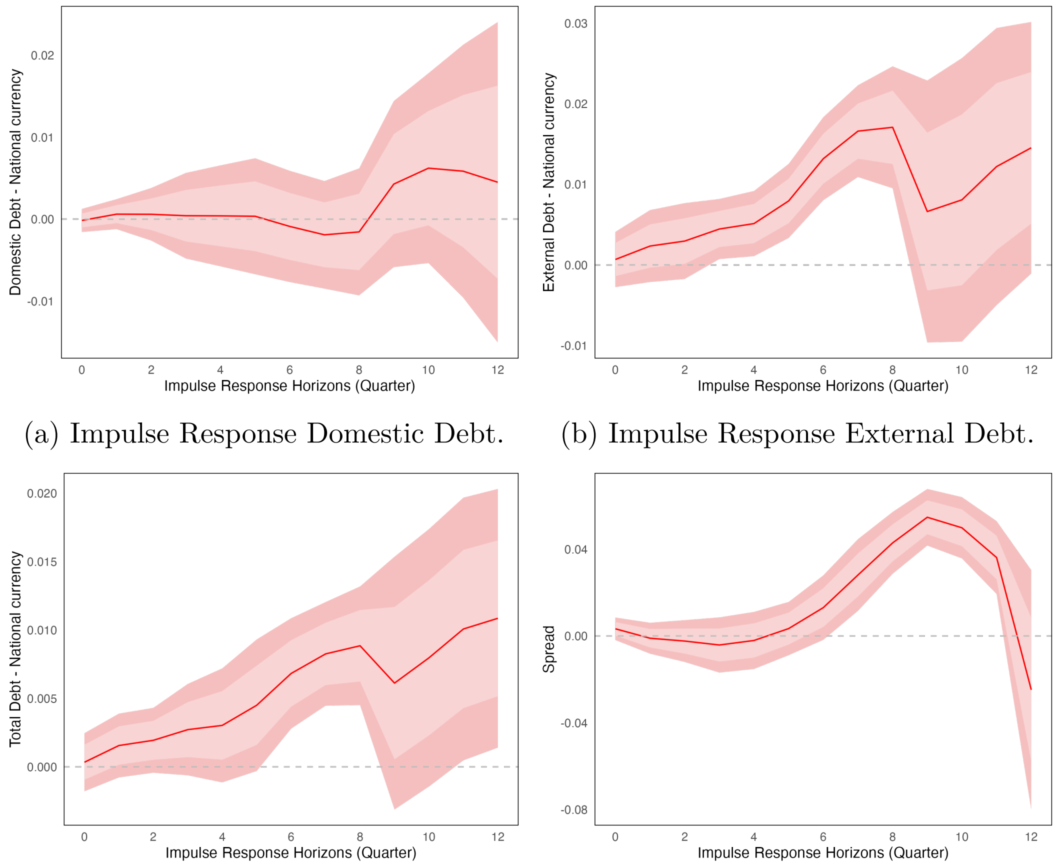
Note: This figure presents Impulse Response Functions (IRFs) reflecting the impact of a positive supply shock on international trade conditions. In panel (a), $\hat{\beta}_1^h$ is shown, obtained through estimating the model described by equation (2), with y_t representing the logarithm of Export Prices. The control vector X_t encompasses the logarithm of Export Prices, the logarithm of Consumer Price Index (CPI) for Food, and the logarithm of FX Debt Value. In panel (b), the estimated $\hat{\beta}_1^h$ is depicted from the model outlined in equation (2), with y_t indicating the logarithm of Import Prices. Here, the control vector X_t includes the logarithm of Import Prices, the logarithm of Consumer Price Index for Food, and the logarithm of FX Debt Value. Panel (c) displays the estimated $\hat{\beta}_1^h$ resulting from estimating the model in equation (2), where y_t denotes the Trade Balance in USD dollars. The control vector X_t encompasses the Trade Balance in USD dollars, the logarithm of FX Debt Value, and the logarithm of Total Debt. Finally, panel (d) showcases the estimated $\hat{\beta}_1^h$ derived from estimating the model in equation (2), where y_t is the logarithm of FX Debt Value. The control vector X_t includes the Trade Balance in USD dollars, the logarithm of FX Debt Value, and the logarithm of Total Debt. The light shaded area corresponds to a 68% confidence interval, while the darker shaded area indicates the 90% confidence interval, both constructed using Newey-West Standard Errors.

Analyzing Figure 13 brings forth several key insights. While it could be argued that integrated and competitive global food commodity markets might lead to a rise in export prices, thereby offsetting increased import costs, our investigation reveals a distinct reality. Examining the model under consideration, this hypothesis is challenged. Panel (a) illustrates our consistent rejection of the hypothesis that Export Prices experience changes across nearly all horizons. Contrarily, panel (b) reveals a noteworthy outcome: the price shock prompts a substantial upswing in Import Prices merely two quarters after initiation, peaking at a 5-quarter increment of import prices by around 2%. These effects eventually dissipate in the long term.

This observation underscores Ghana's vulnerability as a net importer of commodities, particularly during periods of surging food prices. The consequential outcome manifests in a contraction of the terms of trade, as exemplified in panel (c) of Figure 13. Furthermore, the hike in food prices leads to a depreciation of Ghana's currency - panel (d) - exacerbating domestic conditions due to ensuing spikes in import prices, as demonstrated earlier in panel (a) of Figure 12.

Our central proposition, as we delve into subsequent model development, centers on the cumulative impact of deteriorating trade balance coupled with the low elasticity of substitution inherent in food markets. Our central assumption in the model is that a benevolent government in face of this reality drives the trajectory toward increase public debt, since the increase in prices of goods with low elasticity of substitution as food, will lead to a benevolent government to increase public debt to compensate the adverse economic conditions.[6] The subsequent analysis studies the consequences of those shocks to debt and interest rate spreads and it seems to corroborate to the main assumption used in the modeling part developed in Section 5.

[6] Here, we identify two contributing mechanisms driving increased public debt. Primarily, currency depreciation heightens debt service value. Secondarily, given the low elasticity of substitution in food, a benevolent government might augment debt to counterbalance the adverse international conditions. Those assumptions are crucial in the model development presented in Section 5.



(a) Impulse Response Domestic Debt.

(b) Impulse Response External Debt.

(c) Impulse Response Total Debt.

(d) Impulse Response Spread.

Figure 14: Impact of a Positive Shock in International Food Commodity Prices.

Note: This figure presents the Impulse Response Functions (IRFs) that depict the behavior of debt and spread in response to a positive supply shock. Specifically: In panel (a), we illustrate $\hat{\beta}_1^h$, derived from estimating the model as described in equation (2). Here, y_t represents the logarithm of Domestic Debt, and the control vector X_t includes the logarithm of Domestic Debt, the logarithm of External Debt, and the logarithm of the Consumer Price Index (CPI) for Food. Panel (b) displays the estimated $\hat{\beta}_1^h$ obtained from the model outlined in equation (2), where y_t refers to the logarithm of External Debt. In this context, the control vector X_t comprises the logarithm of Domestic Debt, the logarithm of External Debt, and the logarithm of the Consumer Price Index for Food. Moreover, panel (c) depicts the estimated $\hat{\beta}_1^h$ resulting from estimating the model as described in equation (2), with y_t denoting the logarithm of Total Debt. The control vector X_t includes the logarithm of Domestic Debt, the logarithm of External Debt, and the logarithm of the Consumer Price Index for Food. Lastly, panel (d) shows the estimated $\hat{\beta}_1^h$ obtained from estimating the model in equation (2), where y_t is the logarithm of Spread. The control vector X_t comprises the logarithm of Spread and the logarithm of External Debt. The lighter shaded region corresponds to a 68% confidence interval, while the darker shaded area indicates the 90% confidence interval. Both intervals are constructed using Newey-West Standard Errors.

The overarching message from Figure 14 is that while domestic debt remains relatively unaffected by food price shocks, external debt exhibits a distinct response. This becomes evident when analyzing panel (a) of Figure 14, where we observe a lack of evidence to reject a null effect across all horizons. This suggests that Ghana's reliance on international debt markets to support its financial position is substantial. Shifting focus to panel (b), a notable trend emerges: an increase in the price shock corresponds to a rise in Ghana's external debt. This effect is evident two quarters after the shock initiation and becomes more pronounced by the eighth quarter. Eventually, this effect diminishes. The subsequent rise in public debt, combined with a deteriorating international trade position, erodes market confidence in the government's debt repayment capacity. This loss of confidence is reflected in increased spreads, as depicted in panel (d). This mechanism can further contribute to an escalation in external debt burden. In conclusion, as previously highlighted, these interconnected factors mutually reinforce each other, potentially leading to a sovereign default similar to Ghana's experience at the close of 2022.

4.2 Discussions

Overall, we employ these empirical findings to support the modeling we develop in the next section, and as we will see, they are consistent with the predictions of our quantitative model. We view these empirical findings as a complementary result for the theoretical modeling part we develop next. The advantage of using the theoretical model, as opposed to relying solely on the empirical findings we found, is that we are able to identify the causal effects.

Even though we argue that we are using an external instrument to identify the price shocks that are uncorrelated with other macroeconomic conditions, the transmission mechanism discussed above lacks neat identification: the empirical findings might be driven by unobserved factors correlated with the prices that ultimately affect the transmission we have discussed above. It is worth clarifying that the goal of our paper is to highlight the importance of a food crisis for debt distress and to point out that certain scenarios of food crises can increase the probability of a sovereign default. Due to the issue of finding a neat identification strategy in the empirical modeling, in what follows we begin the development of our quantitative analyses, where we can see the mechanisms at hand and compare them with our empirical findings.

5 The toy model: import prices specification

Time is discrete, and the horizon is infinite. We are examining a small open economy in which the government receives an endowment and makes choices regarding consumption between two types of goods: one produced domestically and the other imported. The government also decides the amount of debt to issue. In this economy, there exists a continuum of risk-neutral international financial intermediaries that function as international financial markets. These intermediaries determine the quantity of debt to purchase from the domestic government based on its return relative to a risk-free interest rate, represented as r_t^*. In this first version of the model, we use some simplifying assumptions in order to better illustrate the intuitions. More precisely, in our toy model, the analysis depends on the import prices while in the more complex version, it depends on the terms of trade.

5.1 Environment

There are two types of goods: domestically produced goods, denoted H, and foreign goods denoted F, which are produced abroad and imported. The benevolent government consumes both types of goods according to the following consumption basket:

$$C_t = \left((1-\alpha)^{1/\rho} \, C_{Ht}^{(\rho-1)/\rho} + \alpha^{1/\rho} \, C_{Ft}^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)}, \tag{3}$$

where the parameter ρ denotes the elasticity of substitution between home and foreign goods while α measures the openness of the economy, such that $(1-\alpha)$ is the degree of home bias in preferences. The consumer price index associated to this basket is:

$$P_t = \left((1-\alpha) \, P_{Ht}^{1-\rho} + \alpha \, P_{Ft}^{1-\rho} \right)^{1/(1-\rho)}, \tag{4}$$

which implies that the government split his aggregate consumption C_t between domestic and imported goods according to:

$$C_{Ht} = (1-\alpha)\left(\frac{P_{Ht}}{P_t}\right)^{-\rho} C_t, \tag{5}$$

$$C_{Ft} = \alpha\left(\frac{P_{Ft}}{P_t}\right)^{-\rho} C_t. \tag{6}$$

Finally, the terms of trade can be defined as the ratio of each good price:

$$T_t = \frac{P_{Ft}}{P_{Ht}}. \tag{7}$$

In each period, the government receives a nominal endowment in terms of domestic production Y_t that it can use to consume directly or to trade.

The benevolent government derives its utility from the consumption of both types of goods according to:

$$\mathbb{E}_0 \sum_{i=0}^{\infty} \beta^t u_t(C_{Ht}, C_{Ft}), \tag{8}$$

where $0 < \beta < 1$ is the discount factor, C_{Ht} is the consumption of domestic goods, C_{Ft} is the consumption of foreign goods, and $u(.)$ is strictly increasing and concave.

Taking prices as given, the government chooses how much consumption, borrowing of domestic bonds, denoted B_{Ht}, and whether to default or not on the existing debt in order to maximize the utility of a continuum of households with similar preferences. Notice, if $B_{Ht} > 0$, the government holds positive assets, whereas if $B_{Ht} < 0$, the country is indebted.

Domestic bonds are denominated in local currency. The nominal price of the domestic bond, q_{Ht} is a function of both the amount of debt chosen by the government and of the terms of trade, which under our simplifications, will be equal to the price of the imported good.

The budget constraint in nominal terms is therefore:

$$P_{Ht}C_{Ht} + P_{Ft}C_{Ft} + q_{Ht}(B_{Ht+1}, T_t) \, B_{Ht+1} \leq Y_t + B_{Ht}. \tag{9}$$

In real terms, variables become $y_t \equiv \frac{Y_t}{P_{Ht}}$ and $b_{Ht} \equiv \frac{B_{Ht}}{P_{Ht}}$. Finally, define the gross inflation rate between periods $t-1$ and t as $\Pi_{Ht} \equiv \frac{P_{Ht}}{P_{Ht-1}}$. The budget constraint can be rewritten as:

$$C_{Ht} + T_tC_{Ft} + q_{Ht}(b_{Ht+1}, T_t) \, b_{Ht+1} \, \Pi_{Ht+1} \leq y_t + b_{Ht}. \tag{10}$$

We can characterize households' choices with the following optimality conditions:

$$\frac{u_{cF,t}}{u_{cH,t}} = T_t,$$
$$u_{cH,t} = \frac{1}{q_{Ht}(b_{Ht+1}, T_t)} \beta E_t \Big(\frac{u_{cH,t+1}}{\Pi_{t+1}} \Big).$$

In this toy version, we first assume that the monetary policy at home keeps the price of domestic goods in domestic currency constant, such that $P_{Ht} = P_t = 1$ at each period t. In addition, the trade balance will only be modeled as net imports, which is equivalent mathematically to considering that exports are equal to 0 in this model. This imply that, under our assumptions:

$$C_t = C_{Ht} + P_{Ft} C_{Ft}. \tag{11}$$

Under our simplifications, nominal and real terms are identical, such that the budget constraint can be simplified to:

$$C_{Ht} + P_{Ft} C_{Ft} + q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} \leq y_t + b_{Ht}. \tag{12}$$

Under our simplifications, optimality conditions can be rewritten as:

$$\frac{u_{cF,t}}{u_{cH,t}} = P_{Ft},$$
$$u_{cH,t} = \frac{1}{q_{Ht}(b_{Ht+1}, P_{Ft})} \beta E_t \Big(u_{cH,t+1} \Big).$$

Given our previous results and under our assumptions, the budget constraint can be rewritten as:

$$(1-\alpha) C_t + \alpha P_{Ft} \Big(P_{Ft}\Big)^{-\rho} C_t + q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} \leq y_t + b_{Ht}$$
$$\Leftrightarrow (1-\alpha) C_t + \alpha \Big(P_{Ft}\Big)^{1-\rho} C_t + q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} \leq y_t + b_{Ht}$$
$$\Leftrightarrow C_t \Big((1-\alpha) + \alpha \Big(P_{Ft}\Big)^{1-\rho} \Big) + q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} \leq y_t + b_{Ht}.$$

This finally gives us an expression of C_t:

$$C_t = \frac{y_t + b_{Ht} - q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1}}{\Big((1-\alpha) + \alpha \Big(P_{Ft}\Big)^{1-\rho} \Big)}. \tag{13}$$

The market clearing condition for domestic production is:

$$y_t = C_{Ht}. \tag{14}$$

The trade balance, defined generally as the difference between exports and imports, will be under our assumptions where exports are equal to 0, only equal to minus imports:

$$tb_t = -P_{Ft} C_{Ft}. \tag{15}$$

This implies that an increase in the trade balance implies a decrease in imports.

Using the simplified budget constraint (12) and the market clearing condition (14), we can characterized the trade balance in our environment:

$$\begin{aligned} & C_{Ht} + P_{Ft}C_{Ft} + q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} = y_t + b_{Ht} \\ \Leftrightarrow\; & C_{Ht} + P_{Ft}C_{Ft} + q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} = C_{Ht} + b_{Ht} \\ \Leftrightarrow\; & -P_{Ft}C_{Ft} = q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} - b_{Ht}. \end{aligned}$$

Such that:

$$tb_t = q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} - b_{Ht}. \tag{16}$$

For simplicity, we will refer in our analysis to net imports as the negative of the trade balance.

5.2 Default decisions

Here we assume that the final consumption can be of two types: one when the government does not default and one when the government defaults. We also assume that under the case when the government decides to default, it is excluded from the international credit market and has every period a given probability to regain access to the markets. In addition, in the line of the output loss faced by the government following its default, as modeled by Arellano (2008), we assume that, following its default, the government faces an extra cost in the form of an increase in the price of imports.

Under our simplifying assumptions, consumption in each case is characterized by the two following expressions:

- when the government chooses to repay its debt:

$$C_t = \frac{y_t + b_{Ht} - q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1}}{\Big((1-\alpha) + \alpha\big(P_{Ft}\big)^{1-\rho}\Big)}. \tag{17}$$

- when the government chooses to default:

$$C_t = \frac{y_t}{\Big((1-\alpha) + \alpha\big(\tilde{P}_{Ft}\big)^{1-\rho}\Big)}, \tag{18}$$

  where $\tilde{P}_{Ft} \geq P_{Ft}$.

The timing of the model is as follows:

1. At the beginning of each period, the government decides whether or not to default on its debt. Let V_t^d denotes the value function when the government chooses to default and V_t^p the value function when the government chooses to repay its debt.
2. Default is optimal whenever $V_t^d > V_t^p$. Every time this condition holds, the government chooses to default and therefore does not pay back its past debt.
3. During the default period, the government is excluded from the international markets and, thus cannot borrow. We also assume that the government faces as another punishment an increase in the price paid for its imports.

The problem is therefore such that the government chooses the optimal policy b_{Ht+1} to maximize utility. The expected value from the next period onward and the bond prices q_{Ht} will incorporate the fact that the government can choose to default in the future.

The state in this economy is represented by a vector $s(b_{Ht}, P_{Ft})$. Before we proceed, it is important to recall that foreign creditors in this economy have access to the international credit market and are subject to a constant international interest rate denoted as $r_t^* > 0$. We make the assumption that these creditors are risk neutral and behave in a competitive manner. Consequently, in each period, we can observe their expected profits, defined as ρ_t:

$$\rho_t = q_{Ht}\, b_{Ht+1} - \frac{(1-\delta_t)}{1+r_t}\, b_{Ht+1} = 0,$$

where δ_t is the probability of default. Notice that when $b_{Ht+1} \geq 0$, the probability of default is zero since the government is saving. This no arbitrage condition implies that the equilibrium price of a discounted bond is given by:

$$q_{Ht} = \frac{(1-\delta_t)}{1+r_t}. \tag{19}$$

The probability of default, δ_t, is endogenous to the model and relies on incentives of the government to repay. The probability of default is a function of both b_{Ht+1} and P_{Ft} (under our simplifications), which means that q_{Ht} is also a function of those variables. Finally, the gross interest rate is defined as $r_t \equiv 1/q_{Ht} - 1$ and the spread as $s_t \equiv r_t - r_t^*$. With those elements, we can define the recursive competitive equilibrium.

5.3 Recursive competitive equilibrium

Given the state vector $s(b_H, P_F)$, the government's policy functions b'_H, the price functions q_H, and the consumption functions C, we can determine the equilibrium.

Foreign creditors, being risk-neutral and competitive, determine bond prices based on the following condition:

$$q_H(b'_H, P_F) = \frac{1-\delta(b'_H, P_F)}{1+r}.$$

Now, let $V^0(b_H, P_F)$ be the government's value function at the beginning of the period, with assets b_H and facing import price P_F. The government decides whether to repay or default to maximize utility represented in (8). The value function $V^0(b_H, P_F)$ is defined by the following equation:

$$V^0(b_H, P_F) = \max_{\{C, b'_H\}} \{V^p(b_H, P_F), V^d(P_F)\}, \tag{20}$$

where $V^p(b_H, P_F)$ represents the value function when repaying, and $V^d(P_F)$ represents the value function when defaulting. When the government defaults, it enters a state of autarky but has a constant and exogenous chance to regain access to credit markets every period.

The value function of defaulting is given by:

$$V^d(P_F) = u\left(\frac{y}{\left((1-\alpha)+\alpha(\tilde{P}_F)^{1-\rho}\right)}\right) + \beta \int_{P'_F} \left[\theta\, V^0(0, P'_F) + (1-\theta)\, V^d(P'_F)\right] f(P'_F, P_F)\, dP'_F, \tag{21}$$

where θ represents the probability of re-entering the credit market.

On the other hand, when the government has not defaulted, and therefore has repaid its debt, the value function $V^p(b_H, P_F)$ is given by:

$$V^p(b_H, P_F) = \max_{b'_H} \left\{ u\left(\frac{y + b_H - q_H(b'_H, P_F)\, b_H}{\left((1-\alpha) + \alpha(P_F)^{1-\rho}\right)} \right) + \beta \int_{P'_F} V^0(b'_H, P'_F)\, f(P'_F, P_F)\, dP'_F \right\}. \tag{22}$$

Therefore, the government chooses b'_H to maximize utility, with the decision being made period by period. Additionally, to prevent Ponzi schemes, we assume that the government faces a lower bound on debt defined as $b'_H \geq -z$, not binding in equilibrium.

The policy functions can be characterized by the following sets:

$$A(b_H) = \{P_F \in \mathbb{P} : V^p(b_H, P_F) \geq V^d(P_F)\},$$

and

$$D(b_H) \equiv \tilde{A}(b_H) = \{P_F \in \mathbb{P} : V^p(b_H, P_F) < V^d(P_F)\}.$$

where the set $A(b_H)$ represents the optimal set for remaining in the contract, while the set $D(b_H)$ represents the optimal set for default. Given this information, and denoting the aggregate states of the economy as $s(b_H, P_F)$, we can define a recursive equilibrium as follows:

Definition 1. *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumption $C(s)$; (ii) government's asset holdings $b'_H(s)$, repayment sets $A(b_H)$, and default sets $D(b_H)$; and (iii) the price function for bonds $q_H(b'_H, P_F)$ such that:*
1. Taking as given the government policies, household consumption $C(s)$ satisfies the resource constraint.
2. Taking as given the bond price function $q_H(b'_H, P_F)$, the government policy function $b'_H(s)$, repayment sets $A(b_H)$, and default sets $D(b_H)$ satisfy the government optimization problem.
3. Bonds prices $q_H(b'_H, P_F)$ reflect the government's default probabilities and are consistent with the creditor's expected zero profits.

Furthermore, default probabilities $\delta(b'_H, P_F)$ and default sets $D(b'_H)$ are related by the following equation:

$$\delta(b'_H, P_F) = \int_{D(b'_H)} f(P'_F, P_F)\, dP'_F. \tag{23}$$

In other words, when $D(b'_H) = \emptyset$, equilibrium default probabilities are zero because the government does not choose to default for any realization of the import prices when it has assets b'_H. Similarly, when $D(b'_H) = \mathbb{P}$, default probabilities are equal to one.

The value of remaining in the contract increases with b_H, while the value of default is independent of b_H. Assuming bounded support for import price shocks, there exists a level of assets low enough for which $D(b'_H) = \mathbb{P}$. However, since default only occurs when assets are negative, there is a level of b_H for which the default set is empty.

From equation (23), we can observe that the equilibrium bond price increases with b'_H, indicating that a low discount price for a large loan compensates lenders for the possibility of default. Additionally, bond prices are also dependent on import price shocks.

5.4 Calibration

In the simulations presented below, we employed the Constant Relative Risk Aversion (CRRA) utility function:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where $\sigma = 2$. The risk-free interest rate is set at 1%, which corresponds to the quarterly yield of the US 5-year bond.

The stochastic process for the import price follows the equation:

$$\log P_{Ft+1} = (1-\xi)\mu + \xi \log P_{Ft} + \varepsilon_t,$$

where $E[\varepsilon] = 0$ and $E[\varepsilon^2] = \eta_p^2$. The calibrated values used are: $\xi = 0.94$, $\eta_p = 0.2$, and $\mu = 3.00$. We discretized the shock into a 60-state Markov process, employing the method developed by Tauchen & Hussey (1991). Overall, the parameters of the model are represented in the table below:

Table 1: Model Parameters.

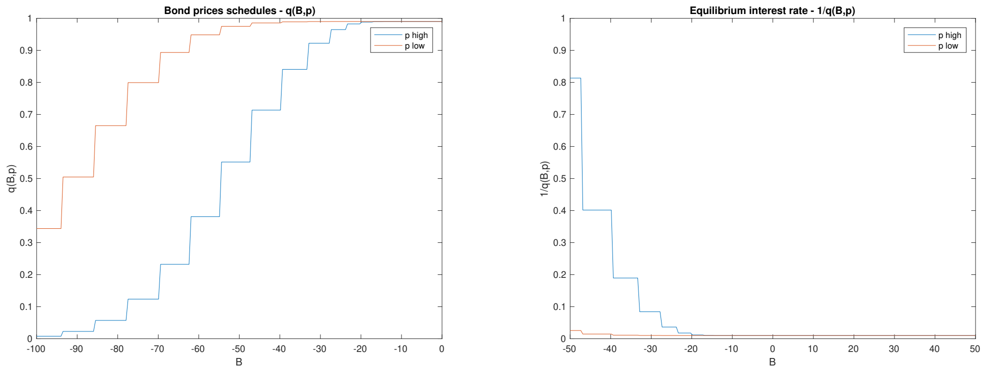
Parameter	**Value**
Risk-free interest rate	$r = 1\%$
Risk aversion	$\sigma = 2$
Stochastic structure	$\xi = 0.94$, $\mu = 3.00$, $\eta_p = 0.2$
Discount factor	$\beta = 0.96$
Probability of reentry	$\theta = 0.2$
Elasticity of substitution among domestic and foreign goods	$\rho = 0.9$
Import share	$\alpha = 0.5$
Import price in default	$\tilde{P}_F = \max(P_{Ft})$
Endowment	$y = 1$

By using the parameters as defined in Table 1, we simulate the model and we obtain the following results for the competitive recursive equilibrium. In what follows we discuss the results. The algorithm to solve the model is discussed in the Appendix A.

It worth noticing that we choose the elasticity of substitution to be below than 1, in order for the goods to be poor substitutes. Indeed, in our framework, we think of imports as food products and more specifically cereals such as maize, rice and wheat. For this type of product that are both used to consume directly and to produce, it is costly and complicated to substitute toward other suppliers, even more under a context as the Ukraine-Russia war which has motivated this project.

5.5 Results for the competitive equilibrium

Figure 15 illustrates the bond price experienced by the borrower in relation to assets b_{Ht}, considering two import price shocks: one that is 20 percent above the trend, identified as high, and another that is 20 percent below the trend, identified as low.



(a) Bond price schedules.

(b) Equilibrium interest rate.

Figure 15: Bond prices and assets.

The left panel of Figure 15 presents the bond price schedules, displaying the bond price faced by the government as a function of assets b_{Ht} for the aforementioned import price shocks. The interpretation of this panel is straightforward: bond prices increase as assets increase. In other words, higher levels of debt (larger $-b_{Ht+1}$) result in greater discounts in the face value, leading to higher interest rates. Notably, a high import price corresponds to more discounting, indicating that foreign creditors anticipate a higher likelihood of default. This fact is further emphasized in Figure 16, where the shaded area represents moments of default. It is evident that, under low import prices, the country never defaults since international conditions are favorable in such cases. Conversely, when import prices reach sufficiently high levels, even with low levels of debt, default becomes optimal for the government. As the debt levels increase, the level of import prices under which default is optimal decreases, showing that for high-debt countries, the import price shocks do not need to be high enough for default to become optimal.

Lastly, it is worth noting that the borrowing constraint is more relaxed during periods of low import prices compared to periods of high import prices. This discrepancy arises due to the preference for default during times when import prices are high and shocks to these prices are persistent. This occurs because a high import price shock today suggests a likelihood of similarly high shocks in the future, prompting the government to default even when debt levels are not sufficiently high. As a result, the interest rate increases in anticipation of this behavior by foreign creditors, as can be seen in the right panel of Figure 15. The model highlighted here exhibits counter-cyclical borrowing constraints.

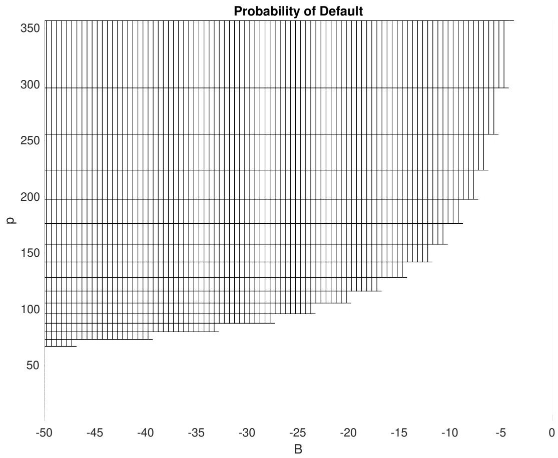
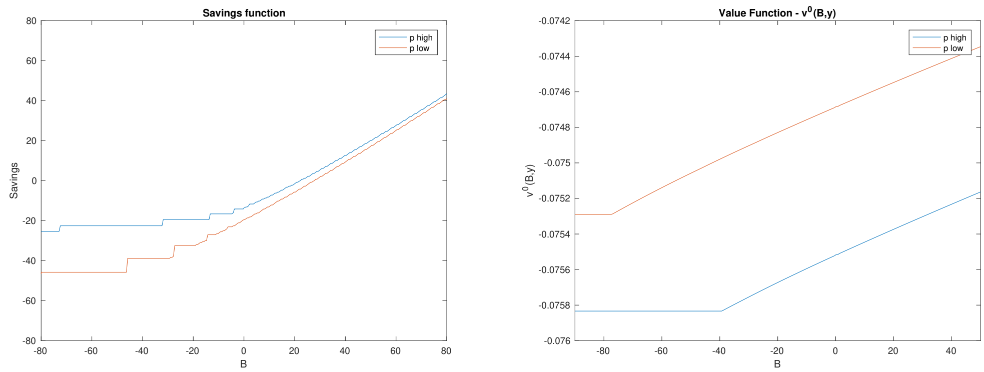


Figure 16: Default probability.

It is also important to highlight that in this model, the government has two tools at its disposal to influence the consumption trajectory in response to import price shocks: borrowing and defaulting. The purpose of debt issuance is to mitigate the impact of income fluctuations on consumption. Moreover, considering that the parameter β is less than the inverse of the risk-free interest rate (i.e., $\frac{\beta}{1+r_t} < 1$), the government always favors the option of borrowing over defaulting.



(a) Savings Function.

(b) Value Function.

Figure 17: Savings and Value Function.

The left panel of Figure 17 illustrates the conditional savings function, considering the non-defaulting scenario, as a function of b_{Ht} for the two previously explained import price levels. When b_{Ht} is small and negative, it is evident that the government prefers to borrow more when import prices increase. However, due to certain unavailable financial contracts, the government is constrained and unable to do so. This tighter borrowing constraint is characterized by significantly higher interest rates compared to the borrowing constraint faced when import prices are low. Notably, for low import prices and high debt levels, the government is allowed to borrow, leading to a decrease in savings. As the government becomes a net saver, the two savings functions converge.

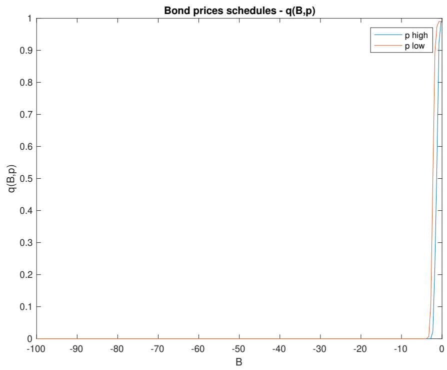
Finally, the right panel of Figure 17 presents the value function in relation to b_{Ht} for the two aforementioned import price levels. It is worth observing that, for both cases, default is chosen for all asset levels below a threshold represented by the flat line. Furthermore, it is important to note that there exists a range of debt levels where default is not chosen when import prices are low, but becomes optimal as import prices increase. This highlights the significance of international conditions in the default decisions of small open economies, particularly those with high levels of indebtedness.

5.6 Analysing the importance of the international conditions

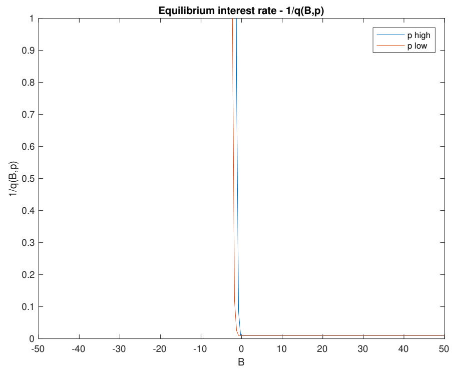
To examine the impact of international conditions, specifically import prices, on the default decisions of small open economies, we can analyze how the equilibrium changes by varying the parameter α, which as explained before, represents the degree of economic openness. We anticipate that as α decreases, the influence of international conditions diminishes. Consequently, the decision to default or not becomes less dependent on the fluctuation of import prices. On one hand, this may appear favorable as it reduces the country's vulnerability to international conditions. However, on the other hand, as α decreases, the government's fiscal buffer diminishes. In the extreme case, when the country is completely closed and unable to access international markets, it can only accumulate positive debt levels in the form of savings, as can be seen in panel (a) of Figure 18. Notice that as the country becomes more open, it also becomes more indebted.

Conversely, as the degree of openness increases, the country gains the ability to borrow more through its greater participation in financial markets and potentially securing substantial international assistance. Moreover, even in scenarios where import prices rise, a more open country will face lower interest rates for the same debt level compared to a lower degree of openness, which can be easily seen by comparing panels (b), (c), and (d) of Figure 18. Finally, by examining Figure 19, we observe a reduction in the regions where default occurs as we increase α, which implies that a country with higher openness is less prone to default.

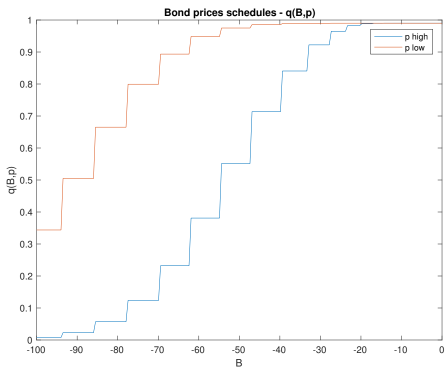
Therefore, the main challenge lies in small highly indebted open economies with limited openness. While a highly indebted country that is more open experiences a decrease in the regions prone to default, the same cannot be said for small open countries with high debt levels. These economies align with the focus of our analysis.



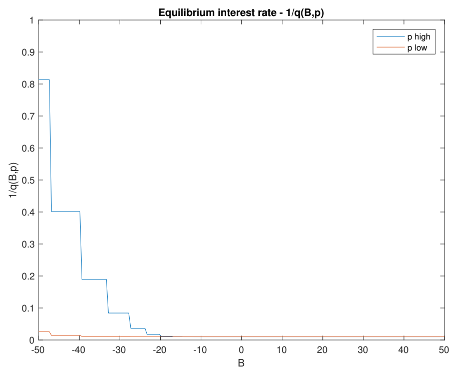
(a) Bond price schedules, $\alpha = 0.01$.



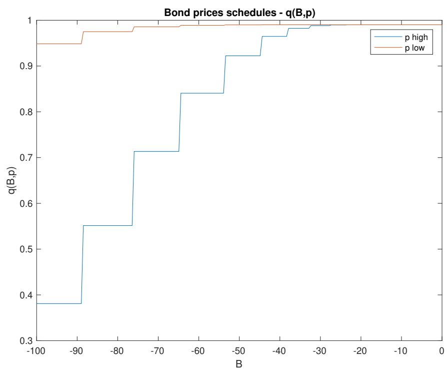
(b) Equilibrium interest rate, $\alpha = 0.01$.



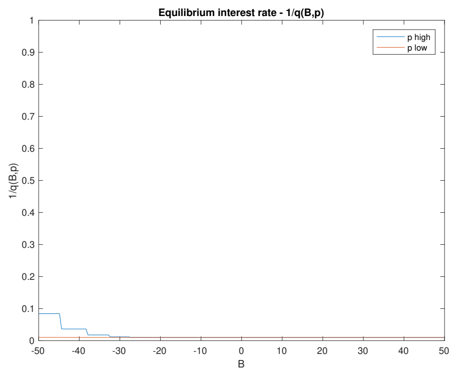
(c) Bond price schedules, $\alpha = 0.5$.



(d) Equilibrium interest rate $\alpha = 0.5$.

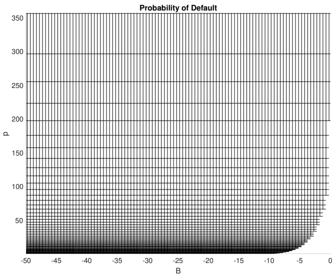


(e) Bond price schedules - $\alpha = 0.99$.

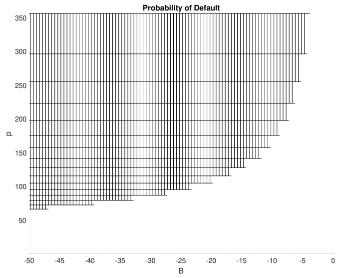


(f) Equilibrium interest rate- $\alpha = 0.99$.

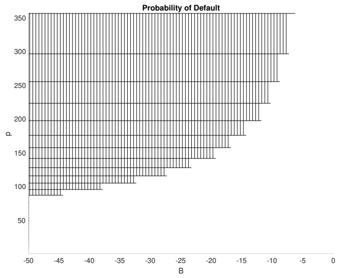
Figure 18: Bond prices and assets.



(a) Default probability, $\alpha = 0.01$.

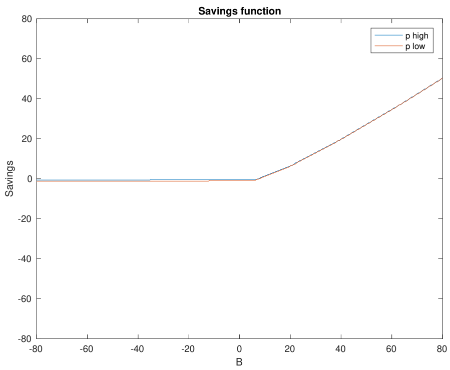


(b) Default probability, $\alpha = 0.5$.

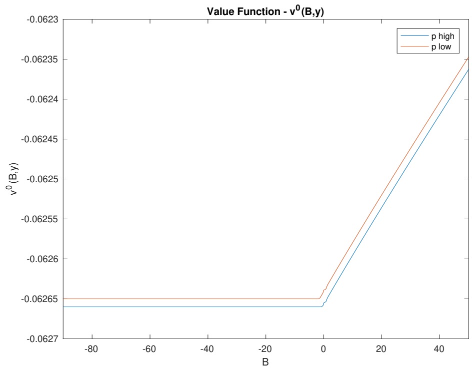


(c) Default probability, $\alpha = 0.99$.

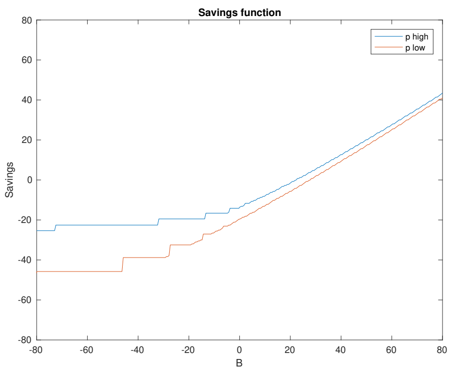
Figure 19: Default Probabilities.



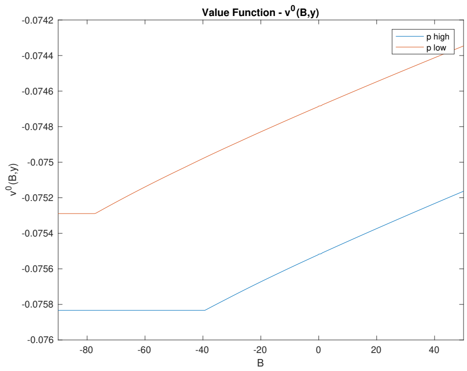
(a) Savings Function, $\alpha = 0.01$.



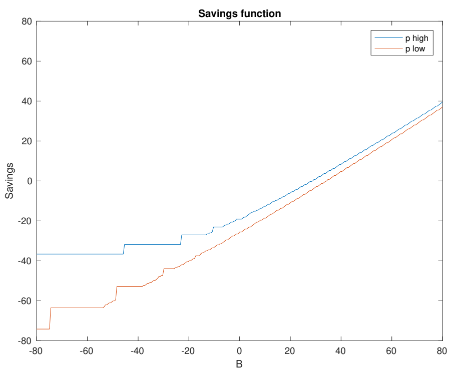
(b) Value Function, $\alpha = 0.01$.



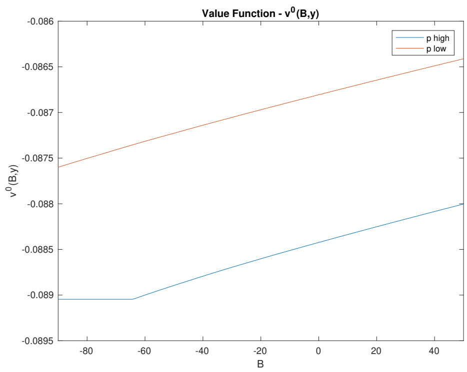
(c) Savings Function, $\alpha = 0.5$.



(d) Value Function, $\alpha = 0.5$.



(e) Savings Function - $\alpha = 0.99$.



(f) Value Function - $\alpha = 0.99$.

Figure 20: Savings and Value Function.

5.7 Analysing the importance of elasticity of substitution

In Appendix B, we analyse the effect of the elasticity of substitution for our results and we argue that our choice $\rho < 1$ is reasonable, especially under our simplifying assumptions.

6 The model: terms of trade specification

Here, we do an extension analyzing the model considering the terms of trade specification. Time is discrete, and the horizon is infinite. We are examining a small open economy in which the government receives an endowment and makes choices regarding consumption between two types of goods: one produced domestically and the other imported. The government also decides the amount of debt to issue. In this economy, there exists a continuum of risk-neutral international financial intermediaries that function as international financial markets. These intermediaries determine the quantity of debt to purchase from the domestic government based on its return relative to a risk-free interest rate, represented as r_t^*. In this second version of the model, we characterize the decisions using a more complex structure based on the terms of trade and relax our previous simplifying assumptions.

6.1 Environment

There are two types of goods: domestically produced goods, denoted H, and foreign goods denoted F, which are produced abroad and imported. The benevolent government consumes both types of goods according to the following consumption basket:

$$C_t = \left((1-\alpha)^{1/\rho}\, C_{Ht}^{(\rho-1)/\rho} + \alpha^{1/\rho}\, C_{Ft}^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)}, \tag{24}$$

where the parameter ρ denotes the elasticity of substitution between home and foreign goods while α measures the openness of the economy, such that $(1-\alpha)$ is the degree of home bias in preferences. The consumer price index associated to this basket is:

$$P_t = \left((1-\alpha)\, P_{Ht}^{1-\rho} + \alpha\, P_{Ft}^{1-\rho} \right)^{1/(1-\rho)}, \tag{25}$$

which implies that the government split his aggregate consumption C_t between domestic and imported goods according to:

$$C_{Ht} = (1-\alpha)\left(\frac{P_{Ht}}{P_t}\right)^{-\rho} C_t, \tag{26}$$

$$C_{Ft} = \alpha\left(\frac{P_{Ft}}{P_t}\right)^{-\rho} C_t. \tag{27}$$

Finally, the terms of trade can be defined as the ratio of each good price:

$$T_t = \frac{P_{Ft}}{P_{Ht}}. \tag{28}$$

In each period, the government receives a nominal endowment in terms of domestic production Y_t that it can use to consume directly or to trade.

The benevolent government derives its utility from the consumption of both types of goods according to:

$$\mathbb{E}_0 \sum_{i=0}^{\infty} \beta^t u_t(C_{Ht}, C_{Ft}), \tag{29}$$

where $0 < \beta < 1$ is the discount factor, C_{Ht} is the consumption of domestic goods, C_{Ft} is the consumption of foreign goods, and $u(.)$ is strictly increasing and concave.

Taking prices as given, the government chooses how much consumption, borrowing of domestic bonds, denoted B_{Ht}, and whether to default or not on the existing debt in order to maximize the utility of a continuum of households with similar preferences. Notice, if $B_{Ht} > 0$, the government holds positive assets, whereas if $B_{Ht} < 0$, the country is indebted.

Domestic bonds are denominated in local currency. The nominal price of the domestic bond, q_{Ht} is a function of both the amount of debt chosen by the government and of the terms of trade, which under our simplifications, will be equal to the price of the imported good.

The budget constraint in nominal terms is therefore:

$$P_{Ht} C_{Ht} + P_{Ft} C_{Ft} + q_{Ht}(B_{Ht+1}, T_t)\ B_{Ht+1} \leq Y_t + B_{Ht}. \tag{30}$$

In real terms, variables become $y_t \equiv \frac{Y_t}{P_{Ht}}$ and $b_{Ht} \equiv \frac{B_{Ht}}{P_{Ht}}$. Finally, define the gross inflation rate between periods $t-1$ and t as $\Pi_{Ht} \equiv \frac{P_{Ht}}{P_{Ht-1}}$. The budget constraint can be rewritten as:

$$C_{Ht} + T_t C_{Ft} + q_{Ht}(b_{Ht+1}, T_t)\ b_{Ht+1}\ \Pi_{Ht+1} \leq y_t + b_{Ht}. \tag{31}$$

We can characterize households' choices with the following optimality conditions:

$$\frac{u_{cF,t}}{u_{cH,t}} = T_t,$$
$$u_{cH,t} = \frac{1}{q_{Ht}(b_{Ht+1}, T_t)} \beta E_t \Big(\frac{u_{cH,t+1}}{\Pi_{t+1}} \Big).$$

Using our previous results, we can rewrite the ratio of each good price with respect to the domestic consumer price index as:

$$\frac{P_{Ht}}{P_t} = \frac{P_{Ht}}{\Bigg(\Big(1-\alpha\Big)\ {P_{Ht}}^{(1-\rho)} + \alpha\ {P_{Ft}}^{(1-\rho)} \Bigg)^{\frac{1}{(1-\rho)}}}$$

$$\Leftrightarrow \frac{P_{Ht}}{P_t} = P_{Ht} \Bigg(\Big(1-\alpha\Big)\ {P_{Ht}}^{(1-\rho)} + \alpha\ {P_{Ft}}^{(1-\rho)} \Bigg)^{\frac{1}{(\rho-1)}}$$

$$\Leftrightarrow \frac{P_{Ht}}{P_t} = \Bigg({P_{Ht}}^{(\rho-1)} \Big(1-\alpha\Big)\ {P_{Ht}}^{(1-\rho)} + {P_{Ht}}^{(\rho-1)} \alpha\ {P_{Ft}}^{(1-\rho)} \Bigg)^{\frac{1}{(\rho-1)}}$$

$$\Leftrightarrow \frac{P_{Ht}}{P_t} = \Bigg(\Big(1-\alpha\Big) \left(\frac{P_{Ht}}{P_{Ht}} \right)^{(1-\rho)} + \alpha \left(\frac{P_{Ft}}{P_{Ht}} \right)^{(1-\rho)} \Bigg)^{\frac{1}{(\rho-1)}}$$

$$\Leftrightarrow \frac{P_{Ht}}{P_t} = \Big(1 - \alpha + \alpha\ T_t^{1-\rho} \Big)^{\frac{1}{\rho-1}}.$$

Similarly:

$$\frac{P_{Ft}}{P_t} = \left(\left(1-\alpha\right) T_t{}^{\rho-1} + \alpha\right)^{\frac{1}{\rho-1}}.$$

This imply that the optimal decision of domestic and imported goods can be rewritten with respect to the terms of trade as:

$$C_{Ht} = (1-\alpha)\left(1-\alpha+\alpha\, T_t{}^{1-\rho}\right)^{\frac{\rho}{1-\rho}} C_t. \tag{32}$$

$$C_{Ft} = \alpha\left(\left(1-\alpha\right) T_t{}^{\rho-1} + \alpha\right)^{\frac{\rho}{1-\rho}} C_t. \tag{33}$$

The budget constraint can therefore be rewritten as:

$$(1-\alpha)\left(1-\alpha+\alpha\, T_t{}^{1-\rho}\right)^{\frac{\rho}{1-\rho}} C_t + T_t\, \alpha\left(\left(1-\alpha\right) T_t{}^{\rho-1} + \alpha\right)^{\frac{\rho}{1-\rho}} C_t + q_{Ht}(b_{Ht+1}, T_t)\, b_{Ht+1}\, \Pi_{Ht+1}$$
$$\leq y_t + b_{Ht}$$

$$\Leftrightarrow C_t\left((1-\alpha)\left(1-\alpha+\alpha\, T_t{}^{1-\rho}\right)^{\frac{\rho}{1-\rho}} + T_t\, \alpha\left(\left(1-\alpha\right) T_t{}^{\rho-1} + \alpha\right)^{\frac{\rho}{1-\rho}}\right) + q_{Ht}(b_{Ht+1}, T_t)\, b_{Ht+1}\, \Pi_{Ht+1}$$
$$\leq y_t + b_{Ht}.$$

This finally gives us an expression of C_t:

$$C_t = \frac{y_t + b_{Ht} - q_{Ht}(b_{Ht+1}, T_t)\, b_{Ht+1}\, \Pi_{Ht+1}}{\left((1-\alpha)\left(1-\alpha+\alpha\, T_t{}^{1-\rho}\right)^{\frac{\rho}{1-\rho}} + T_t\, \alpha\left(\left(1-\alpha\right) T_t{}^{\rho-1} + \alpha\right)^{\frac{\rho}{1-\rho}}\right)}. \tag{34}$$

We denote the export of domestic production to the rest of the world X_t.

The market clearing condition for domestic production is:

$$y_t = C_{Ht} + X_t. \tag{35}$$

The trade balance can therefore be written as:

$$tb_t = P_{Ht} X_t - P_{Ft} C_{Ft}.$$

Which according to our definition of the terms of trade can be rewritten as:

$$tb_t = X_t - T_t C_{Ft}. \tag{36}$$

Using the simplified budget constraint (12) and the market clearing condition (35), we can characterized the trade balance in our environment:

$$C_{Ht} + T_t C_{Ft} + q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} = y_t + b_{Ht}$$
$$\Leftrightarrow C_{Ht} + T_t C_{Ft} + q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} = C_{Ht} + X_t + b_{Ht}$$
$$\Leftrightarrow X_t - T_t C_{Ft} = q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} - b_{Ht}.$$

Such that, as before:

$$tb_t = q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1} - b_{Ht}. \tag{37}$$

6.2 Default decisions

Here we assume that the final consumption can be of two types: one when the government does not default and one when the government defaults. We also assume that under the case when the government decides to default, it is excluded from the international credit market and has every period a given probability to regain access to the markets. In addition, in the line of the output loss faced by the government following its default, as modeled by Arellano (2008), we assume that, following its default, the government faces an extra cost in the form of an increase in the price of imports.

Under our terms of trade specification, consumption in each case is characterized by the two following expressions:

- when the government chooses to repay its debt:

$$C_t = \frac{y_t + b_{Ht} - q_{Ht}(b_{Ht+1}, P_{Ft})\, b_{Ht+1}}{\left((1-\alpha)\left(1-\alpha+\alpha\, {T_t}^{1-\rho}\right)^{\frac{\rho}{1-\rho}} + T_t\, \alpha\left(\left(1-\alpha\right)\, {T_t}^{\rho-1}+\alpha\right)^{\frac{\rho}{1-\rho}}\right)}. \tag{38}$$

- when the government chooses to default:

$$C_t = \frac{y_t}{\left((1-\alpha)\left(1-\alpha+\alpha\, {\tilde{T}_t}^{1-\rho}\right)^{\frac{\rho}{1-\rho}} + \tilde{T}_t\, \alpha\left(\left(1-\alpha\right)\, {\tilde{T}_t}^{\rho-1}+\alpha\right)^{\frac{\rho}{1-\rho}}\right)}. \tag{39}$$

  where $\tilde{T}_t \geq T_t$.

The timing of the model is as follows:

1. At the beginning of each period, the government decides whether or not to default on its debt. Let V_t^d denotes the value function when the government chooses to default and V_t^p the value function when the government chooses to repay its debt.

2. Default is optimal whenever $V_t^d > V_t^p$. Every time this condition holds, the government chooses to default and therefore does not pay back its past debt.

3. During the default period, the government is excluded from the international markets and, thus cannot borrow. We also assume that the government faces as another punishment an increase in the price paid for its imports.

The problem is therefore such that the government chooses the optimal policy b_{Ht+1} to maximize utility. The expected value from the next period onward and the bond prices q_{Ht} will incorporate the fact that the government can choose to default in the future.

The state in this economy is represented by a vector $s(b_{Ht}, T_t)$. Before we proceed, it is important to recall that foreign creditors in this economy have access to the international credit market and are subject to a constant international interest rate denoted as $r_t^* > 0$. We make the assumption that these creditors are risk neutral and behave in a competitive manner. Consequently, in each period, we can observe their expected profits, defined as ρ_t:

$$\rho_t = q_{Ht}\, b_{Ht+1} - \frac{(1-\delta_t)}{1+r_t}\, b_{Ht+1} = 0,$$

where δ_t is the probability of default. Notice that when $b_{Ht+1} \geq 0$, the probability of default is zero since the government is saving. This no arbitrage condition implies that the equilibrium price of a discounted bond is given by:

$$q_{Ht} = \frac{(1-\delta_t)}{1+r_t}. \tag{40}$$

The probability of default, δ_t, is endogenous to the model and relies on incentives of the government to repay. The probability of default is a function of both b_{Ht+1} and T_t, which means that q_{Ht} is also a function of those variables. Finally, the gross interest rate is defined as $r_t \equiv 1/q_{Ht} - 1$ and the spread as $s_t \equiv r_t - r_t^*$. With those elements, we can define the recursive competitive equilibrium.

6.3 Recursive competitive equilibrium

Given the state vector $s(b_H, T)$, the government's policy functions b'_H, the price functions q_H, and the consumption functions C, we can determine the equilibrium.

Foreign creditors, being risk-neutral and competitive, determine bond prices based on the following condition:

$$q_H(b'_H, T) = \frac{1-\delta(b'_H, T)}{1+r}.$$

Now, let $V^0(b_H, T)$ be the government's value function at the beginning of the period, with assets b_H and facing terms of trade T. The government decides whether to repay or default to maximize utility represented in (29). The value function $V^0(b_H, T)$ is defined by the following equation:

$$V^0(b_H, T) = \max_{\{C, b'_H\}} \{V^p(b_H, T), V^d(T)\}, \tag{41}$$

where $V^p(b_H, T)$ represents the value function when repaying, and $V^d(T)$ represents the value function when defaulting. When the government defaults, it enters a state of autarky but has a constant and exogenous chance to regain access to credit markets every period.

The value function of defaulting is given by:

$$V^d(T) = u\left(\frac{y}{g(\tilde{T})}\right) + \beta \int_{T'} \Big[\theta\, V^0(0, T') + (1-\theta)\, V^d(T')\Big]\, f(T', T)\, dT', \tag{42}$$

where θ represents the probability of re-entering the credit market and

$$g(\tilde{T}) = \left((1-\alpha)\Big(1-\alpha+\alpha\, \tilde{T}^{1-\rho}\Big)^{\frac{\rho}{1-\rho}} + \tilde{T}\,\alpha\Big(\Big(1-\alpha\Big)\,\tilde{T}^{\rho-1}+\alpha\Big)^{\frac{\rho}{1-\rho}}\right).$$

On the other hand, when the government has not defaulted, and therefore has repaid its debt, the value function $V^p(b_H, T)$ is given by:

$$V^p(b_H, T) = \max_{b'_H} \left\{ u\left(\frac{y + b_H - q_H(b'_H, T)\, b_H}{g(T)} \right) + \beta \int_{T'} V^0(b'_H, T')\, f(T', T)\, dT' \right\}, \tag{43}$$

where

$$g(T) = \left((1-\alpha)\left(1 - \alpha + \alpha\, T^{1-\rho}\right)^{\frac{\rho}{1-\rho}} + T\, \alpha\left(\left(1-\alpha\right) T^{\rho-1} + \alpha\right)^{\frac{\rho}{1-\rho}} \right).$$

Therefore, the government chooses b'_H to maximize utility, with the decision being made period by period. Additionally, to prevent Ponzi schemes, we assume that the government faces a lower bound on debt defined as $b'_H \geq -z$, not binding in equilibrium.

The policy functions can be characterized by the following sets:

$$A(b_H) = \{T \in \mathbb{P} : V^p(b_H, T) \geq V^d(T)\},$$

and

$$D(b_H) \equiv \tilde{A}(b_H) = \{T \in \mathbb{P} : V^p(b_H, T) < V^d(T)\}.$$

where the set $A(b_H)$ represents the optimal set for remaining in the contract, while the set $D(b_H)$ represents the optimal set for default. Given this information, and denoting the aggregate states of the economy as $s(b_H, T)$, we can define a recursive equilibrium as follows:

Definition 2. *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumption $C(s)$; (ii) government's asset holdings $b'_H(s)$, repayment sets $A(b_H)$, and default sets $D(b_H)$; and (iii) the price function for bonds $q_H(b'_H, T)$ such that:*
1. Taking as given the government policies, household consumption $C(s)$ satisfies the resource constraint.
2. Taking as given the bond price function $q_H(b'_H, T)$, the government policy function $b'_H(s)$, repayment sets $A(b_H)$, and default sets $D(b_H)$ satisfy the government optimization problem.
3. Bonds prices $q_H(b'_H, T)$ reflect the government's default probabilities and are consistent with the creditor's expected zero profits.

Furthermore, default probabilities $\delta(b'_H, T)$ and default sets $D(b'_H)$ are related by the following equation:

$$\delta(b'_H, T) = \int_{D(b'_H)} f(T', T)\, dT'. \tag{44}$$

In other words, when $D(b'_H) = \emptyset$, equilibrium default probabilities are zero because the government does not choose to default for any realization of the import prices when it has assets b'_H. Similarly, when $D(b'_H) = \mathbb{P}$, default probabilities are equal to one.

The value of remaining in the contract increases with b_H, while the value of default is independent of b_H. Assuming bounded support for import price shocks, there exists a level of assets low enough for which $D(b'_H) = \mathbb{P}$. However, since default only occurs when assets are negative, there is a level of b_H for which the default set is empty.

From equation (44), we can observe that the equilibrium bond price increases with b'_H, indicating that a low discount price for a large loan compensates lenders for the possibility of default. Additionally, bond prices are also dependent on import price shocks.

6.4 Calibration

Under construction.

6.5 Results for the competitive equilibrium

Under construction.

6.6 Analysing the importance of international conditions

Under construction.

6.7 Analysing the importance of elasticity of substitution

Under construction.

7 Concluding remarks

The conjunction of the COVID-19 crisis, conflict in Ukraine, and surging global commodity food prices has left a harsh impact on the world's poorest countries. A number of these nations confronting imminent food crises are simultaneously burdened by mounting debt. Subsequent to these debt crises, there has been a rapid escalation in costs associated with food imports, notably affecting countries heavily reliant on imports of essentials such as wheat, rice, and maize. Unlike prior debt crises, the current crisis arises from a surge in commodity prices, with import-dependent economies bearing the brunt.

These dynamics predominantly impact countries with significant debt and limited openness. Our study delves into the link between food crises and debt distress.

Through our model, we explore how food crises interact with debt repayment. The sudden surge in food prices, termed a supply shock, can detrimentally impact a nation's capacity to meet its debt obligations. We underscore the need to distinctly examine shifts in food prices, detached from global economic cycles, to discern their direct influence on debt-related issues. To do this distinction, we use the concept of a harvest shock, which plays a crucial role in understanding food crisis impacts. This shock involves unexpected variations in food commodity supply, and it shapes our understanding of how food prices affect economic conditions. This element allows us to isolate and measure the unique impact of food shocks on economies. Applying this methodology to Ghana, which defaulted in 2022, we find three set of results: (i) when commodity prices suddenly rise, we observe an increase in the cost of imported goods and a rise in overall inflation, (ii) price shocks push up the cost of imports, but they do not affect how much Ghana earns from exports, and (iii) price shock does not seem to impact domestic debt, but it does lead to higher external debt. It also causes interest rate spreads to widen. This last piece of evidence shows that Ghana heavily relies on international markets, and the higher interest rates suggest that the likelihood of the country having trouble repaying its debts is increasing.

To better understand this link we build a small open economy model, where a benevolent government of a country dependent on commodity import decides between consumption of a domestic good, an imported one, and how much debt to issue. Our modeling part shows two main results: (i) import-dependent countries with a low debt can default when faced with high price shocks, (ii) highly-indebted countries can be pushed towards the default zone even for small increases in import prices. This last result is of uttermost importance, since it means that when those countries face a food crisis, they are left with no room for action.

In terms of policy implications, our approach provides insights into addressing debt crises triggered by food-related emergencies. For highly indebted countries grappling with a food crisis, international assistance appears as the primary avenue for relief. The design and implementation of effective assistance strategies remain subjects for future research.

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Appendix

A Algorithm

1. Set up the calibrated value of the parameters summarized in the table above and define the grids of assets, which in our case consist of 400 grids equally spaced between -100 and 100, representing respectively the minimum and maximum value of b_H;

2. Discretize the Markov process into a 60- state space vector using the idea of Tauchen & Hussey (1991);

3. Give some initial guess for the bond price schedule such as $q_H^0(b_H, P_F) = 1/(1+r)$;

4. Given $q_H^0(b_H, P_F)$, solve the optimal policies functions for consumption $C(b_H, P_F)$, asset holdings $b'_H(b_H, P_F)$, repayment sets $A(b_H)$, and default sets $D(b_H)$ through value function iteration. Define the consumption in the two states (default and not default), create the initial guess for all value functions involved in the process - default and not default, give some guess for the expected value given current state - expected value when in default, not in default and when the government had defaulted in the previous period and regain the credit markets. Iterate until convergence for a given q_H^0;

5. Using these sets of default and sets of repayment, compute bond price schedule $q_H^1(b_H, P_F)$ such that lenders break even and compare it to the price schedule of the previous iteration $q_H^0(b_H, P_F)$. If a convergence step is met, $\max\{q_H^0(b_H, P_F) - q_H^1(b_H, P_F)\} < \varepsilon$, you are done, otherwise come back to the step 4.

B Elasticity of substitution

From the optimality condition with respect to the consumption basket, we can rewrite:

$$\frac{C_{Ft}}{C_{Ht}} = \frac{\alpha\left(\frac{P_{Ft}}{P_t}\right)^{-\rho} C_t}{(1-\alpha)\left(\frac{P_{Ht}}{P_t}\right)^{-\rho} C_t},$$

which can be simplified to:

$$\frac{C_{Ft}}{C_{Ht}} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{P_{Ft}}{P_{Ht}}\right)^{-\rho}.$$

This can be rewritten as:

$$C_{Ht} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{P_{Ht}}{P_{Ft}}\right)^{-\rho} C_{Ft}.$$

First, we can characterize the relationship between the two goods by computing the cross-elasticity of demand of the domestic product with respect to the foreign product, denoted $\mathbf{A}$:

$$
\begin{aligned}
&\mathbf{A} = \frac{P_{Ft}}{C_{Ht}} \frac{\partial C_{Ht}}{\partial P_{Ft}} \\
&\Leftrightarrow \mathbf{A} = \frac{P_{Ft}}{C_{Ht}}\, C_{Ft}\Big(\frac{1-\alpha}{\alpha}\Big)\Big(-\rho\Big)\Big(\frac{P_{Ht}}{P_{Ft}}\Big)^{-\rho-1}\Big(\frac{-P_{Ht}}{P_{Ft}^2}\Big) \\
&\Leftrightarrow \mathbf{A} = \Big(\frac{C_{Ft}}{C_{Ht}}\Big)\Big(\frac{1-\alpha}{\alpha}\Big)\Big(\rho\Big)\Big(\frac{P_{Ht}}{P_{Ft}}\Big)^{-\rho-1}\Big(\frac{P_{Ht}}{P_{Ft}}\Big) \\
&\Leftrightarrow \mathbf{A} = \Big(\frac{C_{Ft}}{C_{Ht}}\Big)\Big(\frac{1-\alpha}{\alpha}\Big)\Big(\rho\Big)\Big(\frac{P_{Ht}}{P_{Ft}}\Big)^{-\rho-1+1} \\
&\Leftrightarrow \mathbf{A} = \Big(\rho\Big)\Big(\frac{1}{C_{Ht}}\Big)\Big(\frac{1-\alpha}{\alpha}\Big)\Big(\frac{P_{Ht}}{P_{Ft}}\Big)^{-\rho} C_{Ft} \\
&\Leftrightarrow \mathbf{A} = \Big(\rho\Big)\Big(\frac{C_{Ht}}{C_{Ht}}\Big) \\
&\Leftrightarrow \mathbf{A} = \rho.
\end{aligned}
$$

Given that $\rho > 0$, it means that the two goods are substitutes. This implies that for each 1 percent increase in the price of the imports, there is a ρ percent increase in the demand of domestic production. Theoretically, if $\rho \in (0,1)$, then a change in price of foreign good results in a less than proportionate change in quantity demanded for domestic good, while if $\rho > 1$, a change in price of foreign good results in a more than proportionate change in quantity demanded for domestic good.

But, under our assumption of a constant output, given the market clearing condition stating that domestic production is equal to domestic demand, domestic demand is fixed, which imply that a change in price of foreign good has no impact on the demand for domestic good. Therefore, only the price elasticity of the foreign good matter given that only the demand for the foreign good will be affected.

Under the assumptions of constant price and output equal to 1, we can rewrite our previous result as:

$$
C_{Ft} = \Big(\frac{\alpha}{1-\alpha}\Big) P_{Ft}^{-\rho}.
$$

We can characterize the price elasticity of demand for the foreign good, denoted $\mathbf{B}$:

$$
\begin{aligned}
&\mathbf{B} = \frac{P_{Ft}}{C_{Ft}} \frac{\partial C_{Ft}}{\partial P_{Ft}} \\
&\Leftrightarrow \mathbf{B} = \frac{P_{Ft}}{C_{Ft}}\Big(\frac{\alpha}{1-\alpha}\Big)\Big(-\rho\Big) P_{Ft}^{-\rho-1} \\
&\Leftrightarrow \mathbf{B} = \frac{P_{Ft}}{C_{Ft}}\Big(\frac{\alpha}{1-\alpha}\Big)\Big(-\rho\Big) P_{Ft}^{-\rho}\frac{1}{P_{Ft}} \\
&\Leftrightarrow \mathbf{B} = \Big(-\rho\Big)\frac{1}{C_{Ft}}\Big(\frac{\alpha}{1-\alpha}\Big) P_{Ft}^{-\rho} \\
&\Leftrightarrow \mathbf{B} = \Big(-\rho\Big)\frac{C_{Ft}}{C_{Ft}} \\
&\Leftrightarrow \mathbf{B} = -\rho.
\end{aligned}
$$

This implies that for each 1 percent increase in the price of the imports, there is a ρ percent fall in the demand for foreign good.

To make the analysis easier, we can modify the previous equation such as to make appear net imports in value:

$$P_{Ft}\, C_{Ft} = \left(\frac{\alpha}{1-\alpha}\right) P_{Ft}^{1-\rho}.$$

For simplicity, we define $ni_t \equiv P_{Ft}\, C_{Ft}$. We can characterize the price elasticity of demand for net imports in value denoted $\mathbf{C}$:

$$\begin{aligned}
&\mathbf{C} = \frac{P_{Ft}}{ni_t}\frac{\partial ni_t}{\partial P_{Ft}} \\
&\Leftrightarrow \mathbf{C} = \frac{P_{Ft}}{ni_t}\left(\frac{\alpha}{1-\alpha}\right)\Big(1-\rho\Big)P_{Ft}^{-\rho} \\
&\Leftrightarrow \mathbf{C} = \Big(1-\rho\Big)\frac{1}{ni_t}\left(\frac{\alpha}{1-\alpha}\right)P_{Ft}^{1-\rho} \\
&\Leftrightarrow \mathbf{C} = \Big(1-\rho\Big)\frac{ni_t}{ni_t} \\
&\Leftrightarrow \mathbf{C} = \Big(1-\rho\Big).
\end{aligned}$$

This implies that for each 1 percent change in the price of the imports, there is a $1-\rho$ percent change in net imports in value. If $\rho < 1 \implies 1-\rho > 0$, then for each 1 percent increase in the price of the imports, there is a $1-\rho$ percent increase in net imports in value. As the goods are poor substitutes, the increase in price is higher than the reduction in quantity, such that overall the net imports in value will increase. If $\rho > 1 \implies 1-\rho < 0$, then for each 1 percent increase in the price of the imports, there is a $1-\rho$ percent fall in net imports in value.

We can rewrite our previous result with respect to the trade balance:

$$\begin{aligned}
&P_{Ft}\, C_{Ft} = \left(\frac{\alpha}{1-\alpha}\right) P_{Ft}^{-\rho}\, P_{Ft} \\
&\Leftrightarrow P_{Ft}\, C_{Ft} = \left(\frac{\alpha}{1-\alpha}\right) P_{Ft}^{1-\rho}\ .
\end{aligned}$$

We take the derivative of the total import spending with respect to import prices, denoted $\mathbf{D}$:

$$\begin{aligned}
&\mathbf{D} = \frac{\partial(P_{Ft}C_{Ft})}{\partial P_{Ft}} \\
&\Leftrightarrow \mathbf{D} = \left(\frac{\alpha}{1-\alpha}\right)\Big(1-\rho\Big)P_{Ft}^{-\rho}.
\end{aligned}$$

Here, it is worth noticing that the sign of the derivative of total import spending with respect to the import prices will depend on ρ. Given that $(\frac{\alpha}{1-\alpha}) > 0$, if $\rho < 1 \implies 1-\rho > 0$, then $\frac{\partial(P_{Ft}C_{Ft})}{\partial P_{Ft}} > 0$ while if $\rho > 1 \implies 1-\rho < 0$, then $\frac{\partial(P_{Ft}C_{Ft})}{\partial P_{Ft}} < 0$.

This implies that for each 1 percent change in the price of the imports, there is a $1-\rho$ percent change in the import spending. If $\rho < 1 \implies 1-\rho > 0$, then for each 1 percent increase in the price of the imports, there is a $1-\rho$ percent increase in the import spending (i.e., a $(1-\rho)$ percent fall in the trade balance). If $\rho > 1 \implies 1-\rho < 0$, then for each 1 percent increase in the price of the imports, there is a $1-\rho$ percent fall in the import spending (i.e., a $(1-\rho)$

percent increase in the trade balance).

In our model calibration, we consider that the two goods are poor substitutes ($\rho < 1$), which implies, in theory, that the demand for one good is relatively insensitive to changes in the price of the other good. This assumption appears realistic given that the goods we are considering here are food products and cereals such as wheat, rice, and maize. However, we now consider two alternative calibrations: first where the elasticity of substitution is equal to 1, such that the two goods are perfect substitutes, and second where it is higher than 1, such that the two goods are close substitutes.

(Under construction).