

## APEX Calculus

### 1. Limits

- (a) Graphical and Numerical Explorations
- (b)  $\delta$ - $\epsilon$  Proofs
- (c) Finding Limits Analytically Subtitle: Using known limits to find unknown limits. Contains several “big” theorems.
- (d) One Sided Limits and Continuity
- (e) Limits Involving Infinity

### 2. Derivatives

- (a) Introduction Speed during free fall at a particular time; relate this to tangent line problems.
- (b) Basic Differentiation Rules Power ( $n > 0$ ), trig, log, exponential.  
Average rate of change  $\rightarrow$  inst. rate of change  
?? Numerical derivative approximations? (I think above.)  
Higher order derivatives
- (c) Product and Quotient Rules Define rest of the trig functions
- (d) Chain Rule  $f(x) = a^x$
- (e) Implicit Differentiation
- (f) Derivatives of Inverse Functions Especially inverse trig functions

### 3. Graphical Behavior of Functions

- (a) Extreme Value Thm (including definition of critical numbers, etc.)
- (b) Mean Value Theorem (usually includes Rolle’s Thm)
- (c) 1st Deriv. Test (intervals of incr/decr)
- (d) 2nd Deriv Test (concavity explained, etc.)
- (e) Curve Sketching (recall we’ve already talked about limits at infinity, asymptotes, etc.)

### 4. Application of Derivatives

- (a) Newton’s Method (?) This one is good, but not critical to me at the moment
- (b) Related Rates
- (c) Optimization (hopefully with not too trivial examples)
- (d) Differentials (with an eye towards both approximation and a lead-in to integration)

### 5. Fundamental Theorem of Calculus: Integration

- (a) Antiderivatives
- (b) Finite/Riemann Sums – Area Approximations (summation notation, etc.) Left/Right hand sums
- (c) Riemann Sums and Definite Integrals
- (d) Fundamental Theorem of Calculus
- (e) Numerical Integration

### 6. Integration Techniques

- (a) Integration by Substitution
  - i. Integrals Involving Logarithms
  - ii. Integrals Involving Inverse Trigonometric Functions
- (b) Integration by Parts
- (c) Integrals Involving Trigonometric Functions
- (d) Integrals Involving Partial Fraction Decomposition

- (e) Integrals of Hyperbolic Functions
- (f) L'Hopital's Rule
- (g) Improper Integration

#### 7. Applications of the Definite Integral

- (a) Area Between Curves (place earlier?)
- (b) Volume: Disk Method
- (c) Volume: Shell Method
- (d) Arc Length, Surface Area
- (e) Work
- (f) Fluid Force/Pressure
- (g) Center of Mass

#### 8. Sequences and Series

- (a) Sequences
- (b) Series
- (c) Taylor Polynomials