

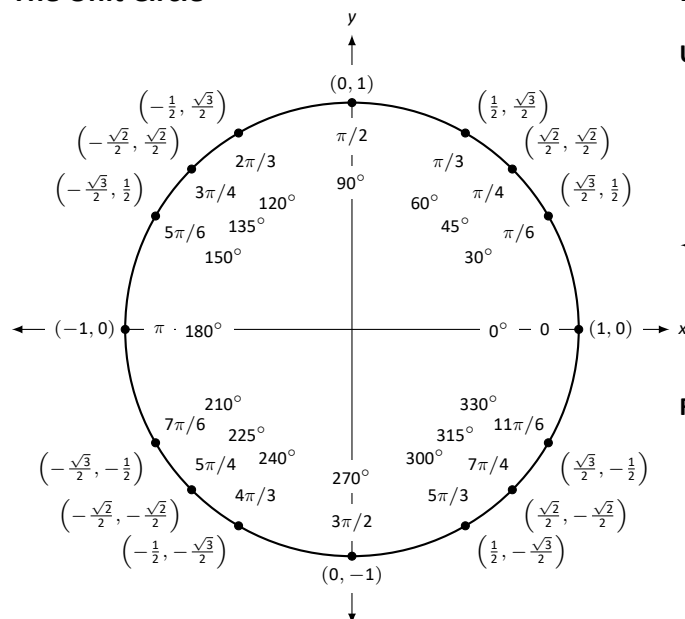
Differentiation Rules

1. $\frac{d}{dx}(cx) = c$
2. $\frac{d}{dx}(u \pm v) = u' \pm v'$
3. $\frac{d}{dx}(u \cdot v) = uv' + u'v$
4. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}(u(v)) = u'(v)v'$
6. $\frac{d}{dx}(c) = 0$
7. $\frac{d}{dx}(x) = 1$
8. $\frac{d}{dx}(x^n) = nx^{n-1}$
9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(a^x) = \ln a \cdot a^x$
11. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
12. $\frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \cdot \frac{1}{x}$
13. $\frac{d}{dx}(\sin x) = \cos x$
14. $\frac{d}{dx}(\cos x) = -\sin x$
15. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
16. $\frac{d}{dx}(\sec x) = \sec x \tan x$
17. $\frac{d}{dx}(\tan x) = \sec^2 x$
18. $\frac{d}{dx}(\cot x) = -\csc^2 x$
19. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
20. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
21. $\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$
22. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$
23. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
24. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
25. $\frac{d}{dx}(\cosh x) = \sinh x$
26. $\frac{d}{dx}(\sinh x) = \cosh x$
27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$
28. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$
29. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$
30. $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$
31. $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$
32. $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$
33. $\frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$
34. $\frac{d}{dx}(\operatorname{csch}^{-1} x) = \frac{-1}{|x|\sqrt{1+x^2}}$
35. $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$
36. $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

Integration Rules

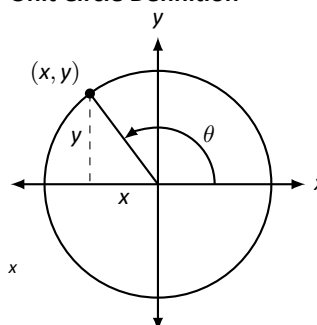
1. $\int c \cdot f(x) dx = c \int f(x) dx$
2. $\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$
3. $\int 0 dx = C$
4. $\int 1 dx = x + C$
5. $\int x^n dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$
6. $\int e^x dx = e^x + C$
7. $\int \ln x dx = x \ln x - x + C$
8. $\int a^x dx = \frac{1}{\ln a} \cdot a^x + C$
9. $\int \frac{1}{x} dx = \ln |x| + C$
10. $\int \cos x dx = \sin x + C$
11. $\int \sin x dx = -\cos x + C$
12. $\int \tan x dx = -\ln |\cos x| + C$
13. $\int \sec x dx = \ln |\sec x + \tan x| + C$
14. $\int \csc x dx = -\ln |\csc x + \cot x| + C$
15. $\int \cot x dx = \ln |\sin x| + C$
16. $\int \sec^2 x dx = \tan x + C$
17. $\int \csc^2 x dx = -\cot x + C$
18. $\int \sec x \tan x dx = \sec x + C$
19. $\int \csc x \cot x dx = -\csc x + C$
20. $\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$
21. $\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$
22. $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$
23. $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
24. $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{|x|}{a}\right) + C$
25. $\int \cosh x dx = \sinh x + C$
26. $\int \sinh x dx = \cosh x + C$
27. $\int \tanh x dx = \ln(\cosh x) + C$
28. $\int \coth x dx = \ln |\sinh x| + C$
29. $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln |x + \sqrt{x^2 - a^2}| + C$
30. $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln |x + \sqrt{x^2 + a^2}| + C$
31. $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$
32. $\int \frac{1}{x\sqrt{a^2 - x^2}} dx = \frac{1}{a} \ln \left(\frac{x}{a + \sqrt{a^2 - x^2}} \right) + C$
33. $\int \frac{1}{x\sqrt{x^2 + a^2}} dx = \frac{1}{a} \ln \left| \frac{x}{a + \sqrt{x^2 + a^2}} \right| + C$

The Unit Circle



Definitions of the Trigonometric Functions

Unit Circle Definition

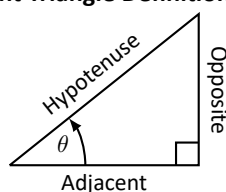


$$\sin \theta = y \quad \cos \theta = x$$

$$\csc \theta = \frac{1}{y} \quad \sec \theta = \frac{1}{x}$$

$$\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}$$

Right Triangle Definition



$$\sin \theta = \frac{O}{H} \quad \csc \theta = \frac{H}{O}$$

$$\cos \theta = \frac{A}{H} \quad \sec \theta = \frac{H}{A}$$

$$\tan \theta = \frac{O}{A} \quad \cot \theta = \frac{A}{O}$$

Common Trigonometric Identities

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \quad \sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Double Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Sum to Product Formulas

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2 \sin\left(\frac{x-y}{2}\right) \cos\left(\frac{x+y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

Power-Reducing Formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Even/Odd Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x$$

$$\sec(-x) = \sec x$$

$$\cot(-x) = -\cot x$$

Product to Sum Formulas

$$\sin x \sin y = \frac{1}{2} (\cos(x-y) - \cos(x+y))$$

$$\cos x \cos y = \frac{1}{2} (\cos(x-y) + \cos(x+y))$$

$$\sin x \cos y = \frac{1}{2} (\sin(x+y) + \sin(x-y))$$

Angle Sum/Difference Formulas

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Areas and Volumes

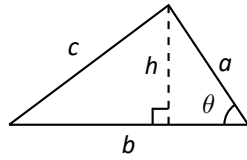
Triangles

$$h = a \sin \theta$$

$$\text{Area} = \frac{1}{2}bh$$

Law of Cosines:

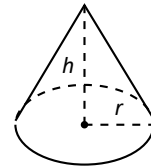
$$c^2 = a^2 + b^2 - 2ab \cos \theta$$



Right Circular Cone

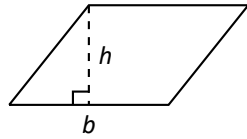
$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Surface Area} = \pi r \sqrt{r^2 + h^2} + \pi r^2$$



Parallelograms

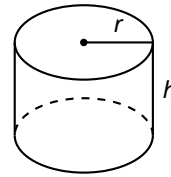
$$\text{Area} = bh$$



Right Circular Cylinder

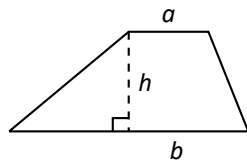
$$\text{Volume} = \pi r^2 h$$

$$\text{Surface Area} = 2\pi rh + 2\pi r^2$$



Trapezoids

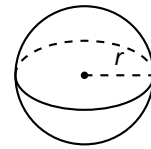
$$\text{Area} = \frac{1}{2}(a + b)h$$



Sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

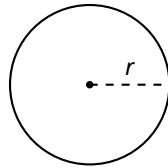
$$\text{Surface Area} = 4\pi r^2$$



Circles

$$\text{Area} = \pi r^2$$

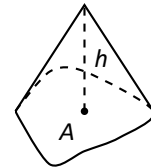
$$\text{Circumference} = 2\pi r$$



General Cone

$$\text{Area of Base} = A$$

$$\text{Volume} = \frac{1}{3}Ah$$

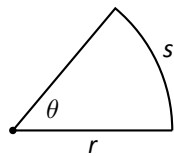


Sectors of Circles

θ in radians

$$\text{Area} = \frac{1}{2}\theta r^2$$

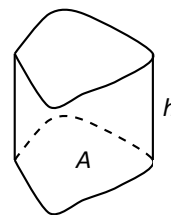
$$s = r\theta$$



General Right Cylinder

$$\text{Area of Base} = A$$

$$\text{Volume} = Ah$$



Algebra

Factors and Zeros of Polynomials

Let $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial. If $p(a) = 0$, then a is a *zero* of the polynomial and a solution of the equation $p(x) = 0$. Furthermore, $(x - a)$ is a *factor* of the polynomial.

Fundamental Theorem of Algebra

An n th degree polynomial has n (not necessarily distinct) zeros. Although all of these zeros may be imaginary, a real polynomial of odd degree must have at least one real zero.

Quadratic Formula

If $p(x) = ax^2 + bx + c$, and $0 \leq b^2 - 4ac$, then the real zeros of p are $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Special Factors

$$\begin{aligned}x^2 - a^2 &= (x - a)(x + a) & x^3 - a^3 &= (x - a)(x^2 + ax + a^2) \\x^3 + a^3 &= (x + a)(x^2 - ax + a^2) & x^4 - a^4 &= (x^2 - a^2)(x^2 + a^2) \\(x + y)^n &= x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + nxy^{n-1} + y^n \\(x - y)^n &= x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \cdots \pm nxy^{n-1} \mp y^n\end{aligned}$$

Binomial Theorem

$$\begin{aligned}(x + y)^2 &= x^2 + 2xy + y^2 & (x - y)^2 &= x^2 - 2xy + y^2 \\(x + y)^3 &= x^3 + 3x^2y + 3xy^2 + y^3 & (x - y)^3 &= x^3 - 3x^2y + 3xy^2 - y^3 \\(x + y)^4 &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 & (x - y)^4 &= x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4\end{aligned}$$

Rational Zero Theorem

If $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients, then every *rational zero* of p is of the form $x = r/s$, where r is a factor of a_0 and s is a factor of a_n .

Factoring by Grouping

$$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d)$$

Arithmetic Operations

$$\begin{aligned}ab + ac &= a(b + c) & \frac{a}{b} + \frac{c}{d} &= \frac{ad + bc}{bd} & \frac{a + b}{c} &= \frac{a}{c} + \frac{b}{c} \\ \left(\frac{a}{b}\right) \left(\frac{c}{d}\right) &= \left(\frac{a}{b}\right) \left(\frac{d}{c}\right) = \frac{ad}{bc} & \left(\frac{a}{b}\right) \frac{c}{c} &= \frac{a}{bc} & \frac{a}{\left(\frac{b}{c}\right)} &= \frac{ac}{b} \\ a \left(\frac{b}{c}\right) &= \frac{ab}{c} & \frac{a - b}{c - d} &= \frac{b - a}{d - c} & \frac{ab + ac}{a} &= b + c\end{aligned}$$

Exponents and Radicals

$$\begin{aligned}a^0 &= 1, \quad a \neq 0 & (ab)^x &= a^x b^x & a^x a^y &= a^{x+y} & \sqrt{a} &= a^{1/2} & \frac{a^x}{a^y} &= a^{x-y} & \sqrt[n]{a} &= a^{1/n} \\ \left(\frac{a}{b}\right)^x &= \frac{a^x}{b^x} & \sqrt[n]{a^m} &= a^{m/n} & a^{-x} &= \frac{1}{a^x} & \sqrt[n]{ab} &= \sqrt[n]{a} \sqrt[n]{b} & (a^x)^y &= a^{xy} & \sqrt[n]{\frac{a}{b}} &= \frac{\sqrt[n]{a}}{\sqrt[n]{b}}\end{aligned}$$

Additional Formulas

Summation Formulas:

$$\sum_{i=1}^n c = cn$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2} \right)^2$$

Trapezoidal Rule:

$$\int_a^b f(x) dx \approx \frac{\Delta x}{2} [f(x_1) + 2f(x_2) + 2f(x_3) + \dots + 2f(x_n) + f(x_{n+1})]$$

$$\text{with Error} \leq \frac{(b-a)^3}{12n^2} [\max |f''(x)|]$$

Simpson's Rule:

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} [f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + \dots + 2f(x_{n-1}) + 4f(x_n) + f(x_{n+1})]$$

$$\text{with Error} \leq \frac{(b-a)^5}{180n^4} [\max |f^{(4)}(x)|]$$

Arc Length:

$$L = \int_a^b \sqrt{1 + f'(x)^2} dx$$

Surface of Revolution:

$$S = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} dx$$

(where $f(x) \geq 0$)

$$S = 2\pi \int_a^b x \sqrt{1 + f'(x)^2} dx$$

(where $a, b \geq 0$)

Work Done by a Variable Force:

$$W = \int_a^b F(x) dx$$

Force Exerted by a Fluid:

$$F = \int_a^b w d(y) \ell(y) dy$$

Taylor Series Expansion for $f(x)$:

$$p_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

Maclaurin Series Expansion for $f(x)$, where $c = 0$:

$$p_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{(n)}(0)}{n!}x^n$$

Summary of Tests for Series:

| Test | Series | Condition(s) of Convergence | Condition(s) of Divergence | Comment |
|--------------------|--|--|--|--|
| n th-Term | $\sum_{n=1}^{\infty} a_n$ | | $\lim_{n \rightarrow \infty} a_n \neq 0$ | This test cannot be used to show convergence. |
| Geometric Series | $\sum_{n=0}^{\infty} r^n$ | $ r < 1$ | $ r \geq 1$ | $\text{Sum} = \frac{1}{1-r}$ |
| Telescoping Series | $\sum_{n=1}^{\infty} (b_n - b_{n+a})$ | $\lim_{n \rightarrow \infty} b_n = L$ | | $\text{Sum} = \left(\sum_{n=1}^a b_n \right) - L$ |
| p -Series | $\sum_{n=1}^{\infty} \frac{1}{(an+b)^p}$ | $p > 1$ | $p \leq 1$ | |
| Integral Test | $\sum_{n=0}^{\infty} a_n$ | $\int_1^{\infty} a(n) \, dn$ is convergent | $\int_1^{\infty} a(n) \, dn$ is divergent | $a_n = a(n)$ must be continuous |
| Direct Comparison | $\sum_{n=0}^{\infty} a_n$ | $\sum_{n=0}^{\infty} b_n$ converges and $0 \leq a_n \leq b_n$ | $\sum_{n=0}^{\infty} b_n$ diverges and $0 \leq b_n \leq a_n$ | |
| Limit Comparison | $\sum_{n=0}^{\infty} a_n$ | $\sum_{n=0}^{\infty} b_n$ converges and $\lim_{n \rightarrow \infty} a_n/b_n \geq 0$ | $\sum_{n=0}^{\infty} b_n$ diverges and $\lim_{n \rightarrow \infty} a_n/b_n > 0$ | Also diverges if $\lim_{n \rightarrow \infty} a_n/b_n = \infty$ |
| Ratio Test | $\sum_{n=0}^{\infty} a_n$ | $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} < 1$ | $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} > 1$ | $\{a_n\}$ must be positive Also diverges if $\lim_{n \rightarrow \infty} a_{n+1}/a_n = \infty$ |
| Root Test | $\sum_{n=0}^{\infty} a_n$ | $\lim_{n \rightarrow \infty} (a_n)^{1/n} < 1$ | $\lim_{n \rightarrow \infty} (a_n)^{1/n} > 1$ | $\{a_n\}$ must be positive Also diverges if $\lim_{n \rightarrow \infty} (a_n)^{1/n} = \infty$ |