3. We find the possible points of inflection by solving $f''(x) = 0$ for x. We find
$$f''(x) = -\frac{30x^3 + 180x^2 - 240}{(x^2 + 2x + 4)^3}.$$
The cubic in the numerator does not factor very "nicely." We instead approximate the roots at $x = -5.759$, $x = -1.305$ and $x = 1.064$.

4. There are no vertical asymptotes.

5. We have a horizontal asymptote of $y = 5$, as $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to\infty} f(x) = 5$.

6. We place the critical points and possible points on a number line as shown in Figure 1.42 and mark each interval as increasing/decreasing, concave up/down appropriately.

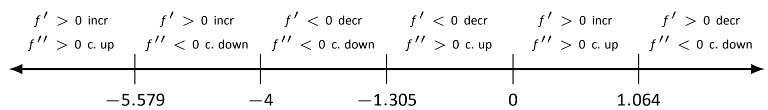

Figure 1.42: Number line for f in Example 19.

7. In Figure 1.43(a) we plot the significant points from the number line as well as the two roots of f, $x = -1$ and $x = 2$, and connect the points with straight lines to get a general impression about the graph. In Figure 1.43(b), we add concavity. Figure 1.43(c) shows a computer generated graph of f, affirming our results.

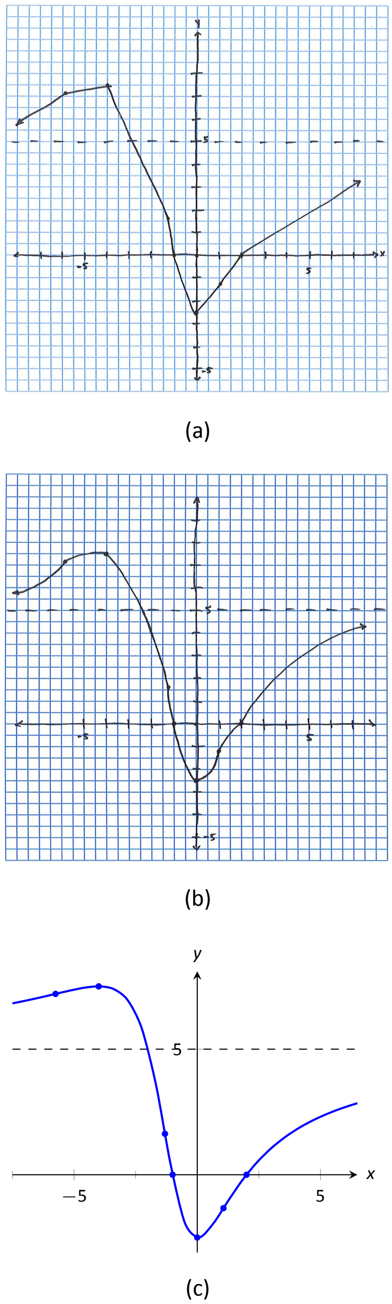

Figure 1.43: Sketching f in Example 19.

In each of our examples, we found a few, significant points on the graph of f that corresponded to changes in increasing/decreasing or concavity. We connected these points with straight lines, then adjusted for concavity, and finished by showing a very accurate, computer generated graph.

Why are computer graphics so good? It is not because computers are "smarter" than we are. Rather, it is largely because computers are much faster at computing than we are. In general, computers graph functions much like most students do when first learning to draw graphs: they plot equally spaced points, then connect the dots using lines. By using lots of points, the connecting lines are short and the graph looks smooth.

This does a fine job of graphing in most cases (in fact, this is the method used for many graphs in this text). However, in regions where the graph is very "curvy," this can generate noticeable sharp edges on the graph unless a large number of points are used. High quality computer algebra systems, such as

Notes: