Regular languages

Names: Diego Araque y Daniel Sanchez

1. Give a recursive definition of the set of strings over {*a*, *b*} that contain at least one *b* and have an even number of *a*'s before the first *b*. For example: *aab*, *bab*, and *aaaabbabababa* are in the set, but *abb*, *aaab* do not

Basis: $b \in \Sigma$

Recursive steps: If z is in Σ, then aaz and za is in Σ

Closure: z can only be obtained from the basis by a finite number of applications

2. Let **X** = {*aa*, *bb*} and **Y** = {**λ**, *b*, *ab*}.
   I. List the strings in set **XY**

XY = {aa, aab, aaab, bb, bbb, bbab}

   II. How many strings of length 6 are there in **X***?

There're 2ˆ3 = 8 strings of length 6.

   III. List the strings in set **Y*** of length three or less

Length 0: {**λ**}
Length 1: {b, ab}
Length 2: {bb, bab, abab, abb}
Length 3: {bbb, bbab, babb, abbb, ababb, ababab, abbab, babab}

From this selection the only ones from length 3 or less are: {**λ**, b, ab, bb, bab, abb, bbb}

   IV. List the strings in set **X*Y*** of length four or less

Y* = {**λ**, b, ab, bb, bab, abb, bbb, bbab, babb, abbb, abab, bbbb}

X* = {**λ**, aa, bb, aaaa, aabb, bbaa, bbbb}

X*Y* = { **λ**, aa, bb, aaaa, aabb, bbaa, bbbb, b, ab, bb, bab, abb, bbb, bbab, babb, abbb, abab, bbbb }

3. Give a recursive definition of the set $\{a^i b^j \mid 0 \le i \le j \le 2i\}$

Basis: λ ∈ Σ
Recursive step: if u is in Σ, then aub and aubb are in Σ
Closure: u can only be obtained from the basis by a finite number of applications

4. Let **L** be the set of strings over $\{a, b\}$ generated by the recursive definition
   I. Basis: $b \in \mathbf{L}$
   II. Recursive step: if u is in **L** the $ub \in \mathbf{L}$, $uab \in \mathbf{L}$, $uba \in \mathbf{L}$ and $bua \in \mathbf{L}$
   III. Closure: a string v is only in **L** if it can be obtained from the basis by a finite number of iterations of the recursive step
   a. List the elements in the sets $\mathbf{L_0}$, $\mathbf{L_1}$, $\mathbf{L_2}$

L_0 : {b}
L_1: {bb, bab, bba}
L_2: {bbb, bbab, bbba, bbba
babb, babab, babba, bbaba
bbab, bbaab, bbaba, bbbaa}

   b. Is the string *bbaaba* in **L**? If so, trace how it is produced. If not, explain why not.

In the set L_2 we have the string bbaab, whis is part of the strin in question. But bbaaba is not in our language because we cannot add an a to that string, because in this language everything starts with a b.

   c. Is the string *bbaaaabb* in **L**? If so, trace how it is produced. If not, explain why not.

In our language is impossible to have more than 2 consecutive a's, therefore it's impossible to have the string in question in the language.

5. Prove, using induction on the length of a string, that $(w^R)^R = w$ for all string $w \in \mathbf{\Sigma}$

$(w^R)^R = (x^R z^R)^R$

$= (x^R z)^R$ since z is just one letter it's reverse is the same

$= (x^R)^R z^R$

$= xz$ the reverse of the reverse of x is equal to x

$= w$ we end up having the same string, the reverse of the reverse of a string is the same string