UNAM
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Evolutionary Computation

Design of a multiobjetive system with Lagrange Multipliers using a restriction function of costs

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1 Introduction

Nowadays, big corporations have a vast division of departments that carry out different tasks. Banks, in particular, have sub-departments depending on the service they offer. For example, they have the legal department, the collecting money department, the loan department, the insurance department, the one that deals with matters of government, logistics, etc. Each department has to make decisions based on studies and data analysis and according to economics, social context, political, financial, etc. Therefore, the company that I propose is a consulting firm for banks that uses evolutionary computation to generate decision-making sets that allow banks to increase their effectiveness in any department that relies on data analysis for their decision-making.

The final project that I present is a compilation of the algorithms developed in task 3 altogether with an implementation of Non-dominated-sorting to generate the Pareto frontiers for the selection of elements. The genetic algorithm is now a multi-objective algorithm that uses a segmentation algorithm (K-Means) to generate clusters from the initial population and perform an analysis on each cluster. This is intended to find more local optima. It also implements parallelism with multithreading so that, simultaneously, each data cluster is analyzed with a computer processor and decrease its analysis time.

For the analysis of the problem presented below, the goal is to optimize a set of functions under a constraint function which I called the budget constraint function. Each function in the set of objective functions refers to a spending function, and what you want to optimize is the expected spending by different departments (insurance, mortgage, and loans), therefore, we have to sum all the objetive functions to get the expected cost.

The infrastructure of the system will continue to be developed after delivering this work since this final project was intended to emphasize more the theory that I implemented.

The competitive advantages proposed in this company are those that refer to the use of technologies in the analysis and construction of solutions such as Amazon Web Services for the treatment of data in the cloud as well as its processing, Artificial Intelligence algorithms to analyze data and the design of graphic web interfaces for the future visualization of the results of the optimization algorithms. On the other hand, a staff with skills in Mathematical, Actuarial and Computer Sciences is projected for a transdisciplinary team that aims to solve these particular optimization problems for companies such as banks or industries.

2 Implementation of the Lagrange Multipliers Method

2.1 Idea

Lagrange multipliers is a calculation and optimization method that seeks to optimize a real function using the vector of gradients and the Lagrange function. This method is used when you have a function $g(x)$ that denotes a constraint under which you want to optimize the function $f(x)$. Therefore, for the purposes of this work, we assume that we have a bank as a client and it wishes to optimize the expected expenses of life insurance, home insurance, health insurance, personal loans, business loans and property mortgages.

Information that is given by the client (bank):

$$\begin{aligned} f_{T|V} &\sim Exp(\lambda = f(x)) \\ f_{T|C} &\sim Exp(\lambda = g(y)) \\ f_{T|S} &\sim Exp(\lambda = h(z)) \\ f_{G|P} &\sim \Gamma(m(u), n(u)) \\ f_{G|E} &\sim \Gamma(a(x), b(x)) \\ f_H &\sim N(r(x), t(x)) \end{aligned}$$

Where T denotes the cost of insurance, $T|V$ is the cost of insurance given that it is life insurance, $T|C$ is the cost of insurance given that it is for each and $T|S$ is the cost of insurance since it is health. On the other hand, $G|P$ refers to the expense since it is a personal loan and $G|E$ is an expense in business loans. These functions are given to us by a financial entity that wants to optimize the expected expenses, so to calculate the expected expense of these random variables we consider the following equations:

$$\mathbb{E}[\textbf{Gasto total}] = \mathbb{E}[T] + \mathbb{E}[G] + \mathbb{E}[H]$$

Using the mathematical expectation, we get the following equations:

$$\mathbb{E}[T] = \mathbb{E}[T|V]\mathbb{P}[V] + \mathbb{E}[T|C]\mathbb{P}[C] + \mathbb{E}[T|S]\mathbb{P}[S]$$

$$\mathbb{E}[G] = \mathbb{E}[G|P]\mathbb{P}[P] + \mathbb{E}[G|E]\mathbb{P}[E]$$

We know the distributions of the conditional random variables, their expectations, and now the client offers us the following information:

$$\mathbb{P}[V] = \frac{2}{5}, \mathbb{P}[C] = \frac{1}{5}, \mathbb{P}[S] = \frac{2}{5}, \mathbb{P}[P] = \frac{2}{9}, \mathbb{P}[E] = \frac{7}{9}$$

And the functions that parameterize the distribution functions of the conditional random variables are the following:

$$\begin{aligned} f(x) = \frac{x^2}{2}\ ; g(y) = (y^2 + 0.5y)^{-1};\ \ h(z) = 0.5z \\ m(u) = 3u + 1;\ \ n(u) = u^3 - 1; a(x) = x; \\ b(x) = 0.5x^2;\ \ r(x) = x^2 + 1 = w(x) \end{aligned}$$

The bank asks us that by calculating the expected values, we optimize the variables x, y, z, u ergo, optimize expected expense subject to the following condition:

$$x + y + z + u \leq 1500$$

If we consider the objective function, which is the sum of the expected values of the expenses (we assume that we know how to calculate the expected value for an Exponential, a Gamma and a Normal distributions):

$$f_{obj} = \frac{2}{5}\frac{2}{x^2} + \frac{1}{5}y^2 + \frac{1}{10}y + \frac{4}{5z} + \frac{2}{9}(\frac{3u+1}{u^3-1}) + \frac{14}{9x} + x^2 + 1$$

Using Lagrange's Method in Evolutionary Computing

We want to optimize the following function f_obj considering the restricción:

$$g(x) = x + y + z + u \leq 1500$$

Now we define the Lagrange Multiplier as follows:

$$\mathscr{L}(x, y, z, u, \lambda) = \frac{2}{5}\frac{2}{x^2} + \frac{1}{5}y^2 + \frac{1}{10}y + \frac{4}{5z} + \frac{2}{9}(\frac{3u+1}{u^3-1}) + \frac{14}{9x} + x^2 + 1 - \lambda(x + y + z + u - 1500)$$

What we are looking for is to optimize this Lagrange multiplier using the gradient vector of the multiplier; looking for vectors that make it converge to zero. As there is the possibility of going to numbers less than 0 since we are "minimizing", the absolute value will be added to each partial derivative so that when it is minimized, the vectors that found by the evolutionary algorithm make each partial derivative converge to zero.

$$|\frac{\partial \mathscr{L}}{\partial x}| = |\frac{-8}{5}x^{-3} + 2x - \frac{14}{9}x^{-2} - \lambda|$$
$$|\frac{\partial \mathscr{L}}{\partial y}| = |\frac{2}{5}y + \frac{1}{10} - \lambda|$$
$$|\frac{\partial \mathscr{L}}{\partial z}| = |\frac{-4}{5}z^{-2} - \lambda|$$
$$|\frac{\partial \mathscr{L}}{\partial u}| = |\frac{2}{9}[\frac{3u^3 - 3 - 3u^2(3u+1)}{(u^3-1)^2}] - \lambda|$$
$$|\frac{\partial \mathscr{L}}{\partial \lambda}| = | - (x + y + z + u - 1500)|$$

Therefore, to optimize simultaneously, we use a multi-objective genetic algorithm where each objective function is the absolute value of each partial derivative from the Lagrange multiplier. Since we are minimizing an absolute value, we will look for values in its domain that make each partial converge to 0 and we will be able to solve our problem.

The goal of this project is to study how the Lagrange multipliers applied to a multiobjective evolutionary algorithm with a budget constraint function. These kind of solutions are those intended to be developed in the company that I'm proposing. In addition, different data analysis methods can be added to have a better overview of the problem that want to be optimized.

The resources of the company would be the acquisition of servers in the cloud in order to process our data in less time and also all the information necessary to carry out the solutions of this kind of problems. The amazon cloud and google tools for the delivery of results and continuous communication with the client.