Algorithms

Robert Sedgewick | Kevin Wayne

http://algs4.cs.princeton.edu

Geometric Applications of BSTs

- 1d range search
- line segment intersection
- kd trees
- interval search trees
- rectangle intersection

2-d orthogonal range search

Extension of ordered symbol-table to 2d keys.

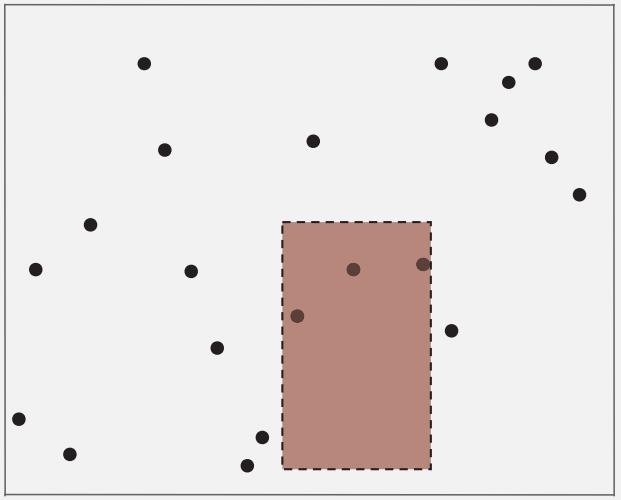
- Insert a 2d key.
- Delete a 2d key.
- Search for a 2d key.
- Range search: find all keys that lie in a 2d range.
- Range count: number of keys that lie in a 2d range.

Applications. Networking, circuit design, databases, ...

Geometric interpretation.

- Keys are point in the plane.
- Find/count points in a given h–v rectangle

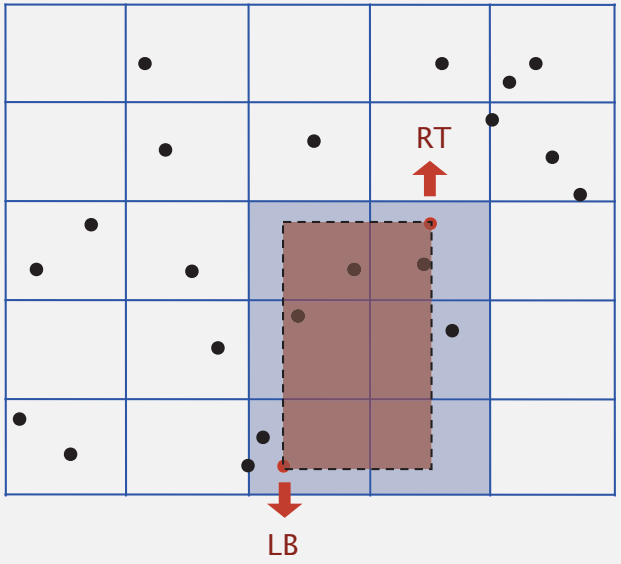
rectangle is axis-aligned



2d orthogonal range search: grid implementation

Grid implementation.

- Divide space into M-by-M grid of squares.
- Create list of points contained in each square.
- Use 2d array to directly index relevant square.
- Insert: add (x, y) to list for corresponding square.
- Range search: examine only squares that intersect 2d range query.



2d orthogonal range search: grid implementation analysis

Space-time tradeoff.

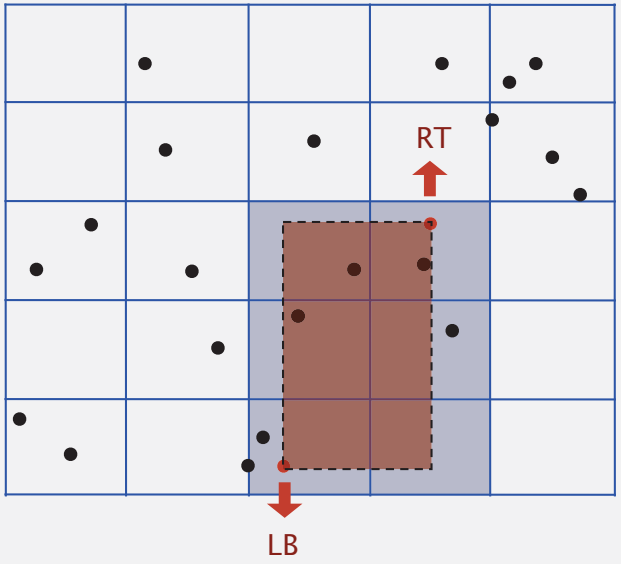
- Space: $M^2 + N$.
- Time: $1 + N / M^2$ per square examined, on average.

Choose grid square size to tune performance.

- Too small: wastes space.
- Too large: too many points per square.
- Rule of thumb: $\sqrt{N}$-by-$\sqrt{N}$ grid.

Running time. [if points are evenly distributed]

- Initialize data structure: N.
- Insert point: 1.
- Range search: 1 per point in range.

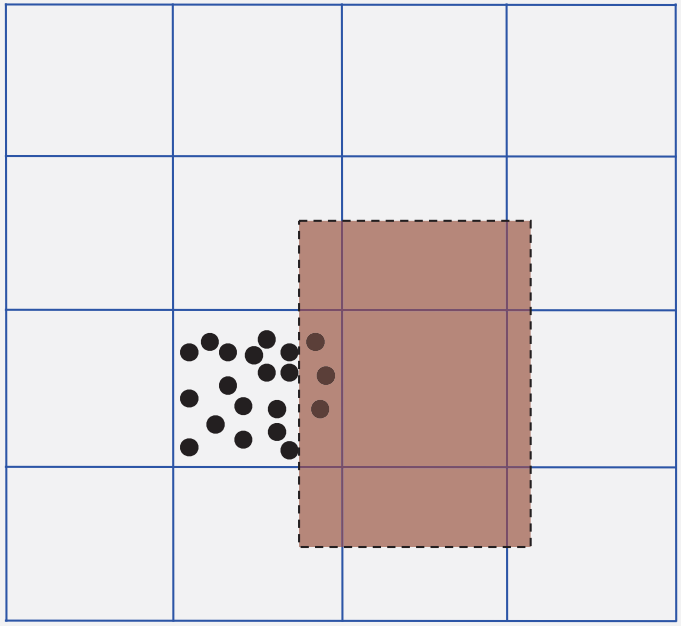


Clustering

Grid implementation. Fast, simple solution for evenly-distributed points.

Problem. Clustering a well-known phenomenon in geometric data.

- Lists are too long, even though average length is short.
- Need data structure that adapts gracefully to data.

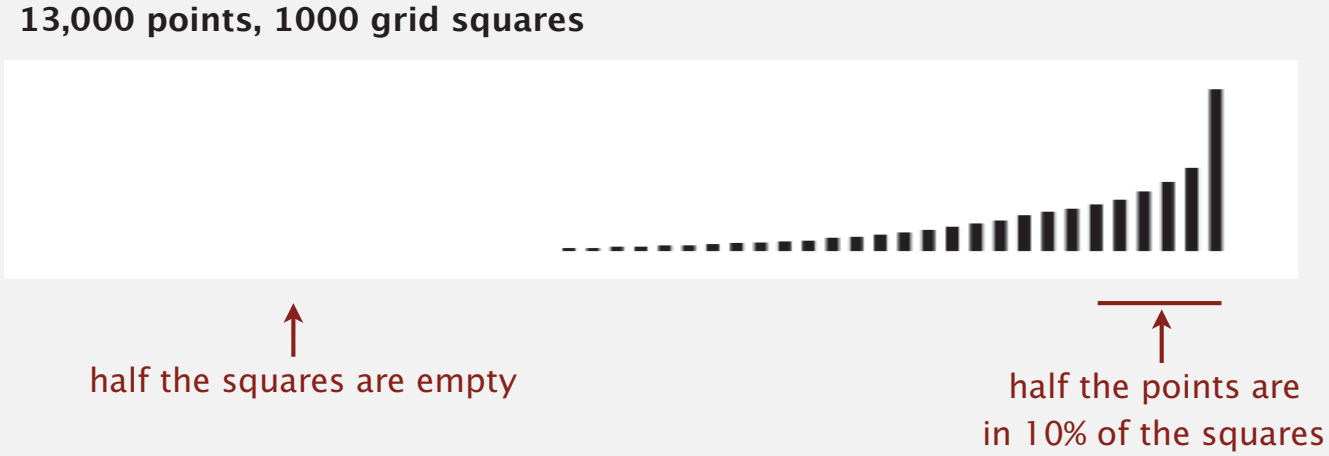


Clustering

Grid implementation. Fast, simple solution for evenly-distributed points.

Problem. Clustering a well-known phenomenon in geometric data.

Ex. USA map data.



Space-partitioning trees

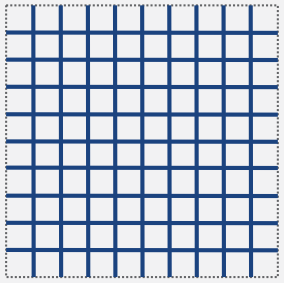
Use a tree to represent a recursive subdivision of 2d space.

Grid. Divide space uniformly into squares.

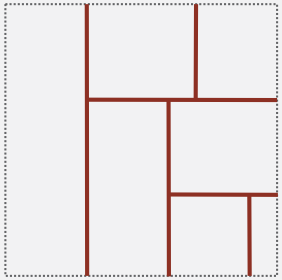
2d tree. Recursively divide space into two halfplanes.

Quadtree. Recursively divide space into four quadrants.

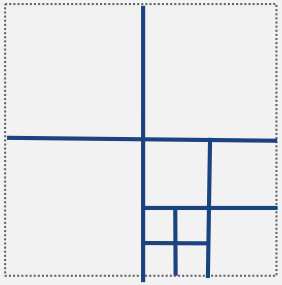
BSP tree. Recursively divide space into two regions.



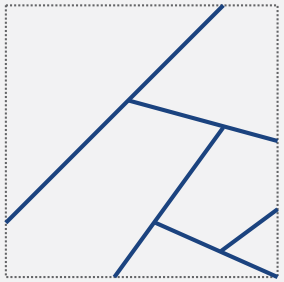
Grid



2d tree



Quadtree

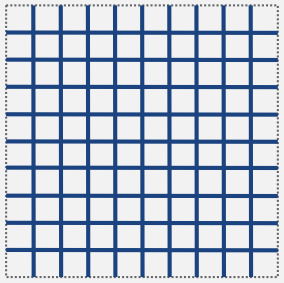


BSP tree

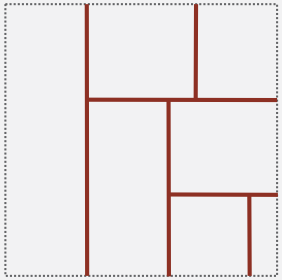
Space-partitioning trees: applications

Applications.

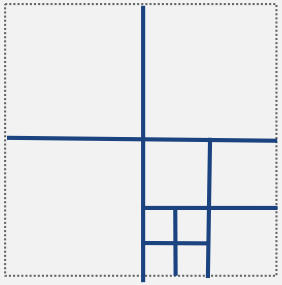
- Ray tracing.
- 2d range search.
- Flight simulators.
- N-body simulation.
- Collision detection.
- Astronomical databases.
- Nearest neighbor search.
- Adaptive mesh generation.
- Accelerate rendering in Doom.
- Hidden surface removal and shadow casting.



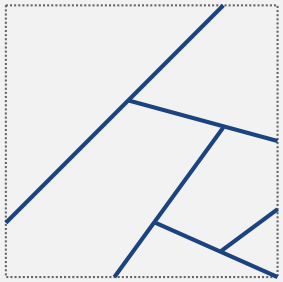
Grid



2d tree



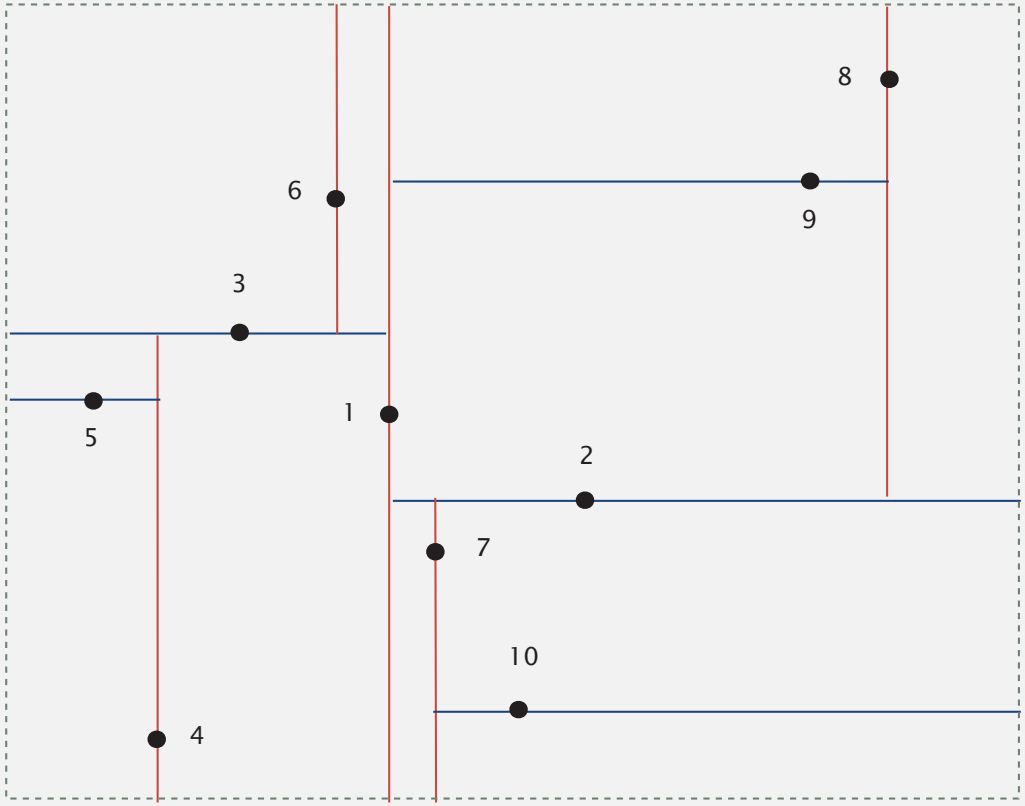
Quadtree

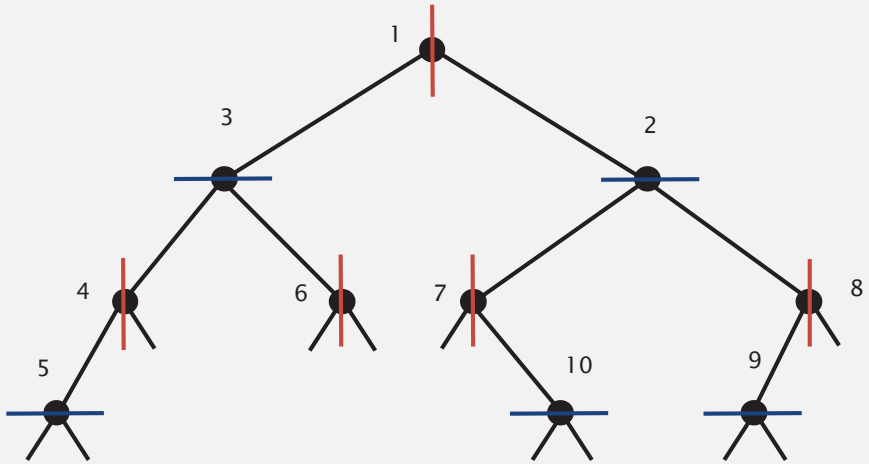


BSP tree

2d tree construction

Recursively partition plane into two halfplanes.

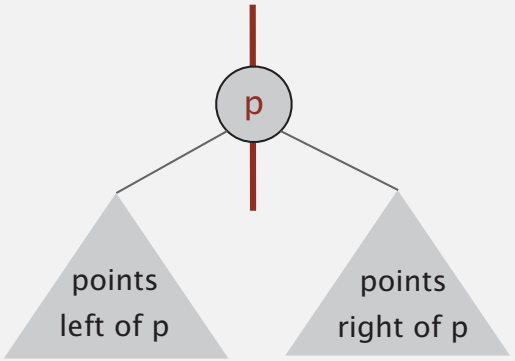


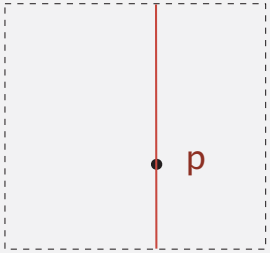


2d tree implementation

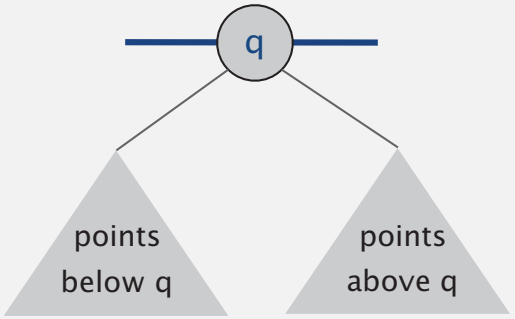
Data structure. BST, but alternate using x- and y-coordinates as key.

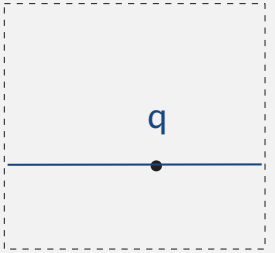
- Search gives rectangle containing point.
- Insert further subdivides the plane.



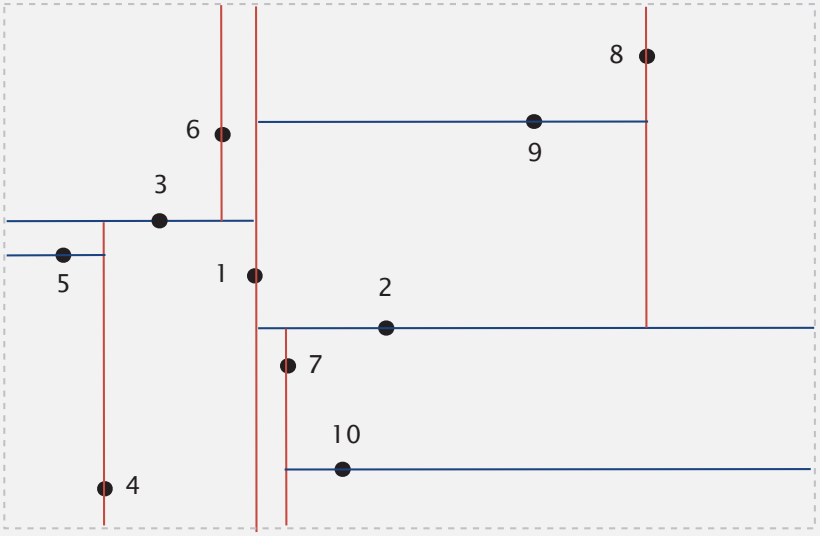


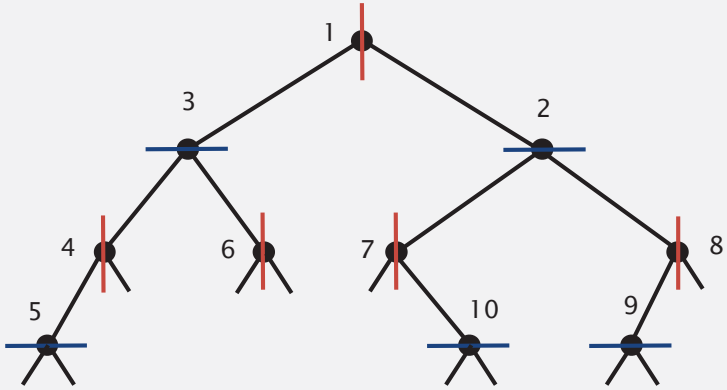
even levels





odd levels

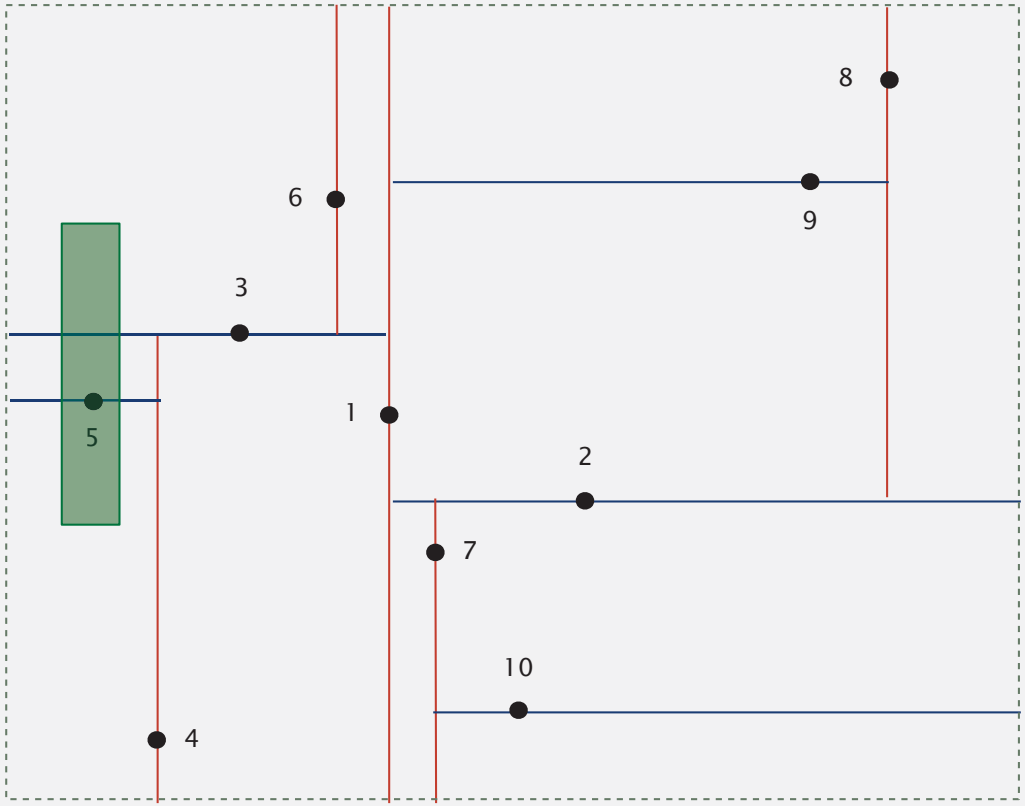


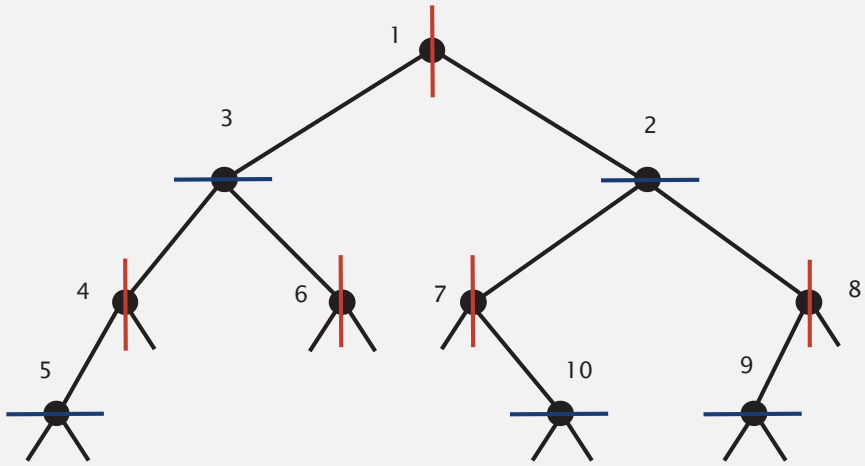


2d tree demo: range search

Goal. Find all points in a query axis-aligned rectangle.

- Check if point in node lies in given rectangle.
- Recursively search left/bottom (if any could fall in rectangle).
- Recursively search right/top (if any could fall in rectangle).





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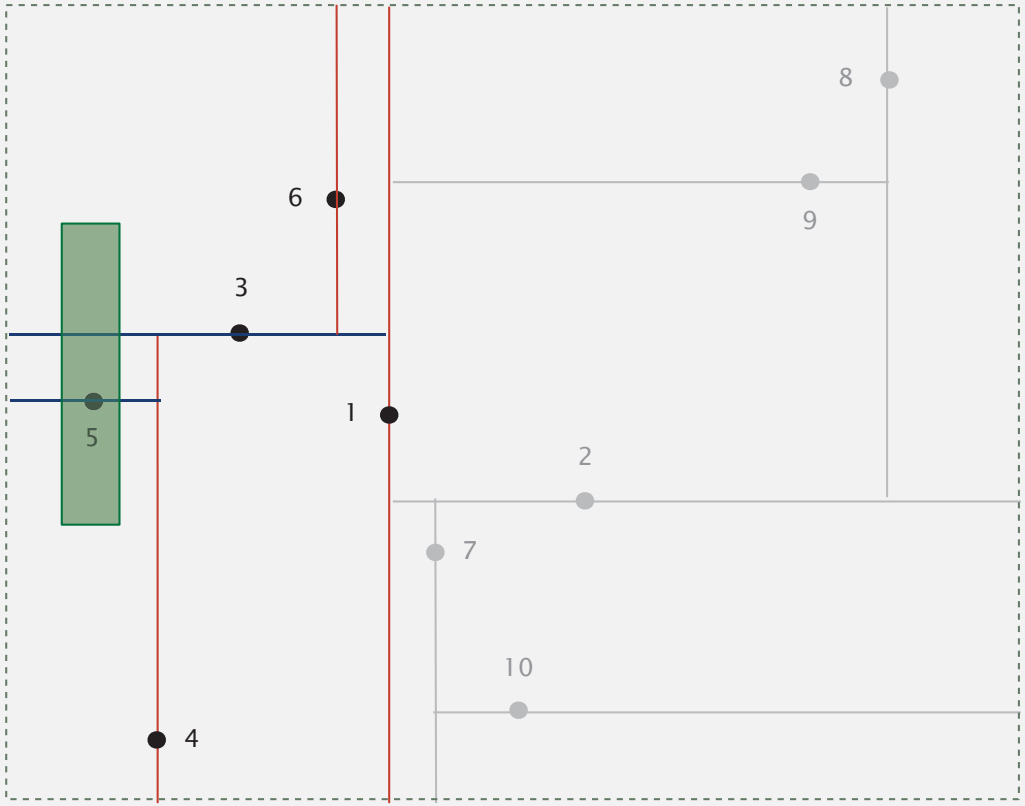
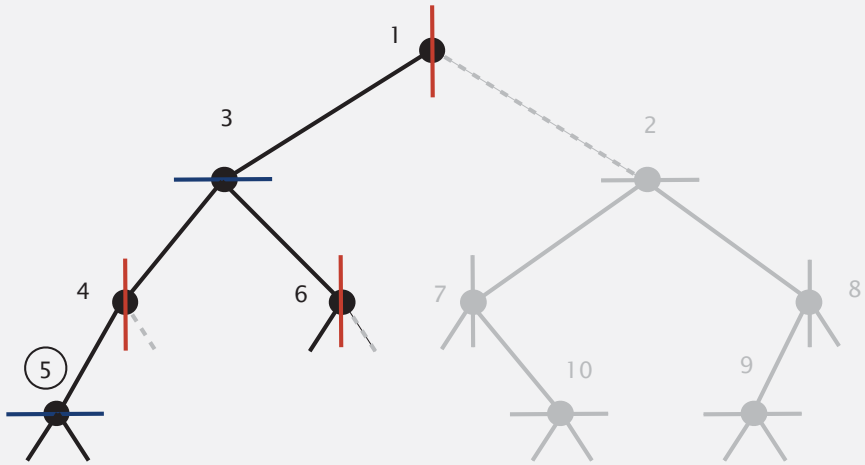


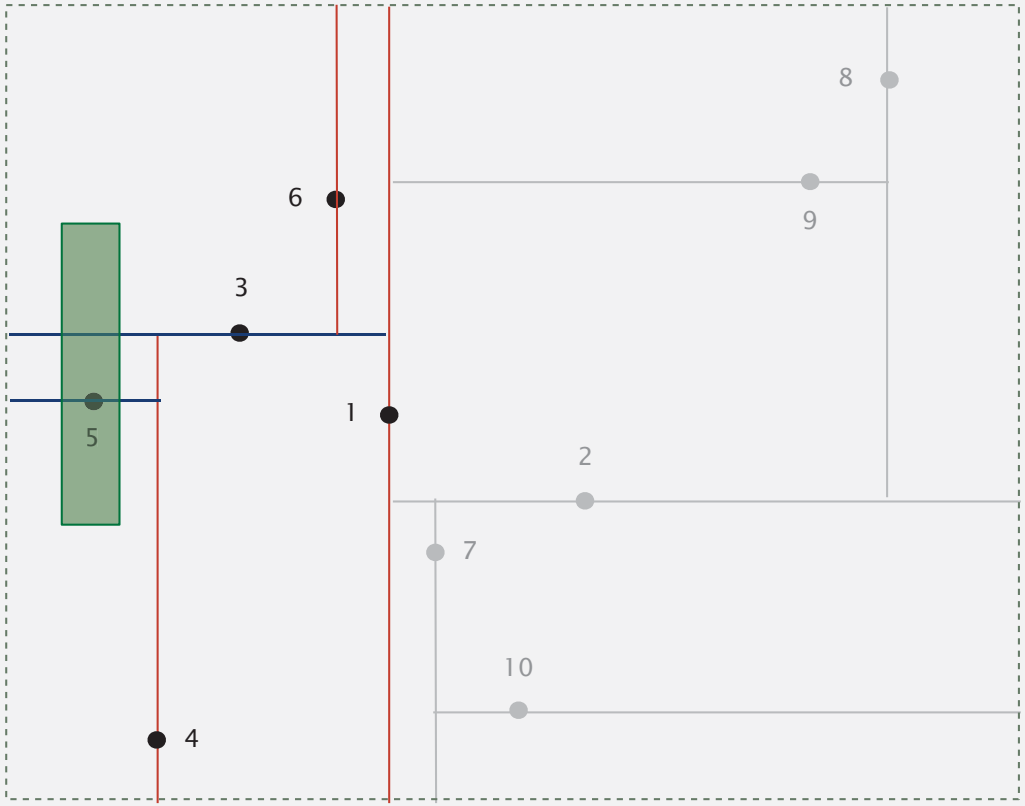


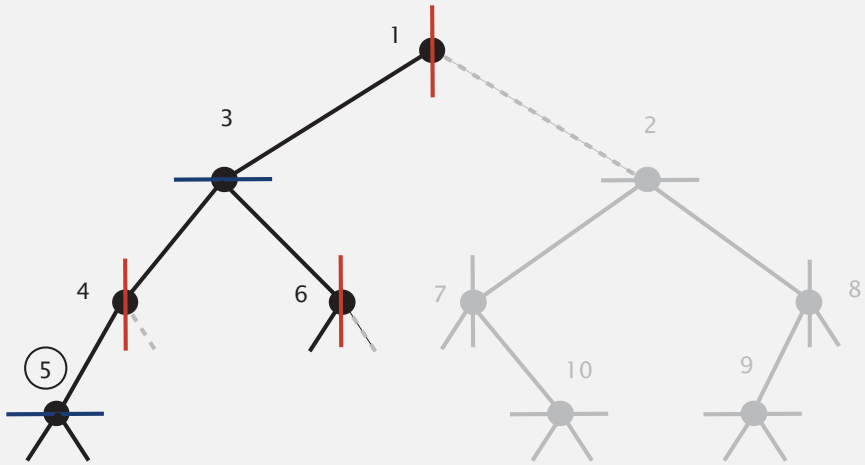
done

Range search in a 2d tree analysis

Typical case. $R + \log N$.

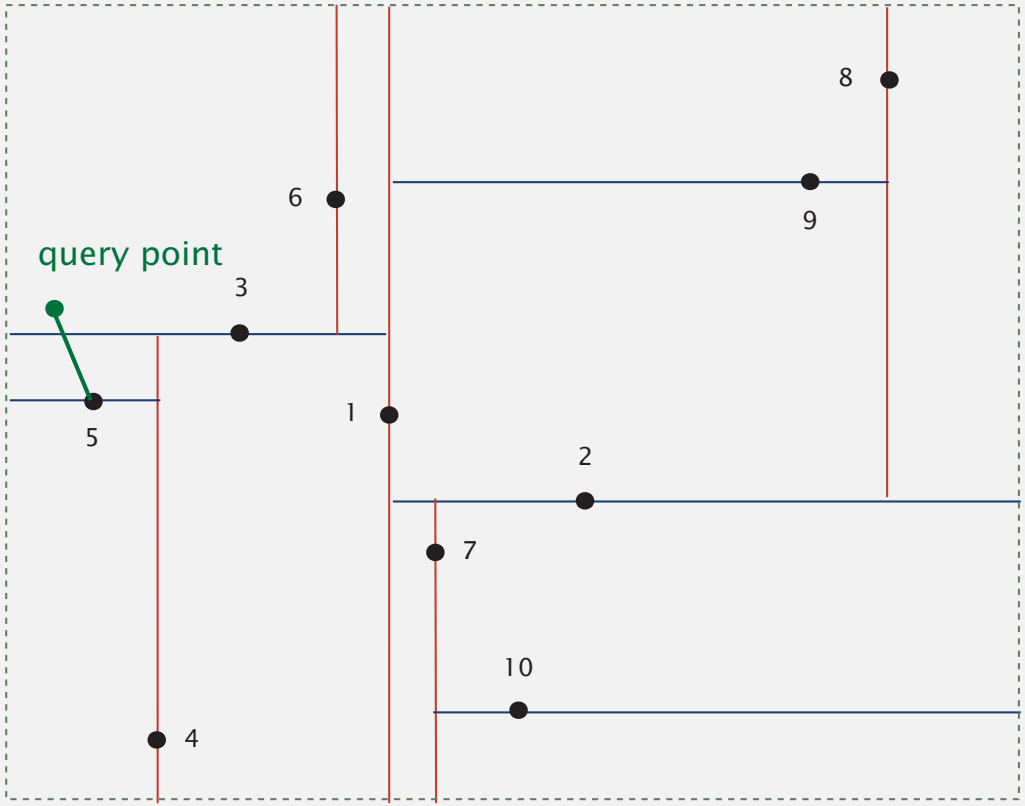
Worst case (assuming tree is balanced). $R + \sqrt{N}$.

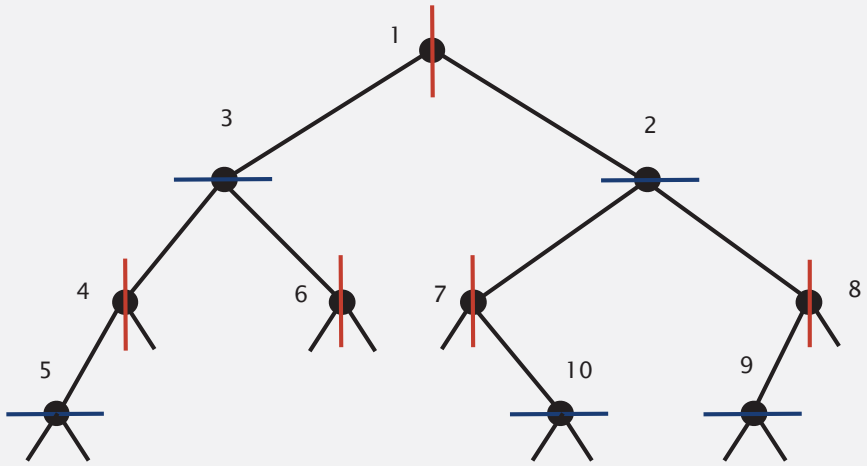




2d tree demo: nearest neighbor

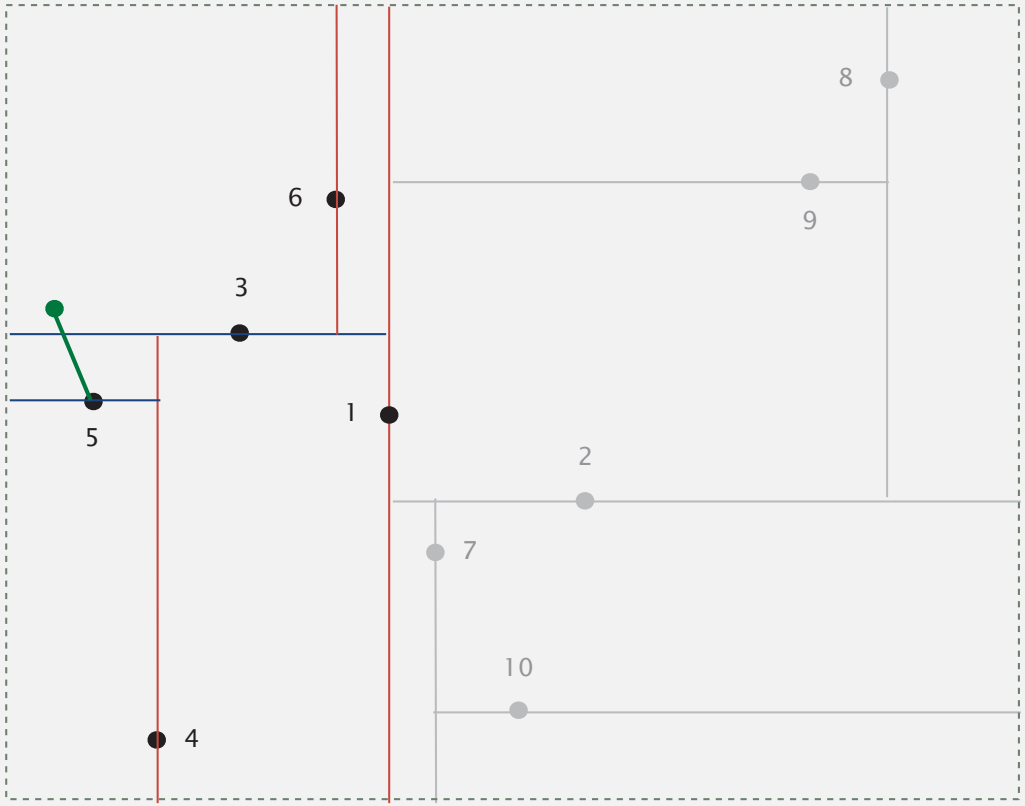
Goal. Find closest point to query point.

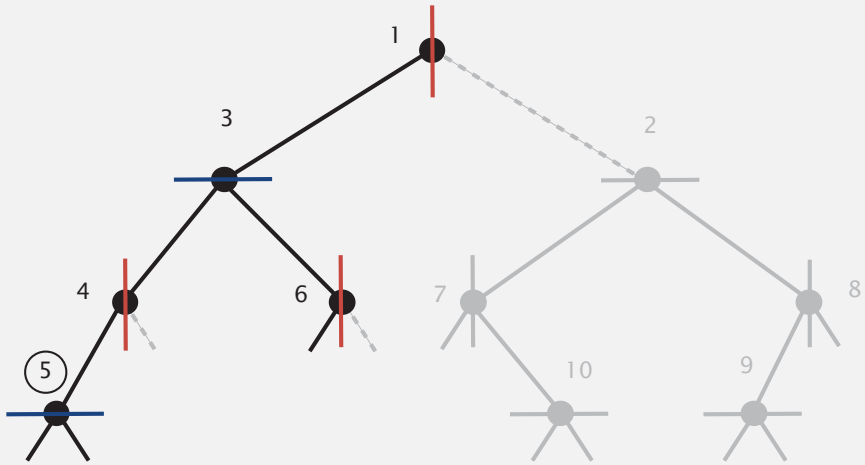




2d tree demo: nearest neighbor

- Check distance from point in node to query point.
- Recursively search left/bottom (if it could contain a closer point).
- Recursively search right/top (if it could contain a closer point).
- Organize method so that it begins by searching for query point.



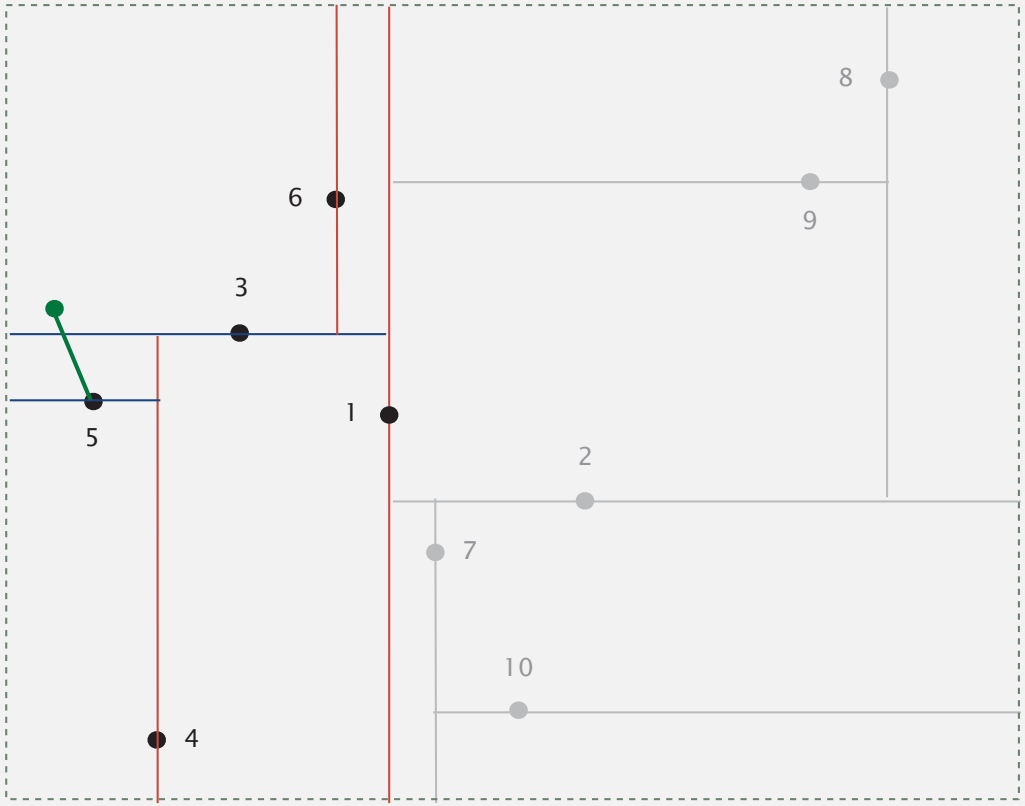


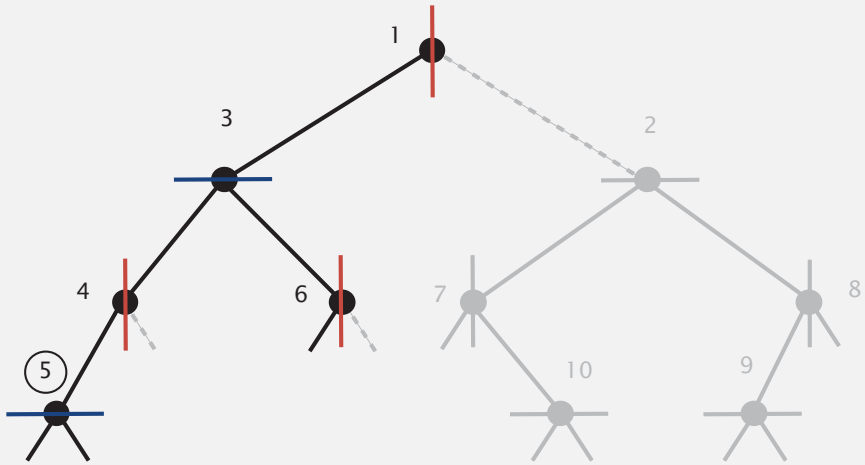
nearest neighbor = 5

Nearest neighbor search in a 2d tree analysis

Typical case. $\log N$.

Worst case (even if tree is balanced). N.





nearest neighbor = 5

Flocking birds

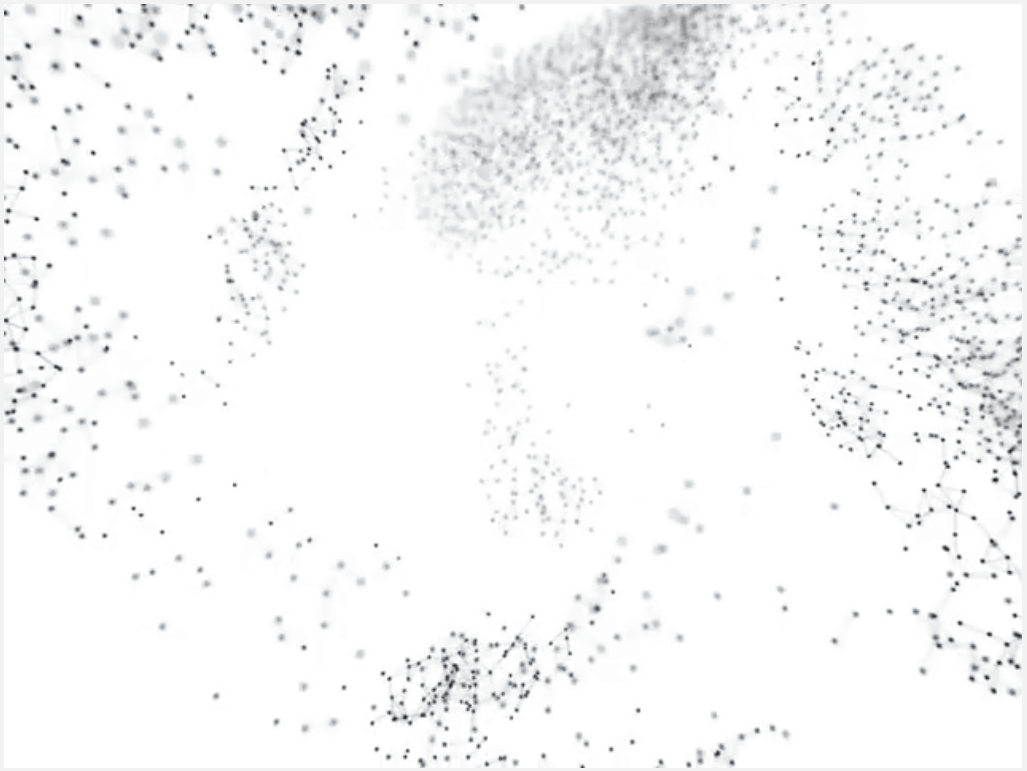
Q. What "natural algorithm" do starlings, migrating geese, starlings, cranes, bait balls of fish, and flashing fireflies use to flock?

http://www.youtube.com/watch?v=XH-groCeKbE

Flocking boids [Craig Reynolds, 1986]

Boids. Three simple rules lead to complex emergent flocking behavior:

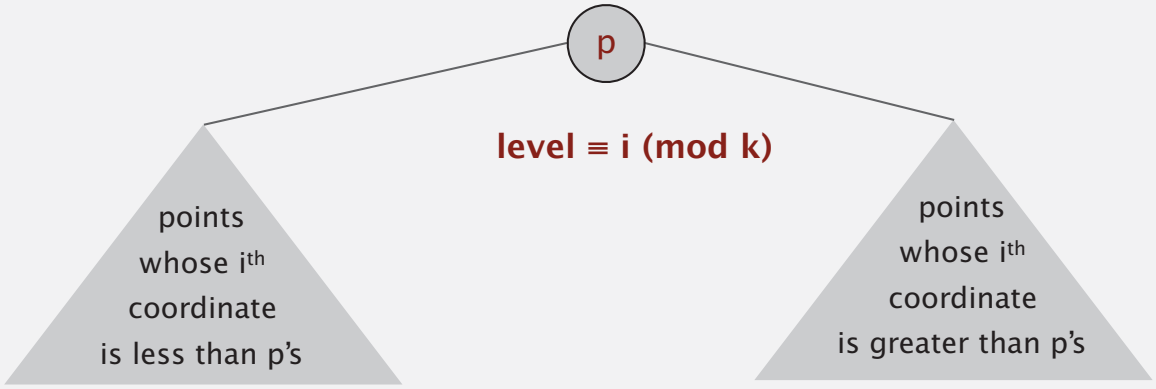
- Collision avoidance: point away from k nearest boids.
- Flock centering: point towards the center of mass of k nearest boids.
- Velocity matching: update velocity to the average of k nearest boids.

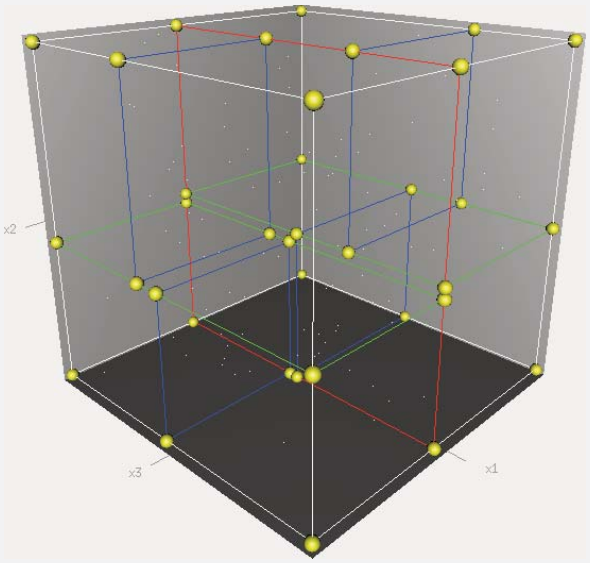


Kd tree

Kd tree. Recursively partition k-dimensional space into 2 halfspaces.

Implementation. BST, but cycle through dimensions ala 2d trees.





Efficient, simple data structure for processing k-dimensional data.

- Widely used.
- Adapts well to high-dimensional and clustered data.
- Discovered by an undergrad in an algorithms class!

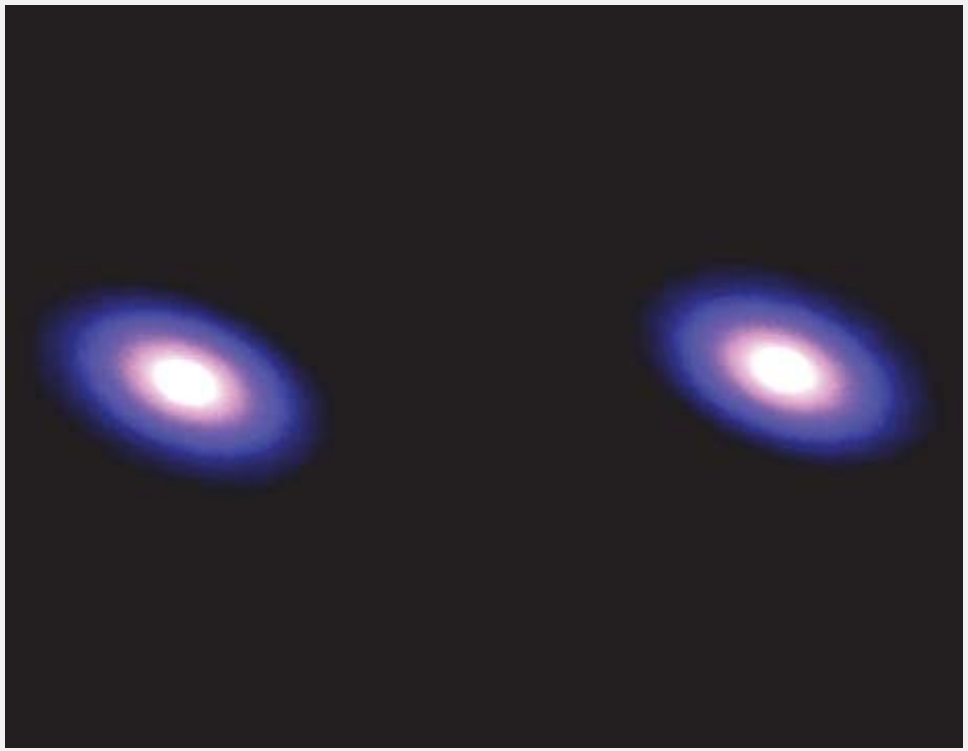
Jon Bentley

N-body simulation

Goal. Simulate the motion of N particles, mutually affected by gravity.

Brute force. For each pair of particles, compute force: $F = \dfrac{G\, m_1\, m_2}{r^2}$

Running time. Time per step is N^2.

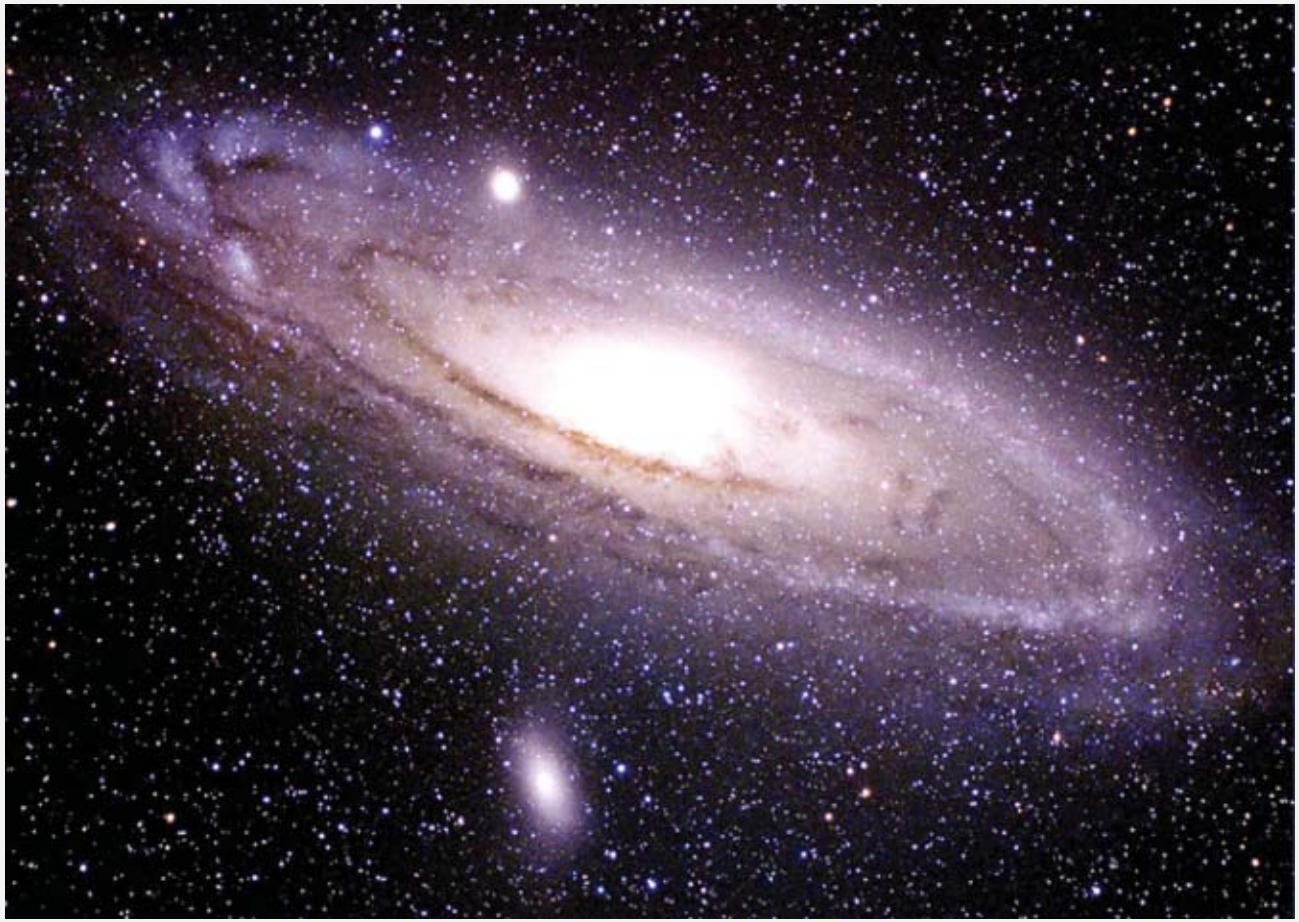


http://www.youtube.com/watch?v=ua7YlN4eL_w

Appel's algorithm for N-body simulation

Key idea. Suppose particle is far, far away from cluster of particles.

- Treat cluster of particles as a single aggregate particle.
- Compute force between particle and center of mass of aggregate.



Appel's algorithm for N-body simulation

- Build 3d-tree with N particles as nodes.
- Store center-of-mass of subtree in each node.
- To compute total force acting on a particle, traverse tree, but stop as soon as distance from particle to subdivision is sufficiently large.

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Vol. 6, No. 1, January 1985

008

AN EFFICIENT PROGRAM FOR MANY-BODY SIMULATION*

ANDREW W. APPEL†

Abstract. The simulation of N particles interacting in a gravitational force field is useful in astrophysics, but such simulations become costly for large N. Representing the universe as a tree structure with the particles at the leaves and internal nodes labeled with the centers of mass of their descendants allows several simultaneous attacks on the computation time required by the problem. These approaches range from algorithmic changes (replacing an $O(N^2)$ algorithm with an algorithm whose time-complexity is believed to be $O(N \log N)$) to data structure modifications, code-tuning, and hardware modifications. The changes reduced the running time of a large problem ($N = 10{,}000$) by a factor of four hundred. This paper describes both the particular program and the methodology underlying such speedups.

Impact. Running time per step is $N \log N$ $\Rightarrow$ enables new research.